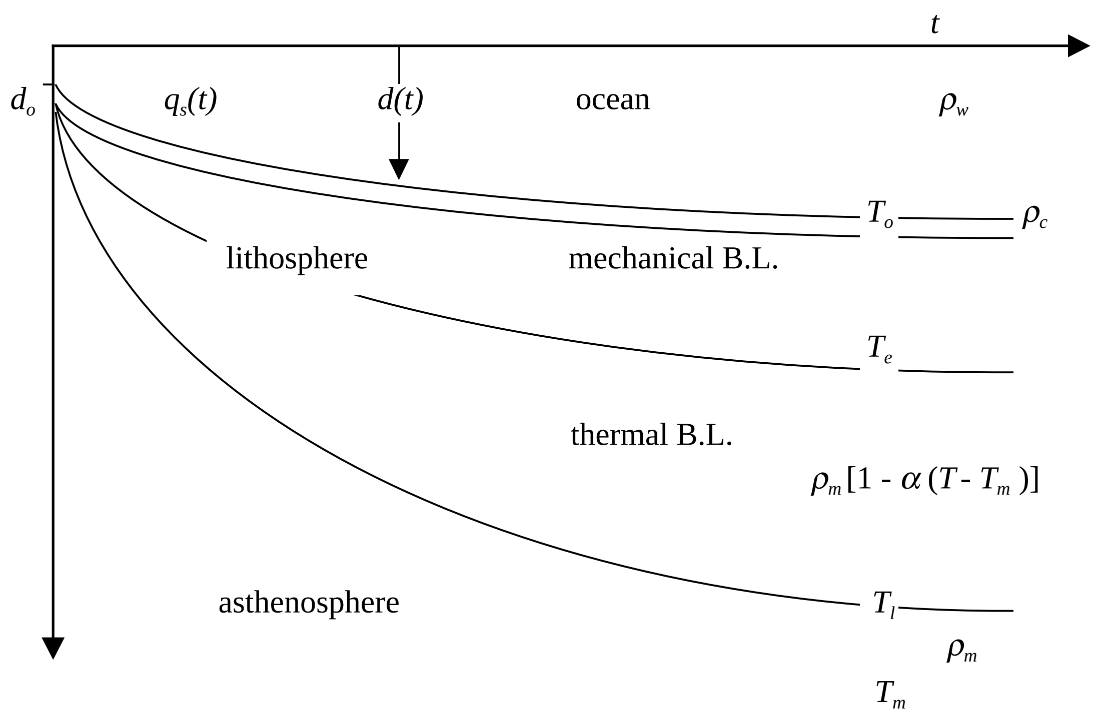
**SUMMARY OF BOUNDARY LAYER COOLING**

Cooling and subduction of the oceanic lithosphere is the primary heat-loss mechanism for the earth. Plate tectonics and mantle flow is mostly driven by the negative buoyancy of the subducting lithosphere. This lithospheric cooling process is expressed as temperature, surface heat flow, seafloor depth, geoid height, gravitational sliding force, lithospheric thickness/strength, and lithospheric buoyancy. The simple half-space cooling model can be used to predict all of these quantities to a high level of confidence. Below is a summary of the simple analytic expressions for the quantities that we have developed in this course. It is wonderful and rare to have such a simple model explain so many observations. Many other branches of earth science are "wandering in the dark" because they lack this fundamental understanding.



| Parameter | Definition | Value |
|---|---|---|
| | | |
| $T_o$ | surface temperature | 0˚C |
| $T_e$ | temp. at base of mechanical boundary layer | 750˚C |
| $T_l$ | temp. at base of thermal boundary later | 1100˚C |
| $T_m$ | mantle temperature | 1300˚C |
| $\kappa$ | thermal diffusivity | 8 x $10^{-7}$ $m^2$ $s^{-1}$ |
| $k$ | thermal conductivity | 3.3 W $m^{-1}$ $C^{-1}$ |
| $\alpha$ | thermal expansion coefficient | 3.1 x $10^{-5}$ $C^{-1}$ |
| $\rho_w$ | seawater density | 1025 kg $m^{-3}$ |
| $\rho_c$ | crustal density | 2800 kg $m^{-3}$ |
| $\rho_m$ | mantle density | 3300 kg $m^{-3}$ |
| $d_o$ | ridge axis depth | 2500 m |
| $G$ | gravitational const. | 6.67 x $10^{-11}$ $m^3$ $s^{-2}$ $kg^{-1}$ |
| $g$ | acceleration of gravity | 9.82 m $s^{-2}$ |
| $E$ | Young's modulus | 6.5 x $10^{10}$ Pa |
| $\nu$ | Poisson's ratio | 0.25 |

