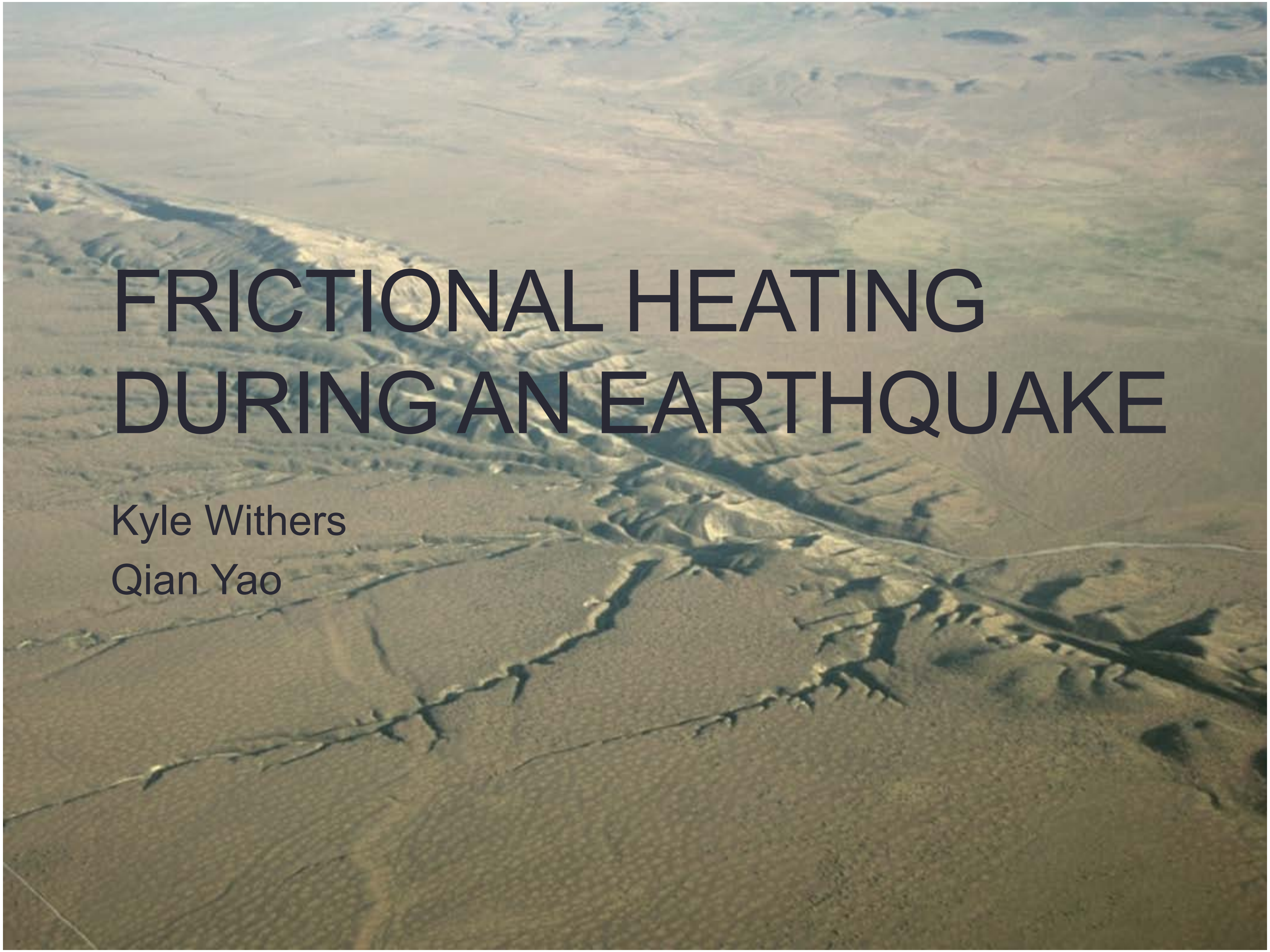

FRICTIONAL HEATING DURING AN EARTHQUAKE

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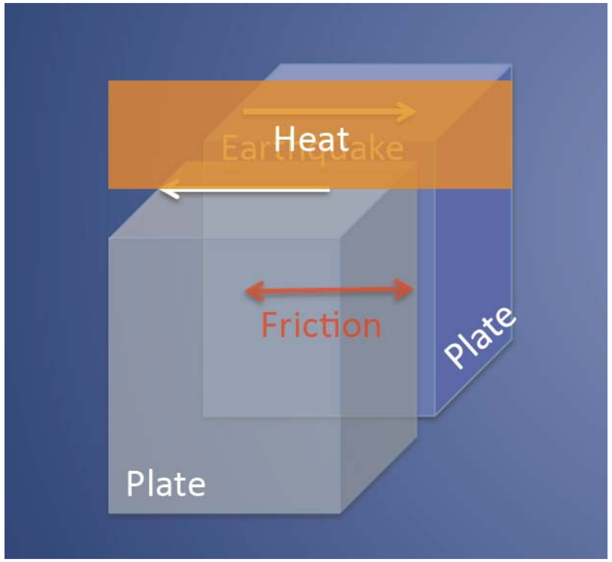
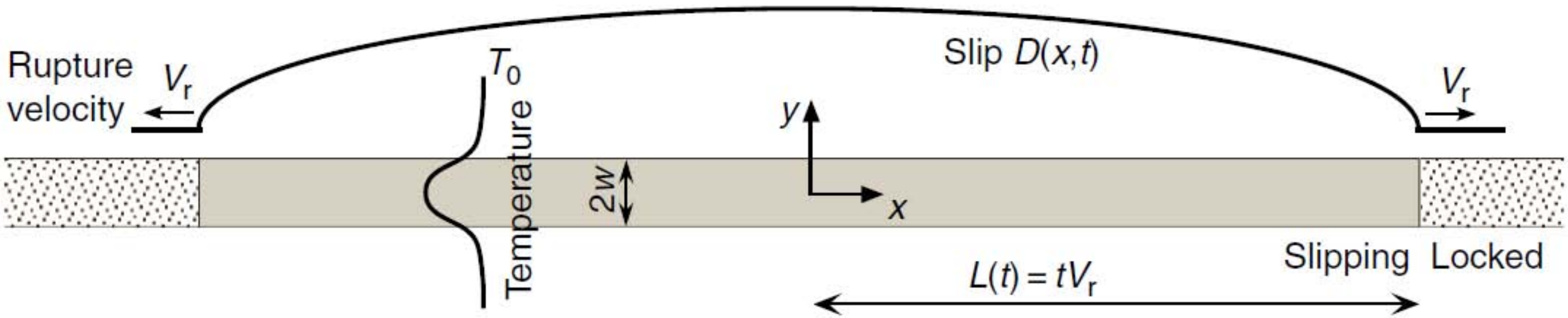
Temperature Change Along Fault

