Keg Cooler Problem

Mike Sly, Thomas Martin

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Beer

- Beer is not the problem, keeping it cold is
- Ground temperature changes seasonally, daily and depending on local geology
- What depth should our cellars be in the cities of San Diego, CA and Bend, OR?
- According to CNN, #3 and #7 for “best beer towns in USA”

http://www.cnn.com/2013/05/06/travel/usa-beer-towns/

Problems with Warm Beer

- Warm beer is not fun
- If you are going to spend ~150 dollars on a keg, might as well give it some love
- Causes it to go flat faster

http://d2qdgu53yc9lue.cloudfront.net/deal/de_1000_lg.jpg

Problems with Cold Beer

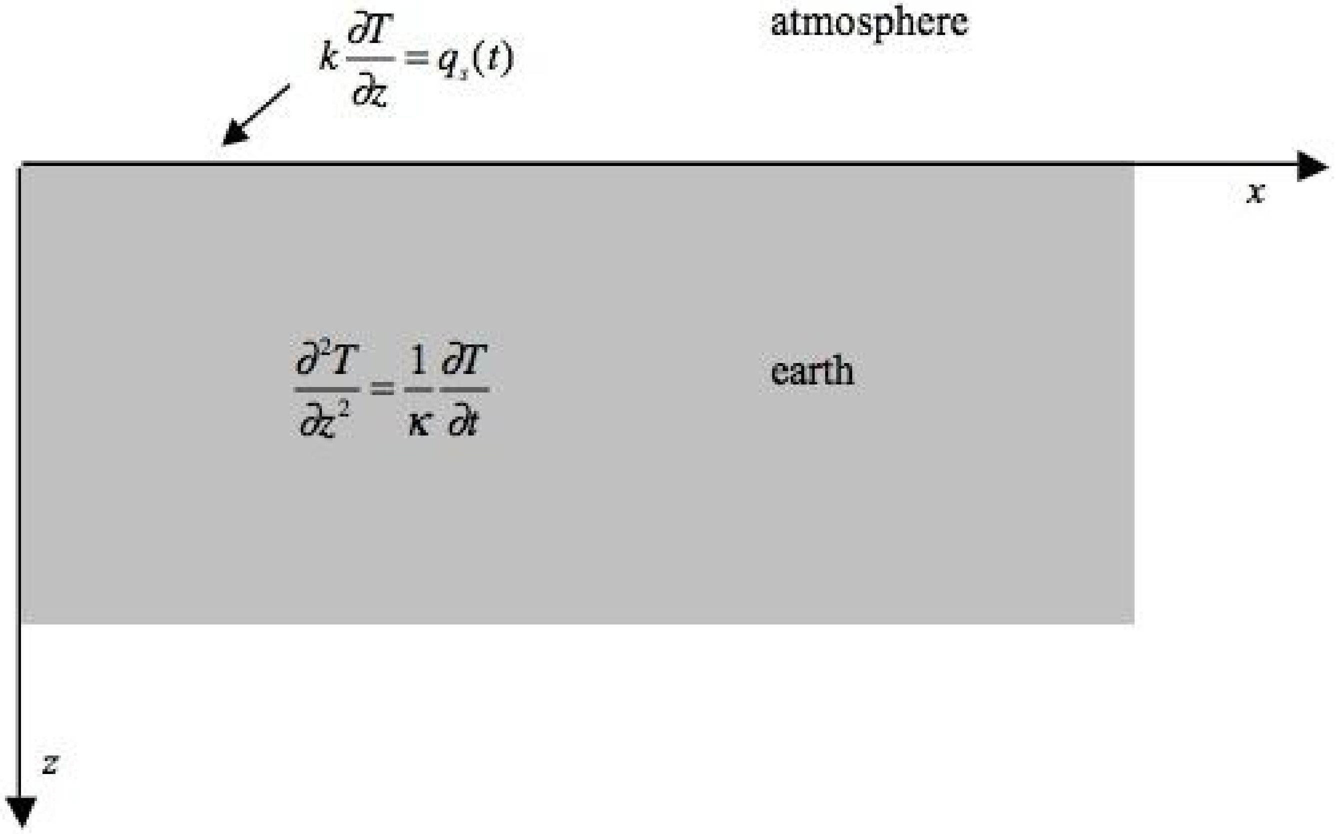
- Running out
- ……..?