***Temperature***

$$T(z,t) = (T_m - T_o)\,\mathrm{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right) + T_o \qquad (1)$$

*Mechanical boundary layer thickness*

$$h_e(t) = 2\sqrt{\kappa t}\,\mathrm{erfc}^{-1}\left(\frac{T_m - T_e}{T_m - T_o}\right) \quad = \quad 5\ \mathrm{km}\ \sqrt{\mathrm{age(Ma)}} \qquad (2)$$

*Thermal boundary layer thickness*

$$h_l(t) = 2\sqrt{\kappa t}\,\mathrm{erfc}^{-1}\left(\frac{T_m - T_l}{T_m - T_o}\right) \quad = \quad 10\ \mathrm{km}\ \sqrt{\mathrm{age(Ma)}} \qquad (3)$$

***Surface Heat Flow***

$$q_s(t) = \frac{k(T_m - T_o)}{\sqrt{\pi \kappa t}} \quad = \quad 480 \text{ mWm}^{-2}[\text{age(Ma)}]^{-1/2} \tag{4}$$

***Seafloor Depth***

$$d(t) = d_o + \frac{2\alpha\rho_m(T_m - T_o)}{(\rho_m - \rho_w)}\sqrt{\frac{\kappa t}{\pi}} \quad = \quad 2500 \;+\; 350 \text{ m } \sqrt{\text{age(Ma)}} \tag{5}$$

***Geoid Height***

$$N(t) = -\frac{2\pi G\alpha\rho_m(T_m - T_o)\kappa}{g}\left\{1 + \frac{2\alpha\rho_m(T_m - T_o)}{\pi(\rho_m - \rho_w)}\right\}t \quad = \quad -0.15 \text{ m age(Ma)} \tag{6}$$

***Gravitational Sliding Force***

$$F_R = \frac{-g^2}{2\pi G}N \tag{7}$$

$$F_R = g\alpha\rho_m(T_m - T_o)\kappa\left\{1 + \frac{2\alpha\rho_m(T_m - T_o)}{\pi(\rho_m - \rho_w)}\right\}t$$

***Flexural Rigidity***

$$D(t) = \frac{Eh_e^3}{12(1-\nu^2)} = \frac{2E}{3(1-\nu^2)}\left\{erfc^{-1}\left(\frac{T_m - T_e}{T_m - T_o}\right)\right\}^3(\kappa t)^{3/2} \tag{8}$$

***Buoyancy***

$$\delta(t) = \int_o^\infty \frac{\rho_m - \rho(z)}{\rho_m}dz = \delta_{comp.} + \delta_{thermal} \;=\; 1.3 \text{ km } - \; 2\alpha(T_m - T_o)\left(\frac{\kappa t}{\pi}\right)^{1/2} \tag{9}$$

# LITHOSPHERIC BUOYANCY

(Oxburgh & Parmentier, 1977)

