Heat Flow Due to Plume

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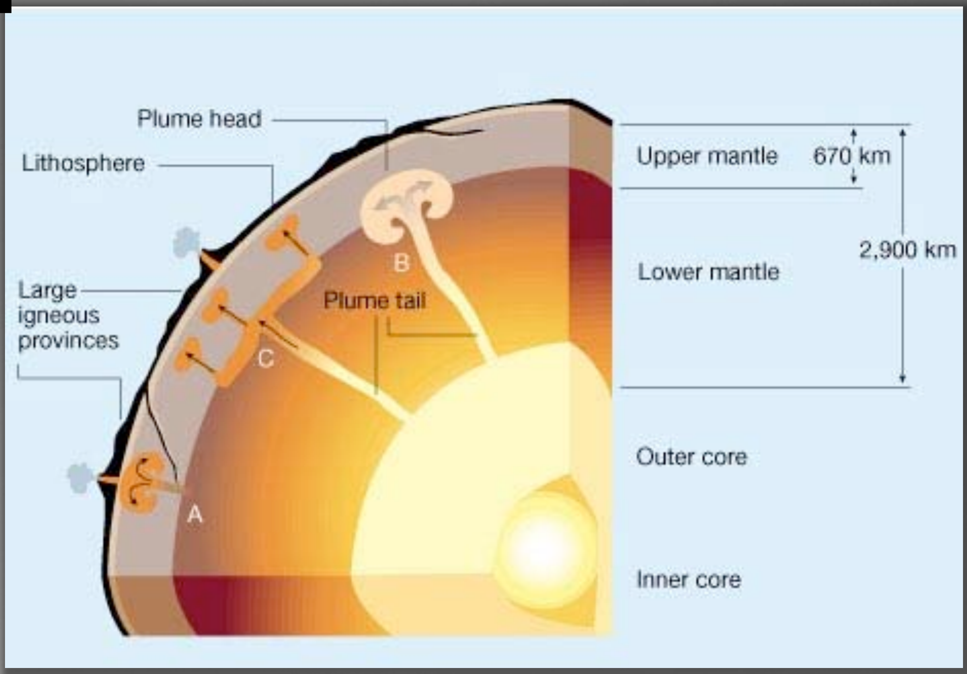
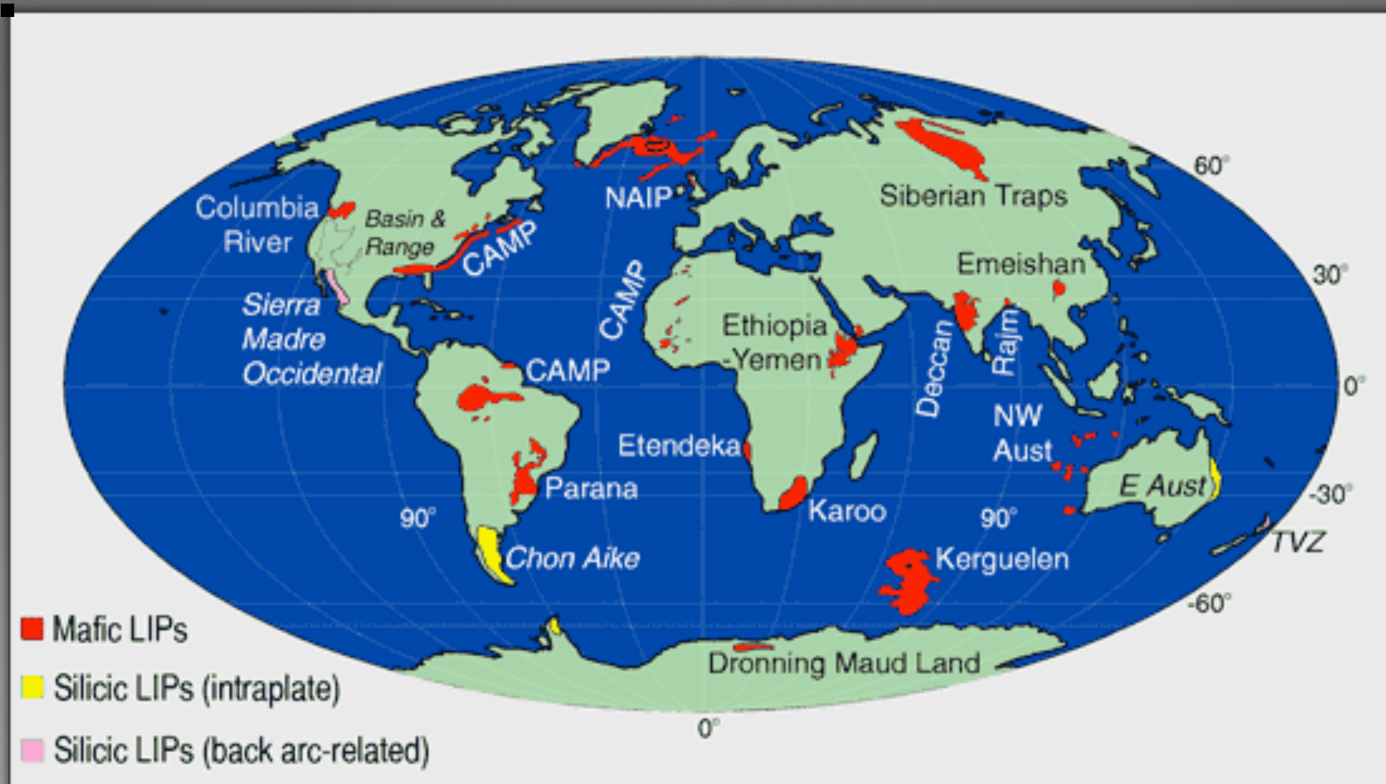
Mantle Plume

- Mantle is heated below the mantle-core boundary
- Plume rises through the mantle like a mushroom cloud with a plume head and plume tail
- Hot, buoyant mantle continues to rise through this plume pipe



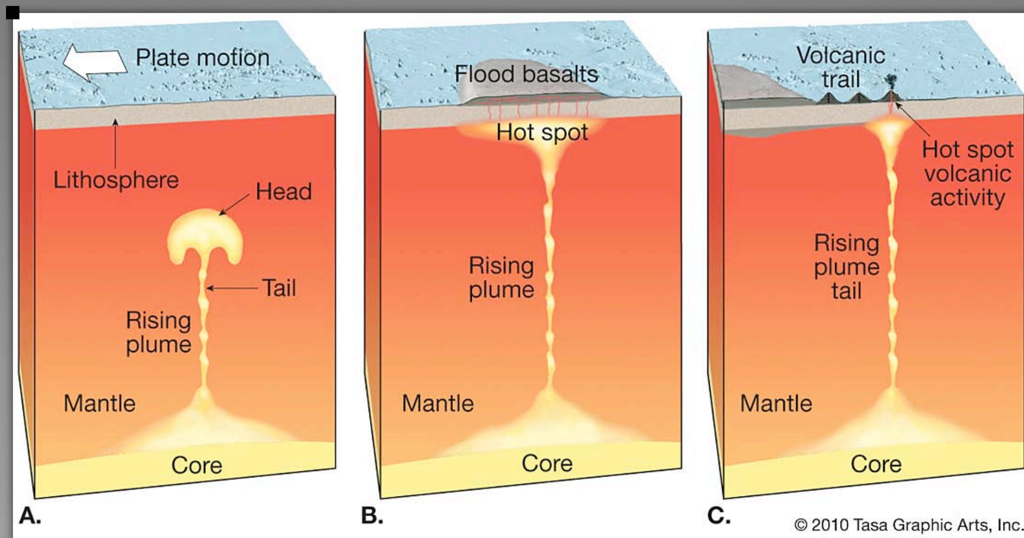
LIPS

- When the plume head reaches the surface, it erupts as a flood basalt
- This forms Large Igneous Provinces and massive dike swarms



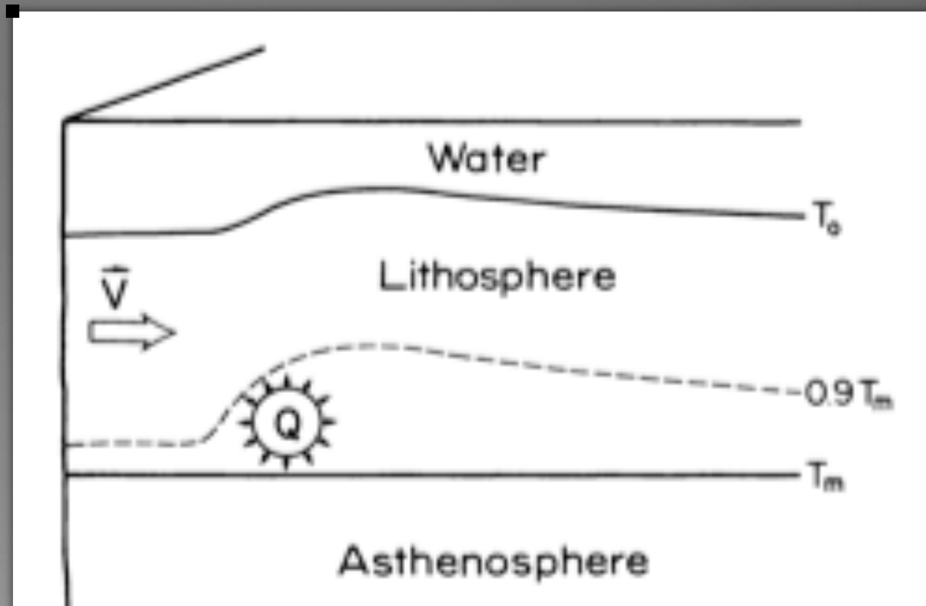
Hot Spot

- After the initial surface breach, the mantle continues to melt the surface
- This creates a volcano at this location
- As the lithosphere moves over the hot spot, a chain of volcanoes is created leaving a hot spot track
- Hotspots can drift over time
  - This could be due to mantle wind



