Modeling Magnetic Anomalies with MATLAB

Geodynamics, FA2012
Nate O'Flaherty
James Holmes

Introduction

- History
  - 1912 – Wegner publicly advocates theory of Continental Drift
    - 1943 – G.G. Simpson publishes rebuttal, Wegner falls out of favor.
  - 1960 – Harry Hess proposes concept of Seafloor Spreading to explain symmetric bathymetry profiles across spreading ridges.
  - 1963 – Vine and Matthews publish results of survey over Carlsberg ridge
    - They found symmetric stripes of magnetically polarized rock running parallel to the spreading ridge.

Earth’s Magnetism

- Geomagnetic Field
  - Earth’s geomagnetic field is a dipole to first-order.
  - Poles slightly diverge from geographic poles over short term, aligned over long term.

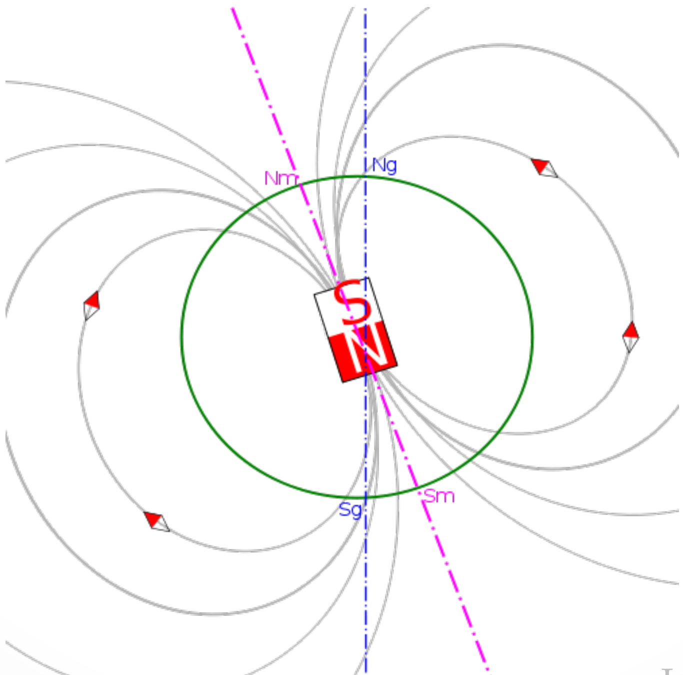


Image: Wikimedia Commons

Marine Geology

- Oceanic Lithosphere
  - Three layers
    - Layer 1 – Sediments
    - Layer 2 – Basalts
      - 2A – Pillow Basalts
      - 2B – Sheeted Dikes
    - Layer 3 – Gabbros (dark, coarse-grain plutonic rock)

Magnetic Anomalies

- What are magnetic anomalies?
  - Spreading ridges extrude magnetite-rich magma at axes.
  - As magma cools below Curie temperature (~500°C), crust magnetizes in direction of geomagnetic field. "Thermo-remnant Magnetism" (TRM).
  - Most TRM recorded in Layer 2a (upper 1000m of oceanic crust).
  - Plate spreading results in magnetized record of intensity and polarity.

