# Heat Flow Across the San Andreas Fault

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# Calculating Shear Stress on the San Andreas Fault

$$\tau(z) = f \sigma_n$$

Where $\sigma_n = \rho_c g z$

| | |
|---|---|
| $\tau(z)$ | shear stress on a locked fault |
| $\sigma_n$ | normal force |
| $f$ | coefficient of static friction |
| $\rho_c$ | crustal density |
| $g$ | acelleration due to gravity |
| $z$ | depth of seismogenic zone |

# Calculating Shear Stress on the San Andreas Fault

San Andreas Fault Assumptions

$f = 0.6$

$\rho_c = 2600 \text{ kg m}^{-3}$

$g = 9.8 \text{ m s}^{-2}$

$z = 12 \text{ km}$

$\tau(z) = 183 \text{ MPa}$

PROBLEM – Average stress drop on a fault during an actual earthquake $\leq$ 10 MPa

# Calculating Shear Stress on the San Andreas Fault

Considerations

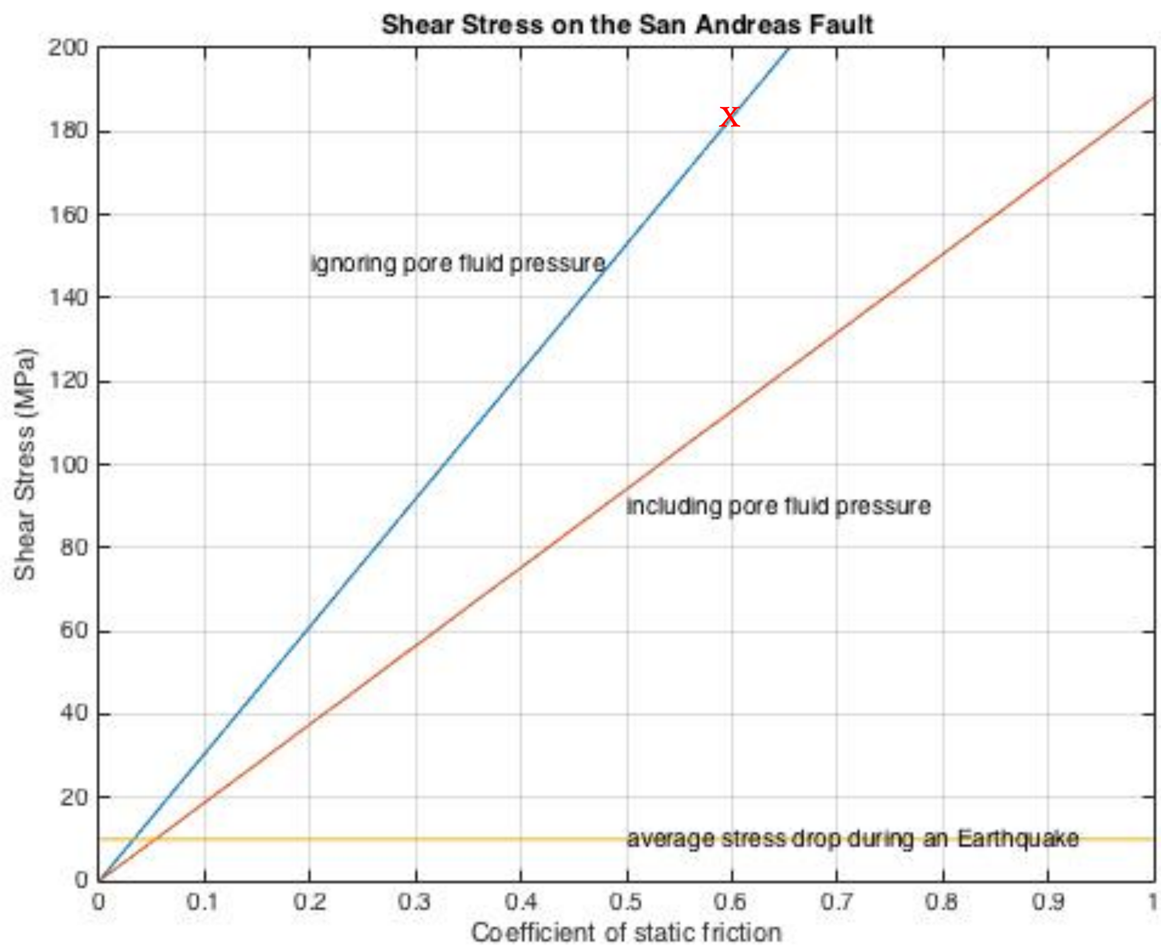
Coefficients of Static Friction

$f$ varies with depth

Pore fluid pressure

$\sigma = \sigma_n - \sigma_{pore\ fluid\ pressure}$ so $\tau(z) = f(\rho_{c} - \rho_w)gz$