Problem

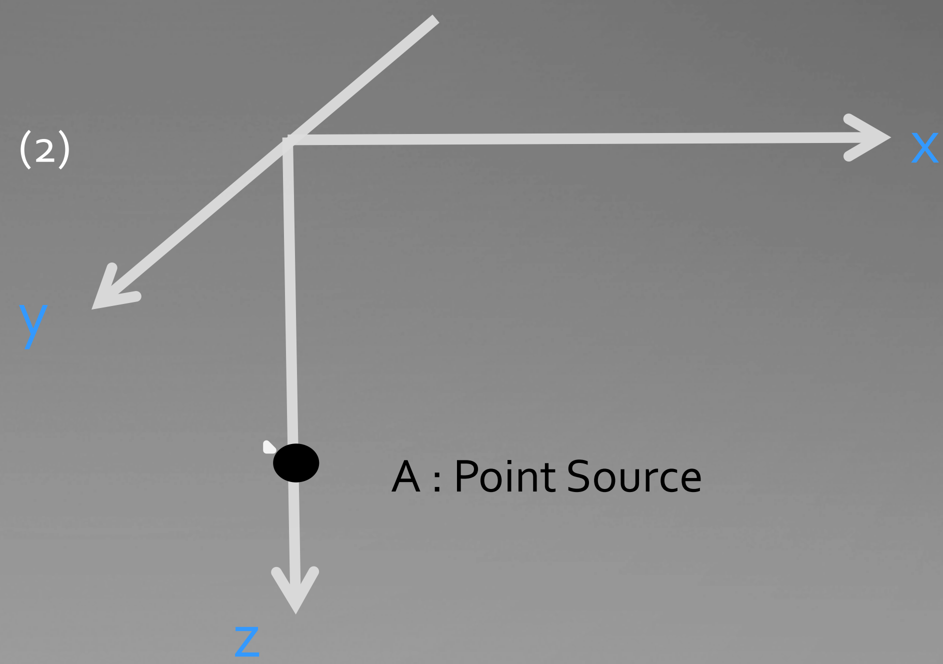
- What is the temperature in the lithosphere as it moves over a hotspot?
- How does velocity of the plate impact the heat distribution from the hot spot?



Derivation

$$PDE: \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad (1)$$

$$PDE: \nabla^2 T = \frac{1}{\kappa}\frac{\partial T}{\partial t} \qquad (2)$$



Boundary Conditions:

- Continuity on the surface
- Infinite half-space

According to Carslaw and Jaeger (1959), the solution to the PDE is:

$$T = \frac{Q}{8(\pi\kappa t)^{3/2}} e^{-[(x-x^1)^2+(y-y^1)^2+(z-z^1)^2]/4\kappa t} \quad (3)$$

Q = Total Heat Flow t=Time
Kappa = Thermal Diffusivity x,y,z = Position in Space
T=Temerature

•During dt' at t', qdt' was emitted at time
•At time t, a point is at (x,y,z)
•At time t', the same point is at (x-u(t-t'),y,z)

Temperature T(x,y,z) due to qdt' emitted at t' is Q=qdt' is:

$$T = \frac{qdt'}{8\rho c[\pi\kappa(t-t')]^{3/2}} e^{\frac{-(x-u(t-t'))^2+y^2+z^2}{4\kappa(t-t')}} \quad (4)$$

At (x,y,z) due to qdt':

$$T = \frac{q}{8\rho c(\pi\kappa)^{3/2}} \int_0^t \frac{e^{\frac{-(x-u(t-t'))^2+y^2+z^2}{4\kappa(t-t')}}}{(t-t')^{3/2}} dt' \quad (5)$$

$$T = \frac{q}{2rk\pi^{3/2}} e^{ux/2\kappa} \int_{R/2\sqrt{\kappa t}}^{\infty} e^{-\varepsilon^2-(u^2R^2/16\kappa^2\varepsilon^2)} d\varepsilon \quad (6)$$

$R = x^2 + y^2 + z^2$

As t -> ∞ steady thermal regime:

$$T = \frac{q}{4\pi kR} e^{-u(R-x)/\kappa} \qquad (7)$$

To satisfy the upper boundary condition in our case,
a negative heat source is added above the upper surface:

$$T = \frac{q}{4\pi k} \left[\frac{e^{\frac{-4(R_1 - x)}{2\kappa}}}{R_1} - \frac{e^{\frac{-4(R_2 - x)}{2\kappa}}}{R_2} \right] \qquad (8)$$

$$Q(x,y,z) = k\frac{dT}{dz} \quad (9) \qquad R_1 = (x^2 + y^2 + z^2)^{\frac{1}{2}} \quad (10)$$

$$\frac{dR_1}{dz} = \frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{1}{2}}(2z) = \frac{z}{R_1} \quad (11)$$

$$\frac{1}{R_2} = \frac{1}{(x^2 + y^2 + z^2)^{\frac{1}{2}}} \quad (12)$$

$$\frac{dR_2}{dz} = -\frac{1}{2}(x^2 + y^2 + z^2)^{-\frac{3}{2}}(2z) = \frac{-z}{R_1^{-3}} \quad (13)$$

$$T(x,y,z)=\frac{q}{4\pi k}\frac{1}{R_1}e^{-u(R_1-x)/2\kappa} \tag{14}$$

$$\frac{dT}{dz}=\frac{q}{4\pi k}\left[-\frac{z}{R_1^3}e^{-u(R_1-x)/2\kappa}+\frac{1}{R_1}e^{\frac{ux}{2\kappa}}\left(-\frac{u}{2\kappa}\right)\left(\frac{z}{R_1}\right)e^{\frac{-uR_1}{2\kappa}}\right] \tag{15}$$

$$Q=k\frac{dT}{dz}=\frac{q}{4\pi R_1^2}\left[-\frac{z}{R_1}e^{-u(R_1-x)/2\kappa}+e^{-u(R_1-x)/2\kappa}\left(-\frac{uz}{2\kappa}\right)\right] \tag{16}$$

$$Q=k\frac{dT}{dz}=\frac{qz_0}{4\pi R_1^2}e^{-u(R_1-x)/2\kappa}\left(-\frac{1}{R_1}-\frac{u}{2\kappa}\right) \tag{17}$$

$$Q = k\frac{dT}{dz} = -\frac{qz}{4\pi R_1^2}e^{-u(R_1-x)/2\kappa}\left(\frac{1}{R_1}+\frac{u}{2\kappa}\right) \tag{18}$$

Z_0 = depth of source
Kappa = 0.005 cal/cm sec °C
K = 0.01

Boundary Conditions:
•Continuity on surface
•Infinite half space

Assumptions

- Only steady state conductive heat transfer is considered
- Amount of heat generated by the hot spot is constant through time and is generated at a discreet point
- Ignoring presence of ocean
- Material through which the heat propagates is homogenous and without water content
- Ignoring sedimentation over time
- There is no lithospheric thinning or melting over time
- Effects of isostacy are ignored