http://3.bp.blogspot.com/_wjyA0QX5Gog/TSYVeWqbssI/AAAAAAAAASw/UkPABeiNRds/s1600/snow-refrigerator-2-Awesome-Beer-Fridge.jpeg

Proper Temperature

- 4 Degrees C to store kegged beer (the fridge)
- But for longer term storage, 7 Degrees C is acceptable (~45F)
- Dark is also a plus for bottles
- Unlike wine, 5-10 years should be max storing length

The Problem



$$k\frac{\partial T}{\partial z} = q_s(t)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t}$$

Math Time

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t}$$

$$T(z,t) = T_o + \Delta T(z,t)$$

$$\Delta T = Z(z)\tilde{T}(t)$$

$$\tilde{T}(t) = cos(\omega t) + sin(\omega t)$$

$$T(z,t) = T_o + Z_1(z)cos(\omega t) + Z_2(z)sin(\omega t)$$

More Math

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t}$$

$$\frac{1}{\kappa}\frac{\partial T}{\partial t} = -\frac{\omega}{\kappa} Z_1 sin(\omega t) + \frac{\omega}{\kappa} Z_2 cos(\omega t)$$

$$\frac{\partial^2 T}{\partial z^2} = \frac{d^2 Z_1}{dz^2} cos(\omega t) + \frac{d^2 Z_2}{dz^2} sin(\omega t)$$

$$\left(\frac{d^2 Z_1}{dz^2} - \frac{\omega}{\kappa} Z_2\right) cos(\omega t) + \left(\frac{d^2 Z_2}{dz^2} + \frac{\omega}{\kappa} Z_1\right) sin(\omega t) = 0$$

Coupled 2nd Order ODEs

$$\frac{d^2 Z_1}{dz^2} - \frac{\omega}{\kappa} Z_2 = 0$$

$$\frac{d^2 Z_2}{dz^2} + \frac{\omega}{\kappa} Z_1 = 0$$

$$Z_1 = -\frac{\kappa}{\omega} \frac{d^2 Z_2}{dz^2}$$

$$\frac{d^2}{dz^2} \left(\frac{\kappa}{\omega} \frac{d^2 Z_2}{dz^2} \right) + \frac{\omega}{\kappa} Z_2 = 0$$

$$\frac{d^4 Z_2}{dz^4} = -\frac{\omega^2}{\kappa^2} Z_2$$

Solution for Z

$$Z_2 = ce^{\alpha z}$$

$$\alpha^4 = -\frac{\omega^2}{\kappa^2}$$

$$\alpha^2 = i\frac{\omega}{\kappa}$$

$$\alpha = \pm\sqrt{i\frac{\omega}{\kappa}}$$

$$\alpha = \pm\left(\frac{1+i}{\sqrt{2}}\right)\sqrt{\frac{\omega}{\kappa}}$$

Solution for Z

$$Z_2 = e^{-z\gamma}\left(c_1 e^{-iz\gamma} + c_2 e^{iz\gamma}\right)$$

$$\gamma = \sqrt{\frac{\omega}{2\kappa}}$$

$$Z_2 = e^{-z\gamma}\left[b_1 cos\left(z\gamma\right) + b_2 sin\left(z\gamma\right)\right]$$

$$b_1 = c_2 - c_1$$

$$b_2 = c_1 + c_2$$

Similarly

$$Z_1 = e^{-z\gamma}\left[b_3 cos\left(z\gamma\right) + b_4 sin\left(z\gamma\right)\right]$$

$$b_3 = b_2$$

$$b_4 = -b_1$$

Final T

$b_1 = -b_4 = 0$ $b_2 = b_3 = \Delta T$

$$Z_2 = e^{-z\gamma} \left[\Delta T sin\left(z\gamma\right)\right]$$

$$Z_1 = e^{-z\gamma} \left[\Delta T cos\left(z\gamma\right)\right]$$

$$T = T_o + \Delta T e^{z\gamma} \left[cos\left(\omega t\right) cos\left(z\gamma\right) + sin\left(\omega t\right) sin\left(z\gamma\right)\right]$$

