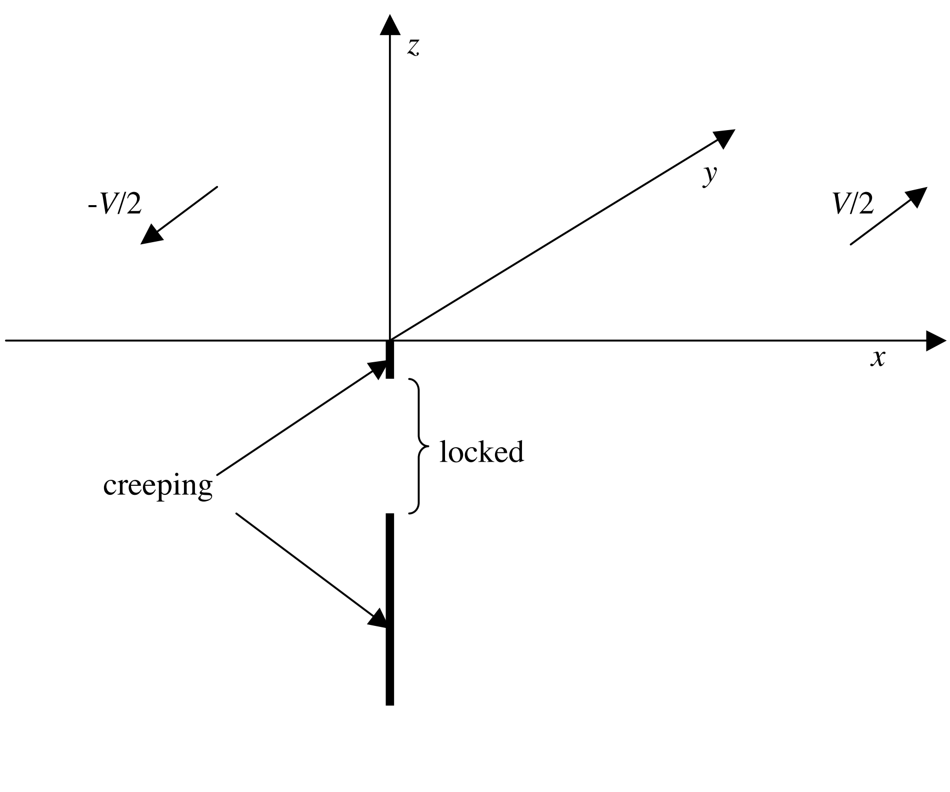
ELASTIC SOLUTIONS FOR STRIKE-SLIP FAULTING

(Reference: Cohen, S. C., Numerical Models of Crustal Deformation in Seismic Zones, *Advances in Geophysics, v. 41*, p. 134-231, 1999)

Today's lecture will be the mathematical development of the deformation and strain pattern due to strike-slip deformation on a partially locked fault. The notes come from Chapter 8 of Turcotte & Schubert but I'll focus on section 8-6 through 8-9. While I'll follow the overall theme of Chapter 8, I'll deviate in two respects. First I'll use a co-ordinate system with the *z*-axis pointed upward to be consistent with my previous notes on gravity, magnetics, and heat flow. Second I'll develop the solution using fourier transforms to be consistent with my previous notes.

Interseismic Strain Buildup

The first objective is to derive an expression for the surface displacement $v(x)$ and surface strain $\delta v/\delta x$ for the model shown below. A constant velocity V is applied at an elastic half space. There is a fault in the half space with locked and creeping sections. Because of free slip on the faults, the strain will be concentrated near the fault.

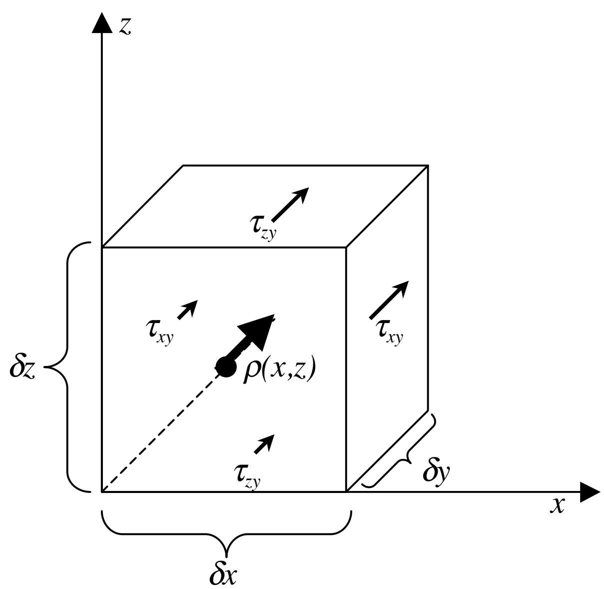


The approach will be as follows:

I. *Develop the force balance from basic principles.*
II. *Establish the line-source Green's function for an elastic full space.*
III. *Establish the screw-dislocation Green's function for an elastic full space.*
IV. *Use the method of images to construct a half-space solution.*
V. *Integrate the line sources to develop the solutions found in the literature.*
VI. *Compute the geodetic moment accumulation rate for an arbitrary slip distribution.*
VII. *Inclined fault plane*
VIII. *Matlab examples*
IX. *Depth-averaged stress above a dislocation*

I. Force Balance

Consider the forces acting on the infinitely-long square rod depicted below. The body force per unit volume of rod must be balanced by tractions on the sides of the rod.



The equation for this force balance is

$$\left[\tau_{xy}(x+\delta x)-\tau_{xy}(x)\right]\delta y\delta z+\left[\tau_{zy}(z+\delta z)-\tau_{zy}(z)\right]\delta x\delta y=\rho(x,z)\delta x\delta y\delta z \tag{1}$$

where τ_{xy} and τ_{zy} are the shear tractions on the side and top of the box, respectively and *ρ(x,y)* is the body force which depends only on *x* and *z*. Dividing through by δx δy δz and taking the limit as all three go to zero, to arrive at:

$$\frac{\partial\tau_{xy}}{\partial x}+\frac{\partial\tau_{zy}}{\partial z}=\rho(x,z) \tag{2}$$

Given the following relationship between stress and displacement, the differential equation reduces to Poisson's equation

$$\tau_{xy} = \mu \frac{\partial v}{\partial x} \qquad (3)$$
$$\tau_{zy} = \mu \frac{\partial v}{\partial z}$$

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} = \frac{1}{\mu}\rho(x,z) \qquad (4)$$

where μ is the shear modulus and v is the displacement in the y-direction.