Models

```
[X,Y,Z] = meshgrid
(-50000000:1000000:100000000,-50000000:1000000:100000000,0:1000000:4000000);
[x,y] = size(X);
q = 0.5;          %Source strength (HFU, microcal cm-2 s-1)
z0 = 5000000;      %Depth to source (cm), value from Birch (1980)
u = 3.17*(10^-8);  %Plate velocity (cm s-1), value from Birch (1980)
K = 5000;          %Thermal conductivity (microcal cm-1 s-1 C-1)
k = 0.01;          %Kappa value (cm2 s-1), value from Birch (1980)

R = sqrt(X.^2+Y.^2+(Z-z0).^2);
T = (q/(4*pi*K.*R)).*exp((-u*(R-X))/(2*k));

figure(1), clf
isosurface(X,Y,Z,T)
title('3D Temperature about a Hotspot at Depth 50 km, v = 1.0 cm/yr'), xlabel('X Distance (cm)'),
ylabel('Y Distance (cm)'), zlabel('Z Distance (cm)')
camlight
lighting gouraud
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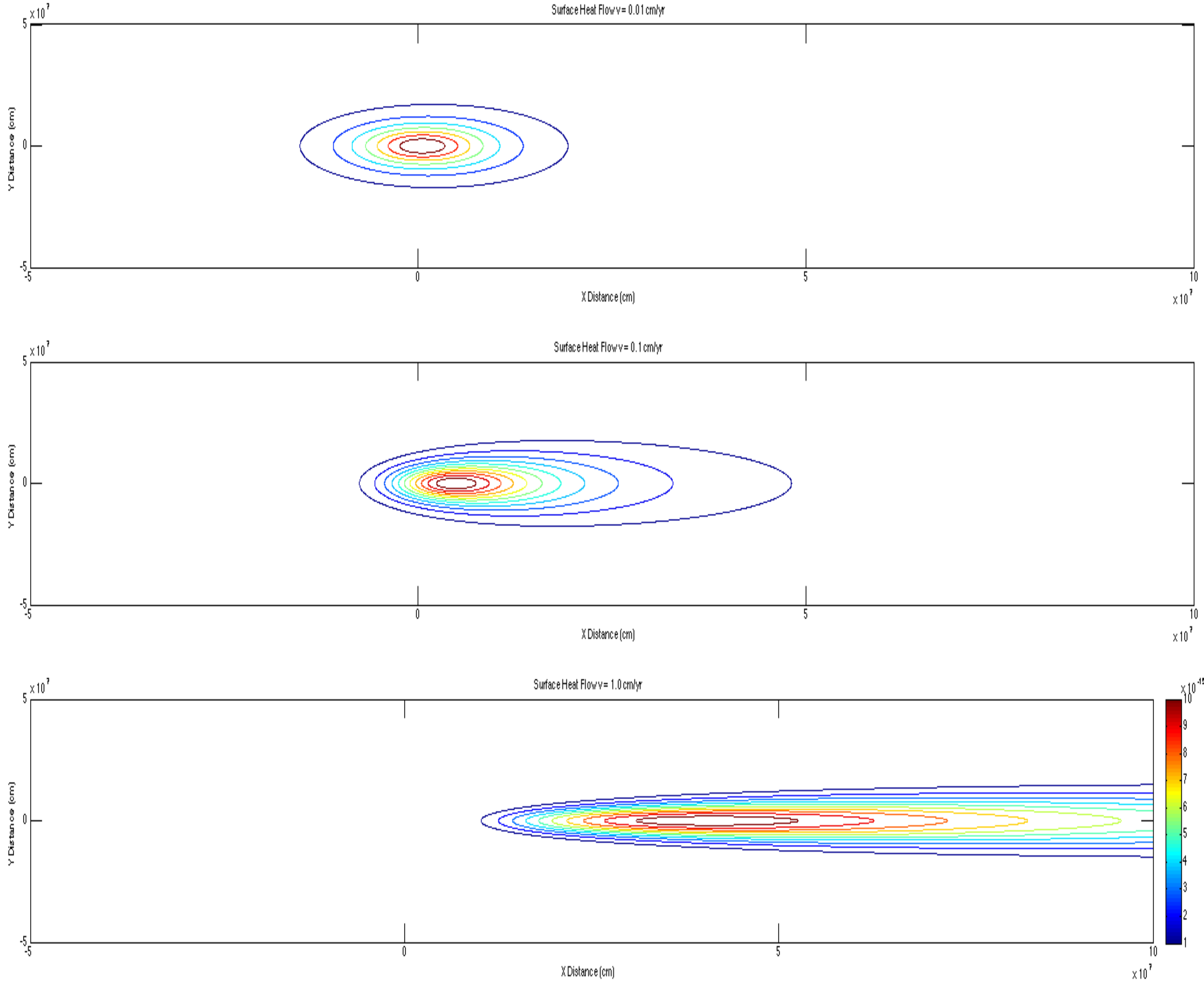


Surface Heat Flow v = 0.01 cm/yr
Surface Heat Flow v = 0.1 cm/yr
Surface Heat Flow v = 1.0 cm/yr