Diagram of oceanic lithosphere

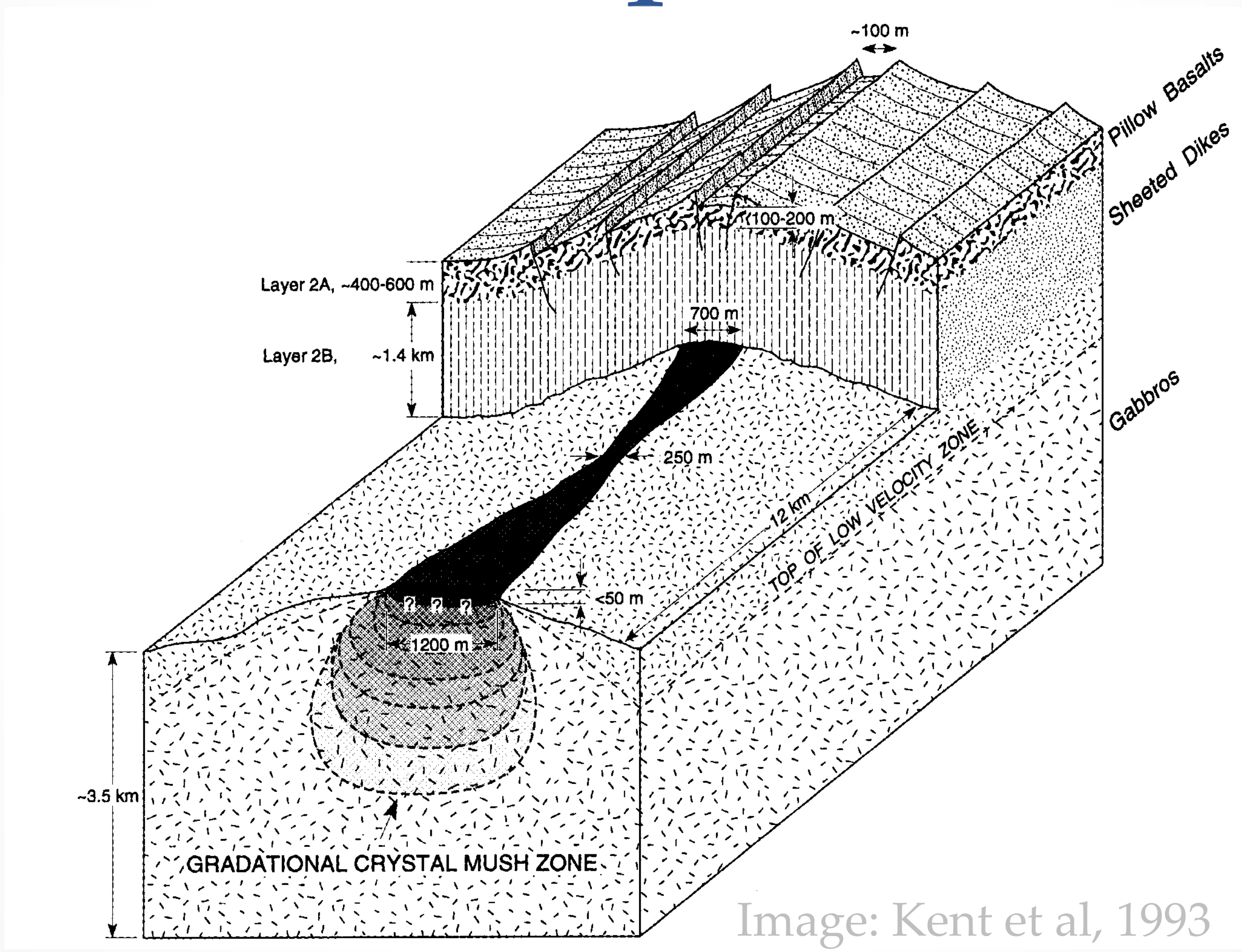


Image: Kent et al, 1993

Diagram of oceanic lithosphere

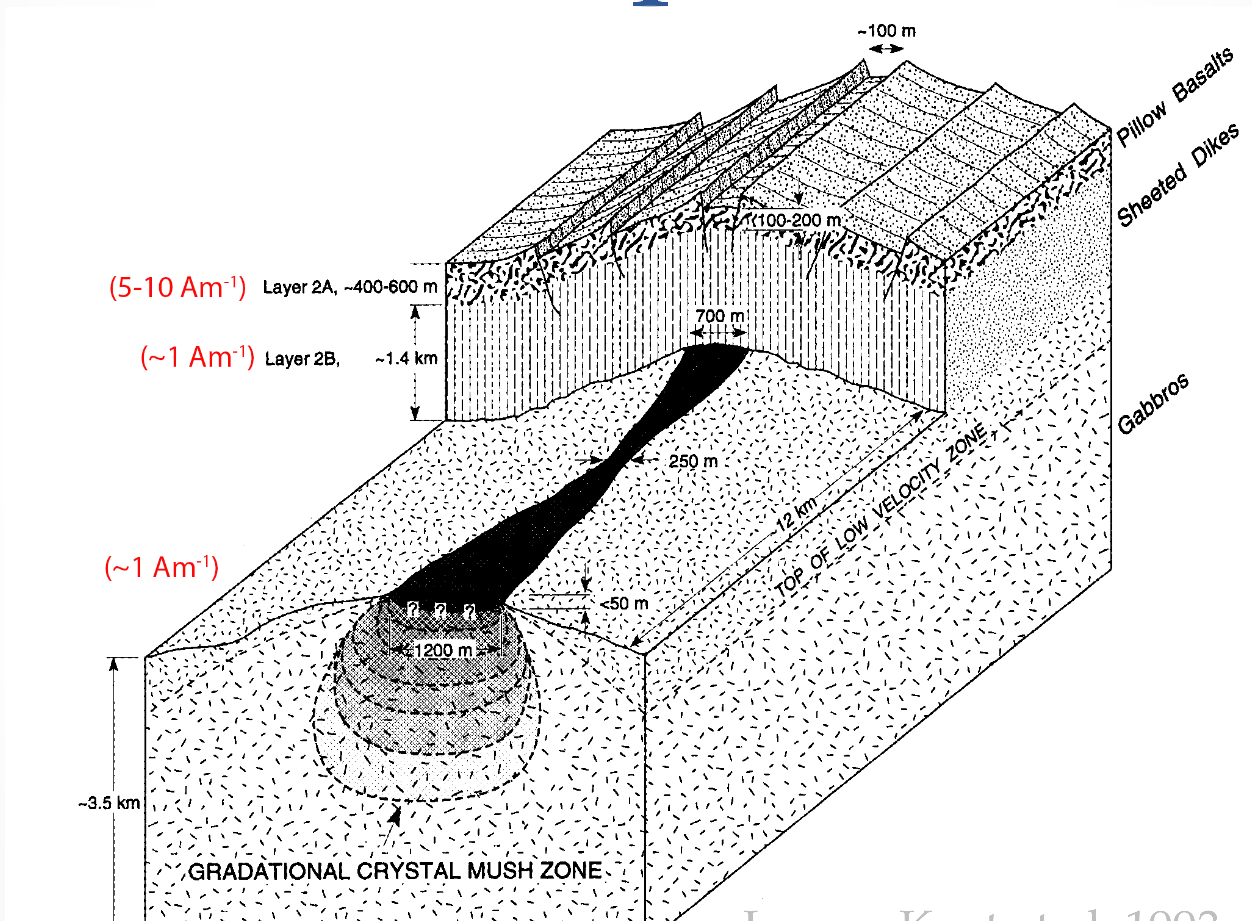
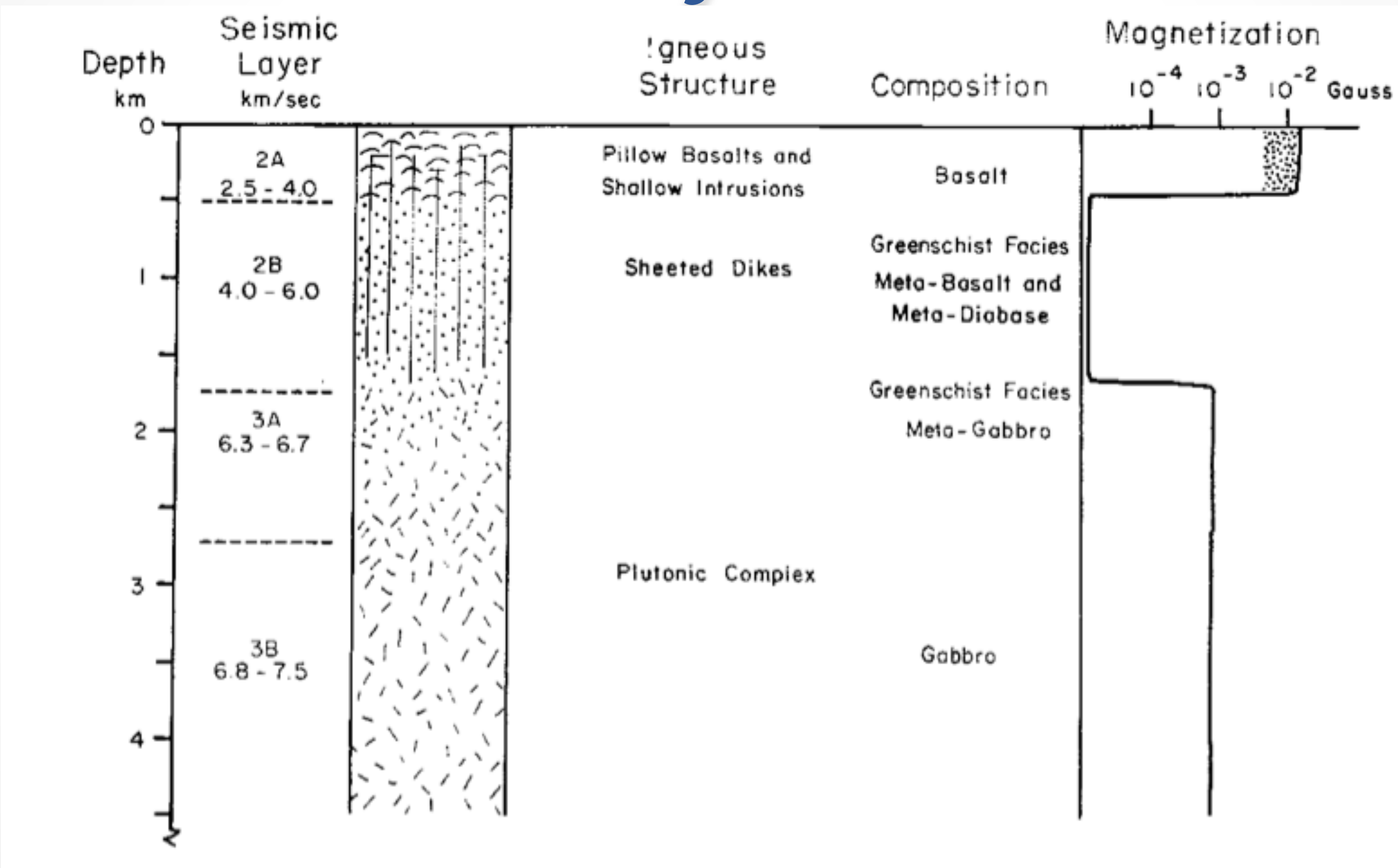
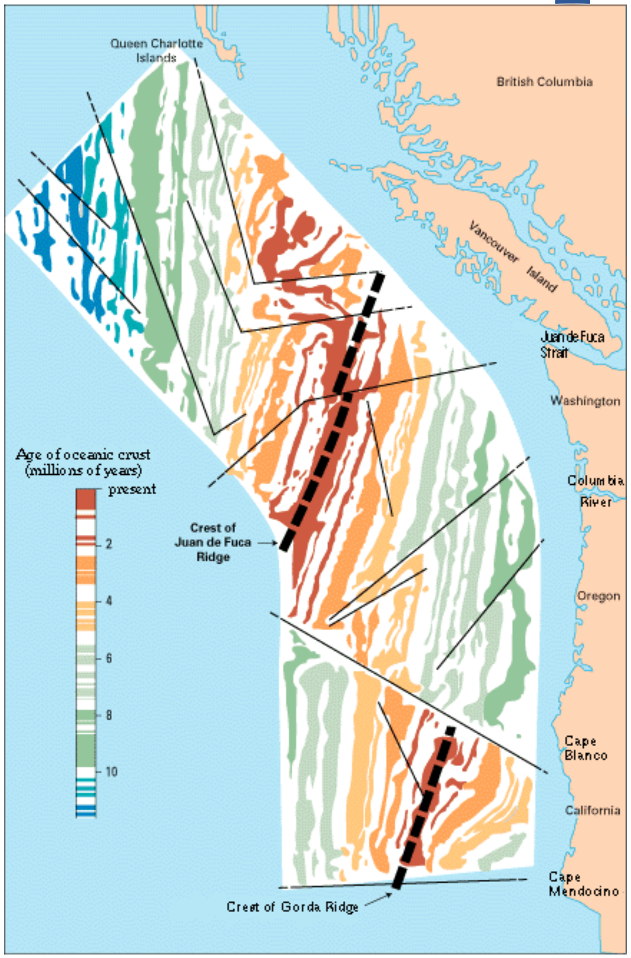