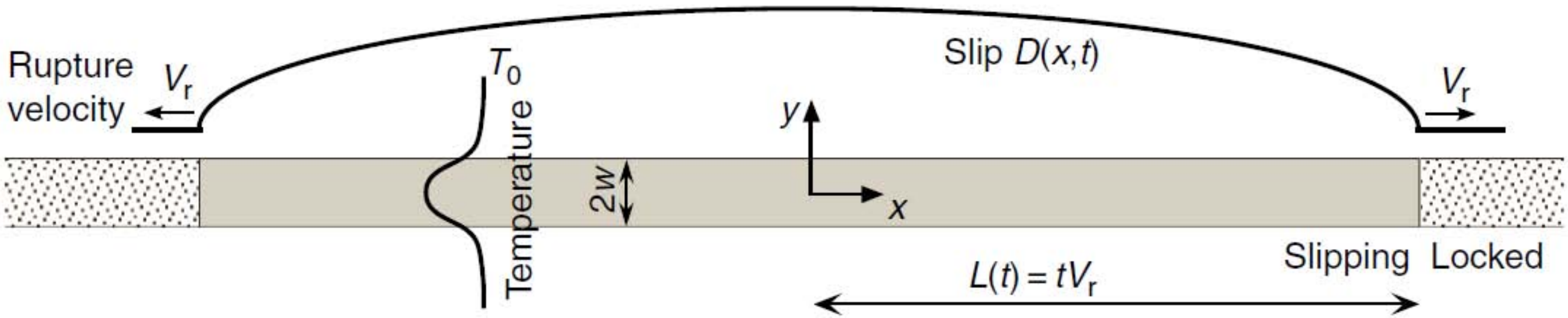
- Mode II (plain strain) crack rupturing bilaterally at a constant speed v_r
- Idealize earthquake ruptures as shear cracks
- Most energy is converted to heat during an earthquake





Importance of Frictional Heating

- Rapid slip during seismic instabilities may substantially raise temperature on a fault surface.
- Coseismic frictional heating can strongly affect the dynamic friction on the slipping interface, and thereby the seismic radiation, efficiency, and stress drop



Starting and Final Equation

- 1-D Diffusion Equation
- Ignore heat conduction in slip-parallel direction

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho}$$

$$\Delta T(y,t) = \frac{1}{2c\rho\sqrt{\pi\kappa}} \int_0^t \int_{-\infty}^{\infty} e^{\frac{(y-\xi)^2}{4\kappa(\tau-t)}} \frac{Q(\xi,\tau)}{\sqrt{(t-\tau)}} d\xi dt$$

Derivation Definitions

- Definition of terms
  - T - rock temperature
  - y – fault perpendicular coordinate
  - κ – thermal diffusivity
  - ρ – density
  - c – heat capacity
  - Q – rate of frictional heat generation

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho}$$

Deriving the Solution

- Boundary Conditions

T=T0=0, Q=0 when t<=0

$$\frac{\partial T}{\partial t} - \kappa \frac{\partial^2 T}{\partial y^2} = \frac{Q}{c\rho}$$

Fourier transforming both sides:

$$F[\frac{\partial T}{\partial t}] - F[\kappa \frac{\partial^2 T}{\partial y^2}] = F[\frac{Q}{c\rho}]$$

Deriving the Solution

$$\frac{\partial \tilde{T}}{\partial t} = -\alpha^2 \kappa \tilde{T} + \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \frac{Q(y,t)}{c\rho} e^{-i\alpha y} dy$$

Or:

$$\frac{\partial \tilde{T}}{\partial t} = -\alpha^2 \kappa \tilde{T} + \frac{\tilde{Q}(\alpha,t)}{c\rho}$$

Deriving the Solution

$$\frac{\partial \tilde{T}}{\partial t} = -\alpha^2 \kappa \tilde{T} + \frac{\tilde{Q}(\alpha,t)}{c\rho}$$

This is a common ODE form:

$$y'(x) + p_0(x)y(x) = f(x)$$

where: $p_0(x) = \alpha^2 \kappa$ $\quad f(x) = \frac{\tilde{Q}(\lambda,t)}{c\rho}$