II. Line-Source Green's Function

We can generate the solution to an arbitrary source distribution by first developing the line-source Green's function. Consider a line source at a depth of $-a$. The differential equation is:

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial z^2} = \frac{A}{\mu}\delta(\mathrm{x})\delta(z+a) \qquad (5)$$

where A is the source strength having units of force/length or force/length/time if this will respresent an interseismic velocity. The boundary conditions for this second-order, partial differential are that v must vanish as both $|x|$ and $|z|$ go to infinity. The 2-dimensional forward and inverse fourier transforms are defined as

$$F(\mathbf{k}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(\mathbf{x}) e^{-i2\pi(\mathbf{k}\bullet\mathbf{x})} d^2\mathbf{x} \qquad (6)$$

$$f(\mathbf{x}) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(\mathbf{k}) e^{i2\pi(\mathbf{k}\bullet\mathbf{x})} d^2\mathbf{k}$$

where $\mathbf{k}=(k_x, k_y)$ and $\mathbf{x} = (x, y)$. Take the 2-dimensional fourier transform of the differential equation (5).

$$-(2\pi)^2\left(k_x^2 + k_z^2\right)V(\mathbf{k}) = \frac{A}{\mu} e^{i2\pi k_z a} \qquad (7)$$

so the solution in the fourier domain is

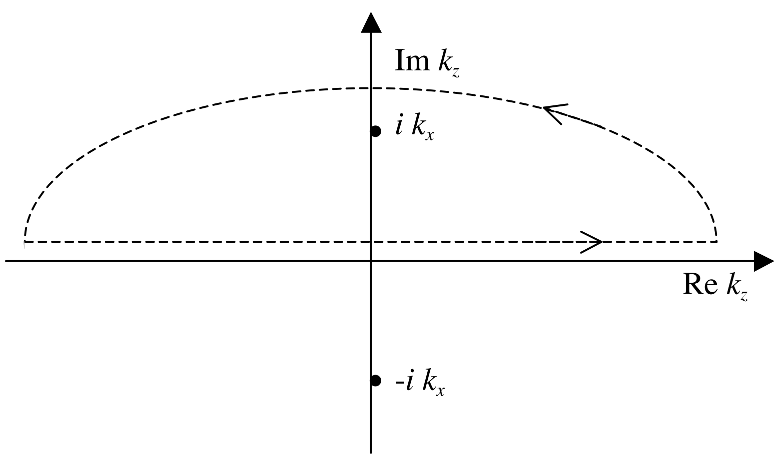
$$V(\mathbf{k}) = \frac{-Ae^{i2\pi k_z a}}{\mu(2\pi)^2\left(k_x^2 + k_z^2\right)} \qquad (8)$$

Now we need to take the inverse fourier transform with respect to k_z and make sure the solution goes to zero as |z| goes to infinity. The integral is

$$V(k_x,z)=\frac{-A}{\mu(2\pi)^2}\int_{-\infty}^{\infty}\frac{e^{i2\pi k_z(z+a)}}{\left(k_x^2+k_z^2\right)}dk_z \tag{9}$$

First consider the case $k_x>0,\ z+a>0$. We can factor the denominator and recognize integrand will vanish for large positive z if we close the contour in the upper hemisphere.

$$V(k_x,z)=\frac{-A}{\mu(2\pi)^2}\oint\frac{e^{i2\pi k_z(z+a)}}{\left(k_z+ik_x\right)\left(k_z-ik_x\right)}dk_z \tag{10}$$



From the Cauchy integral formula, we know that for any analytic function the following holds for a counterclockwise path surrounding the pole.

$$\oint\frac{f(z)}{z-z_o}dz=i2\pi f(z_o) \tag{11}$$

In this case with the pole at ik_x, the result is simply

$$V(k_x,z)=\frac{-i2\pi A}{\mu 4\pi^2}\frac{e^{-2\pi k_x(z+a)}}{i2k_x} \quad = \quad \frac{-A}{2\mu}\frac{e^{-2\pi k_x(z+a)}}{2\pi k_x} \tag{12}$$

Next consider $k_x<0,\ z+a>0$. In this case we must close the integration path in the lower hemisphere to satisfy the boundary conditions; during the integration the only contribution will be from the -ik_x pole. The overall result is to replace k_x by $|k_x|$.

$$V(k_x,z)=\frac{-A}{2\mu}\frac{e^{-2\pi|k_x|(z+a)}}{2\pi|k_x|} \tag{13}$$

Note this is exactly the same as the gravity solution. The Greens function is the inverse cosine transform of equation (13) or $\ln(r^2)$. The final result is

$$v(x,z)=\frac{-A}{4\pi\mu}\ln\left[x^2+(z+a)^2\right] \tag{14}$$

III. Screw Dislocation for Line Source Green's Function

In order to produce a fault plane with strike-slip displacement, we need to construct a line-source screw dislocation. This can be accomplished by abitting equal but opposite line source dislocations as shown in the diagram below.

A simple way of constructing the screw source is to take the derivative of the line source Green's function in a direction normal to the fault plane. So we need to develop the Green's function for the following differential equation.

$$\nabla^2 v_{screw}=\delta(z+a)\left[\delta(x+dx)-\delta(x)\right]/dx=\delta(z+a)\frac{\partial}{\partial x}\delta(x) \tag{15}$$

To do this we take the derivative of the line source Green's function in equation 14.

$$v_{screw}(x,z)=\frac{-A}{4\pi\mu}\frac{\partial}{\partial x}\ln\left[x^2+(z+a)^2\right]=\frac{-Ax}{2\pi\mu}\left[x^2+(z+a)^2\right]^{-1} \tag{16}$$

So the Green's function for a line-source screw dislocation at depth is:

$$v_{screw}(x,z)=\frac{-A}{2\pi\mu}\frac{x}{\left[x^2+(z+a)^2\right]} \tag{17}$$

IV. Surface boundary condition: Method of images

The surface boundary condition is that the shear stress τ_{zy} must be equal to zero but the full-space result provides a non-xero result. This boundary condition will be satisfied if we place an image source at $z = a$. When the combined source and image are evaluated at the surface $z = 0$, the result is to double the strength of the Green's function.