Image: Kent et al, 1993

Strength of anomaly by layer



Magnetic anomalies record seafloor spreading



How do we determine the anomaly?

- Magnetization vector is combination of TRM and current dipole components.
  - $\mathbf{M} = \mathbf{M}_{\mathbf{TRM}} + \mathbf{M}_{\mathbf{I}}$
- Magnetic anomaly vector is parallel to the magnetization direction.
  - $\Delta\mathbf{B}(\mathbf{r}) = \mu_0\mathbf{M}f(\mathbf{r})$
- Using ship-towed magnetometer, we measure total magnetic field, so we must remove Earth's field.
  - $\mathbf{B} = \mathbf{B}_{\mathbf{e}} + \Delta\mathbf{B}$
- However, most magnetometers give us a scalar field.
  - $|\mathbf{B}| = (|\mathbf{B}_e|^2 + 2\mathbf{B}_e\bullet\Delta\mathbf{B} + |\Delta\mathbf{B}|^2)^{1/2}$

How do we determine the anomaly?

- A huge difference in intensity.
  - Earth's dipole is ~ 50,000nT
  - Anomaly is ~ 300nT
- Therefore $|\Delta\mathbf{B}|^2 \sim 0$, and we can generalize $|\mathbf{B}|$ further:
  - $|\mathbf{B}| \cong |\mathbf{B}_e|(1 + \Delta\mathbf{B}\bullet\mathbf{B}_e \,/\, |\mathbf{B}_e|)$
- 30-seconds of algebra later…
  - $A = |\mathbf{B}| - |\mathbf{B}_e| = \Delta\mathbf{B}\bullet\mathbf{B}_e \,/\, |\mathbf{B}_e|$
  - A is our measured scalar anomaly

Mathematically modeling anomalies

- The magnetic anomaly is the negative gradient of the magnetic potential.
  - $\Delta \mathbf{B} = -\nabla U$
- The potential satisfies Poisson's equation within the source layer and Laplace's equation above it.
  - $\nabla^2 U = 0 \qquad , z \neq z_0$
  - $\nabla^2 U = \mu_0 \nabla \bullet \mathbf{M} \qquad , z = z_0$
- We can then generalize the equation to:

$$\nabla^2 U = \mu_o \left[\frac{\partial}{\partial x} M_x p(x) \delta() + \frac{\partial}{\partial y} M_y p(x) \delta() + \frac{\partial}{\partial z} M_z p(x) \delta() \right]$$

Mathematically modeling anomalies

- The magnetic anomaly is the negative gradient of the magnetic potential.
  - $\Delta \mathbf{B} = -\nabla U$
- The potential satisfies Poisson's equation within the source layer and Laplace's equation above it.
  - $\nabla^2 U = 0 \qquad , z \neq z_0$
  - $\nabla^2 U = \mu_0 \nabla \bullet \mathbf{M} \qquad , z = z_0$
- We can then generalize the equation to:

$$\nabla^2 U = \mu_o \left[\frac{\partial}{\partial x} M_x p(x) \delta() + \frac{\partial}{\partial y} M_y p(x) \delta() + \frac{\partial}{\partial z} M_z p(x) \delta() \right]$$

- No vertical variation!

Mathematically modeling anomalies

- To solve our equation we need to:
  - Implement a forward Fourier Transform.
  - Solve for $U(\mathbf{k})$
  - Then inverse Fourier Transform (via Cauchy-Residue Thm.) with respect to k_z.
    - Poles of integrand at $\pm ik_x$
    - Path integral around poles
- When TRM has component parallel to dipole field (parallel to ridge axis) = No field anomaly.
- Assume spreading ridge at magnetic pole so that dipole field lines are parallel to 'z', with no 'x' component.