Integration Factor: $I(x) = e^{\int \alpha^2 \kappa dt} = e^{\alpha^2 \kappa t}$

$$\tilde{T}(\alpha,t) = e^{-k\alpha^2 t} \int_0^t \frac{Q(\alpha,t)}{c\rho} e^{-k\alpha^2 \tau} d\tau$$

Deriving the Solution

$$F^{-1}[\tilde{T}(\alpha,t)] = \Delta T = \frac{1}{c\rho}\int_0^t F^{-1}[e^{-k\alpha^2(t-\tau)}\tilde{Q}(\alpha,t)]d\tau$$

gives:

$$\Delta T = \frac{1}{c\rho}\int_0^t F^{-1}[\tilde{Q}(\alpha,t)] \cdot F^{-1}[e^{-k\alpha^2(t-\tau)}]d\tau$$

Since the Fourier Transform of a Gaussian is:

$$F[ae^{-bk^2}] = \frac{a}{\sqrt{2b}}e^{-\frac{m^2}{4b}}$$ giving:

$$\Delta T = \frac{1}{c\rho}\int_0^t \frac{1}{\sqrt{2\pi}}Q(\alpha,t) \cdot \frac{1}{\sqrt{2\kappa(t-\tau)}}e^{-\frac{y^2}{4\kappa(t-\tau)}}d\tau$$

Deriving the Solution

- Giving the solution as:

$$\Delta T = \frac{1}{2c\rho\sqrt{\kappa\pi}} \int_0^t \int_{-\infty}^{\infty} e^{\frac{(y-\xi)^2}{4\kappa(t-\tau)}} \frac{Q(\xi,\tau)}{\sqrt{(t-\tau)}} d\xi d\tau$$

Rate of Frictional Heat Generation

$$\Delta T = \frac{1}{2c\rho\sqrt{\kappa\pi}} \int_0^t \int_{-\infty}^{\infty} e^{\frac{(y-\xi)^2}{4\kappa(t-\tau)}} \frac{Q(\xi,\tau)}{\sqrt{(t-\tau)}} d\xi d\tau$$

$$Q(x,y,t) = \frac{\sigma_f(x)}{2w(x)} \frac{\partial D(x,t)}{\partial t} \text{ for t>0 } \text{ and } |y| < w$$

$$Q = 0 \text{ for } |y| > w$$

D – crack displacement
v - slip velocity, derivative of D with respect to time
t – slip duration
2w – fault zone thickness
μ – coefficient of friction
σ_n – fault normal stress

Deriving the Solution

$$\Delta T = \frac{1}{2c\rho\sqrt{\kappa\pi}} \int_0^t \int_{-w}^{w} e^{\frac{(y-\xi)^2}{4\kappa(t-\tau)}} \frac{1}{\sqrt{(t-\tau)}} \frac{\mu\sigma_n}{2w} \frac{\partial D}{\partial t} d\xi d\tau$$

Using the substitution:

$$x = \frac{(y-\xi)}{2\sqrt{\kappa(t-\tau)}}$$

Giving:

$$\Delta T = \frac{1}{2c\rho\sqrt{\kappa\pi}} \int_0^t \int_{\frac{(y-w)}{2\sqrt{\kappa(t-\tau)}}}^{\frac{(y+w)}{2\sqrt{\kappa(t-\tau)}}} e^{-x^2} \frac{1}{\sqrt{(t-\tau)}} \frac{\mu\sigma_n}{2w} 2\sqrt{\kappa(t-\tau)} \frac{\partial D}{\partial t} d\xi d\tau$$

Deriving the Solution

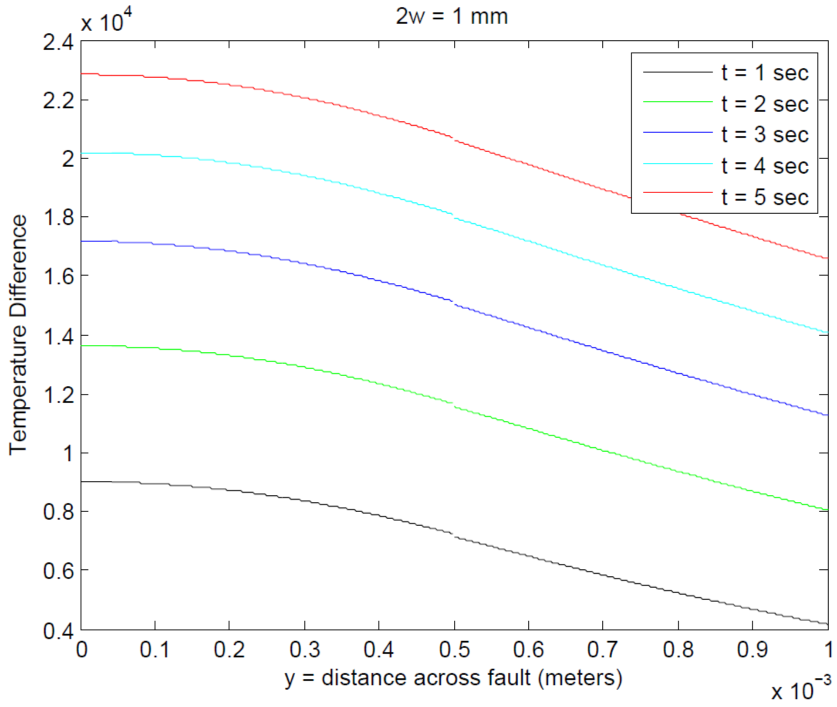
$$\Delta T = \frac{1}{4wc\rho}\int_0^t \frac{2}{\sqrt{\pi}}\left[\int_0^{\frac{(y+w)}{2\sqrt{\kappa(t-\tau)}}} e^{-x^2}\,dx - \int_0^{\frac{(y-w)}{2\sqrt{\kappa(t-\tau)}}} e^{-x^2}\,dx\right]\frac{\mu\sigma_n}{2w}\frac{\partial D}{\partial t}\,d\xi\,d\tau$$