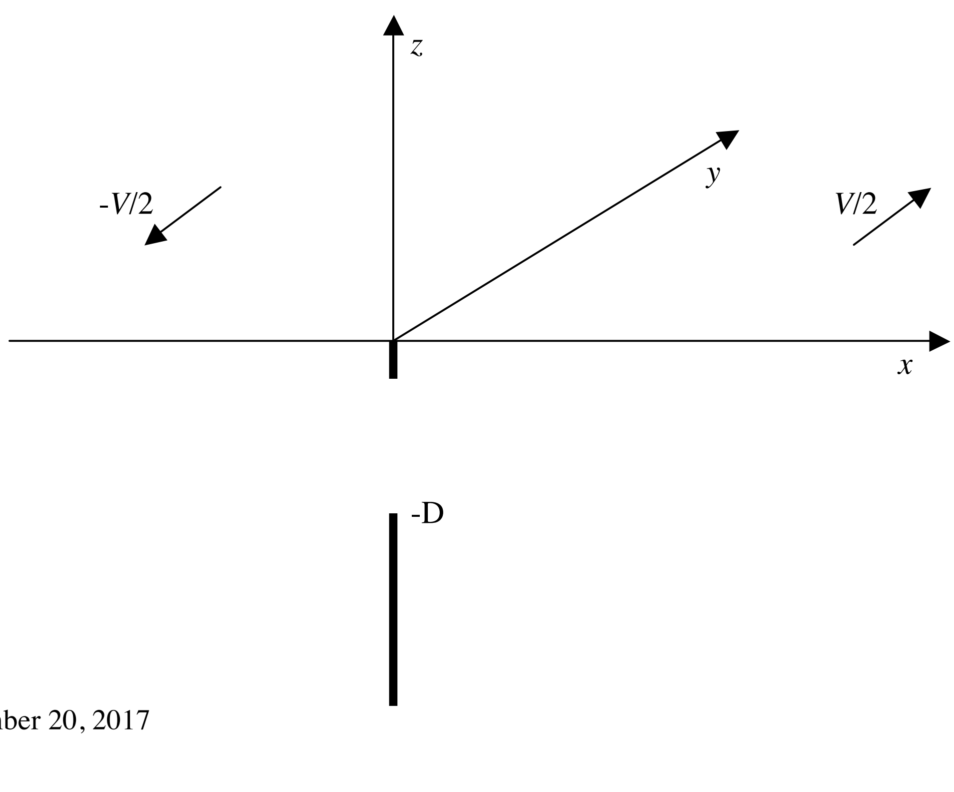
$$v(x,z) = \frac{-A}{2\pi\mu} x \left\{ \left[x^2 + (z+a)^2 \right]^{-1} + \left[x^2 + (z-a)^2 \right]^{-1} \right\}$$

$$v(x,0) = \frac{-A}{\pi\mu} \frac{x}{\left[x^2 + a^2 \right]} \qquad (18)$$

V. Vertical integration of line source to create a fault plane

The final step in the development is to integrate the line-source screw dislocation over depth. We consider three cases: (1) deep slip to represent interseismic deformation above a locked fault, (2) shallow slip to represent shallow creep, and (3) shallow slip on a stress-free crack to represent an earthquake.

Case 1. First consider a fault that is free-slip between a depth *-D* and infinity. This is the solution considered by Savage [Savage, J. C., Equivalent strike-slip cycles in half-space and lithosphere-asthenosphere earth models, *J. Geophys. Res., v. 95*, p. 4873-4879, 1990.].



The integral of the line source Green's function is

$$v(x) = \frac{-A}{\pi\mu} \int_{-\infty}^{-D} \frac{x}{x^2 + z^2} dz \tag{19}$$

To integrate (19) make the following substitution

$$\eta = -xz^{-1} \quad \text{so} \quad d\eta = xz^{-2} dz \tag{20}$$

The integral becomes

$$v(x) = \frac{-A}{\pi\mu} \int_{0}^{x/D} \frac{1}{1+\eta^2} d\eta \quad = \quad \frac{-A}{\pi\mu} \tan^{-1}\left(\frac{x}{D}\right) \tag{21}$$

We know that $v(\pm\infty) = \pm V/2$ so $A = -V\mu$. Note that A has units of force per unit area times a velocity. This corresponds to a moment rate per area of fault. The familiar results for displacement and shear stress are

$$v(x) = \frac{V}{\pi} \tan^{-1} \frac{x}{D}$$

$$\tau_{xy} = \frac{\mu V}{\pi D} \frac{1}{1+\left(\frac{x}{D}\right)^2} \tag{22}$$

Consider the extreme cases of a completely unlocked fault so D=0. The displacement field will be a step function and the stress will be everywhere zero except at the origin where it will be infinite.

Case 2. Next consider a fault that is free-slip between the surface and a depth -d. In this case the integral is

$$v(x) = \frac{V}{\pi} \int_{-x/d}^{0} \frac{1}{1+\eta^2} d\eta \quad = \quad \frac{V}{\pi} \tan^{-1}\eta \; \Big|_{-x/d}^{\infty} \tag{23}$$

There are two cases depending on whether x is positive or negative.

$$v(x) = \frac{V}{\pi}\left(\frac{\pi}{2} - \tan^{-1}\frac{x}{d}\right) \quad x>0 \tag{24}$$

$$v(x) = \frac{V}{\pi}\left(\frac{-\pi}{2} - \tan^{-1}\frac{x}{d}\right) \quad x<0$$

By combining these, the displacement and shear stress are

$$v(x) = \mathrm{V}\left[H(x) - 1/2\right] - \frac{\mathrm{V}}{\pi}\tan^{-1}\frac{x}{d}$$

$$\tau_{xy} = \mu \mathrm{V}\left[\delta(x) - \frac{1}{\pi d}\frac{1}{1+\left(\frac{x}{d}\right)^2}\right] \tag{25}$$

If the fault is completely unlocked so *d* goes to infinity, the displacement becomes a step and the shear stress is infinite at the origin in agreement with our concepts of a free-slipping fault.

Case 3. – The third case considered also has shallow slip between depth –*d* and the surface. However, in this case we consider a so-called *crack model* where the slip versus depth function results in zero stress on the fault. This derivation will lead to the crack solution given in equation 8-110 of Turcotte and Schubert [2002]. The Case-2 solution has uniform slip with depth. This leads to a stress singularity at the base of the fault. In contrast, the model in Turcotte and Schubert has a stress-free crack imbedded in a pre-stressed elastic half space. Using the Green's function developed above it can be shown that the two solutions are in fundamental agreement. The only difference is related to the slip versus depth function.