$$\delta = \int_0^\infty \left[\frac{\rho_m - \rho(z)}{\rho_m}\right] dz$$

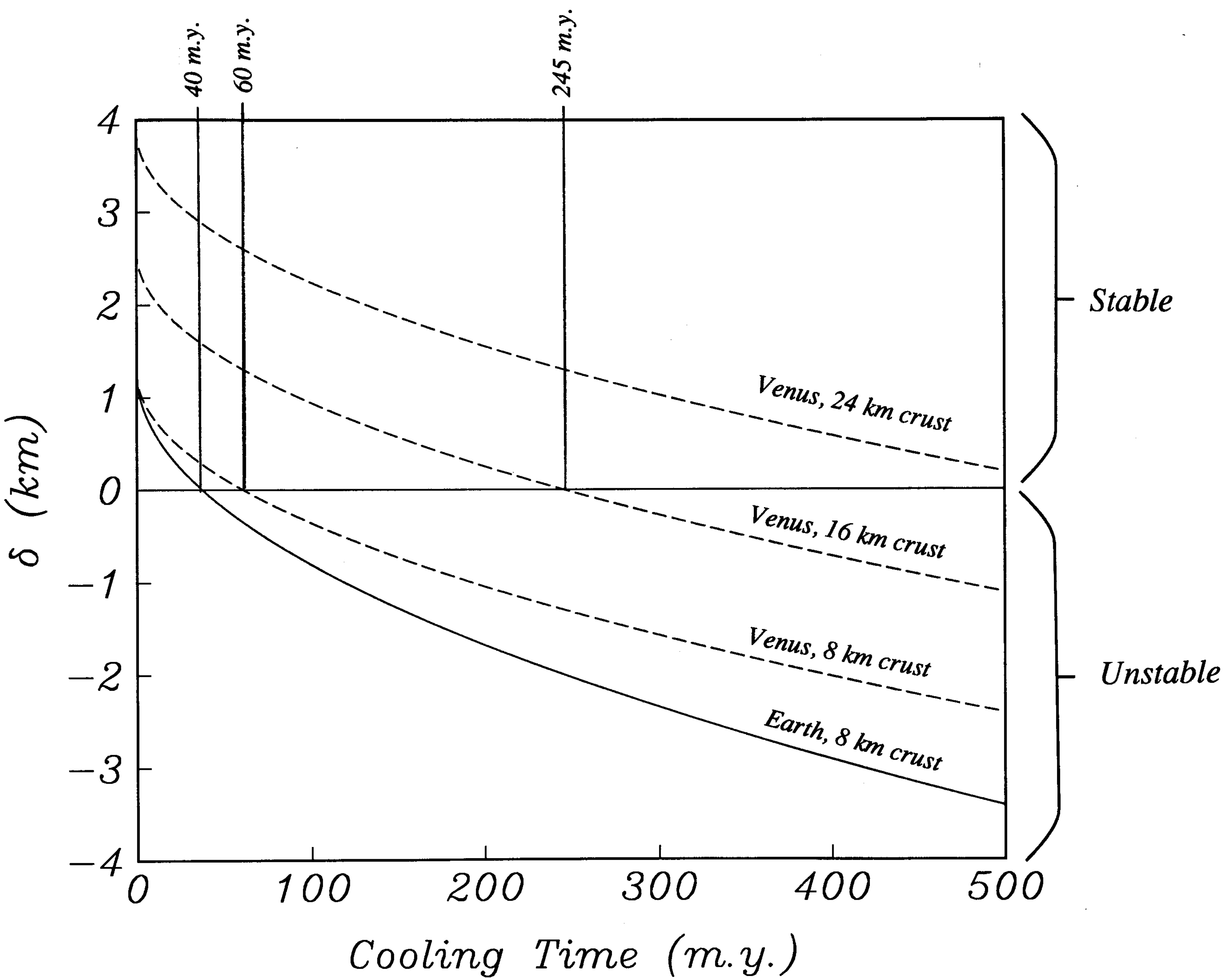
| | | |
|---|---|---|
| $\rho(z)$ | - | lithospheric density |
| $\rho_m$ | - | undepleted mantle density |
| $\delta$ | - | density defect thickness |
| | | $> 0$ stable |
| | | $< 0$ unstable |

$$\delta_{total} = \delta_{comp} + \delta_{thermal}$$

$\delta_{comp}$ = light crust + depleted mantle (assumes spreading)

$$\delta_{thermal} = -2\alpha\,(T_m - T_s)\sqrt{\frac{\kappa t}{\pi}}$$

| | Earth | Venus |
|---|---|---|
| $\delta_{comp}$ | 1.3 km | ? |
| $T_s$ | 0°C | 455°C |
| $T_m$ | 1200°C | 1400°C |
| $\alpha$ | $3.1 \times 10^{-5}\ C^{-1}$ | $3.1 \times 10^{-5}\ C^{-1}$ |
| $\kappa$ | $8.0 \times 10^{-7}\ m^2\ s^{-1}$ | $8.0 \times 10^{-7}\ m^2\ s^{-1}$ |


40 m.y.
60 m.y.
245 m.y.
Stable
Unstable
Venus, 24 km crust
Venus, 16 km crust
Venus, 8 km crust
Earth, 8 km crust
δ (km)
4
3
2
1
0
−1
−2
−3
−4
0
100
200
300
400
500
Cooling Time (m.y.)