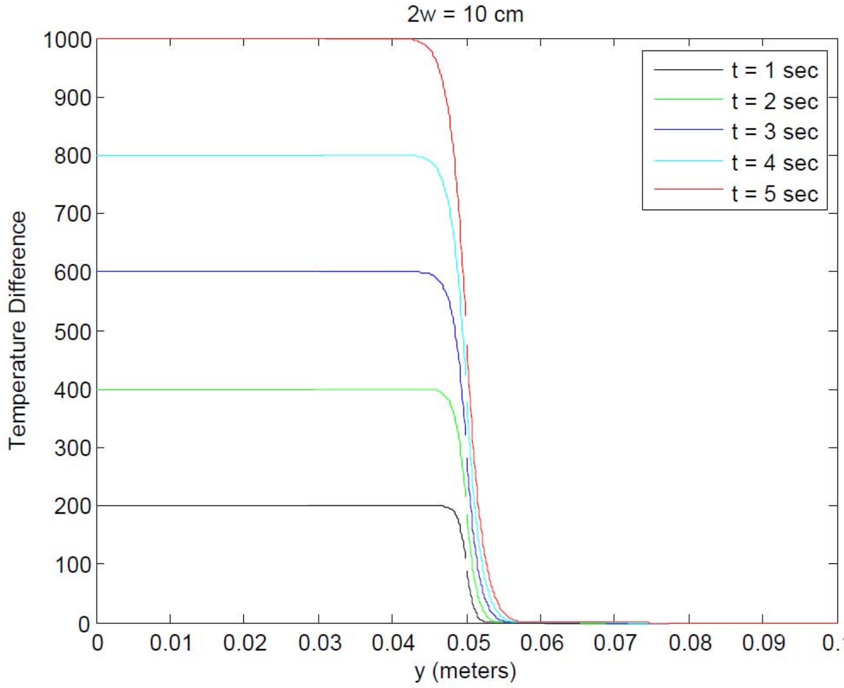
$$\Delta T = \frac{1}{4wc\rho}\int_0^t erf\left[\frac{y+w}{2\sqrt{\kappa(t-\tau)}}\right] - erf\left[\frac{y-w}{2\sqrt{\kappa(t-\tau)}}\right]\frac{\mu\sigma_n}{2w}\frac{\partial D}{\partial t}\,d\tau$$

Parameters

- Definition of terms
  - T - rock temperature
  - y – fault perpendicular coordinate
  - κ – thermal diffusivity: 1 x 10^-6 m^2/s
  - ρ – density: 3000 kg/m^3
  - c – heat capacity: 1000 J/K
  - Q – rate of frictional heat generation
  - D – crack displacement
  - v - slip velocity: 1 m/s
  - t – slip duration: 5 seconds
  - μ – coefficient of friction: 0.6
  - σ_n – fault normal stress: 100 Mpa
  - 2w – fault zone thickness: 1 mm and 10 cm