From the dislocation theory developed in equation (19), the *y*-displacement as a function of distance from the fault is given by

$$v(x) = \frac{1}{\pi}\int_{-d}^{0}\frac{s(z)x}{x^2+z^2}dz \tag{26}$$

where z is depth, x is distance from the fault, $s(z)$ is the slip versus depth, and $v(x)$ is the displacement. Now consider the two slip versus depth functions between the surface and $-d$.

$$s_1 = S$$
$$s_2 = S\left(1 - z^2/d^2\right)^{1/2} \tag{27}$$

The first slip function is constant with depth while the second corresponds to the stress-free crack and has the form provided in equation (8-93) in T&S. Using the approach described above, the integral of the constant slip with depth s_1 is

$$v(x) = \frac{S}{\pi}\left(\frac{x}{|x|}\frac{\pi}{2} - \tan\frac{x}{d}\right). \tag{28}$$

The integral of the slip function for the crack model s_2 is given by

$$v(x) = \frac{S}{\pi}x\int_{-d}^{0}\frac{\left(1 - z^2/d^2\right)^{1/2}}{x^2+z^2}dz = \frac{S}{\pi}x\int_{0}^{d}\frac{\left(1 - z^2/d^2\right)^{1/2}}{x^2+z^2}dz\,. \tag{29}$$

Now we let $x' = x/d$ and $z' = z/d$ so the integral becomes

$$v(x') = \frac{S}{\pi}x'\int_{0}^{1}\frac{\left(1 - z'^2\right)^{1/2}}{x'^2+z'^2}dz'\,. \tag{30}$$

This integral can be performed in Matlab using the following code with the symbolic toolbox.

```
%
clear
syms x positive
syms z
arg=sqrt(1-z*z)/(x*x+z*z);
int(arg,z,0,1)
%
% ans=-1/2*pi*(x-(x^2+1)^(1/2))/x
%
```

Note that the integrand contains x'^2 so the results for positive and negative x' are identical. Therefore in the integrated result, the x' should be replaced by $|x'|$. The result is

$$v(x') = \frac{S}{\pi}x'\frac{\pi}{2|x'|}\left[\left(1+x'^2\right)^{1/2} - |x'|\right]. \tag{31}$$

Finally substitute for x' and we arrive at

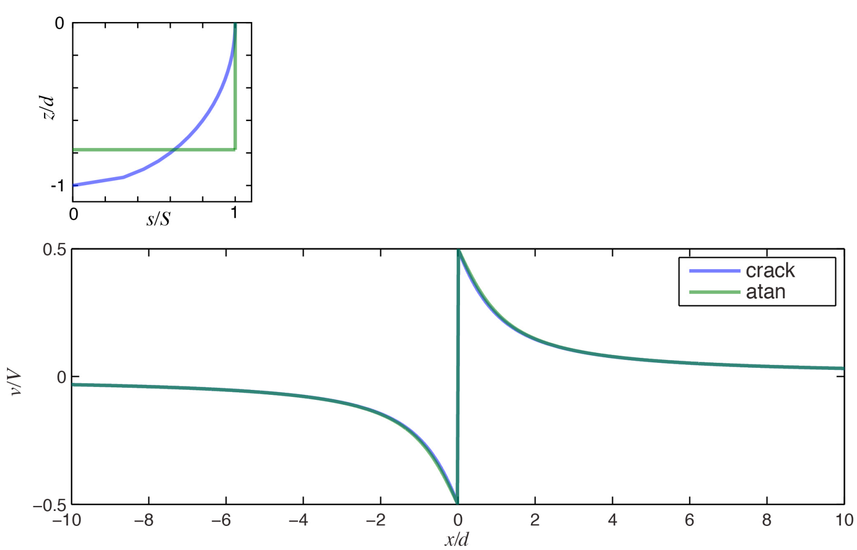
$$v(x') = \frac{x}{|x|}\frac{S}{2}\left[\left(1+\frac{x^2}{d^2}\right)^{1/2} - \frac{|x|}{d}\right]. \tag{32}$$

This matches equation (8-110) given in Turcotte and Schubert.

One can now make a direct comparison between the displacement versus distance for the two slip functions to note their similarities and differences. However, note that the arctangent slip function will have a larger seismic moment (i.e., slip integrated over depth) than the crack model slip function. The magnitude of the difference is found by integrating the slip versus depth for the two cases. For the arctangent function the integrated slip is simply Sd. For the crack model the integrated slip is

$$Sd\int_0^1 (1-z^2)^{1/2} dz = Sd\pi / 4 \ . \tag{33}$$

The following plot compares the two displacement functions when the depth of faulting for the arctangent model is reduced $\pi / 4$ by so the moments are matched; at this scale the plots are nearly identical. This illustrates the fact that measurements of displacement versus distance across a fault are not very sensitive to the shape of the slip versus depth function although they do provide an important constraint on the overall seismic moment. In the next section we highlight this issue that geodetic measurements of surface displacement are relatively insensitive to the shape of the slip versus depth function but provide a good estimate of the overall seismic moment.



VI. Geodetic Moment Accumulation Rate

The geodetic moment accumulation rate M per unit length of fault L is given by the well known formula

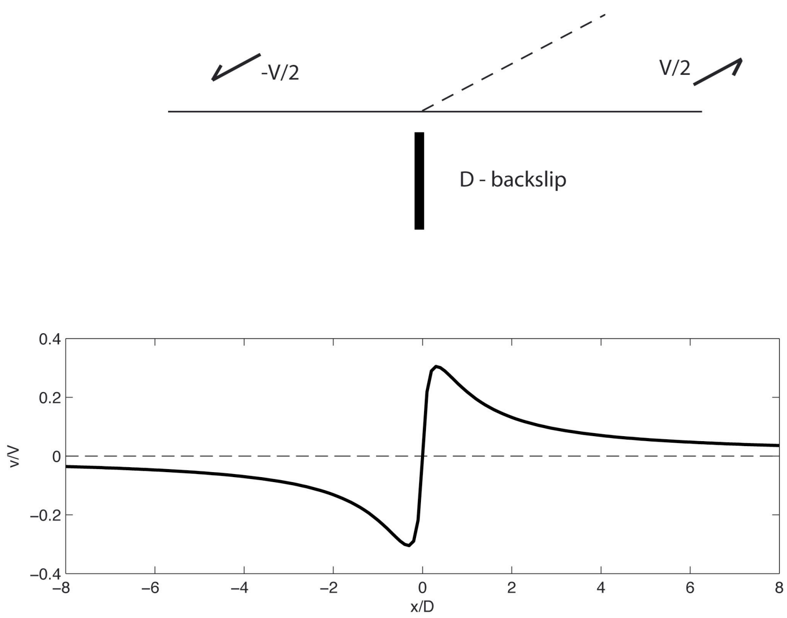
$$\frac{M}{L} = \mu SD \tag{34}$$

where D is the thickness of the locked zone and S is the slip deficit rate or backslip rate used in block models. This is the standard formula provided in all the seismology textbooks although they usually consider the co-seismic moment release due to co-seismic slip. Here we are considering the gradual accumulation of geodetic moment during the interseismic period. These moments must balance over many earthquake cycles. In the general case where the slip rate s varies with depth z, the moment rate M is given by