Y Distance (cm)
X Distance (cm)
x 10^7
x 10^-15

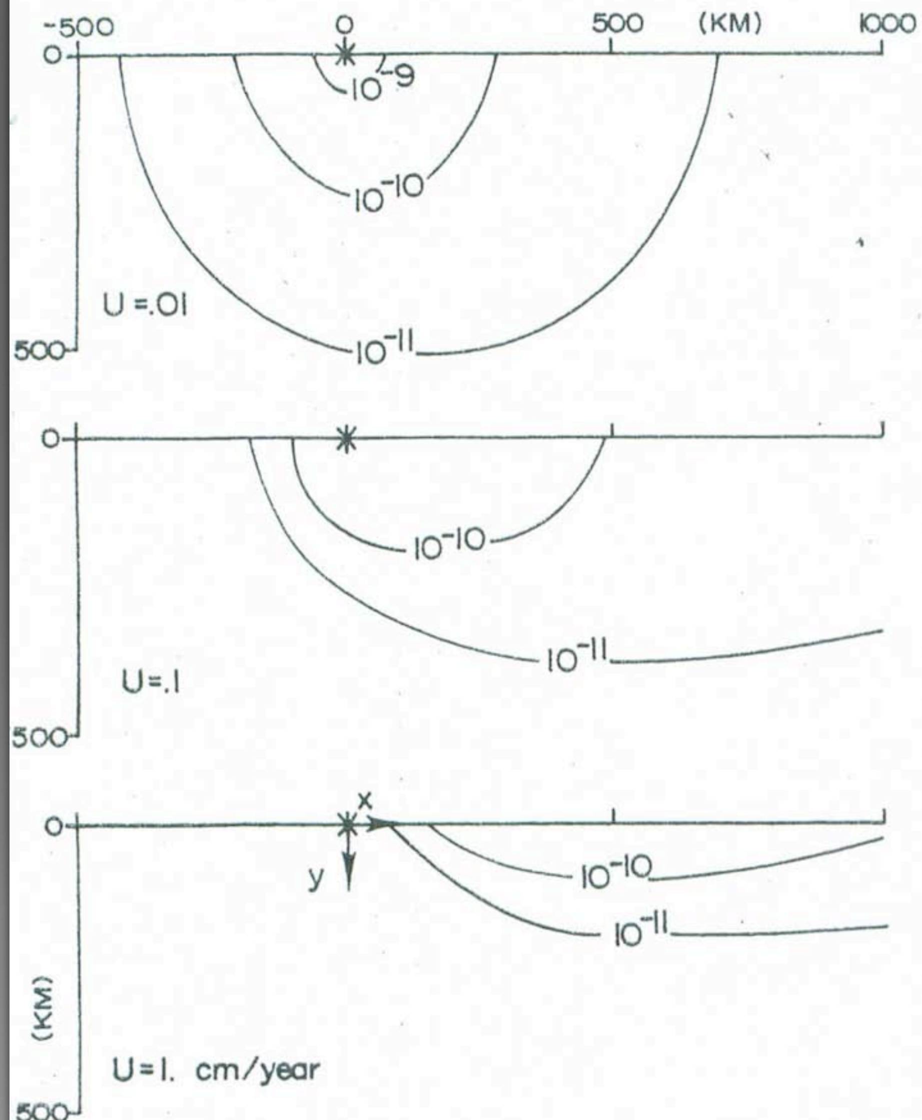
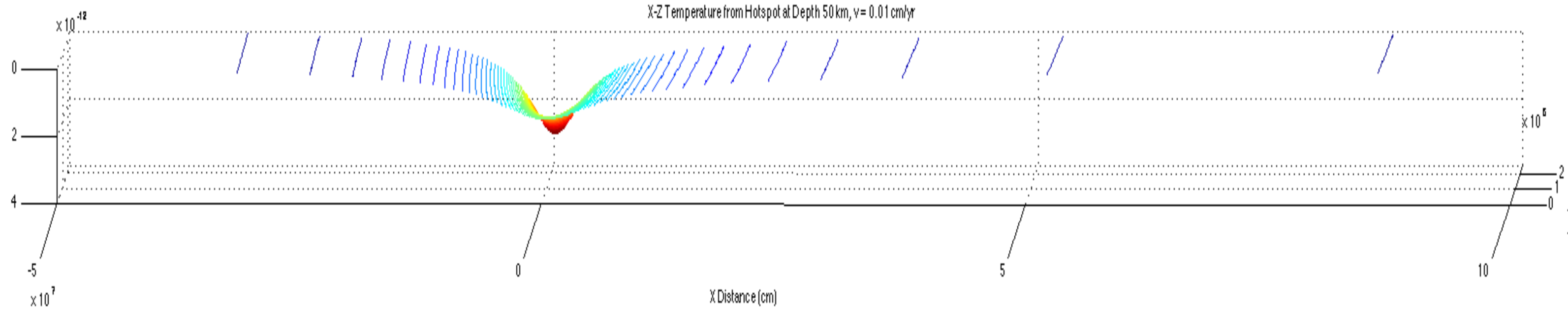
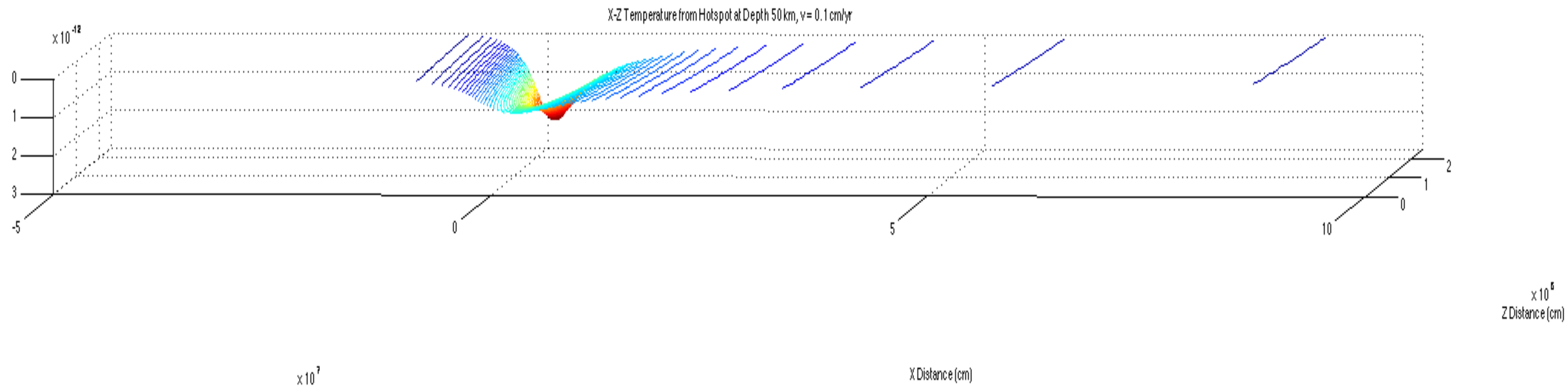
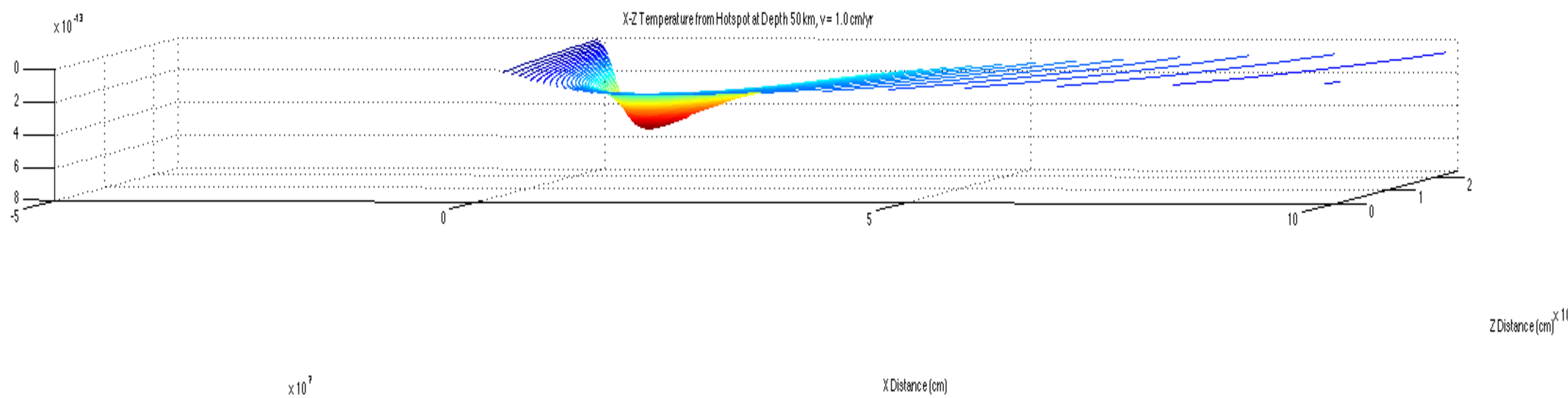


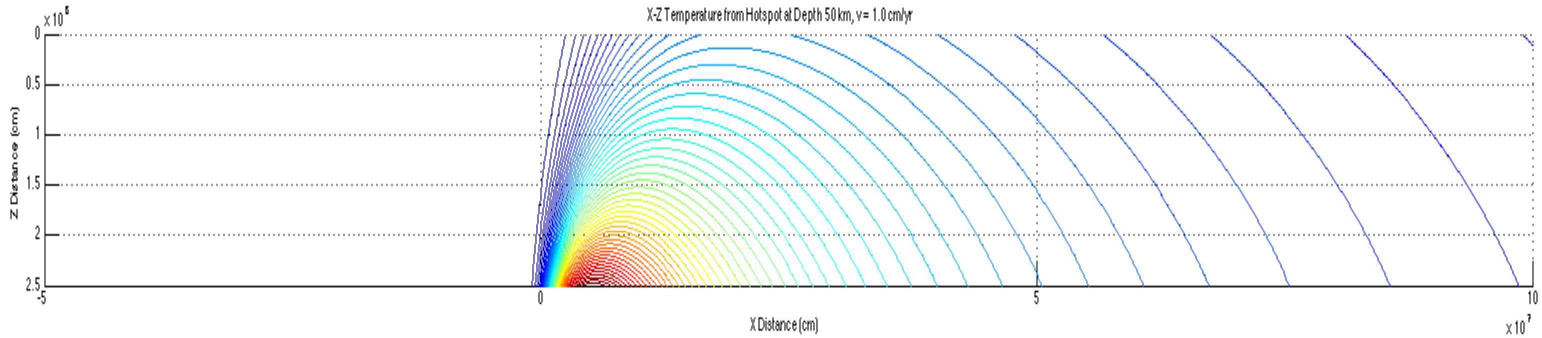
Fig. 1. Surface heat flow for unit source (1 cal/s). Contours are in heat flow units (1 HFU = μcal/cm² s). At 10 cm/yr the highest contour (10^{-11} HFU) falls beyond x = 1000 km. Stars mark projection of the source at the surface. The flow of the medium is to the right ($+x$ direction). Contours are derived from values calculated on 100 × 100 km grid.

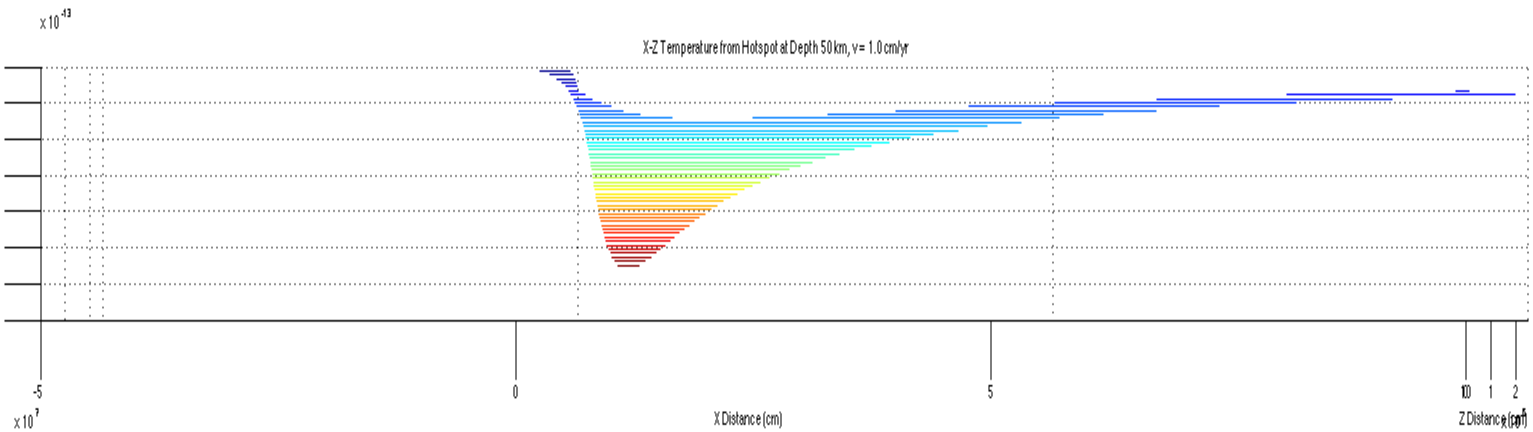
Birch,1980

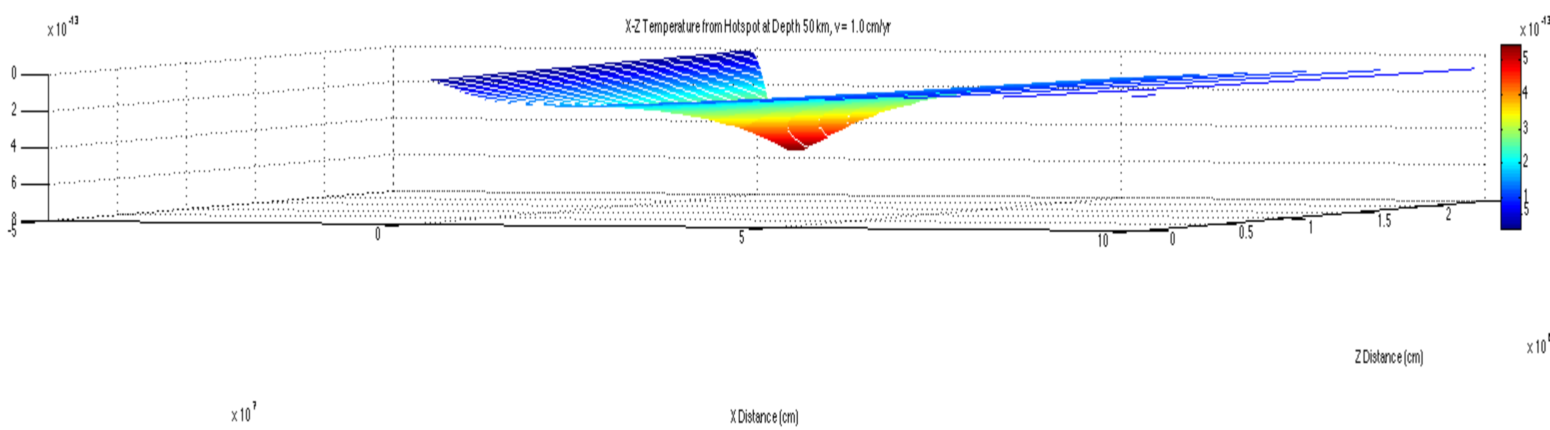

X-Z Temperature from Hotspot at Depth 50 km, v = 0.01 cm/yr
X Distance (cm)
Z Distance (cm)


X-Z Temperature from Hotspot at Depth 50 km, v = 0.1 cm/yr
Z Distance (cm)
X Distance (cm)