# Calculating Shear Stress



# Derivation of a Line Source of Heat to get Heat Flow Anomaly

- We want to model the heat flow off of the San Andreas Fault as a line source of heat that dissipates as one moves away from the fault
- We need to get an equation that can be used to calculate the heat flow anomaly for different coefficients of static friction given an equation for a temperature anomaly
- Method
  - Start with an equation for a temperature anomaly and take a Fourier Transform
  - Take the inverse Fourier Transform in two dimensions to receive an easier temperature function to work with
  - Use the relationship between temperature and heat flow to get our heat flow anomaly equation

# Derivation of Heat Flow from a Line Source

Given:

$$\nabla^2 T = \frac{1}{k} Q(x,z) = \frac{1}{k}\delta(x)\delta(z+a)$$

$$\nabla^2 T = \frac{d^2T}{dx^2} + \frac{d^2T}{dz^2}$$

$$T(x,0) = 0$$

$$\lim_{|z|\to\infty} T(x,z) = 0$$

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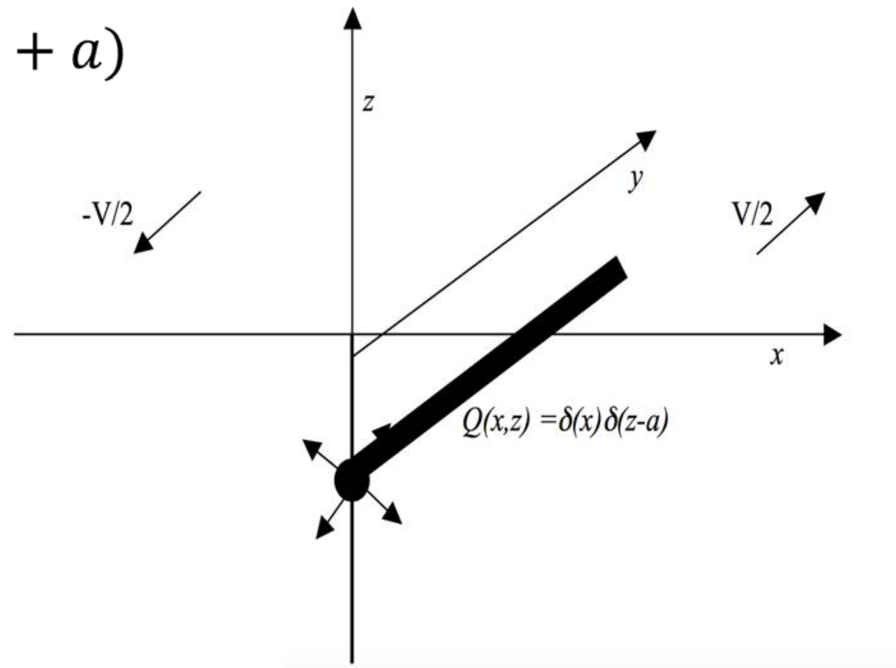



T = Temperature Anomaly
k= Thermal Conductivity = 3.3W/mK
Q =Heat Generation

# Step 1: Take Fourier Transform with Respect to x and z

$$\frac{d^2T}{dx^2} + \frac{d^2T}{dz^2} = \frac{1}{k}\delta(x)\delta(z+a)$$

- Use the derivative property of Fourier Transforms: $\Im\left[\frac{\partial f}{\partial x}\right] = i2\pi k F(k)$

$$[(i2\pi k_x)^2 + (i2\pi k_z)^2][T(k_x,k_z)] = ft\big(\delta(x)\delta(z+a)\big)$$

- Use property for the transform of a delta function: $F[\delta(x-x_o)] = e^{-i2\pi f(x_o)}$

$$F[\delta(x)] = 1$$

$$4\pi^2(k_x^2 + k_z^2)T(k_x,k_z) = \int_{-\infty}^{\infty}\delta(x)\,e^{i2\pi k_x x}dk_x \int_{-\infty}^{\infty}\delta(z+a)\,e^{i2\pi k_z(z+a)}dk_z$$