$$\frac{M}{L} = \mu \int_{-D_m}^{0} s(z)dz \tag{35}$$

where D_m is the maximum slip depth. The objective of the following analysis is to show that the total moment rate per unit length of fault can be measured directly from geodetic data; no slip vs. depth model is needed. The only assumptions are that the strike-slip fault is 2-D and the earth behaves as an elastic half space. From the dislocation theory developed in equation (26), the y-velocity as a function of distance from the fault is given by

$$v(x) = \frac{1}{\pi} \int_{-D_m}^{0} \frac{s(z)x}{x^2 + z^2} dz \ . \tag{36}$$



Schematic of surface velocity due to uniform backslip rate over a depth D.

Next we guess that the integral of the displacement rate times distance from the x-origin is a proxy for the moment accumulation rate. We call this proxi Q and later show how it is related to the moment rate *M*. We integrate to an upper limit *W* and then take the limit as $W \to \infty$.

$$Q = \lim_{W\to\infty}\left[\frac{1}{W}\int_0^W xv(x)dx\right] = \lim_{W\to\infty}\left[\frac{1}{\pi W}\int_0^W\int_{-D_m}^0 s(z)\frac{x^2}{x^2+z^2}dzdx\right]. \qquad (37)$$

After re-arranging the order of integration one finds

$$Q = \frac{1}{\pi}\int_{-D_m}^0 s(z)\lim_{W\to\infty}\left(\frac{1}{W}\int_0^W \frac{x^2}{x^2+z^2}dx\right)dz\,. \qquad (38)$$

The integral over *x* can be done analytically.

$$\frac{1}{W}\int_0^W \frac{x^2}{x^2+z^2}dx = \frac{x}{W} - \frac{z}{W}\tan^{-1}\frac{x}{z}\bigg|_o^W = 1 - \frac{z}{W}\tan^{-1}\frac{W}{z} \qquad (39)$$

In the limit as $W \to \infty$ the second term on the right side is zero because *z* has an upper bound of D_m so the total integral is simply 1. Overall we find this proxy is

$$Q = \frac{1}{\pi}\int_{-D_m}^0 s(z)dz \qquad (40)$$

Comparing equation (37) with equation (40) it is clear that the geodetic moment can be directly related to the integral of the displacement times the distance from the origin. Note we have extended the integral to both sides of the fault to enable the use if geodetic measurements on both sides.

$$\frac{M}{L} = \lim_{W\to\infty}\left[\frac{\mu\pi}{W}\int_{-W}^W xv(x)dx\right] \qquad (41)$$

As a check we can insert equation (36) into equation (41) and make sure we arrive at equation (35).

$$\frac{M}{L} = \lim_{W\to\infty}\left[\frac{\mu\pi}{W}\int_{-W}^W x\left(\frac{1}{\pi}\int_{-D_m}^0 \frac{s(z)x}{x^2+z^2}dz\right)dx\right] = \mu\int_{-D_m}^0 s(z)\left(\lim_{W\to\infty}\frac{1}{W}\int_{-W}^W \frac{x^2}{x^2+z^2}dx\right)dz \qquad (42)$$

We perform the integral over x first and multiply by 2 after changing the limits because the integrand is symmetric about $x=0$.

$$2\int_0^W \frac{x^2}{x^2+z^2}dx = 2\left(x - z\tan^{-1}\frac{x}{z}\right)\bigg|_0^W \qquad (43)$$

In the limit as $W \to \infty$ the final result is

$$\lim_{W\to\infty} \frac{2}{W}\left(W - z\tan^{-1}\frac{W}{z} \right) = 2, \qquad \text{for } z_{\max} = D_m << W \tag{44}$$

The the moment accumulation rate is

$$\frac{M}{L} = 2\mu \int_{-D_m}^{0} s(z)dz \,. \tag{45}$$

This agrees with our original estimate of moment except for a factor of 2. The corrected formula for the moment accumulation rate is

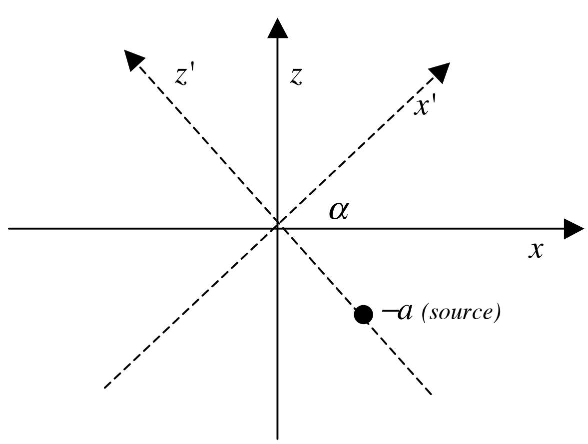
$$\frac{M}{L} = \lim_{W\to\infty} \frac{\mu\pi}{2W} \int_{-W}^{W} xv(x)dx \,. \tag{46}$$

The main utility of this formula is to demonstrate that geodetic measurements of *y*-displacement rate rate across an infinitely-long strike slip fault provide a direct estimate of the geodetic moment rate. It is unnecessary to attempt the unstable inverse problem to calculate slip versus depth and then integrate this function.

VII. Inclined fault plane

Now consider a model where the fault plane is not perpendicular to the free surface of the earth. The angle α between the vertical and the fault plane will introduce an asymmetry in the model. In later notes we'll consider the effect of frictional heating on an inclined fault on the surface heat flux.

To develop this solution we'll start with the surface displacement due to a screw dislocation. We'll integrate over depth and rotate from the inclined frame into the horizontal frame. Finally we'll introduce the image source to reconcile the free surface boundary condition. From equation (17) we have



$$v(x^{'}, z^{'}) = \frac{-A}{2\pi\mu} \frac{x^{'}}{\left[x^2 + \left(z^{'} + a \right)^2 \right]}. \tag{47}$$

The rotation from the x, z frame to the x', z' frame is

$$\begin{aligned} x^{'} &= x\cos\alpha + z\sin\alpha \\ z^{'} &= -x\sin\alpha + z\cos\alpha \end{aligned} \tag{48}$$

Also note that $D = D'\cos\alpha$.