X-Z Temperature from Hotspot at Depth 50 km, v = 1.0 cm/yr
Z Distance (cm)
X Distance (cm)


X-Z Temperature from Hotspot at Depth 50 km, v= 1.0 cm/yr
Z Distance (cm)
X Distance (cm)


X-Z Temperature from Hotspot at Depth 50 km, v= 1.0 cm/yr
X Distance (cm)
Z Distance (cm)


X-Z Temperature from Hotspot at Depth 50 km, v= 1.0 cm/yr
Z Distance (cm)
X Distance (cm)

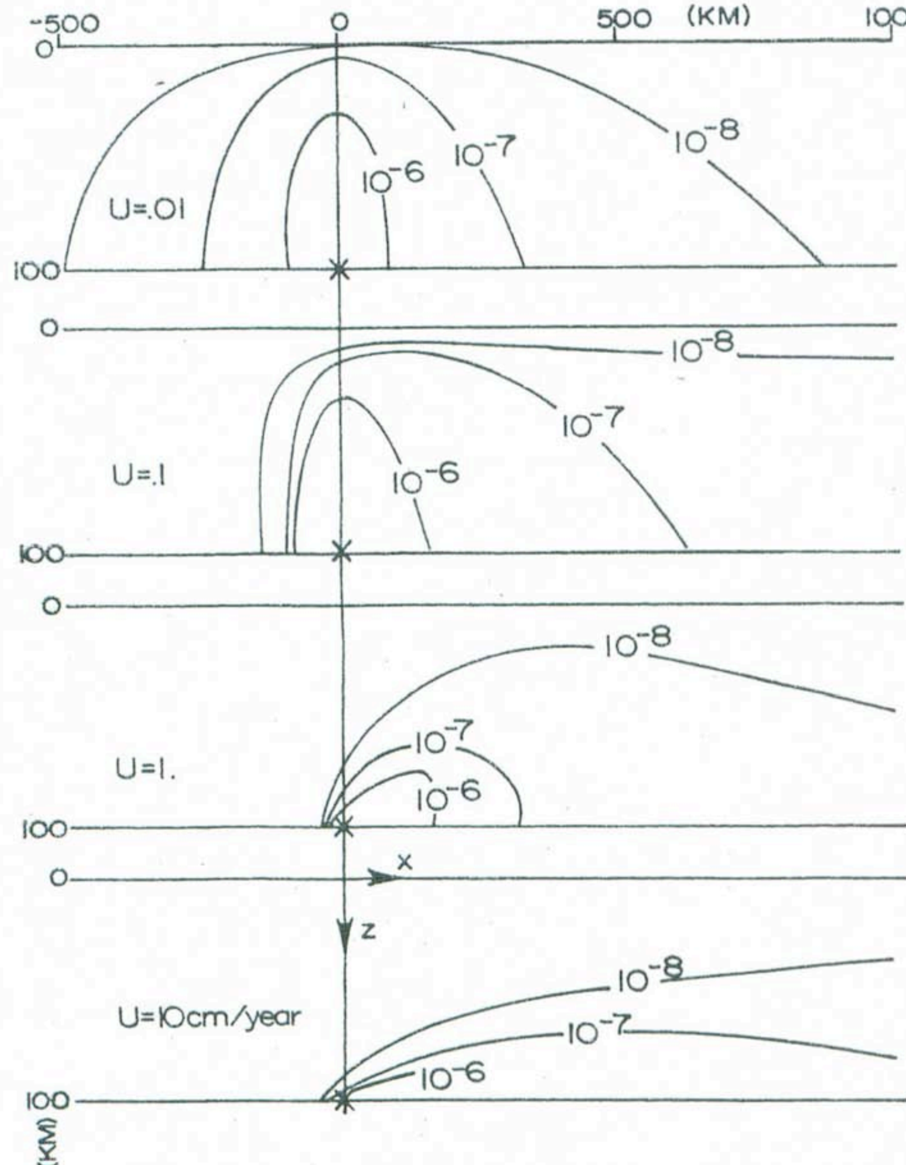
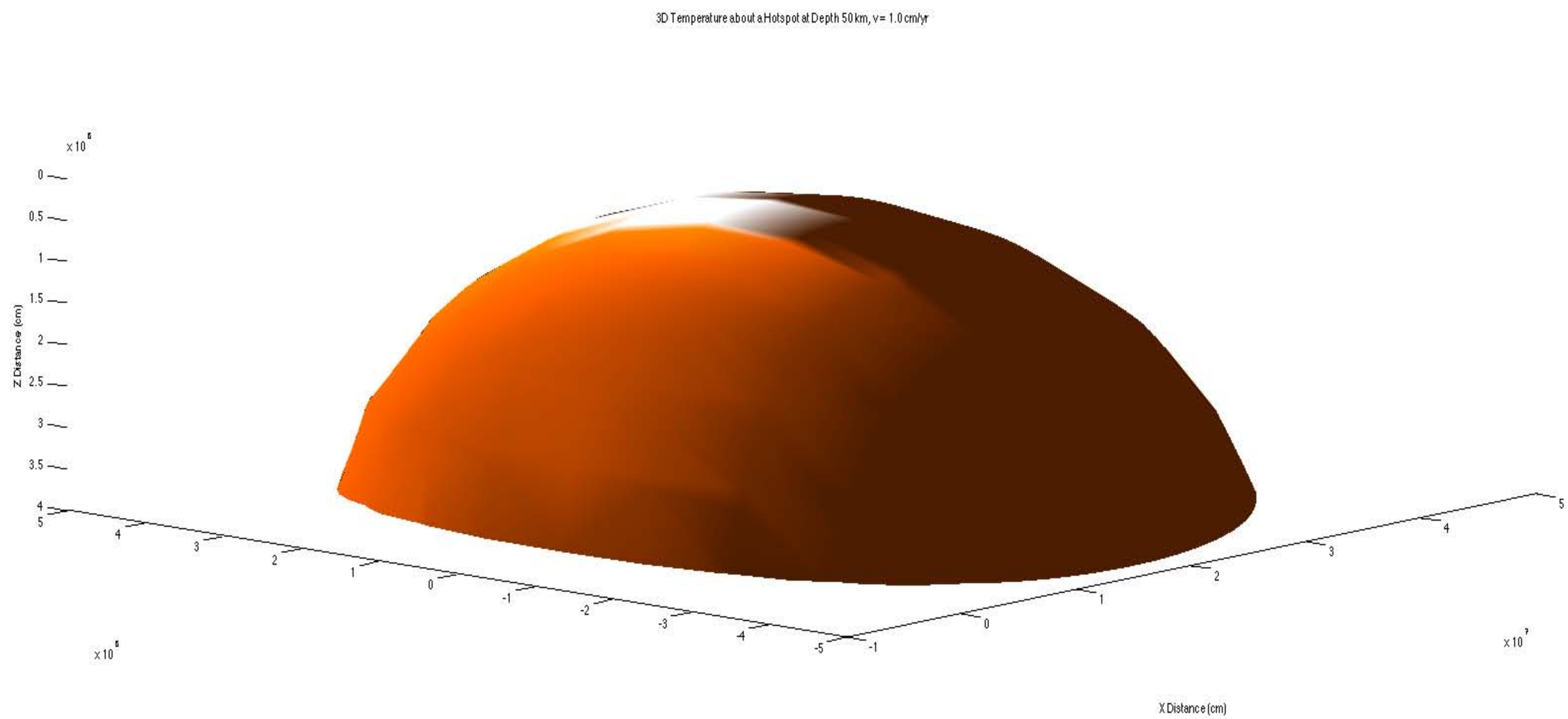
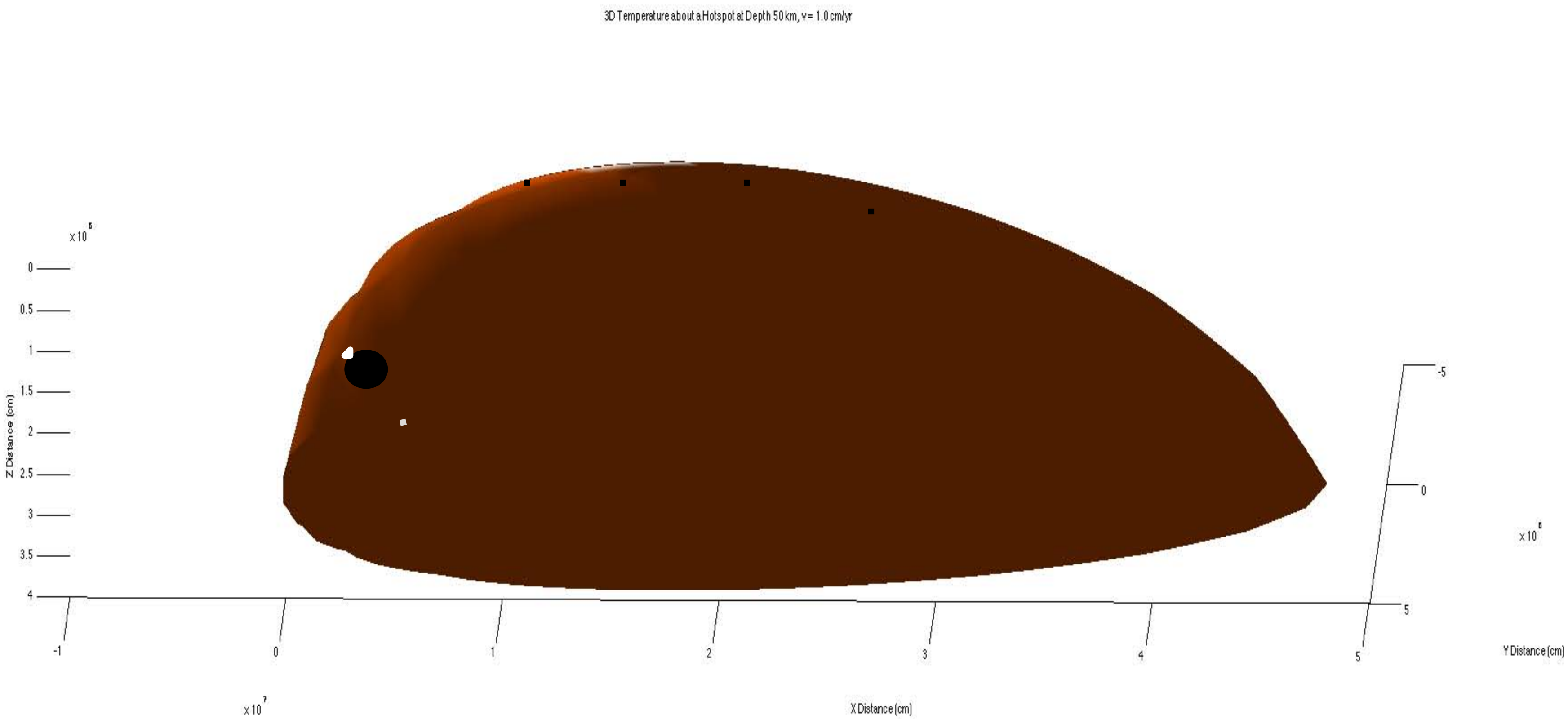


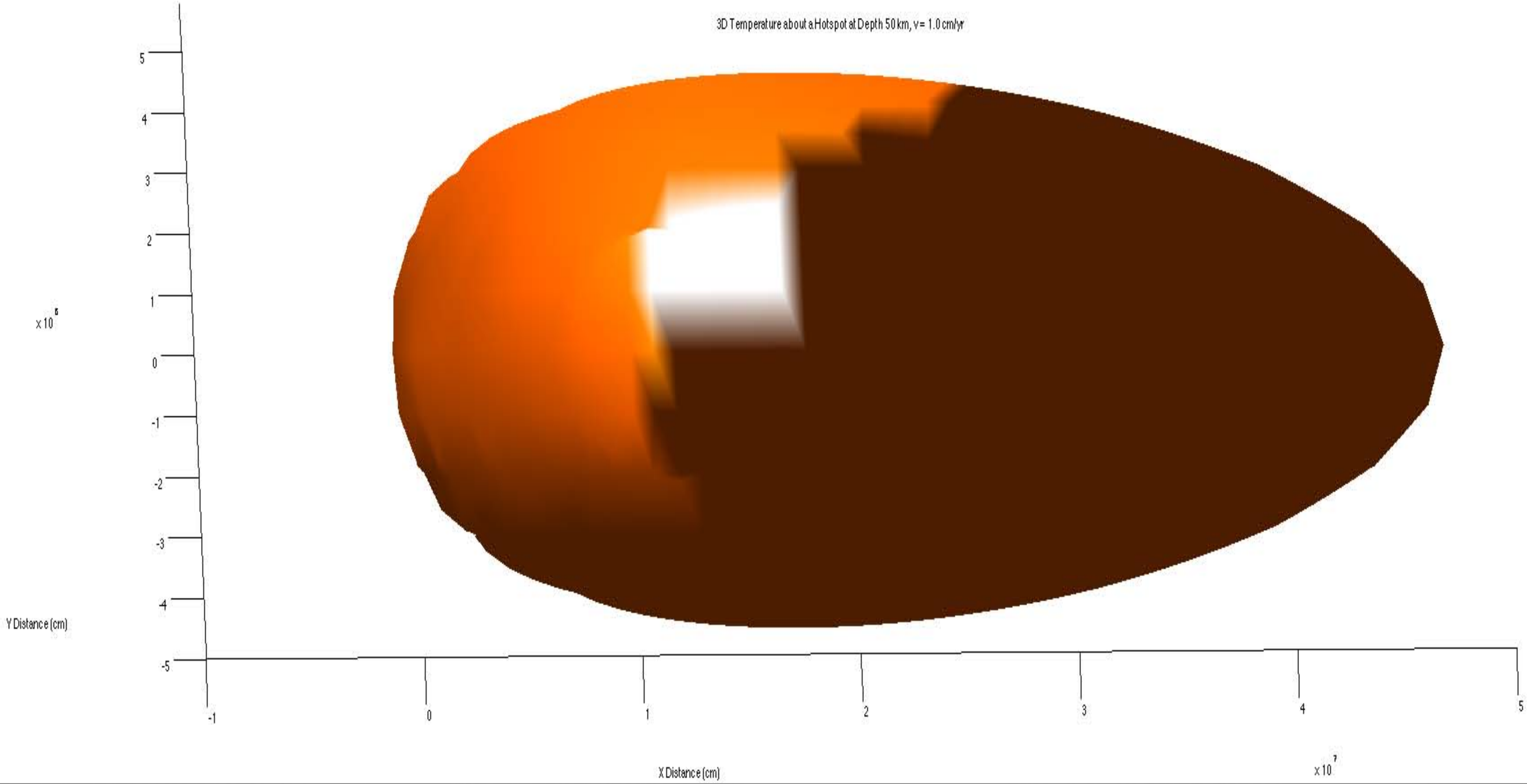
Fig. 2. Isotherms in vertical plane ($y = 0$) for unit source (1 cal/s). Contours are in degrees Celsius. Stars mark the source. The flow of the medium is to the right ($+x$ direction). Contours are derived from values calculated on 100-km (in x) by 25-km (in z) grid.

Birch, 1980


3D Temperature about a Hotspot at Depth 50 km, v= 1.0 cm/yr
Z Distance (cm)
X Distance (cm)

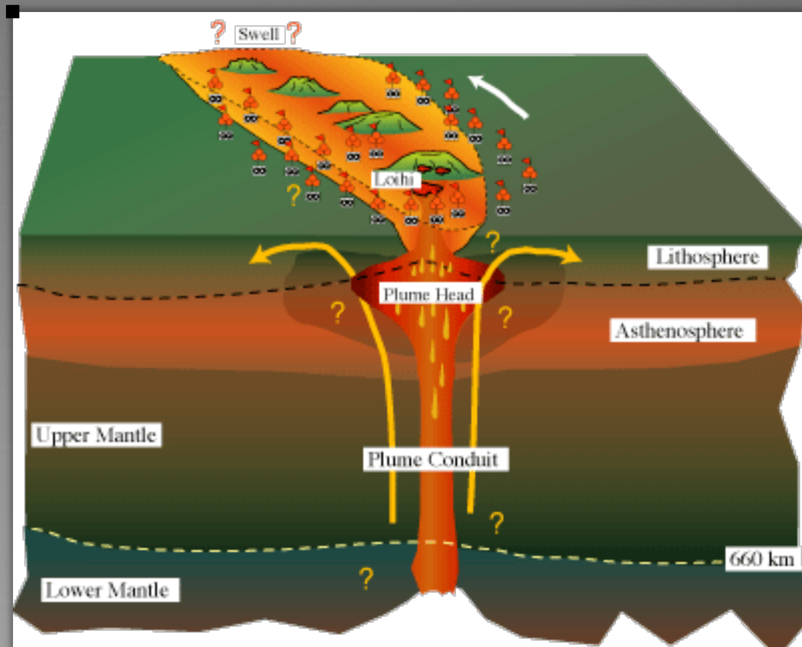