- Which when simplified equals:

$$-4\pi^2(k_x^2 + k_z^2)T(k_x,k_z) = 1 * e^{-i2\pi k_z a}$$

- Rearranging for T yields:

$$T(k_x,k_z) = \frac{e^{-i2\pi k_z a}}{-4\pi^2(k_x^2 + k_z^2)}$$

# Step 2: Take the Inverse Fourier Transform with Respect to $k_z$

$$T(k_x, k_z) = \frac{e^{-i2\pi k_z a}}{-4\pi^2(k_x^2 + k_z^2)}$$

Note that: $(k_x^2 + k_z^2) = (k_z + ik_x)(k_z - ik_x)$

$$T(k_x, z) = \frac{-1}{4\pi^2}\int_{-\infty}^{\infty} \frac{e^{i2\pi k_z(z+a)}}{(k_z + ik_x)(k_z - ik_x)} dk_z$$

# Cauchy Residue Theorem

- If a function is analytic, integrating around complex poles on a closed loop will equal zero

$$\oint f(z)dz = 0$$

- If we have complex poles in the denominator of the function, the Cauchy Residue theorem states

$$\oint \frac{f(z)}{z - z_0} = i2\pi\, f(z_0)$$

# Step 2: Take the Inverse Fourier Transform with Respect to $k_z$

$$T(k_x, z) = \frac{-1}{4\pi^2}\int_{-\infty}^{\infty} \frac{e^{i2\pi k_z(z+a)}}{(k_z + ik_x)(k_z - ik_x)} dk_z$$

$$\oint \frac{f(z)}{z - z_0} = i2\pi\, f(z_0)$$

If $k_z = ik_x$, considering $k_x > 0$ and $z > -a$

$$T(k_x, z) = (i2\pi)\frac{-1}{4\pi^2}\frac{e^{i^2 2\pi k_x(z+a)}}{(ik_x + ik_x)}$$

$$T(k_x, z) = \frac{-1}{4\pi}\frac{e^{-2\pi k_x(z+a)}}{k_x}$$

If $k_z = -ik_x$, considering $k_x < 0$ and $z > -a$

$$T(k_x, z) = (i2\pi)\frac{-1}{4\pi^2}\frac{e^{-i^2 2\pi k_x(z+a)}}{(-ik_x - ik_x)}$$

$$T(k_x, z) = \frac{1}{4\pi}\frac{e^{2\pi k_x(z+a)}}{k_x}$$

Combine the last two equations using the absolute value of $k_x$

$$T(k_x, z) = \frac{-1}{4\pi}\frac{e^{-2\pi |k_x|(z+a)}}{|k_x|}$$

# Step 3: Take the Inverse Fourier Transform with Respect to $k_x$

$$T(x,z) = \frac{-1}{4\pi}\int_{-\infty}^{\infty} \frac{e^{-2\pi|k_x|(z+a)}}{|k_x|} e^{i2\pi k_x x} dk_x$$

- Using the derivative property of Fourier Transforms

$$\frac{\partial T\,(x,z)}{\partial z} = i2\pi k_z \frac{-1}{4\pi}\int_{-\infty}^{\infty} \frac{e^{-2\pi|k_x|(z+a)}}{|k_x|} e^{i2\pi k_x x} dk_x$$

$$\frac{\partial T\,(x,z)}{\partial z} = \frac{1}{2}\int_{-\infty}^{\infty} e^{-2\pi|k_x|(z+a)}\, e^{i2\pi k_x x} dk_x$$

$$\frac{\partial T\,(x,z)}{\partial z} = \frac{-(z+a)}{4\pi[x^2 + (z+a)^2]}$$

# Step 3: Take the Inverse Fourier Transform with Respect to $k_x$

- Integrating with respect to z

$$T\ (x,z) = \frac{-1}{4\pi}\ln\left([x^2 + (z+a)^2\right]^{\frac{1}{2}}$$

- Because this does not satisfy our boundary conditions, we must add a heat sink at z = a, giving us our final equation that satisfies all boundary conditions.

$$T\ (x,z) = \frac{-1}{4\pi}\left[\ln([x^2 + (z+a)^2]^{\frac{1}{2}} - \ln([x^2 + (z-a)^2]^{\frac{1}{2}}\right]$$

$$T(x,z) = \frac{-1}{4\pi}\, ln\left(\frac{(x^2 + (z+a)^2)}{(x^2 + (z-a)^2)}\right)^{\frac{1}{2}}$$

