

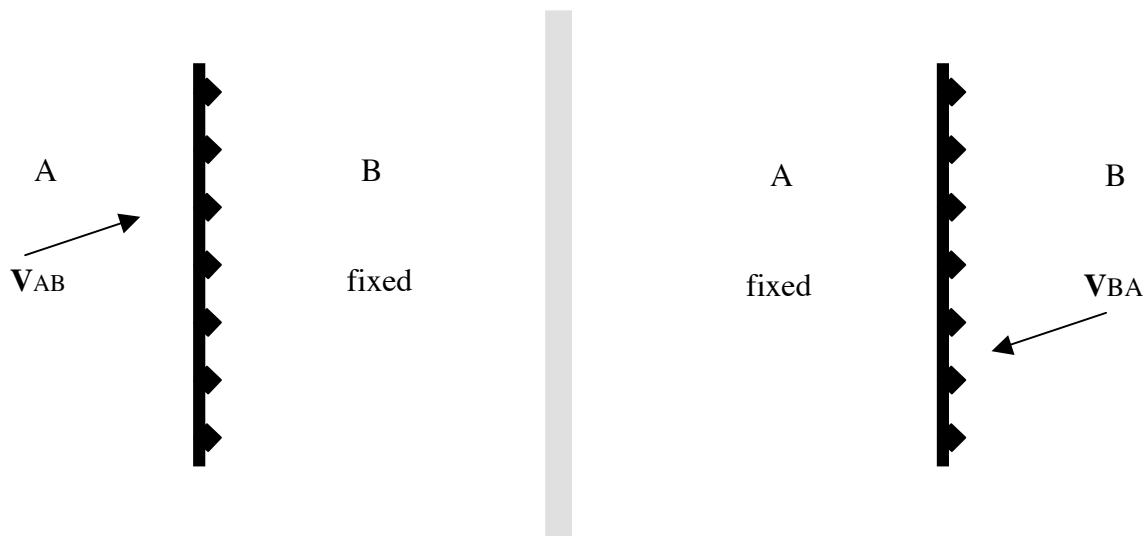
## PLATE KINEMATICS

(Copyright 2010, David T. Sandwell)

(Reference - The Solid Earth, C.M.R. Fowler, Cambridge University Press, 1990, Chapter 2)

### Plate Motions on a Flat Earth

Plate tectonic theory describes the motions of rigid plates on a spherical earth. However, when considering the relative motions very close to the plate boundary or at a triple junction, it is OK to use a flat earth approximation. We'll begin with the flat earth case and then move on to the spherical case. Consider 2 plates A and B which have a subduction zone boundary between them such as the Nazca and South American plates. In this analysis all plate motions are relative so one can either consider plate B as fixed or plate A as fixed and draw the relative vector velocity between them as shown in the diagram below.



$V_{AB}$  – velocity vector of plate A relative to plate B.

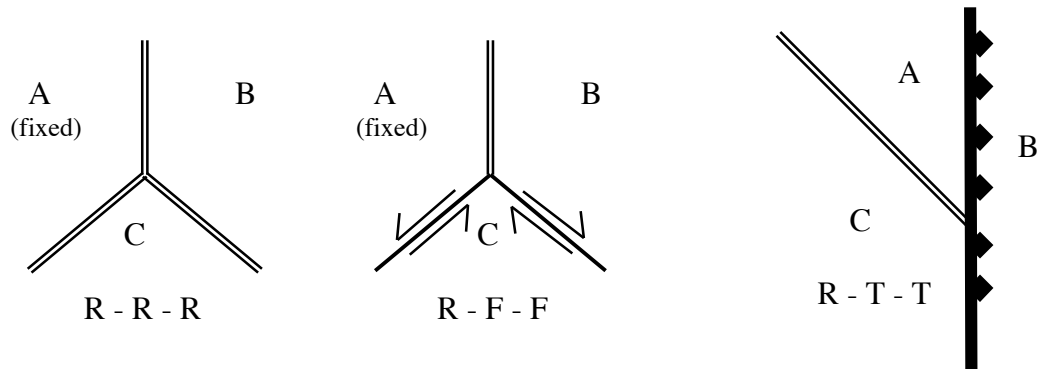
$V_{BA}$  – velocity vector of plate B relative to plate A.

$$V_{AB} = -V_{BA}$$

$$V_{AB} = V_x i + V_y j$$

## Triple Junction

A triple junction is the intersection of three plate boundaries. The most common types of triple junctions are ridge-ridge-ridge (R-R-R), ridge-fault-fault (R-F-F), and ridge-trench-trench (R-T-T).