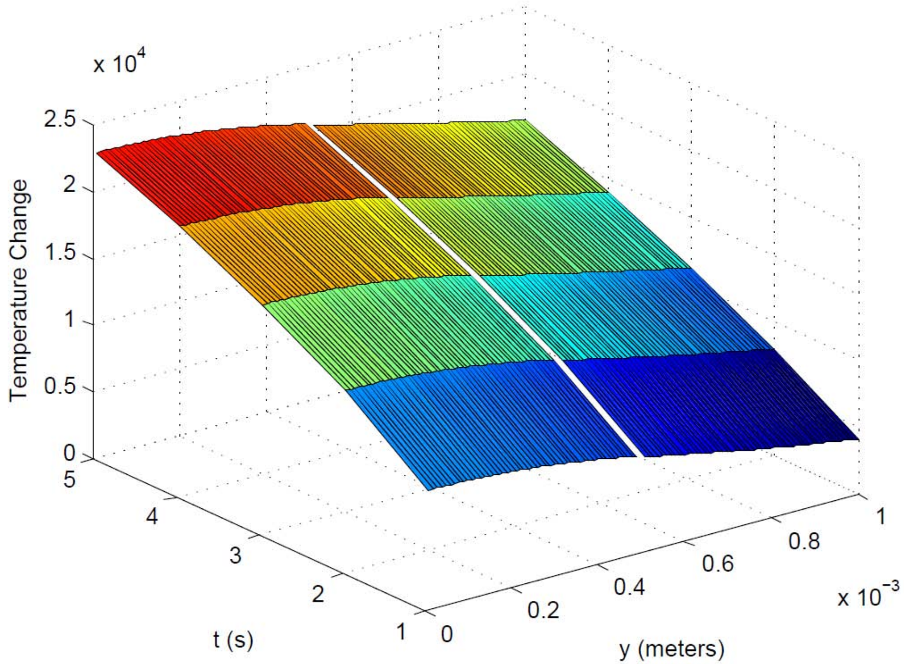
Results

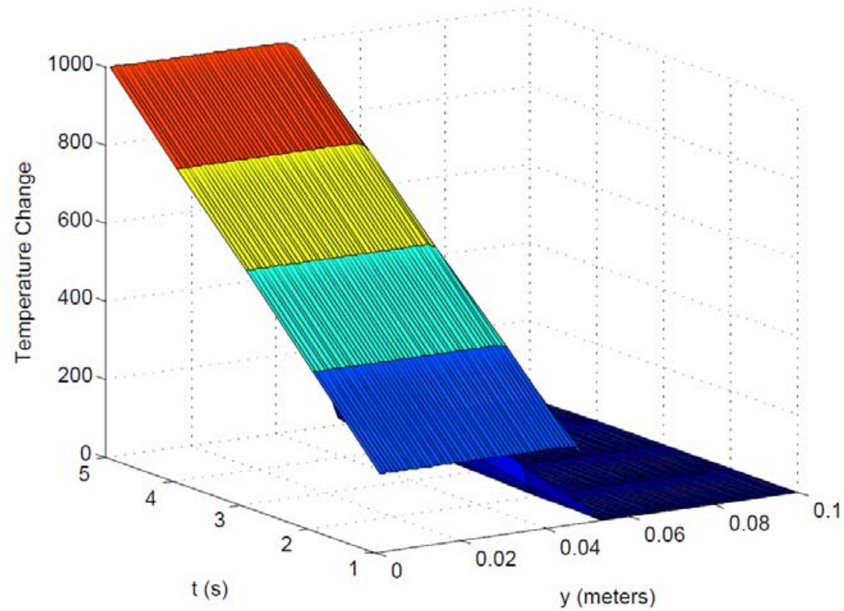




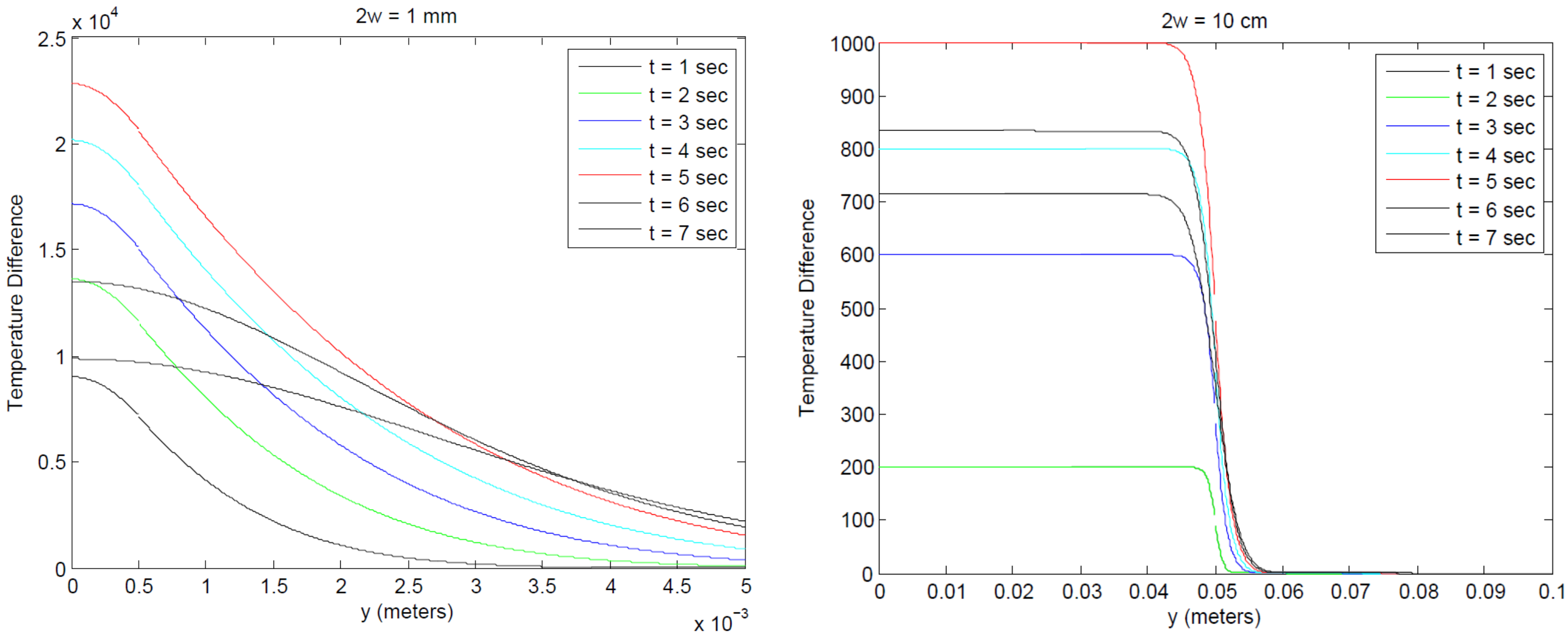
The thicker the crack width, the smaller maximum increase in temperature. Also, the decay across the fault is much more rapid for thicker crack widths.