# Step 4, take our temperature anomaly and find heat flow

- We have temperature in terms of z and x, now we need to find the heat flow q by integrating with respect to a

$$q(x,z,a) = -k\,\frac{\partial T}{\partial z} = \frac{1}{4\pi}\left(\frac{z+a}{x^2+a^2} - \frac{z-a}{x^2+a^2}\right)$$

- To satisfy our boundary conditions, we need to set z equal to zero, thus we get

$$q(x,a) = \frac{1}{4\pi}\frac{2a}{x^2+a^2}$$

# Aside: Green's Function

- Now we must use a Green's function to evaluate heat flow over our heat source by using the following relation:

$$q(x) = \int q(x,a) f(a)\, da$$

- For us, f(a) is the heat generated due to frictional heating, which we also have:

$$f(a) = v \cdot \tau = v \mu \rho_c g a$$

  - Where v is the mean velocity of the plate: San Andreas Fault: 35mm/year

# Step 4, take our temperature anomaly and find heat flow

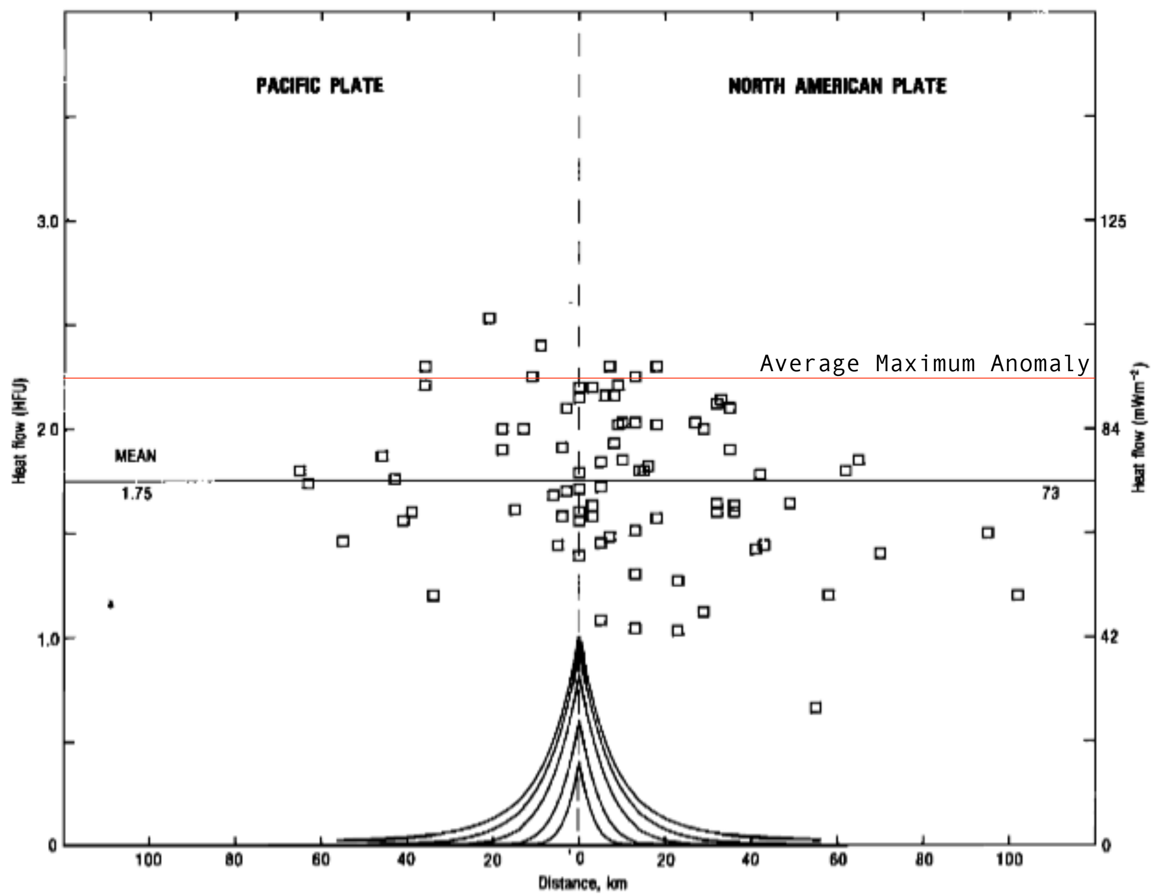
- Substitute q(x,a) and f(a) into our equation for q(x) and get the following:

$$q(x) = \frac{2v\mu\rho_c g}{4\pi}\int_{a_1}^{a_2} \frac{a^2}{x^2 + a^2}\ da$$

- Finally, use the integral identity for an arc tangent function to get the final expected surface heat flow anomaly!