Each type of plate boundary has rules about relative velocities:

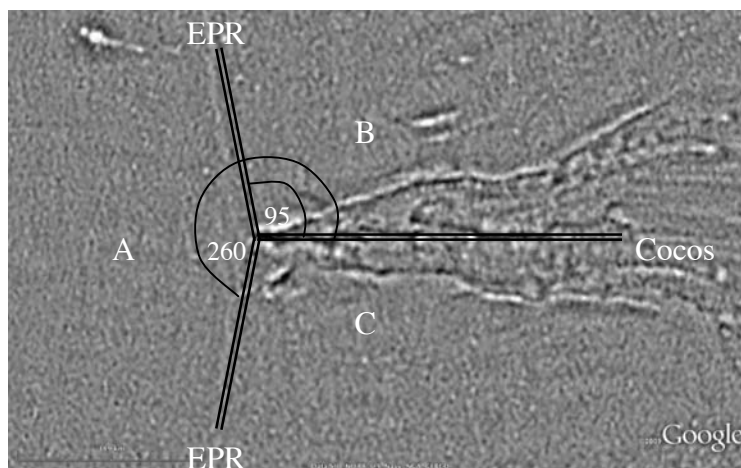
- i) ridge - relative velocity must be divergent and is usually perpendicular to the ridge.
- ii) transform fault - relative velocity must be parallel to the fault
- iii) trench - relative velocity must be convergent but no direction is preferred

All triple junctions must satisfy a velocity condition such that the vector sum around the plate circuit is zero.

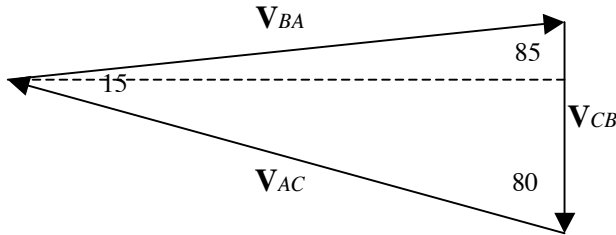
$$\mathbf{V}_{BA} + \mathbf{V}_{CB} + \mathbf{V}_{AC} = 0 \quad (1)$$

In the real world we usually can map the geometry of the spreading ridges, transform faults and trenches but cannot always measure the relative velocities. The triple junction closure equation (1) can be used to solve for spreading velocities given the triple junction geometry, the rules, and at least one relative plate velocity.

*Example: Galapagos Triple Junction - RRR*



Given the above geometry and one spreading velocity  $|\mathbf{V}_{AC}| = 120$  mm/yr, what are the other two spreading rates?



sum of interior angles =  $180^\circ$

Solving for the lengths of the vectors we find  $|\mathbf{V}_{BA}| = 118.45$  mm/yr and  $|\mathbf{V}_{CB}| = 31.12$  mm/yr. One of these flat-earth triple junction closure problems will be on the quiz. The map on the next page shows the other triple junctions. As an exercise, use a bathymetric map (e.g., Google Earth and this KMZ-file [ftp://topex.ucsd.edu/pub/global\\_topo\\_1min/global\\_topo\\_1min\\_V13.1\\_terra.kmz](ftp://topex.ucsd.edu/pub/global_topo_1min/global_topo_1min_V13.1_terra.kmz)) to determine the geometry of another triple junction and use Table 1 (below) to calculate the spreading rate at one of the ridges. The next section develops the mathematics for calculation of plate motions on a spherical earth.