As before consider free slip between a depth of $-D'$ and minus infinity.

$$v(x^{'},z^{'}) = \frac{\mathrm{V}}{2\pi}\int_{-\infty}^{-D^{'}} \frac{x^{'}}{x^{2}+\left(z^{'}+a^{'}\right)^{2}}da^{'} \tag{49}$$

let $\eta = z'+a'$ so $d\eta = da'$.

$$v(x^{'},z^{'}) = \frac{\mathrm{V}}{2\pi}\int_{-\infty}^{-z'-D'} \frac{x^{'}}{x^{2}+\eta^{2}}d\eta \tag{50}$$

We have performed this integration before (equations 19-20) so it is not repeated here. The result is

$$v(x^{'},z^{'}) = \frac{\mathrm{V}}{2\pi}\tan^{-1}\left(\frac{x^{'}}{D^{'}+z^{'}}\right) \tag{51}$$

To match the surface boundary condition, we introduce an image source extending from $+D'$ to infinity but along an image fault inclined at an angle of $-\alpha$ with respect to the vertical. The displacement from the image is

$$v_{image}(x^{"},z^{"}) = \frac{\mathrm{V}}{2\pi}\tan^{-1}\left(\frac{x^{"}}{D^{"}-z^{"}}\right) \tag{52}$$

Finally combining the source and the image and substituting x and z we find

$$v(x,z) = \frac{\mathrm{V}}{2\pi}\left\{\tan^{-1}\left(\frac{x\cos\alpha + z\sin\alpha}{D^{'} - x\sin\alpha + z\cos\alpha}\right) + \tan^{-1}\left(\frac{x\cos\alpha - z\sin\alpha}{D^{'} - x\sin\alpha - z\cos\alpha}\right)\right\} \tag{53}$$

Now calculate the displacement at $z = 0$ and substitute $D' = D/\cos\alpha$.

$$v(x) = \frac{V}{\pi}\tan^{-1}\left(\frac{x\cos^2\alpha}{D - x\sin\alpha\cos\alpha}\right) \tag{54}$$

If one plots this solution there are two differences from the vertical strike-slip fault case. First, the displacement pattern is shifted along the *x*-axis by an amount $D\tan\alpha$. Therefore one can identify a dipping fault by recognizing that the position of the fault based on geodetic measurements is shifted from the position of the fault trace based on field geology.

The second difference is that the solution given in equation (54) solution does not match the far-field boundary conditions of +/- V/2. The hanging wall has more displacement than the foot wall. In the extreme case of a near horizontal fault plane, the hanging wall has the full displacement +V while the foot wall has none. This is to be expected because in our model is driven by a force couple. One can "correct" this asymmetry by subtracting a constant α from the arctangent in (54). It is left as an exercise for the reader to show the final solution is

$$v(x) = \frac{V}{\pi}\left[\tan^{-1}\left(\frac{x\cos^2\alpha}{D - x\sin\alpha\cos\alpha}\right) - \alpha\right] \tag{55}$$

We see that for α=0, this matches the previous solution, equation 22. Also, we can superimpose several of these solutions to simulate any combination of shallow and deep slip. The stress is the shear modulus times the *x*-derivative of the displacement. After a little algebra one finds.

$$\tau_{xy} = \frac{\mu V}{\pi D_\alpha}\left[1 + \left(\frac{x\cos^2\alpha}{D_\alpha}\right)^2\right]^{-1}\left[\cos^2\alpha + \frac{x\cos^3\alpha\sin\alpha}{D_\alpha}\right] \tag{56}$$

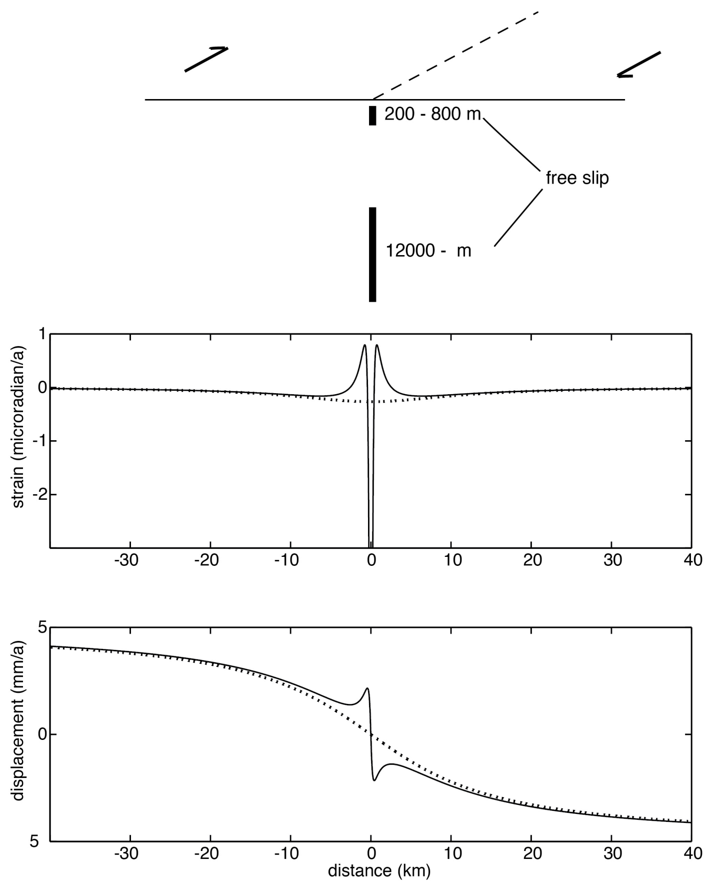
where $D_\alpha = D - x\sin\alpha\cos\alpha$.

VIII. Matlab Examples

The first example is a matlab program to calculate the strain and displacement fields due to a vertical strike-slip fault with free-slip on both shallow and deep fault planes.