$$q(x) = \frac{2v\mu\rho_c g}{4\pi}\left[a - x\tan^{-1}\left(\frac{a}{x}\right)\right]\Big|_{a_1}^{a_2}$$

# Observed Heat Flow Measurements

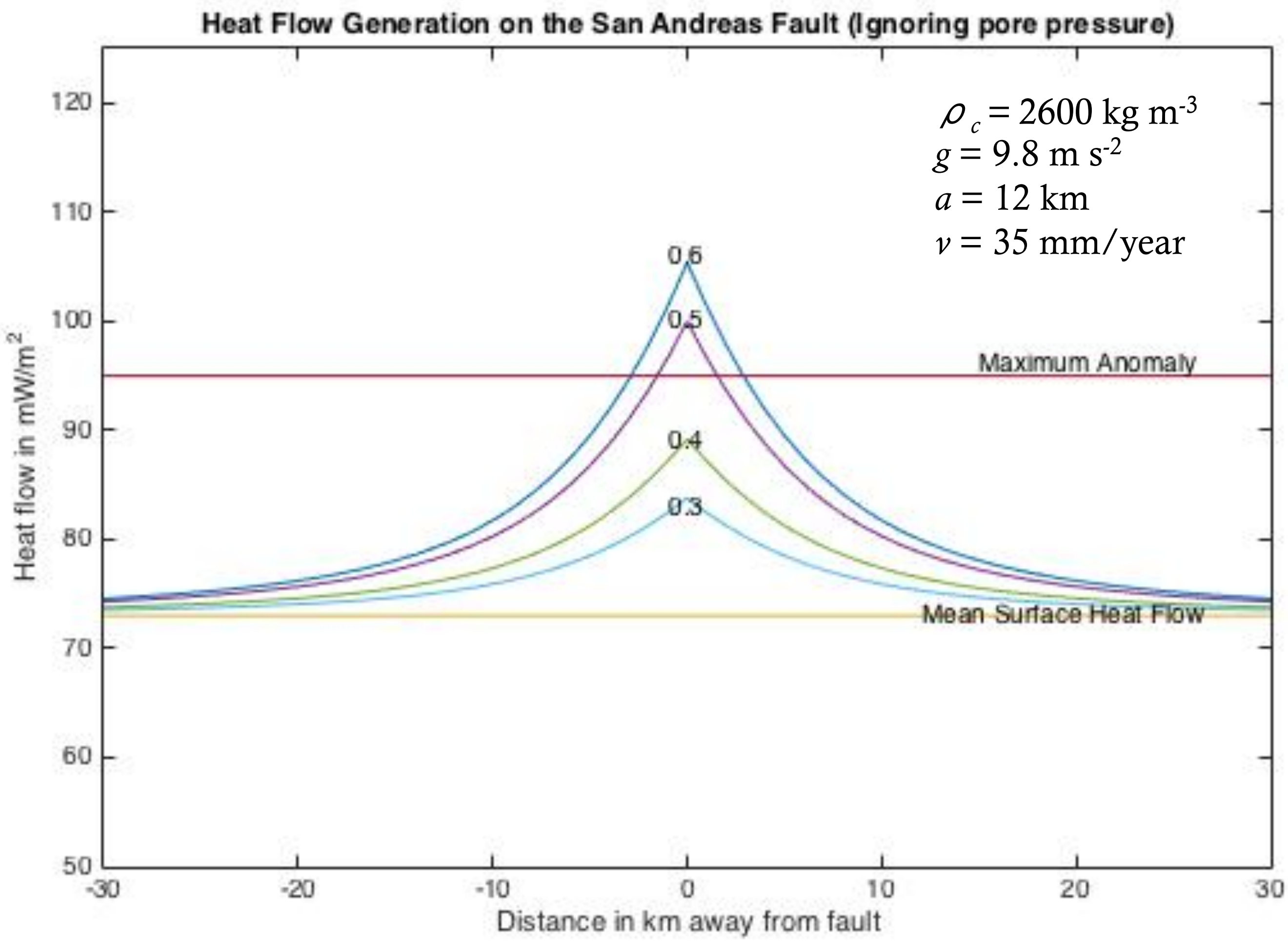



Observed heat flow
By Lachenbruch
& Sass (1980)