Mathematically modeling anomalies

- Because scalar anomaly is:

$$A = \frac{\Delta \vec{B} \cdot \vec{B}_e}{\left| \vec{B}_e \right|}$$

- Only the z-component is non-zero, so scalar anomaly is:

$$A(k, z) = \frac{\mu_o M_z}{2} p(k) 2\pi \left| k \right| e^{-2\pi |k| (z - z_o)}$$

Observed Anomaly = Reversal Pattern * Earth Filter

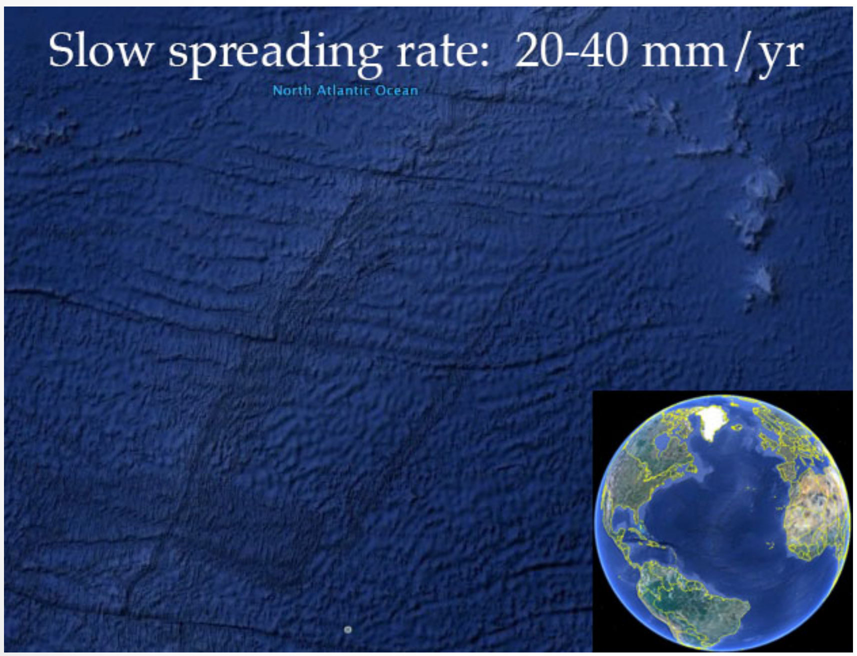
Mathematically modeling anomalies

- But wait! That's not all:
  - Now, rotate the spreading ridge down to an arbitrary latitude.
  - X-component now non-zero.
  - We get a skewness factor, θ .
    - Depends on latitude and orientation of ridge when crust cools.
- Final model:

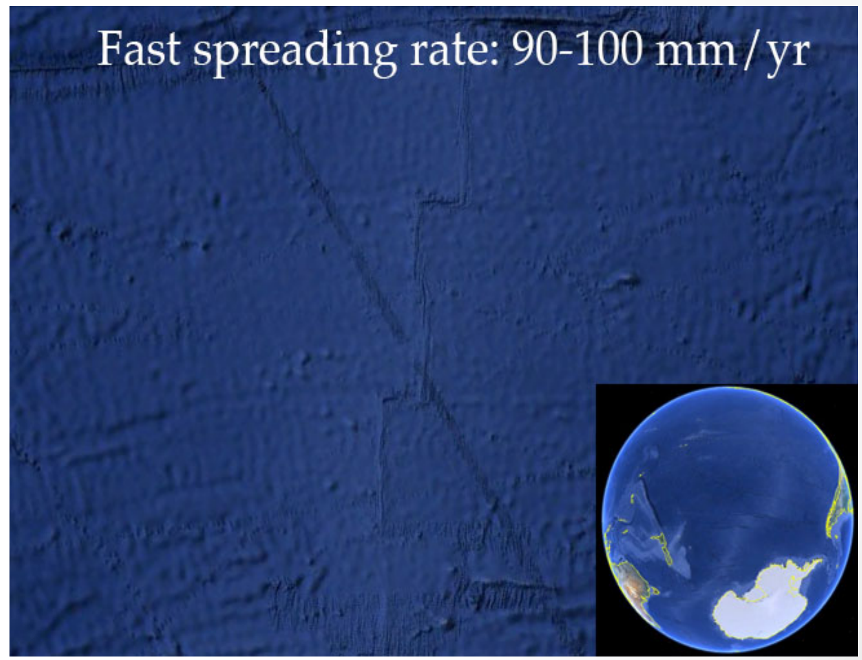
$$A(k,z) = C\mu_o p(k) e^{i\theta \frac{k}{|k|}} 2\pi \left|k\right| e^{2\pi|k|z_o}$$