$$T = T_o + \Delta T e^{-z\gamma} cos\left(\omega t - z\gamma\right)$$

Attenuation Depth

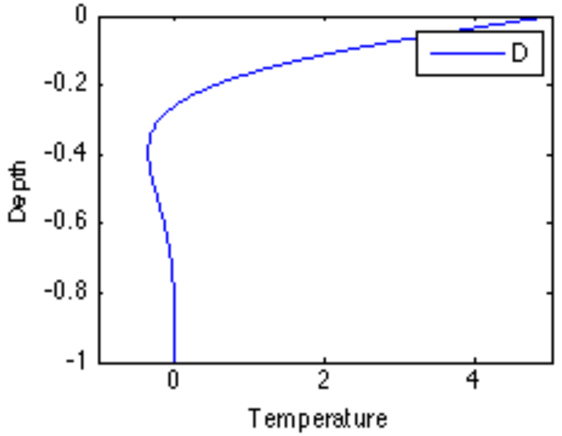
$$d_\omega = \sqrt{\frac{2\kappa}{\omega}} \qquad \kappa = \frac{k}{\rho c}$$

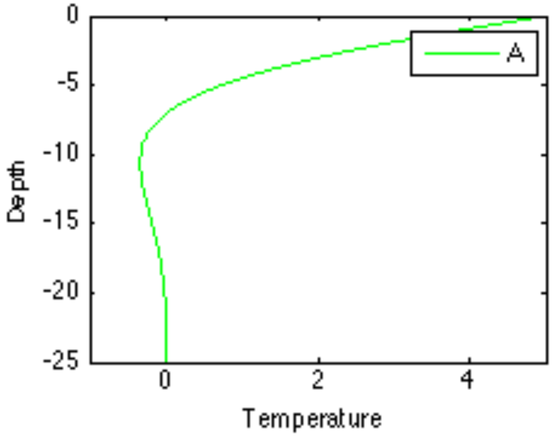
Attenuation depth is related to thermal conductivity, density, specific heat, and frequency.

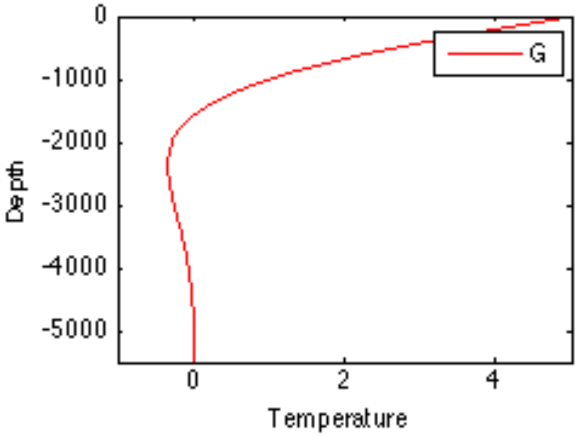
- $d_{\omega d} = 0.05\, m$
- $d_{\omega a} = 1.4\, m$
- $d_{\omega g} = 316\, m$

$$\kappa = 1\, mm^2/s$$

Attenuation Depth







$$\kappa = 1\, mm^2/s$$

$$T_o = 0°C$$

$$\Delta T = 5°C$$

Surface Heat Flux

$$k\frac{\partial T}{\partial z} = q_s(t)$$

$$q_s(t) = -k\Delta T\gamma\left[sin\left(\omega t\right) - cos\left(\omega t\right)\right]$$

$$q_s(t) = \Delta q\left[sin\left(\omega t\right) - cos\left(\omega t\right)\right] \qquad \Delta q = -k\Delta T\gamma$$