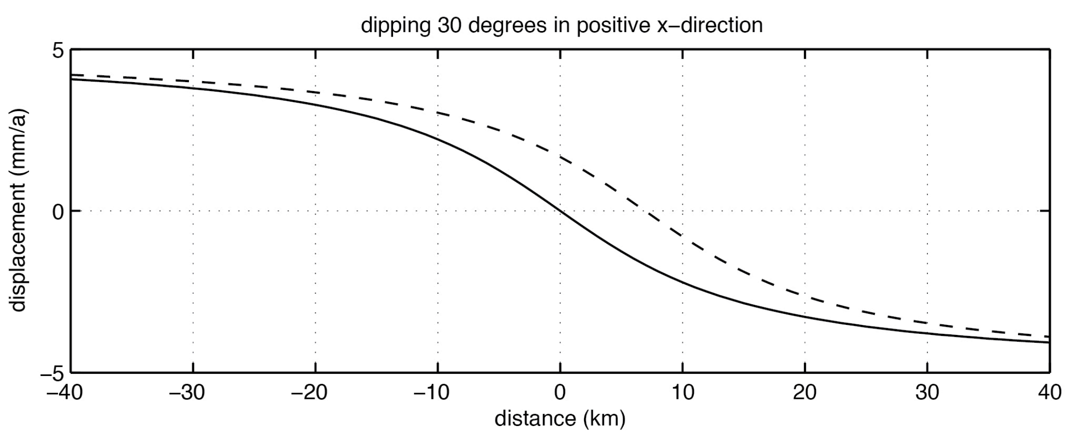
```
%
%  program to generate displacement and strain for a screw
%  dislocation. fault slip occurs both shallow and deep.
%
clear
clf
hold off
```

```
%%
V=-.01;
D=12000.;
d=800.;
d0=200;
x = -40000:8:40000;
xp = x/1000.;
%
%  this first model has shallow creep between depths of d0 and d
%
v1 = (V/pi)*(atan(x/d0)-atan(x/d));
dv1 = (V/(pi*d0))*1./(1.+(x/d0).^2) - (V/(pi*d))*1./(1.+(x/d).^2);
%
%  this second model has free-slip for depths greater than D.
%
v2 = (V/pi)*atan(x/D);
dv2 = (V/(pi*D))*1./(1.+(x/D).^2);
%
subplot(2,1,2);plot(xp,(v1+v2)*1000,xp,v2*1000,':');xlabel('distance
(km)');ylabel('displacement (mm/a)')
subplot(2,1,1);plot(xp,1.e6*(dv1+dv2),xp,1.e6*dv2,':');ylabel('strain
(microradian/a)'); axis([-40,40,-3,1])
```



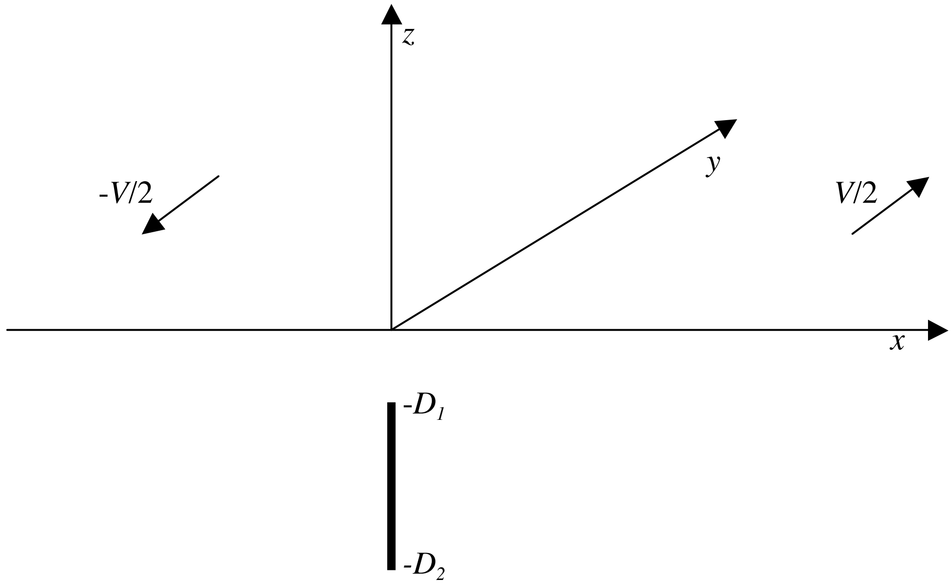
The second example is a matlab program to illustrate the effect of fault dip which simply shifts the arctangent function by an amount $D \tan \alpha$. In this example the shift is 6.9 km.

```
%
%   Compute the displacement due to a dipping fault using equation (34).
%   Note the function atan2() must be used.
%
V=-10;
alph=30*pi/180.;
D=12;
x=-40:40;
%
%
cosa=cos(alph);
sina=sin(alph);
num=x.*cosa*cosa;
dem=D-x.*sina*cosa;
vel0=V*atan2(x,D)/pi;
vel1=V*(atan2(num,dem)/pi-alph/pi);
subplot(2,1,1);plot(x,vel0,x,vel1,'--');
xlabel('distance (km)');ylabel('displacement (mm/a)');
title('dipping 30 degrees in positive x-direction')
grid
%
```



IX. Depth-averaged stress above dislocation

One issue related to the computation of stress at depth is the calculation of the depth-averaged stress due to deep slip. Using the model one could compute the stress at a range of levels between the surface of the earth and the locking depth D. However, this could require a significant amount of computer time if, for example on were doing a full 3-D numerical calculation. Instead of performing a time consuming numerical integration, we would like to establish a characterstic depth where the stress is close in magnitude to the depth-averaged stress. The diagram of the 2-D dislocation follows. The model is the same as discussed above but we have a finite-depth slip plane to avoid singularities in the analytic integration. Unfortunately the singularity remained!



In the previous sections we developed the formula for the displacement $V(x,z)$ and stress $\tau_{xy}(x,z)$ at any depth above the locking depth D. They are:

$$V(x,z)=\frac{V}{2\pi}\left\{\tan^{-1}\left(\frac{x}{D-z}\right)+\tan^{-1}\left(\frac{x}{D+z}\right)\right\} \tag{57}$$

$$\tau_{xy}(x,z)=\frac{\mu V}{2\pi}\left\{\frac{D-z}{x^2+(D-z)^2}+\frac{D+z}{x^2+(D+z)^2}\right\} \tag{58}$$

If we confine the dislocation to lie between depths D_1 and D_2 then the stress above D_1 is

$$\tau_{xy}(x,z)=\tau_{xy}(x,z,D_1)-\tau_{xy}(x,z,D_2) \tag{59}$$

What we would like to know the depth-averaged stress above the dislocation given by

$$\overline{\tau}_{xy}(x)=\frac{1}{D_1}\int_0^{D_1}\tau_{xy}(x,z)dz \tag{60}$$

We'll first do the integration for a general locking depth $D > D_1$ and then consider specific cases.

$$\bar{\tau}_{xy}(x)=\frac{\mu V}{2\pi D_1}\int_0^{D_1}\left\{\frac{D-z}{x^2+(D-z)^2}+\frac{D+z}{x^2+(D+z)^2}\right\}dz \tag{61}$$

To begin, multiply the numerator and denominator of the integrand by $1/x^2$. So the integration over the first term becomes

$$\frac{1}{x}\int_0^{D_1}\frac{\frac{D-z}{x}}{1+\left(\frac{D-z}{x}\right)^2}dz \tag{62}$$.