Results (t=1:5 s)

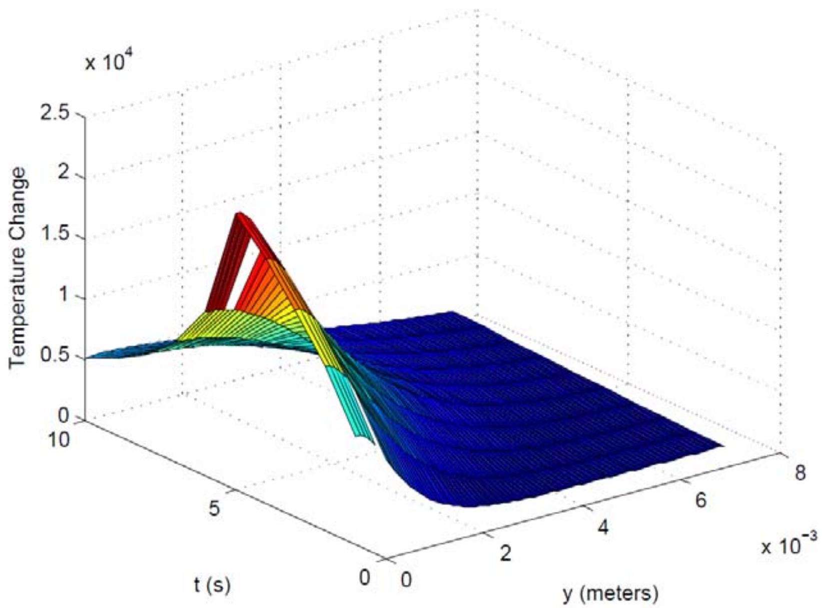


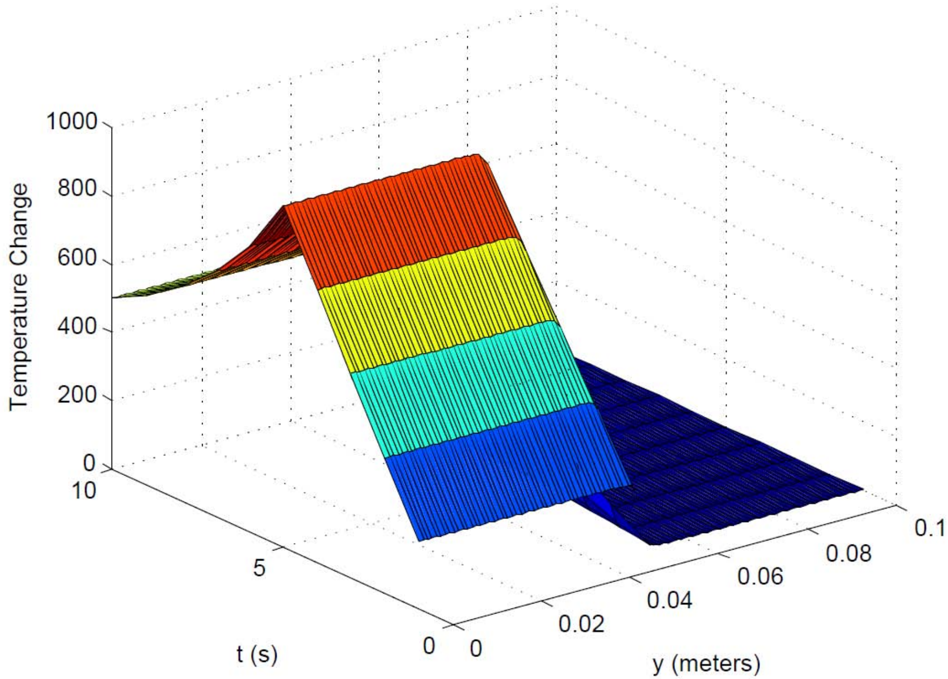


After Rupture (t =1:7 s)



After Rupture (t = 1:10 s)





Self-similar Crack Propagation

$$D(x,t) = a(t)\epsilon D(\chi),$$

$$\frac{\partial D}{\partial t} = V_r \epsilon \left[D(\chi) - \chi \frac{\partial D}{\partial \chi} \right].$$