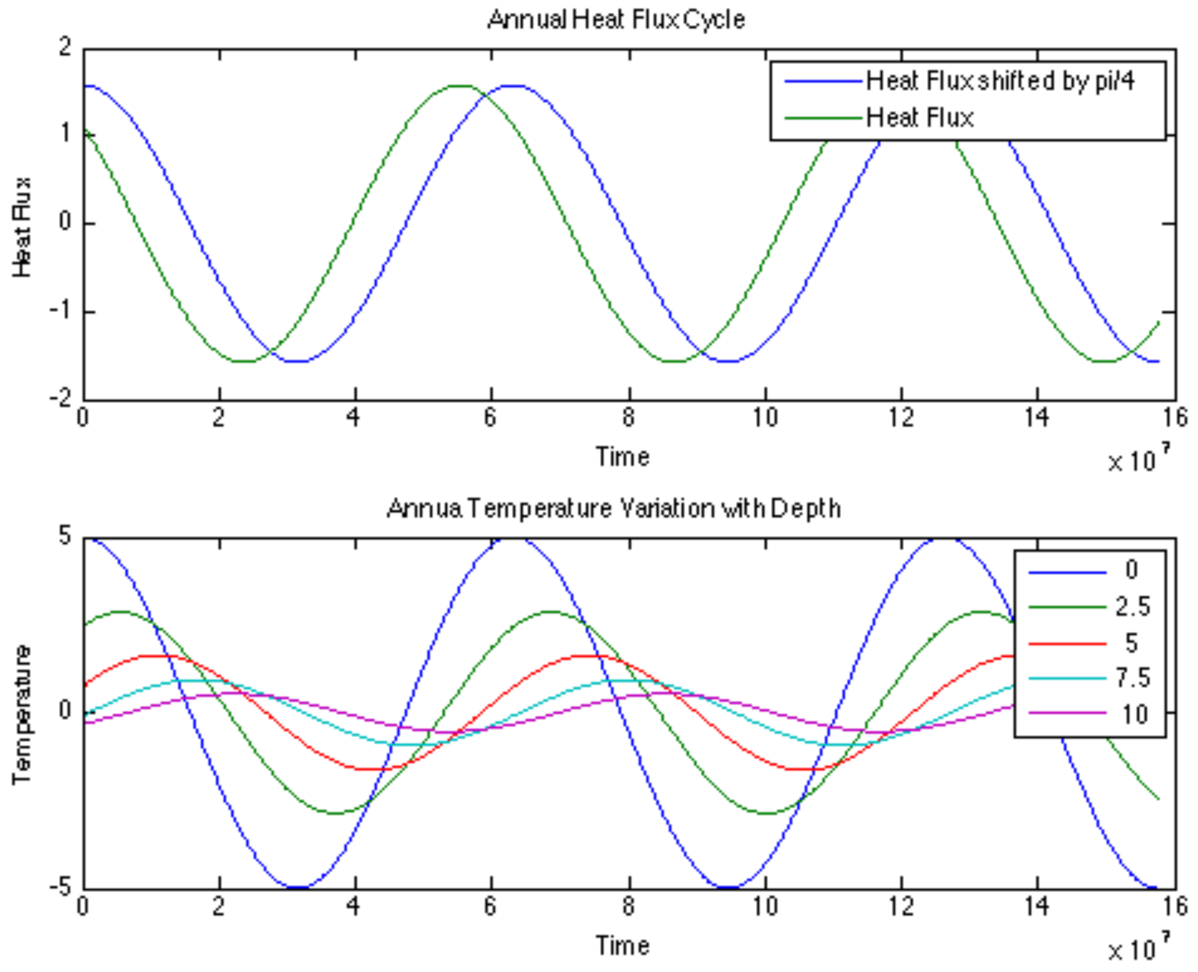
- There is a delay of π/4 between a change in heat flux at the surface and the change in temperature due to it.

Time Variation

$$\omega = \frac{2\pi}{\tau}$$

- Diurnal Variation
  - $\tau = 1\, day$
  - $\omega = 7.27 \times 10^{-5}\, rad/s$
- Annual Variation
  - $\tau = 1\, year$
  - $\omega = 1.99 \times 10^{-7}\, rad/s$
- Glacial Variation
  - $\tau = 100,000\, years$
  - $\omega = 1.99 \times 10^{-12}\, rad/s$