Next make the substitution $u=\frac{D-z}{x}$ so $dz=-xdu$. After changing the limits the integral becomes

$$-\int_{D/x}^{(D-D_1)/x}\frac{u}{1+u^2}du \tag{63}$$

We can recognize this as a $\ln(1+u^2)$ function so the result is

$$\frac{1}{2}\ln\left[\frac{1+\left(\frac{D}{x}\right)^2}{1+\left(\frac{D-D_1}{x}\right)^2}\right] \tag{64}$$

After performing the similar analysis on the second term in the integration we arrive at the depth-averaged stress

$$\bar{\tau}_{xy}(x)=\frac{\mu V}{4\pi D_1}\ln\left\{\frac{1+\left(\frac{D+D_1}{x}\right)^2}{1+\left(\frac{D-D_1}{x}\right)^2}\right\} \tag{65}$$

Now in the case of a single dislocation extending to infinite depth the depth-averaged stress is

$$\bar{\tau}_{xy}(x)=\frac{\mu V}{4\pi D_1}\ln\left\{1+\left(\frac{2D_1}{x}\right)^2\right\} \tag{66}$$

There is a problem with this result because the depth-averaged stress evaluated at $x=0$ is infinite. Therefore we cannot achieve our objective of selecting a depth where the stress versus distance function will be close to the depth-averaged stress unless we ignore the stress within some distance (e.g., 500 m) of the origin. The singularity occurs because the displacement is a step function at the locking depth, which has an infinite derivative. In the real world, the shape of the step will be smoother and limited by the finite strength of the material. Also the real world lithosphere has a finite thickness. Lets simulate a plate by limiting the depth of the dislocation to a second depth D_2. For this case we find the depth-averaged stress is given by the following formula

$$\bar{\tau}_{xy}(x) = \frac{\mu V}{4\pi D_1} \ln\left\{ \frac{\left[x^2 + (2D_1)^2\right]\left[x^2 + (D_2 - D_1)^2\right]}{x^2\left[x^2 + (D_2 + D_1)^2\right]} \right\} \qquad (67)$$

Note this equation has the correct limits. If $D_2 = D_1$ then the stress is zero and if $D_2 \to \infty$ then the stress is the same as the infinite-depth case provided in equation (65). Unfortunately this depth-averaged stress is still singular at $x=0$ (Sylvain Barbot wins the bet). However by plotting the relevant functions versus distance from the x-origin one see that the singularity is extremely localized. If we stay more than 500 m from the singular point then we find that the rms difference between the stress at a particular depth and the depth-averaged stress is minimum when the evaluation depth is 0.63 times the locking depth. The figure below shows a comparison between three stress computations.

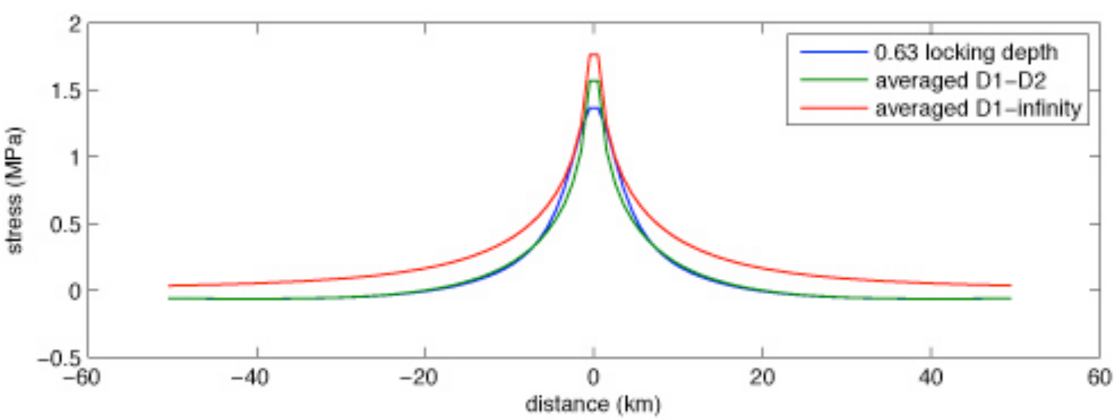


Figure Shear stress versus distance from the fault. Stress was not calculated at the origin. The parameters for the models are provided in the Matlab script below. The blue curve is the shear stress (equation 38) calculated at 0.63 times the locking depth D1. This should be compared with the green curve which is the depth-averaged stress for the same model (equation 46). The red curve is the model for a dislocation that extends to infinite depth (equation 45).

```
%
%   code to calculate the stress and depth-averaged stress due to a dislocation
%
%   set the parameters
%
```

```
V=1;
D1=10;
D2=50;
z=.63*D1;
C=V/(2.*pi);
mu=30.;
%
%   compute the displacement and stress profile
%
x=-50.5:1:50;
%
%   compute the displacement profile
%
V1 =C*(atan2(x,(D1-z))+atan2(x,D1+z));
V2 =C*(atan2(x,(D2-z))+atan2(x,D2+z));
%
%   compute stress at a constant depth
%
T1 =mu*C*((D1-z)./(x.*x+(D1-z)^2) + (D1+z)./(x.*x+(D1+z)^2));
T2 =mu*C*((D2-z)./(x.*x+(D2-z)^2) + (D2+z)./(x.*x+(D2+z)^2));
%
%   compute depth-averaged stress - averaged from 0 to D1
%
TA          =           mu*0.5*C*(log(           (x.*x+(2.*D1)^2).*(x.*x+(D2-
D1)^2)./(x.*x.*(x.*x+(D2+D1)^2))))/D1;
%
%   compute the depth-averaged dtress for an infinite-depth dislocation
%
TA2=mu*0.5*C*log(1.+(2.*D1./x).^2)/D1;
%
%  plot all the results
%
%subplot(2,1,1),plot(x,V1-V2);
%xlabel('distance (km)');ylabel('displacement (m)');
subplot(2,1,2),plot(x,T1-T2,x,TA,x,TA2);
legend('0.63 locking depth','averaged D1-D2','averaged D1-infinity')
xlabel('distance (km)');ylabel('stress (MPa)');
```