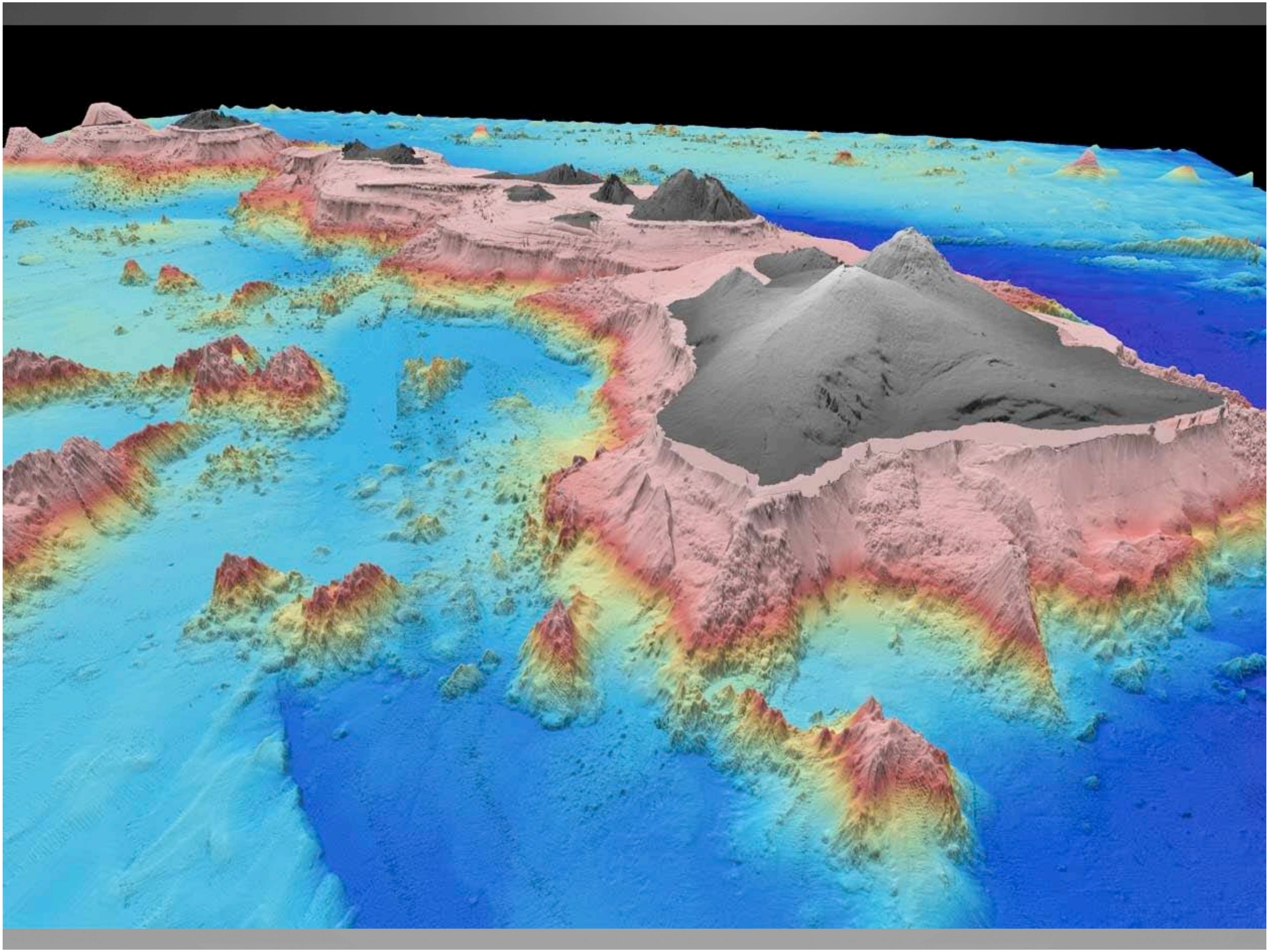
3D Temperature about a Hotspot at Depth 50 km, v= 1.0 cm/yr
x 10^6
0
0.5
1
1.5
2
2.5
3
3.5
4
Z Distance (cm)
-1
0
1
2
3
4
5
x 10^7
X Distance (cm)
-5
0
5
x 10^6
Y Distance (cm)


3D Temperature about a Hotspot at Depth 50 km, v= 1.0 cm/yr
5
4
3
2
1
0
-1
-2
-3
-4
-5
x 10^5
Y Distance (cm)
-1
0
1
2
3
4
5
x 10^7
X Distance (cm)

Hawaiian Island Chain

- Hawaii is a chain of volcanic islands generated by a mantle hot spot
- The hot spot is believed to be generated by a deep mantle plume





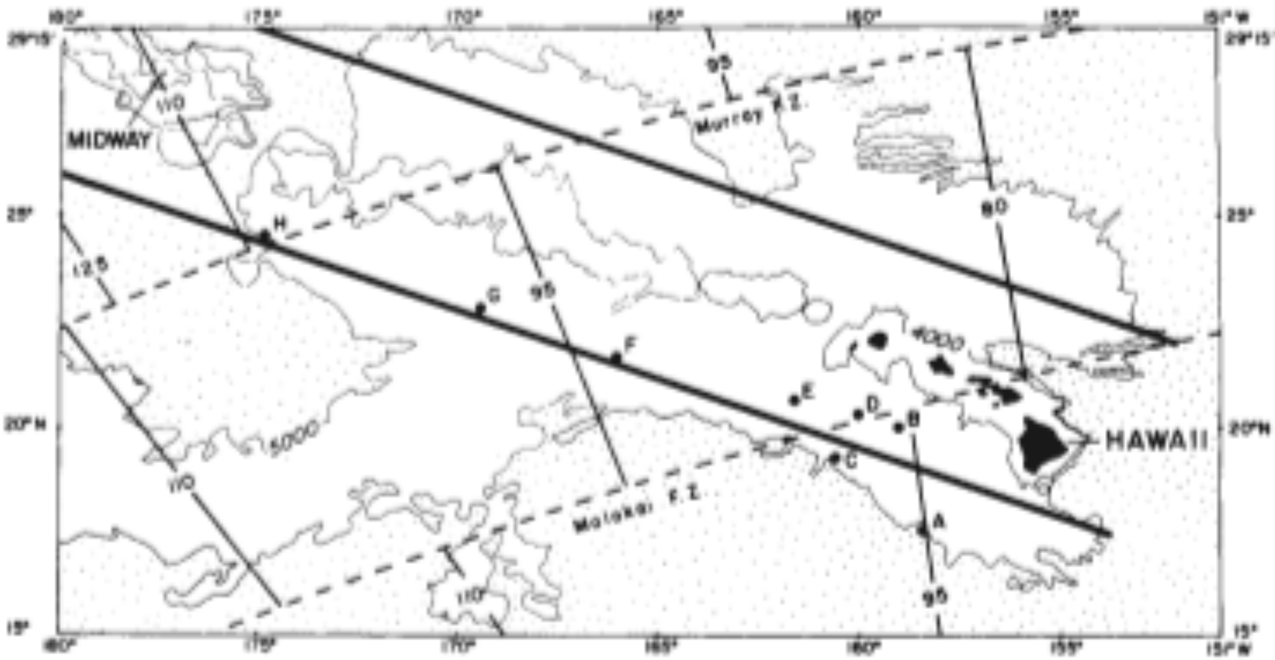


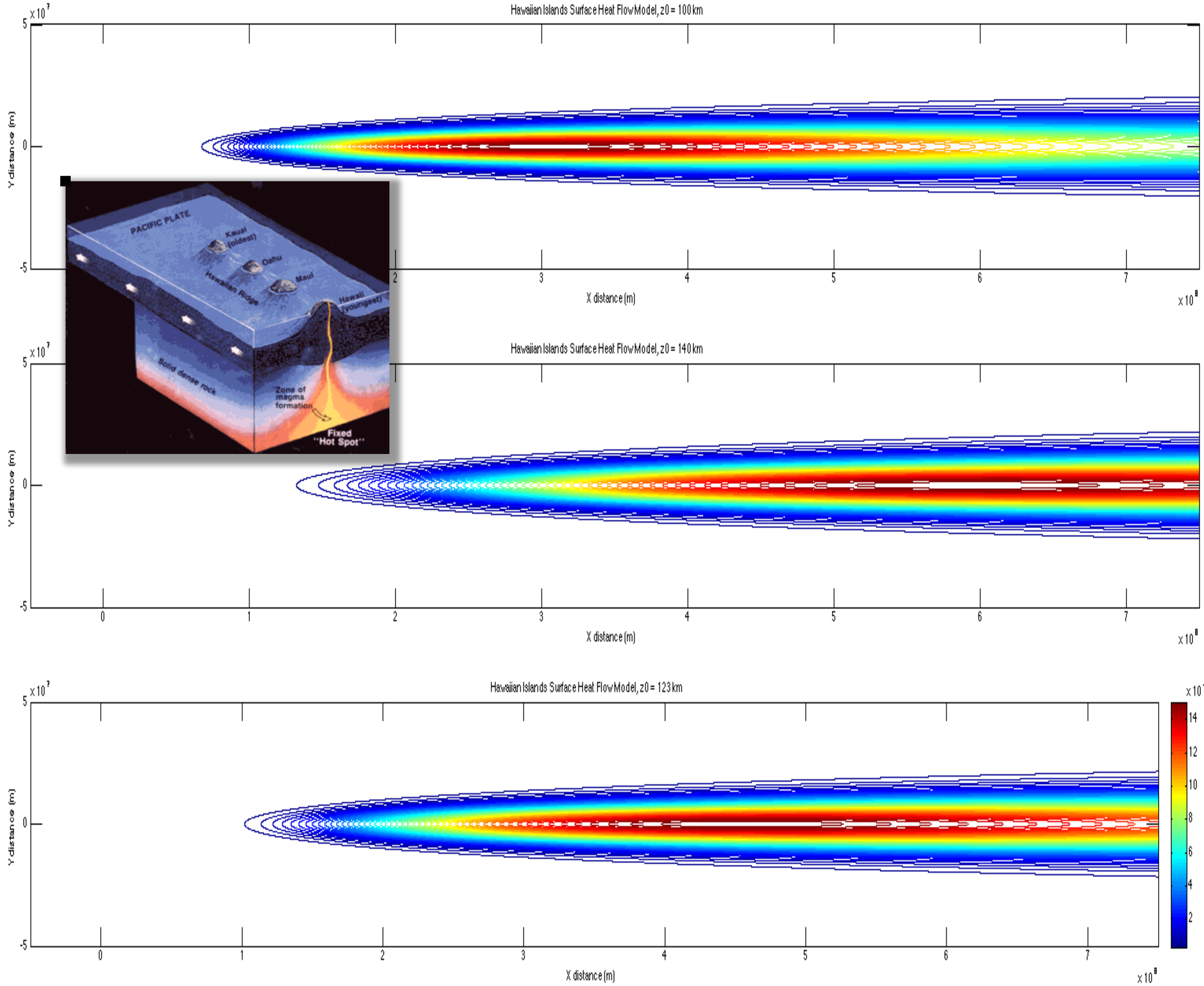
Fig. 1 Bathymetric map of the central Pacific with contours in metres. Isochrons (in Myr) and fracture zones (dashed lines) adapted from ref. 12. The extent of Hawaiian Swell is approximately indicated by the 5,000-m contour. ●, Heat flow site locations. Average depths along thick, black lines are plotted in Fig. 3.