Modeling actual ridges

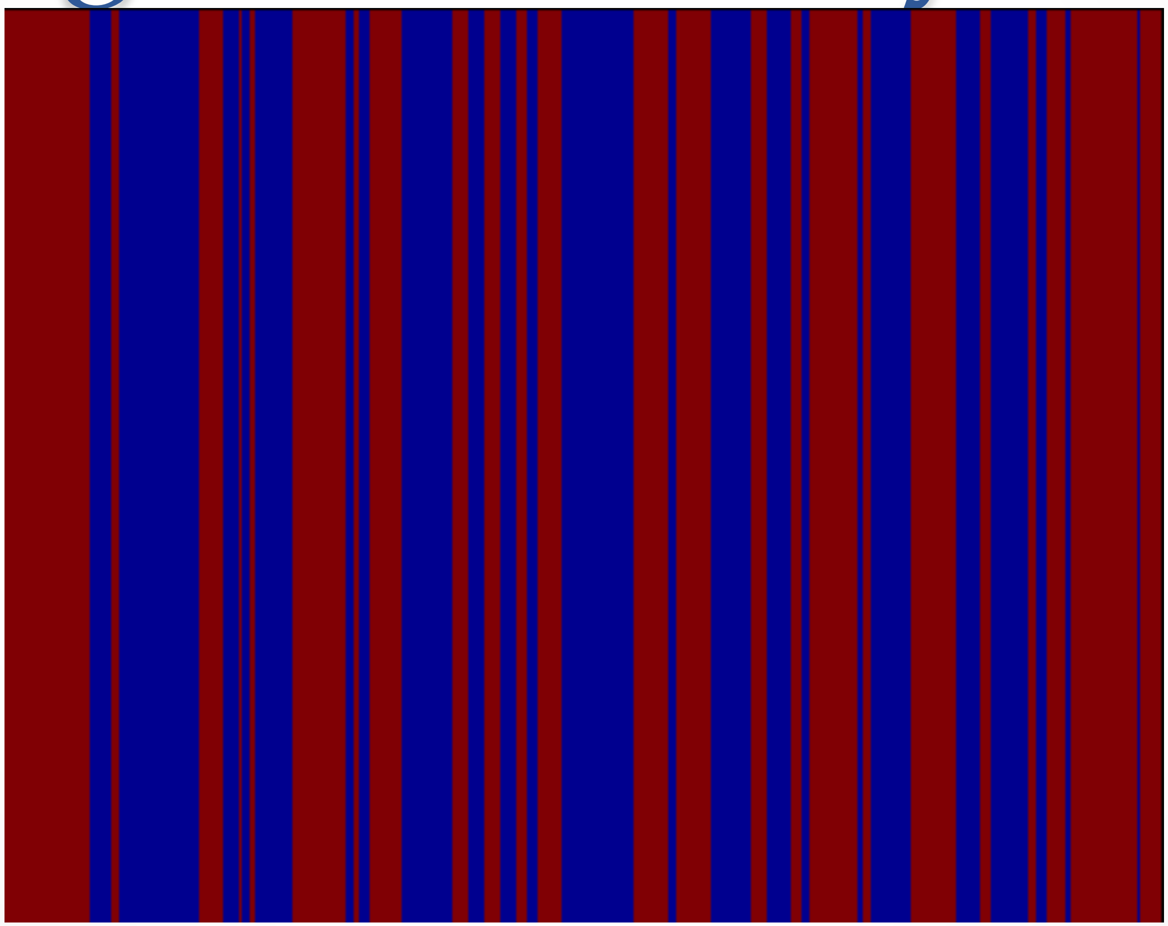
Mid-Atlantic Ridge



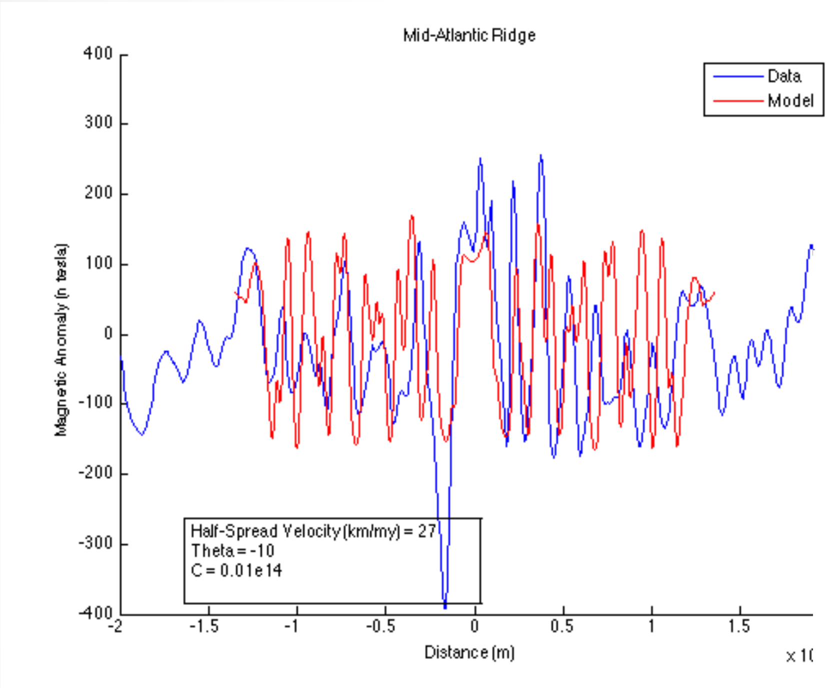
Pacific-Antarctic Ridge

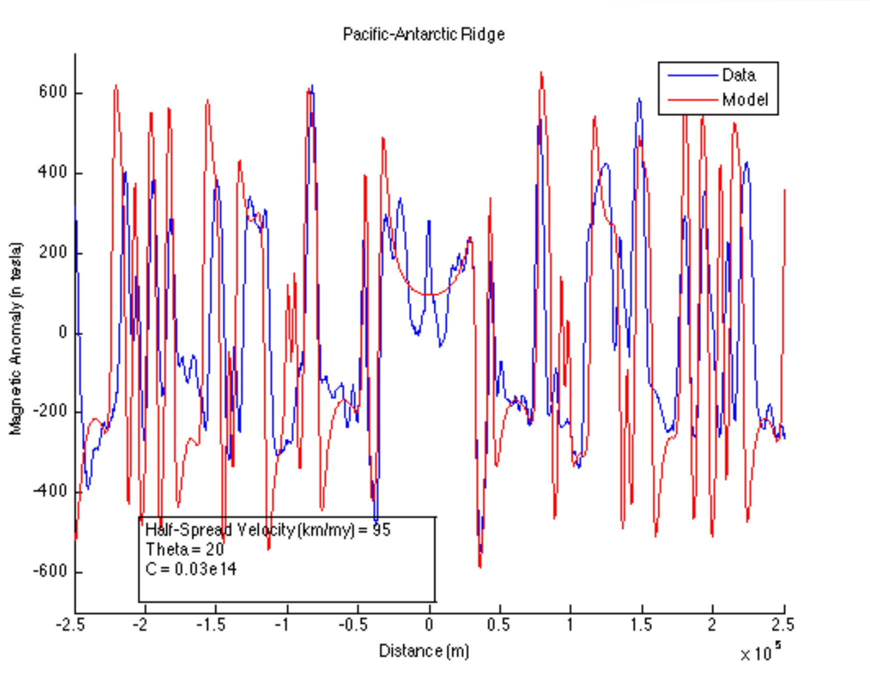