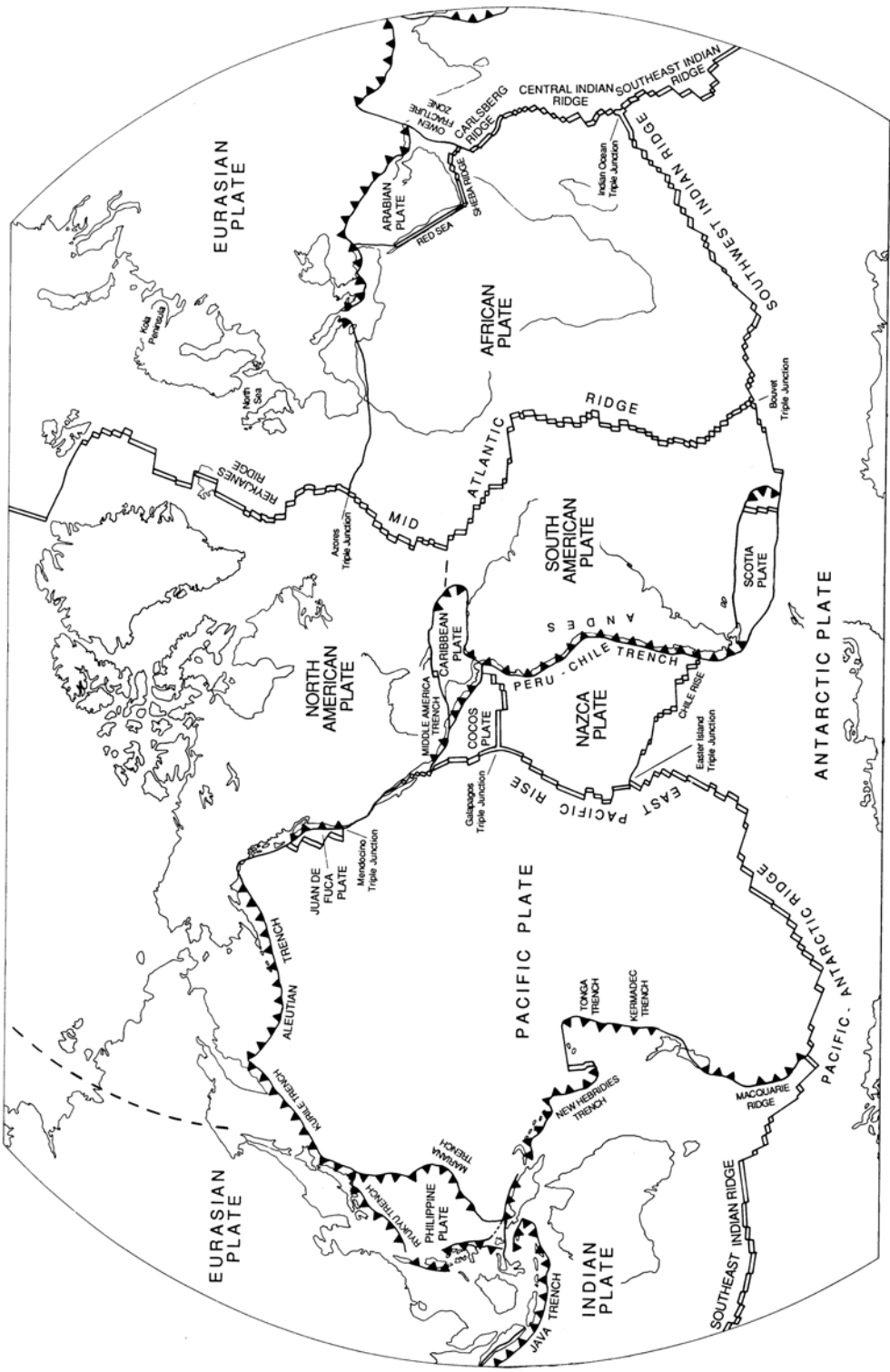


Figure 2.2. The major tectonic plates, midocean ridges, trenches and transform faults.

## Plate Motions on a Sphere

(Minster, J-B, T.H. Jordan, Present-day plate motions, *J. Geophys. Res.*, 85, p. 5331-5354, 1978.

DeMets et al., Current plate motions, *Geophys. J. R. Astr. Soc.*, 101, p. 425-478, 1990.

DeMets, C., R. G. Gordon, and D. F. Argus, Geologically current plate motions, *Geophys. J. Int.*, 181, 1-80, 2010)

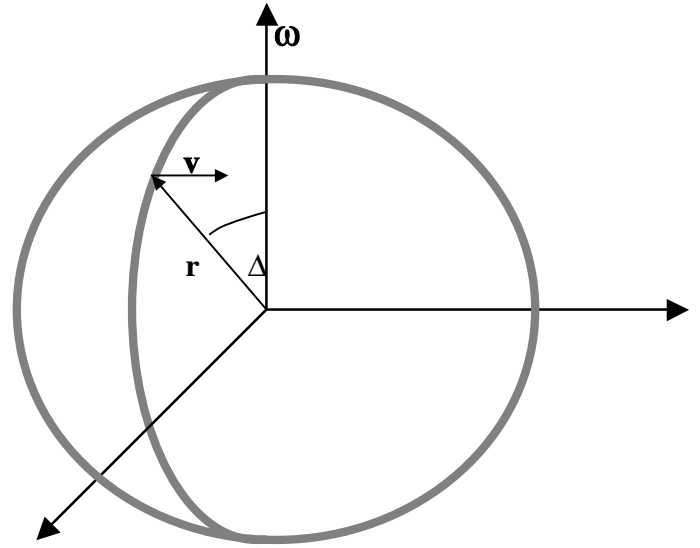
Given:

$\boldsymbol{\omega}$  – angular velocity vector  $\left(\frac{\text{rad}}{\text{s}}\right)$

$\mathbf{r}$  – position on earth (m)

Calculate:

$\mathbf{v}$  – velocity vector at position  $\mathbf{r}$   $\left(\frac{\text{m}}{\text{s}}\right)$



Of course, the velocity of the plate must be tangent to the surface of the earth so the velocity is the cross product of the position vector and the angular velocity vector.

$$\mathbf{v} = \boldsymbol{\omega} \times \mathbf{r} \quad (2)$$

or

$$\mathbf{v} = \hat{i}(\omega_y z - \omega_z y) - \hat{j}(\omega_x z - \omega_z x) + \hat{k}(\omega_x y - \omega_y x) \quad (3)$$

where  $i, j$ , and  $k$  are unit vectors. The magnitude of the velocity is given by

$$|\mathbf{v}| = |\boldsymbol{\omega}| |\mathbf{r}| \sin(\Delta) \quad (4)$$

where  $\Delta$  is the angle between the position vector and the angular velocity vector. It is given by the following formula.

$$\cos(\Delta) = \frac{\boldsymbol{\omega} \cdot \mathbf{r}}{|\boldsymbol{\omega}| |\mathbf{r}|} \quad (5)$$

The formulas above assume that the angular velocity vector and the position vector are provided in Cartesian coordinates. However, usually they are specified in terms of

latitude and longitude. Thus one must transform both vectors. The usual case is to calculate the relative velocity between two plates somewhere along their common boundary. Table 1 (next page) lists the pole position and rates of rotation for relative motion between plate pairs shown in the figure. The Cartesian position of a point along the plate boundary is

$$\begin{aligned}x &= a \cos \theta \cos \phi \\y &= a \cos \theta \sin \phi \\z &= a \sin \theta\end{aligned}\tag{6}$$

where  $\theta$  is latitude and  $\phi$  is longitude. It is helpful to memorize the conversion from latitude longitude to the Cartesian co-ordinate system where the  $x$ -axis runs from the center of the earth, to a point at  $0^\circ$  latitude and  $0^\circ$  longitude (i.e. the Greenwich meridian), the  $y$ -axis runs through a point at  $0^\circ$  latitude and  $90^\circ$  east longitude and the  $z$ -axis runs along the spin-axis to the north pole.

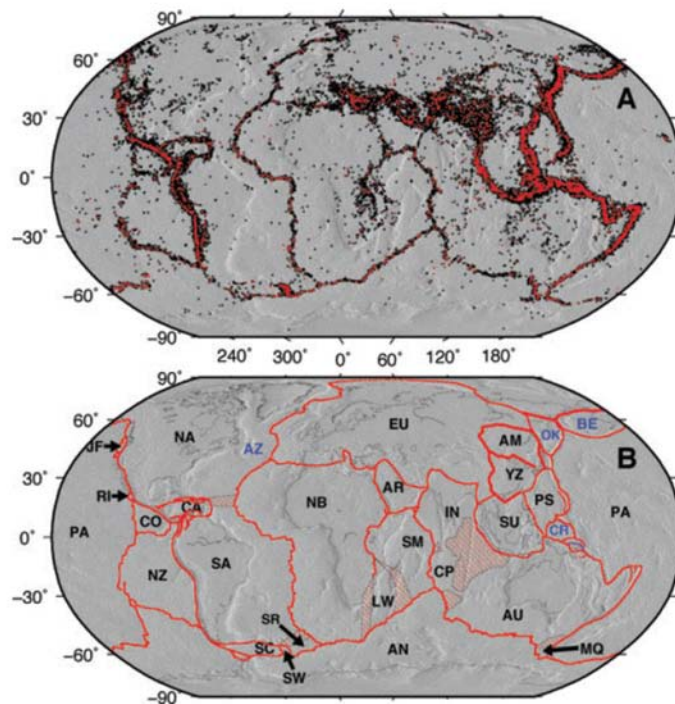
Similarly the pole positions must be converted from geographic co-ordinates  $(\theta_p, \phi_p)$  into the Cartesian system