Nondimensional along-fault coordinate $\chi = \frac{x}{tV_r}$,

Nondimensional fault thickness $\bar{w} = \sqrt{\frac{2}{\kappa t}} w.$

nondimensional temperature $\theta = \frac{T - T_0}{\hat{T}}$

where

$$\hat{T} = \frac{\sigma_d V_r \epsilon}{c\rho} \sqrt{\frac{t}{\pi \kappa}}$$

Linear Elastic Fracture Mechanics Approximation

$$D(x,t) = a(t)\epsilon\sqrt{1-\chi^2}, t > 0, |\chi| < 1$$

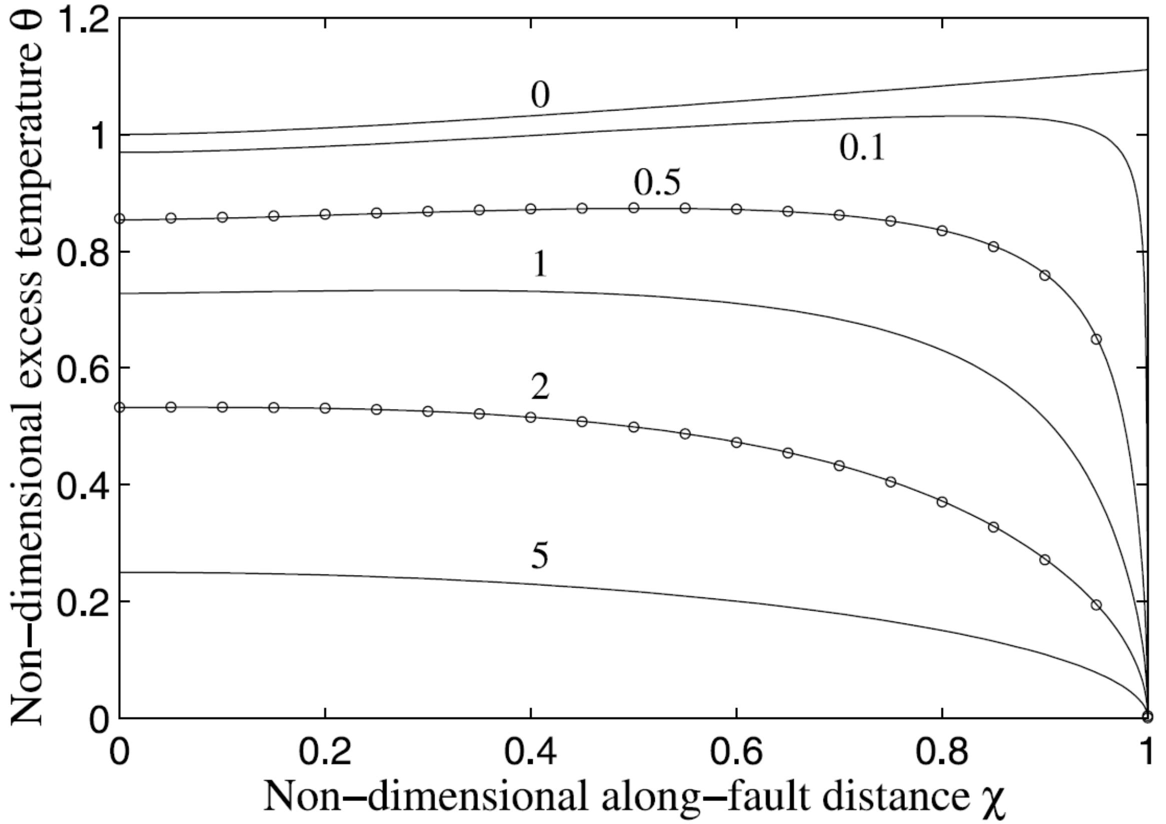
$$\frac{\partial D}{\partial t} = \frac{V_r \epsilon}{\sqrt{1-\chi^2}}$$

$$y = 0$$

$$\theta(\chi) = \frac{\sqrt{\pi}}{\bar{w}\sqrt{2}} \int_{\chi}^{1} \operatorname{erf}\left[\frac{\bar{w}}{2\sqrt{2(1-\xi)}}\right] \frac{\xi d\xi}{\sqrt{\xi^2 - \chi^2}}$$