Magnetic Polarity Data

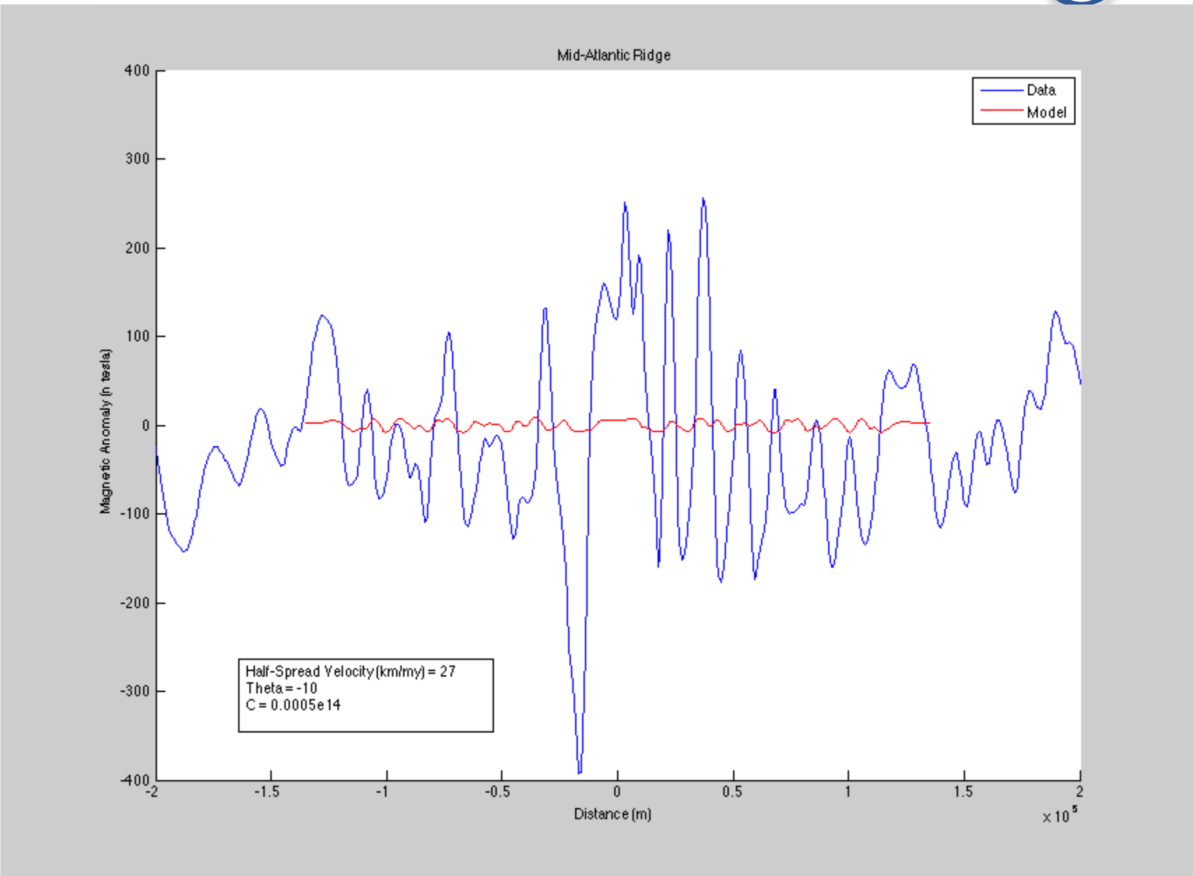


"Closest" Fits



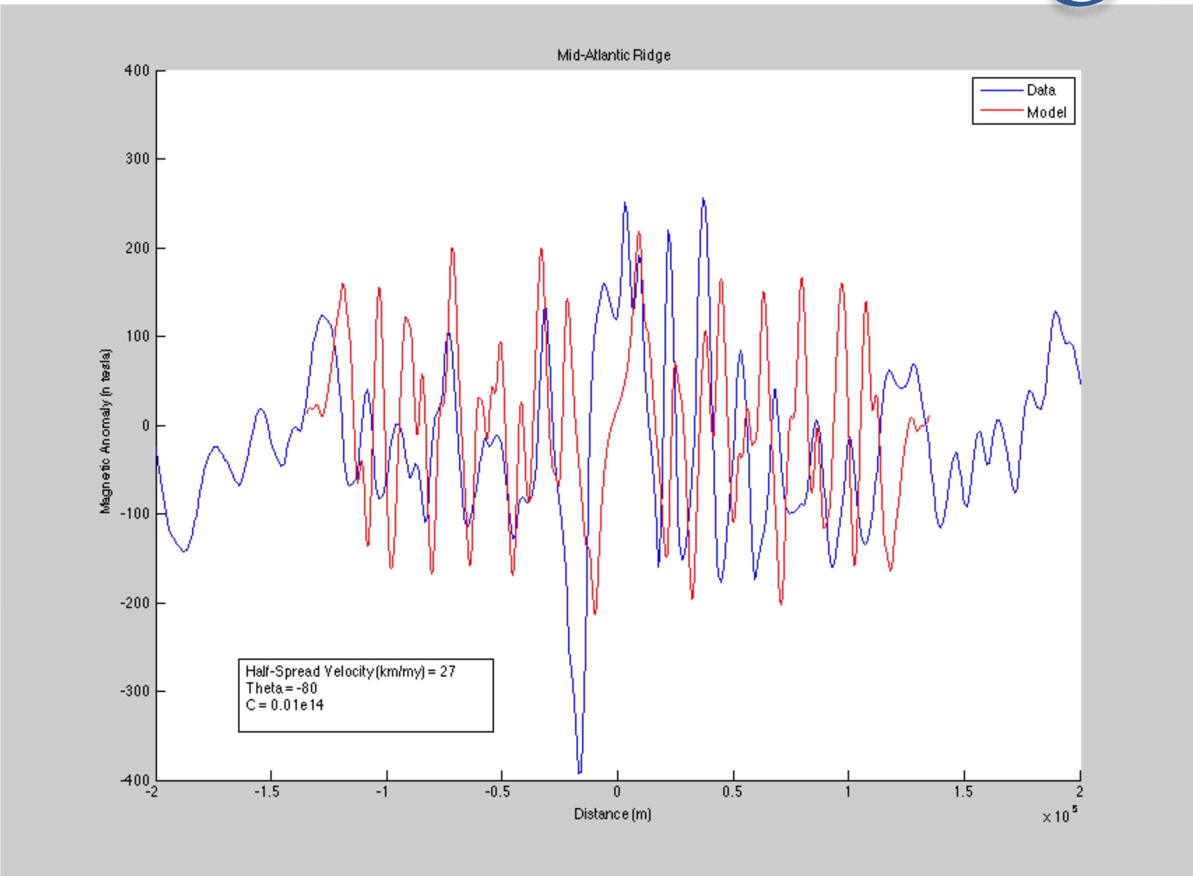


Mid-Atlantic Ridge



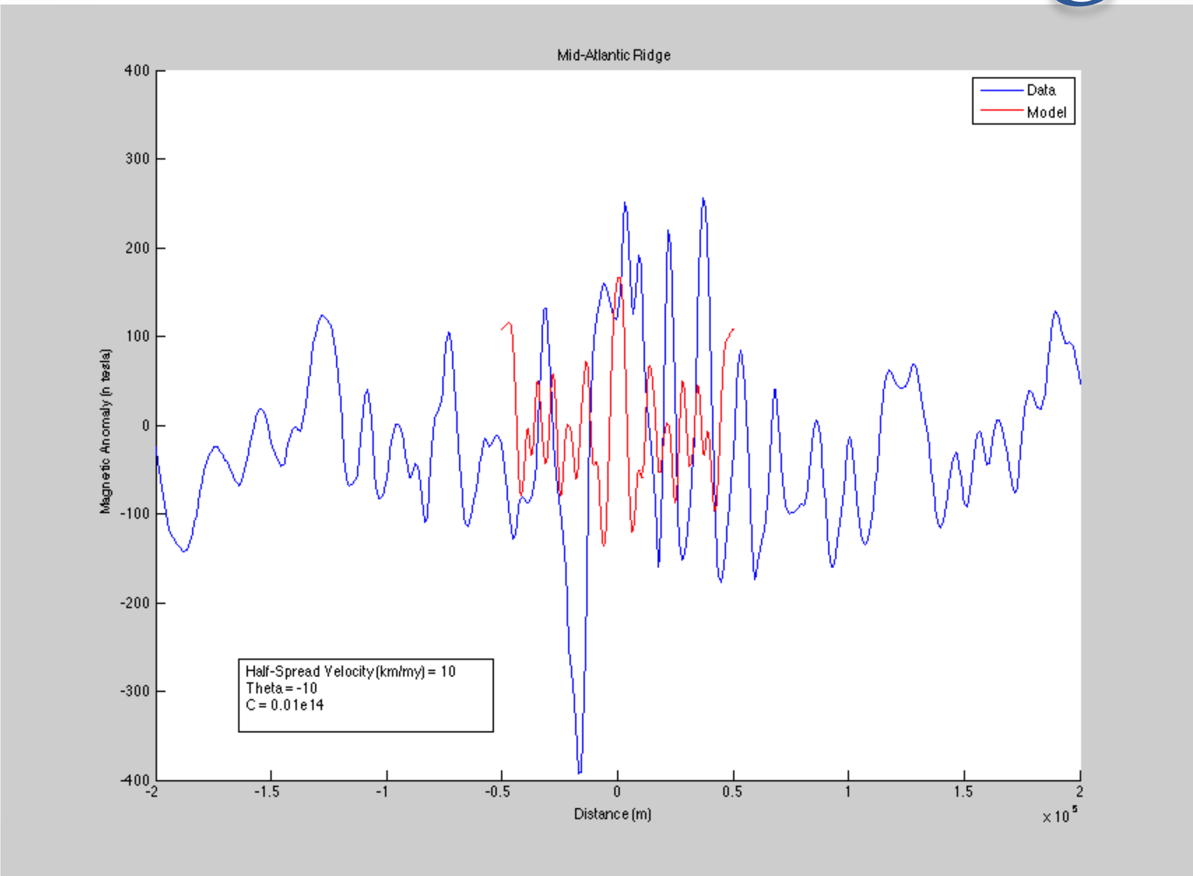