Table 1 Summary of heat flow data on the Hawaiian Swell

Site	Location	N	$\bar{K}$ ($Wm^{-1} K^{-1}$)	$\bar{Q}$ ($mW m^{-2}$)	s.d.	95% confidence limits ($mW m^{-2}$)	t_c (Myr)	t_H (Myr)
A	17°40′ N 158°20′ W	11	0.73	52.9	3.3	±2.3	95	2.1
B	20°00′ N 159°00′ W	8	0.77	51.5	2.0	±1.8	96	3.8
C	19°20′ N 160°25′ W	18	0.77	54.5	3.5	±1.8	98	5.0
D	20°15′ N 160°02′ W	4	0.74	58.7	3.4	±6.2	86?	5.0
E	20°30′ N 161°30′ W	8	0.81	58.0	2.9	±2.6	88	6.8
F	21°45′ N 166°00′ W	10	0.86	57.4	4.5	±3.4	94	12.1
G	22°56′ N 169°36′ W	18	0.95	58.3	4.0	±2.1	97	16.3
H	24°41′ N 174°38′ W	18	0.96	58.8	4.5	±2.3	109?	22.5

N, number of gradient measurements. $\bar{K}$, mean site thermal conductivity (corrected to *in situ* conditions). $\bar{Q}$, weighted mean heat flow with sample standard deviation s.d. t_c, Estimated crustal age. t_H, Inferred time since reheating. Crustal ages were estimated using the isochron map in ref. 12. Time