$$\begin{aligned}\omega_x &= |\boldsymbol{\omega}| \cos \theta_p \cos \phi_p \\ \omega_y &= |\boldsymbol{\omega}| \cos \theta_p \sin \phi_p \\ \omega_z &= |\boldsymbol{\omega}| \sin \theta_p\end{aligned}\tag{7}$$

where  $|\boldsymbol{\omega}|$  is the magnitude of the rotation vector provided in Table 1. There are two ways to compute the magnitude of the velocity. One could compute the cross product of the rotation vector and the position vector (equation 2). Then the magnitude of the velocity is

$$|\mathbf{v}| = (v_x^2 + v_y^2 + v_z^2)^{1/2}\tag{8}$$

A second approach is to calculate the angle  $\Delta$  between the position vector and the angular velocity vector using equation 5 and then use that value in equation 4 to calculate the magnitude of the velocity. Indeed, both Fowler and Turcotte & Schubert use this second approach. However, they use the rather cumbersome spherical trigonometry to calculate the angle  $\Delta$ . Since I don't remember the spherical trigonometry formulas, I prefer to use equation 4 above after converting everything to Cartesian coordinates.



**Figure 1.** (a) Epicentres for earthquakes with magnitudes equal to or larger than 3.5 (black) and 5.5 (red) and depths shallower than 40 km for the period 1967–2007. Hypocentral information is from the U.S. Geological Survey National Earthquake Information Center files. (b) Plate boundaries and geometries employed for MORVEL. Plate name abbreviations are as follows: AM, Amur; AN, Antarctic; AR, Arabia; AU, Australia; AZ, Azores; BE, Bering; CA, Caribbean; CO, Cocos; CP, Capricorn; CR, Caroline; EU, Eurasia; IN, India; JF, Juan de Fuca; LW, Lwandle; MQ, Macquarie; NA, North America; NB, Nubia; NZ, Nazca; OK, Okhotsk; PA, Pacific; PS, Philippine Sea; RI, Rivera; SA, South America; SC, Scotia; SM, Somalia; SR, Sur; SU, Sundaland; SW, Sandwich; YZ, Yangtze. Blue labels indicate plates not included in MORVEL. Patterned red areas show diffuse plate boundaries.

**Table 1.** Best-fitting angular velocities and covariances.

Plate pair	N <sub>Data</sub> R/T/S	Angular velocity		
		Lat. (°N)	Long. (°E)	$\omega$ (deg Myr <sup>-1</sup> )
AU-AN	167/19/0	11.3	41.8	0.633
CP-AN	35/1/0	17.2	32.8	0.580
LW-AN	16/6/0	-1.2	-33.6	0.133
NB-AN	59/4/0	-6.2	-34.3	0.158
NZ-AN	60/21/0	33.1	-96.3	0.477
PA-AN	48/10/0	-65.1	99.8	0.870
SM-AN	29/2/0	11.2	-56.7	0.140
EU-NA	453/5/0	61.8	139.6	0.210
NB-NA	161/4/0	79.2	40.2	0.233
AR-NB	45/0/0	30.9	23.6	0.403
CO-NZ	88/2/0	1.6	-143.5	0.636
CO-PA	61/6/0	37.4	-109.4	2.005
JF-PA	27/1/0	-0.6	37.8	0.625
NZ-PA	42/15/0	52.7	-88.6	1.326
RI-PA	26/3/0	25.7	-104.8	4.966
NB-SA	99/27/0	60.9	-39.0	0.295
SW-SC	18/4/0	-32.0	-32.2	1.316
AR-SM	51/5/0	22.7	26.5	0.429
CP-SM	56/10/0	16.9	45.8	0.570
IN-SM	113/2/22	22.7	30.6	0.408
AN-SR	9/2/0	85.7	-139.3	0.317
NB-SR	25/2/0	70.6	-60.9	0.346

[MORVEL - Demets *et al.*, 2010]

### *Velocity Azimuth*

We know that the velocity vector is tangent to the sphere. Given the Cartesian velocity components from equation 3, we would like to compute the latitude  $v_\theta$  and longitude  $v_\phi$  components of velocity. Begin by taking the time derivative of equation 6.