$$A(k, z) = C\mu_0 2\pi |k| e^{-2\pi|k|z} e^{i\theta sgn(k)} p(k)$$

Mid-Atlantic Ridge



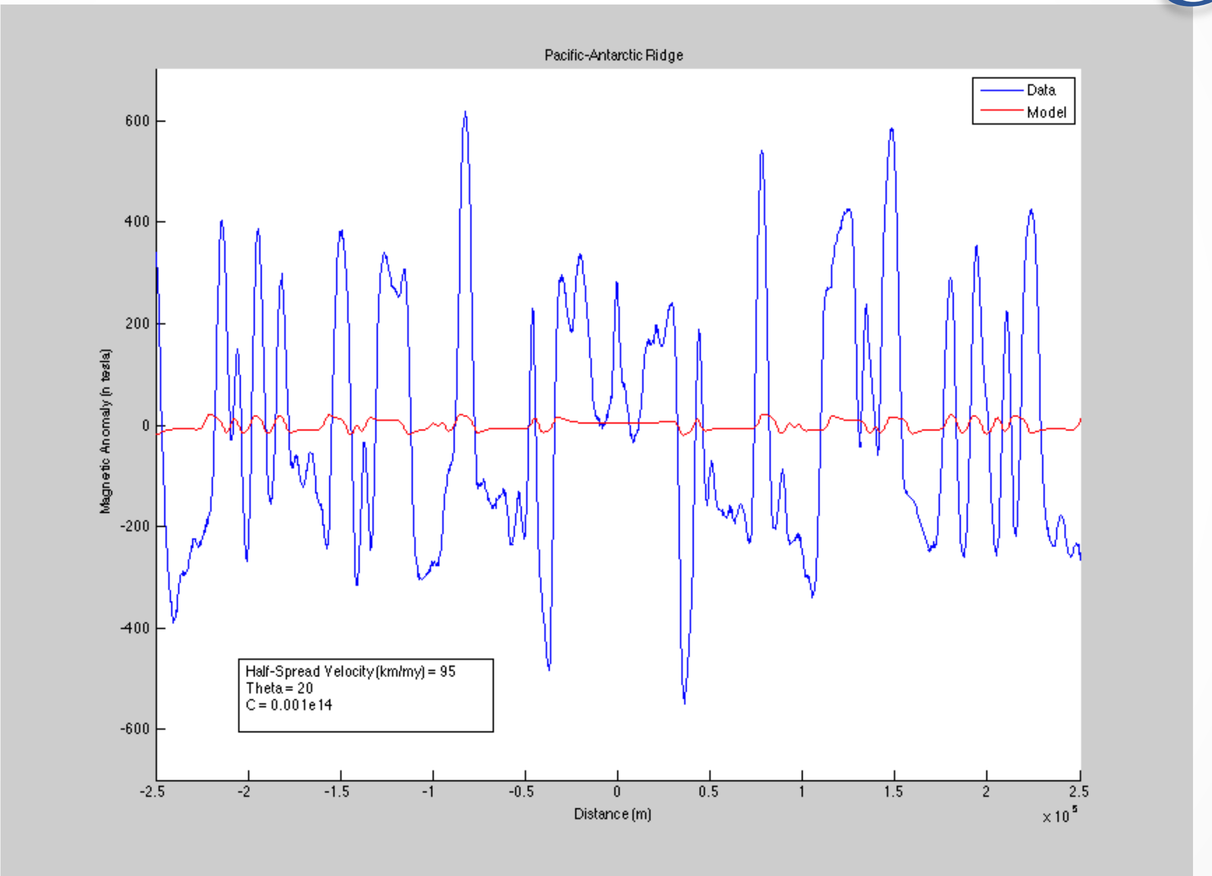
$$A(k, z) = C\mu_0 2\pi |k| e^{-2\pi |k| z} e^{i\theta sgn(k)} p(k)$$