since reheating was determined by projecting the site locations onto a line joining Hawaii and Midway and assuming an age of 27 Myr for Midway[13].

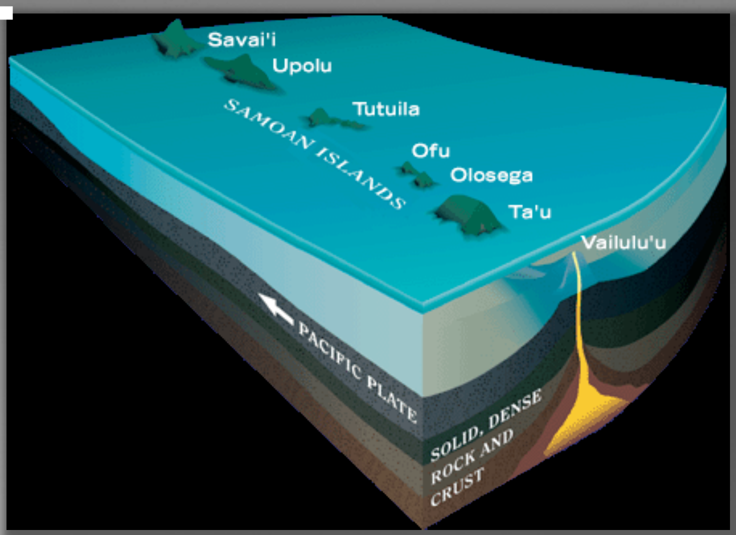
Detrick,1981

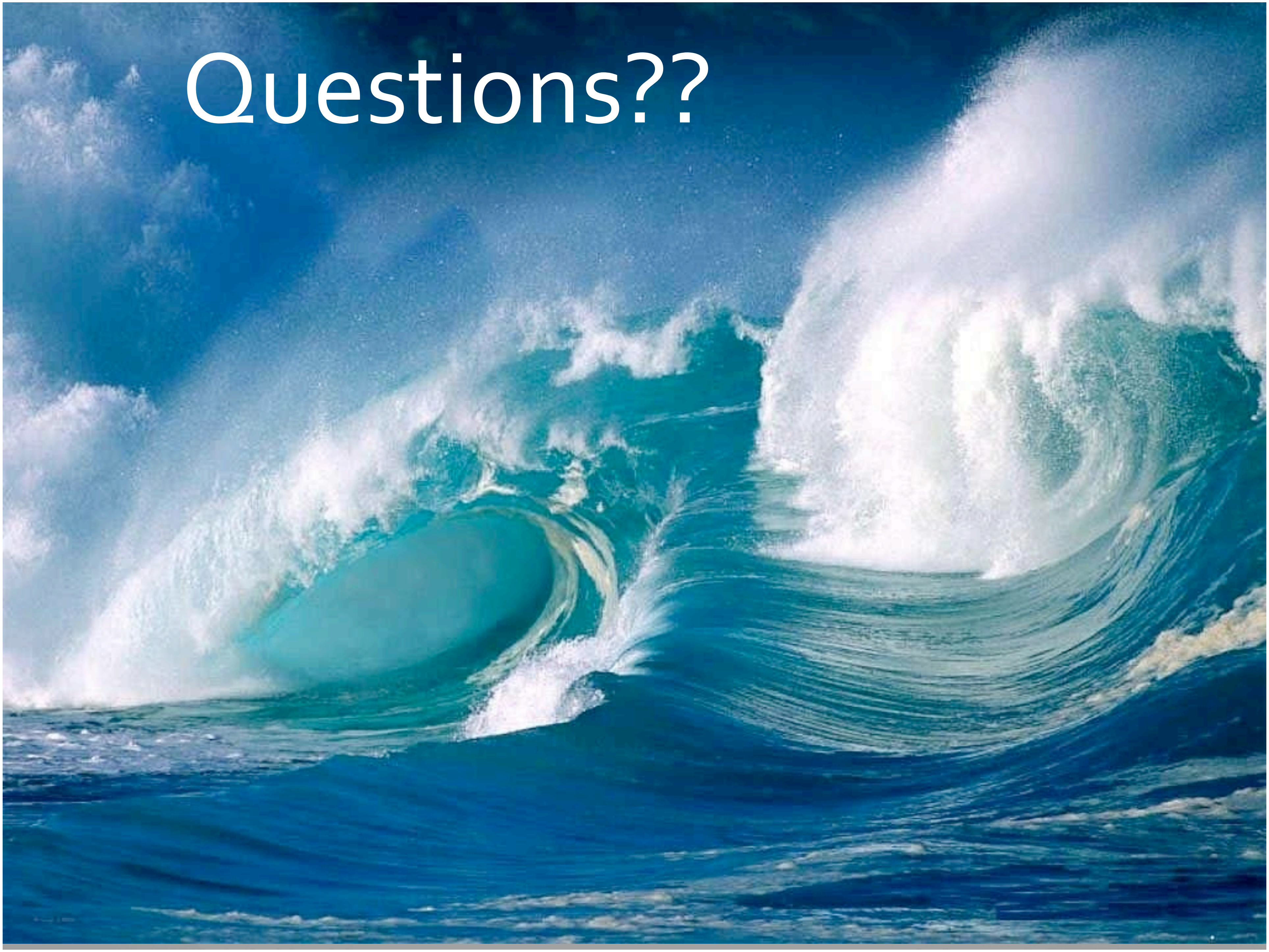


Hawaiian Islands Surface Heat Flow Model, z0 = 100 km
Y distance (m)
X distance (m)
x 10^7
x 10^8
PACIFIC PLATE
Kauai (oldest)
Oahu
Maui
Hawaiian Ridge
Hawaii (youngest)
Solid dense rock
Zone of magma formation
Fixed "Hot Spot"
Hawaiian Islands Surface Heat Flow Model, z0 = 140 km
Y distance (m)
X distance (m)
x 10^7
x 10^8
Hawaiian Islands Surface Heat Flow Model, z0 = 123 km
Y distance (m)
X distance (m)
x 10^7
x 10^8
x 10^-12

Conclusions

- This is a very simple model, but it does reflect what we see in Hawaii
- This model can be used to model heat flow in a variety of different spaces
- Additional variables can be added to make the model more realistic




Questions??