$$v_x = a(-\cos\phi \sin\theta \ v_\theta - \cos\theta \sin\phi \ v_\phi) \quad (9)$$

$$v_y = a(-\sin\phi \sin\theta \ v_\theta + \cos\theta \cos\phi \ v_\phi)$$

$$v_z = a(\cos\theta \ v_\theta)$$

From the last equation in (9), we can solve for the latitude velocity component.

$$v_\theta = \frac{v_z}{a \cos\theta} \quad (10)$$

Now plug  $v_\theta$  into either the  $v_x$  or  $v_y$  equation and solve for  $v_\phi$ .

$$v_\phi = \frac{v_y + v_z \sin\phi \tan\theta}{a \cos\theta \cos\phi} \quad (11)$$

If this equation turns out to be singular, then use the  $v_x$  equation.

$$v_\phi = -\frac{v_x + v_z \cos\phi \tan\theta}{a \cos\theta \sin\phi} \quad (12)$$

*Recipe for Computing Velocity Magnitude*

In summary, to calculate the magnitude of the velocity:

- i) Transform lat, lon into  $\mathbf{x} = (x, y, z)$  unit vector using equation 5.
- ii) Transform pole lat and lon into  $\mathbf{x}_p = (x_p, y_p, z_p)$  unit vector
- iii)  $\cos\Delta = \mathbf{x} \cdot \mathbf{x}_p$
- iv)  $v = \omega a \sin\Delta$

Example: Given the rotation pole between the Pacific and Nazca plates, calculate the spreading rate at  $-20^\circ$   $113.5^\circ\text{W}$ . \*\*\*One of these calculations will be on the quiz\*\*\*

<u>POLE</u>			<u>POINT</u>	
52.7	-88.6	$1.326 \times 10^{-6}$ deg/yr	20.0°S 113.5°W	
52.7	271.4	$2.314 \times 10^{-8}$ rad/yr	-20.0	246.5
$x_p = 0.0148$			$x = -0.375$	
$y_p = -0.606$			$y = -0.862$	
$z_p = 0.795$			$z = -0.342$	

$$\cos\Delta = \mathbf{x} \cdot \mathbf{x}_p = (-0.0056 + .522 - .272) = .244$$

$$\Delta = 75.8^\circ \quad v = \omega a \sin\Delta = 142.9 \text{ mm/yr}$$

## Triple Junctions on a Sphere

Triple junction closure on a sphere is similar to triple junction closure on a flat earth except that the sum of the rotation vectors must be zero.

$$\boldsymbol{\omega}_{BA} + \boldsymbol{\omega}_{CB} + \boldsymbol{\omega}_{AC} = 0$$

*Example: Galapagos Triple Junction*

Given the rotation vectors of the Cosos plate relative to the Pacific plate and the Pacific plate relative to the Nazca plate, calculate the spreading rate at 2°N, 260°E.

$$\boldsymbol{\omega}_{CP} + \boldsymbol{\omega}_{NC} + \boldsymbol{\omega}_{PN} = 0$$

$$\boldsymbol{\omega}_{NC} = -\boldsymbol{\omega}_{CP} - \boldsymbol{\omega}_{PN}$$

$$\mathbf{v}_{NC} = \boldsymbol{\omega}_{NC} \times \mathbf{r}(\theta, \phi)$$

$|\mathbf{v}|$  - magnitude of spreading rate

## Hot Spots and Absolute Plate Motions

So far we have only considered relative plate motions because there was no way to tie the positions of the plates to the mantle. One method of making this connection and thus determining absolute plate motions is to assume that “hot spots” remain fixed with respect to the lower mantle.

- A. A hot spot is an area of concentrated volcanic activity. There is a subset of hot spots that have the following characteristics:
- B. They produce linear volcanic chains in the interiors of the plates.
- C. The youngest volcanoes occur at one end of the volcanic chain and there is a linear increase in age away from that end.
- D. The chemistry of the erupted lavas is significantly different from lava erupted at mid-ocean ridges or island arcs.
- E. Some hotspots are surrounded by a broad topographic swell about 1000 m above the surrounding ocean basin.

These features are consistent with a model where the plates are moving over a relatively fixed mantle plume. After identifying the linear volcanic chains associated with the mantle plumes, it has been shown that the relative motions among hot spots is about 10 times less than the relative plate motions.