Mid-Atlantic Ridge



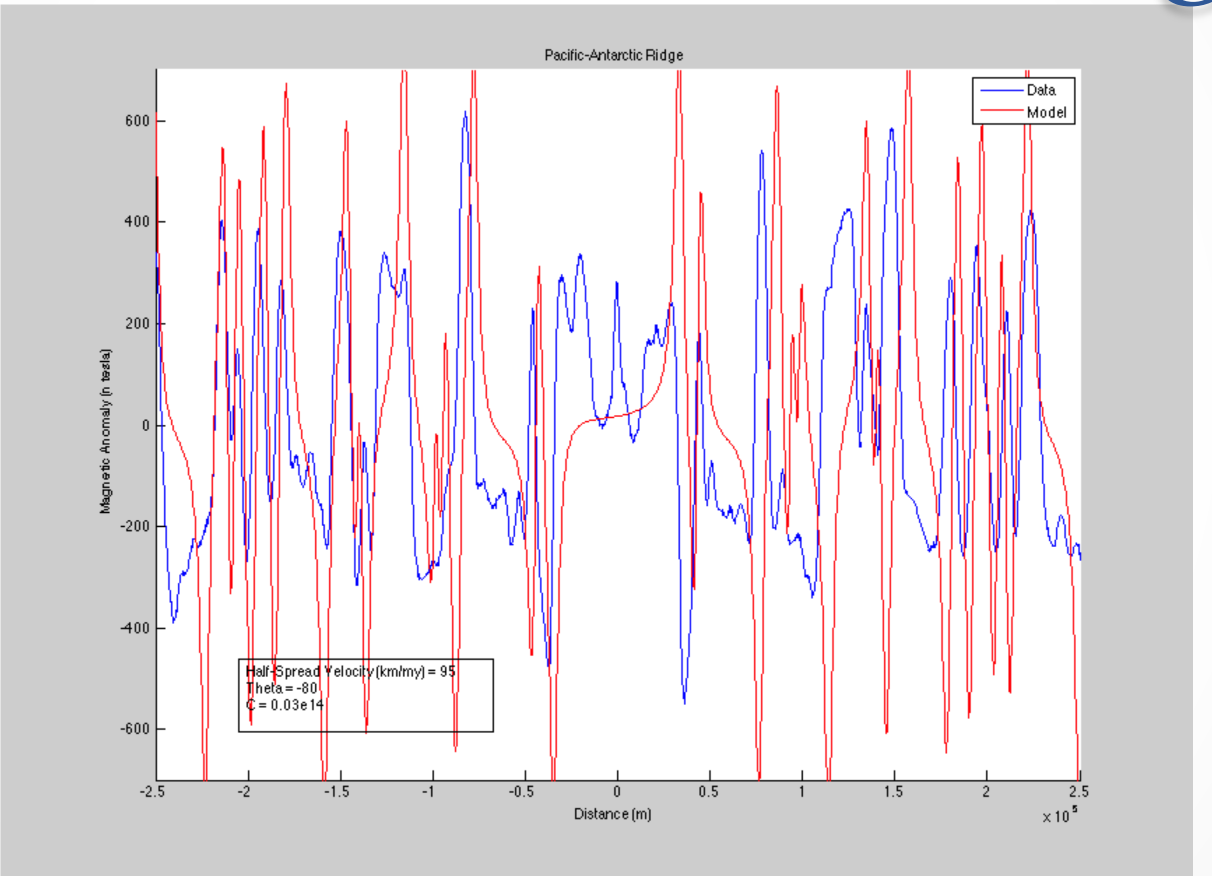
$$A(k,z) = C\mu_0 2\pi |k| e^{-2\pi|k|z} e^{i\theta sgn(k)} p(k) \qquad k = \frac{-nx/2 : nx/2-1}{L} = \frac{-nx/2 : nx/2-1}{v*dt}$$

Pacific-Antarctic Ridge



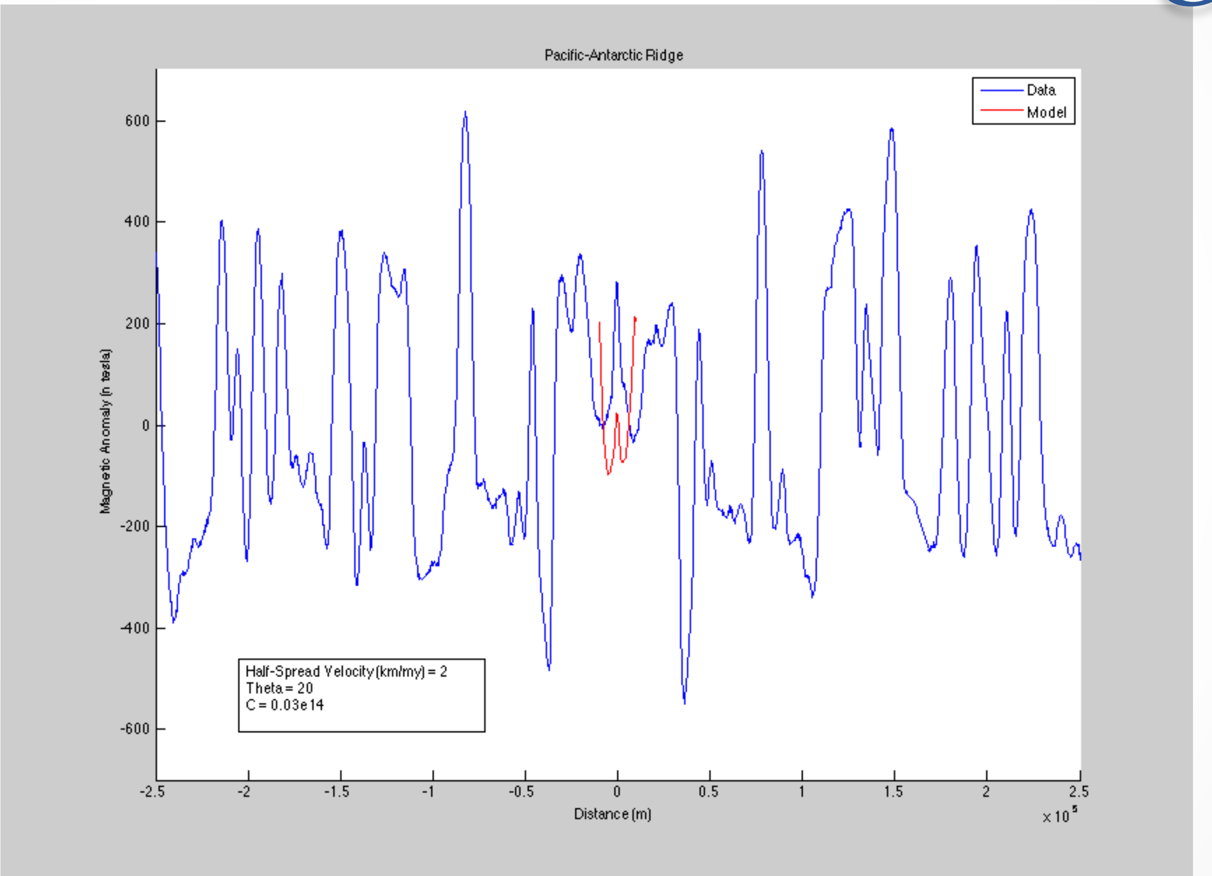
$$A(k,z) = C\mu_0 2\pi |k| e^{-2\pi|k|z} e^{i\theta sgn(k)} p(k)$$

Pacific-Antarctic Ridge



$$A(k,z) = C\mu_0 2\pi|k|e^{-2\pi|k|z}e^{i\theta sgn(k)}p(k)$$

Pacific-Antarctic Ridge



$$A(k,z) = C\mu_0 2\pi|k|e^{-2\pi|k|z}e^{i\theta sgn(k)}p(k) \qquad k = \frac{-nx/2 : nx/2-1}{L} = \frac{-nx/2 : nx/2-1}{v * dt}$$

Questions?