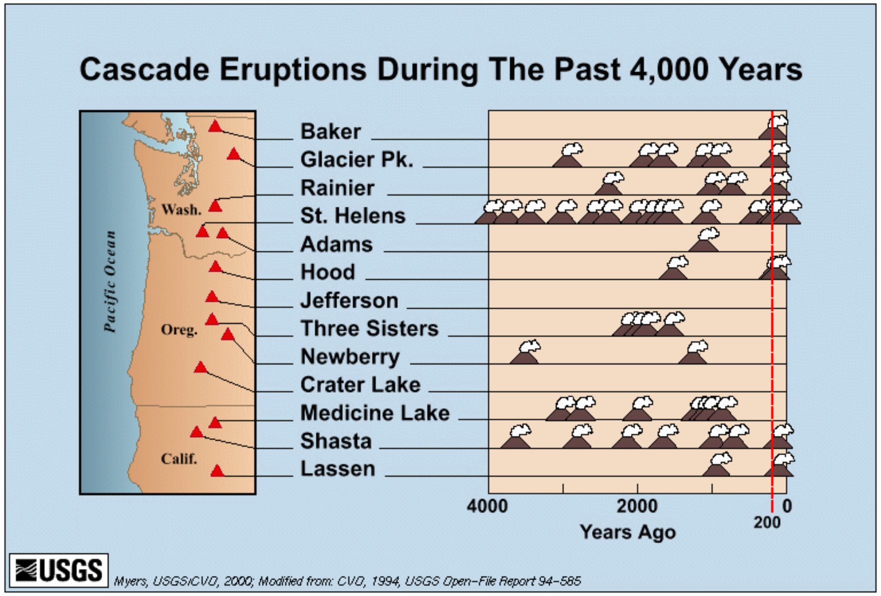
Time Variation



$$\kappa = 1\,mm^2/s$$
$$T_o = 0°C$$
$$\Delta T = 5°C$$

Necessary Constants (Bend/Basalt)

- Mean Temperature
    - 8 °C
- Density
    - Basalt: 2900 kg*m^{-3}
- Specific heat
    - Basalt: 0.84 kJ*(kg*K)$^{-1}$
- Heat Capacity
    - Basalt: 1.5 W*(m*K)$^{-1}$



Cascade Eruptions During The Past 4,000 Years

Myers, USGS/CVO, 2000; Modified from: CVO, 1994, USGS Open-File Report 94-585

http://vulcan.wr.usgs.gov/Imgs/Gif/Cascades/EruptiveHistory/cascades_eruptions_4000yrs.gif

Results (Bend)

- $T_o = 8°C$ $\Delta T = 16°C$

- To reach a temperature of 7°C we have to dig ~8.2 m
  - really deep
- Very little temperature fluctuation
  - Too deep for diurnal
  - Not deep enough to be affected by glacial
  - Small (~20%) of surface variation for annual
- Too deep for realistic storage

Necessary Constants (San Diego/Sandstone)

- Mean Temperature
  - 17.6 °C
- Density
  - Sandstone: 2650 kg*m^{-3}
- Specific Heat
  - Sandstone: 0.92 kJ*(kg*K)$^{-1}$
- Heat Capacity
  - Sandstone: 4 W*(m*K)$^{-1}$

http://farm4.static.flickr.com/3305/3314611913_c8e89bf6b9.jpg

Results (San Diego)

- $T_o = 17.6°C$ $\Delta T = 8°C$
- Unfortunately the lowest temperature attainable is 17°C
    - still too hot
- Just use a refrigerator because the ground just doesn’t get cold enough

Head North

- Need an overall colder location
- Several cities in Alaska as well as Canada provide the necessary climate
  - Juneau, AK; Skagway, AK; Calgary, AB; etc
- All these locations would allow a keg cooler to be built within 3 m of the surface providing temperatures around 7°C or colder
- Don't go too far north (frozen beer = bad)

http://www.muckleyphoto.com/data/photos/198_1SDSM270_San_Diego_snow_view.jpg

Three Sisters (Cascades)

http://brewpublic.com/wp-content/uploads/2009/06/bend-june-2009-108.jpg

THANKS!

http://cdn.lightgalleries.net/4bd5ec082e28a/images/BEVERAGE-DRINKS-PHOTOGRAPHY-ZACK-BURRIS-CHICAGO-BEER-MUGS-TOASTING-1.jpg