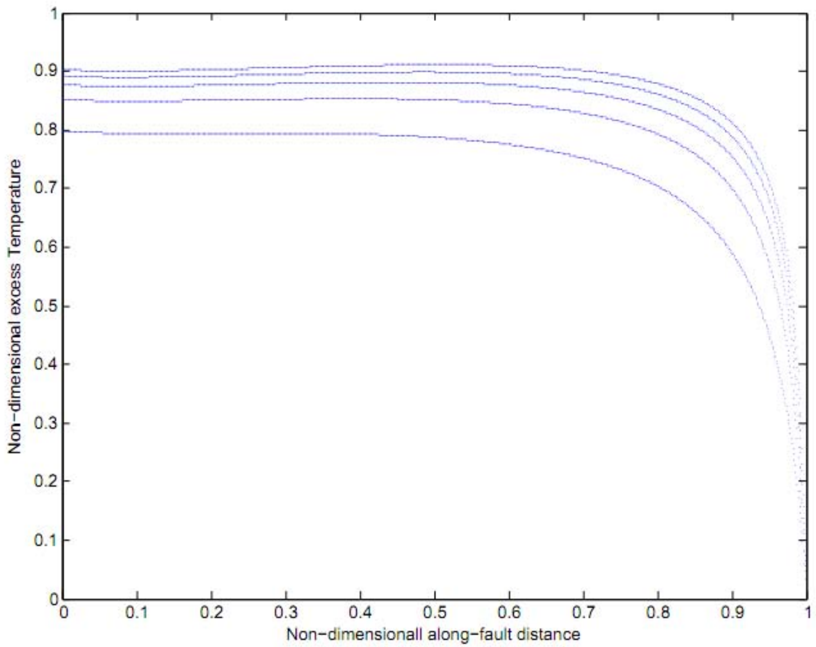
Lawn, 1993

Change of Temperature Along Fault

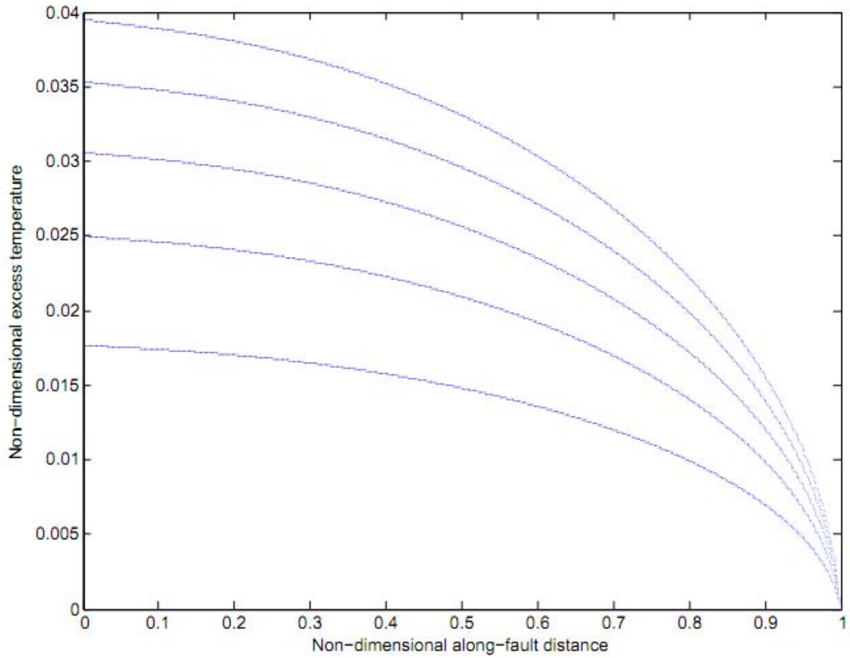


For thicker faults or at early stage of rupture, the temperature increase along the fault is proportional to the amount of slip.
For thin faults, or later stages of rupture , the temperature distribution is quite different. The maximum is somewhere between crack center and crack tip.

Along Fault Temperature Distribution



1mm

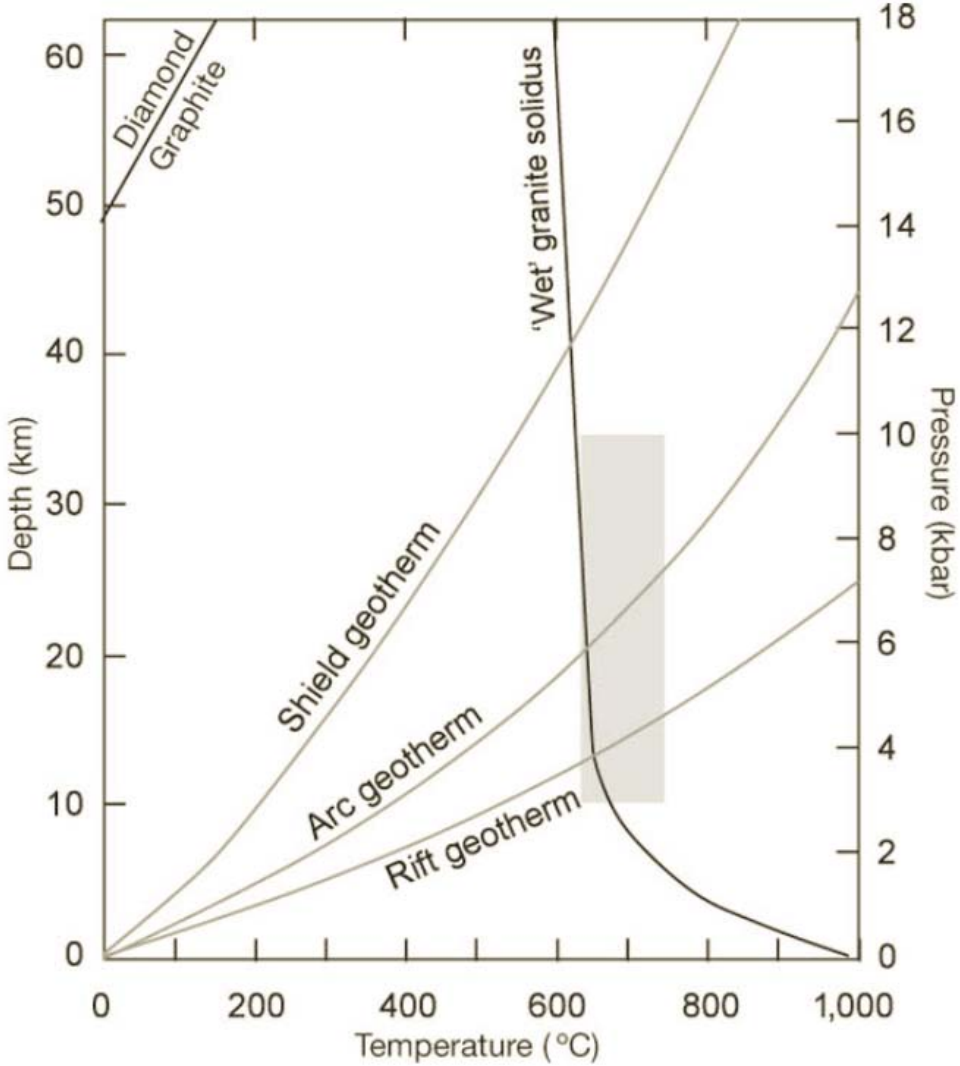


10cm

With constant thickness, for one point, the temperature will goes up during the rupture

For the same plane along fault, the temperature varies with the distance with the crack center during the rupture.

Implications



Rock tends to melt between 870-1270 K

- easily melts in analysis
- Basaltic compositions: 1220 K
- Granitic compositions: 970 K

Conclusions

- High temperatures will be generated by frictional heating only if the fault zone is relatively thin. These temperatures are sufficient to cause melting.
- Thick faults are considered to be greater than 1 cm in width. Here, thermal conduction is not important during faulting, and frictional heat is uniformly distributed across the width of the fault.
- The onset of frictional melting may give rise to substantial increases in the effective fault strength due to an increase in the effective fault contact area, and high viscosity of silicate melts near solidus.

Conclusions

- Possible effects of viscous braking on the earthquake rupture dynamics include
- 1) delocalization of slip and increases in the effective fracture energy
- 2) transition from a crack-like to a pulse like rupture propagation or
- 3) ultimate rupture arrest

References

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- Fialko, Y. (2004), Temperature fields generated by the elastodynamic propagation of shear cracks in the Earth, J. Geophys. Res., 109, B01303, doi: 10.1029/2003JB002497.
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