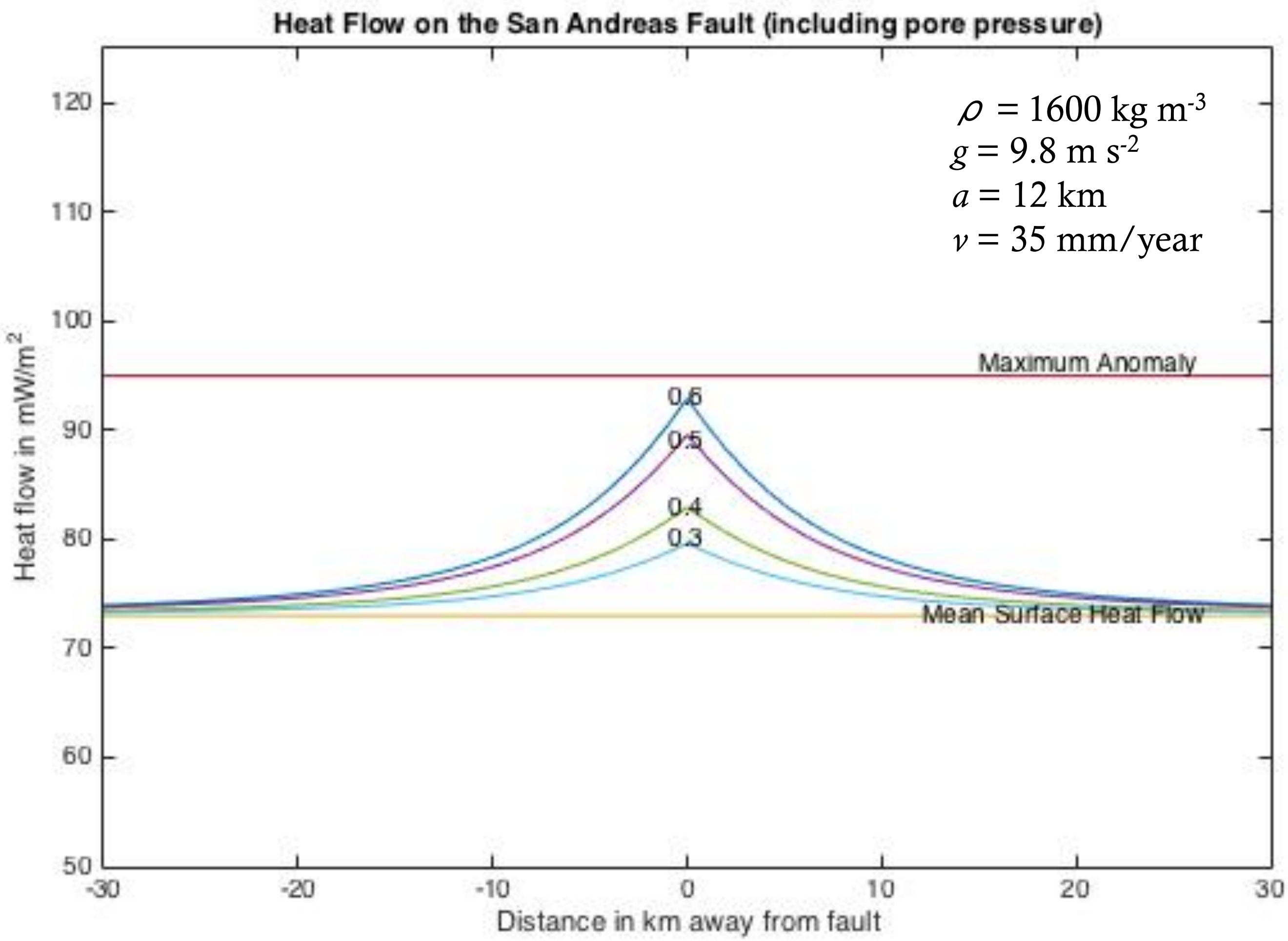
Mean Surface
Heat Flow
- 73 mW $m^{-2}$

Average Maximum
Anomaly
– 94 mW $m^{-2}$

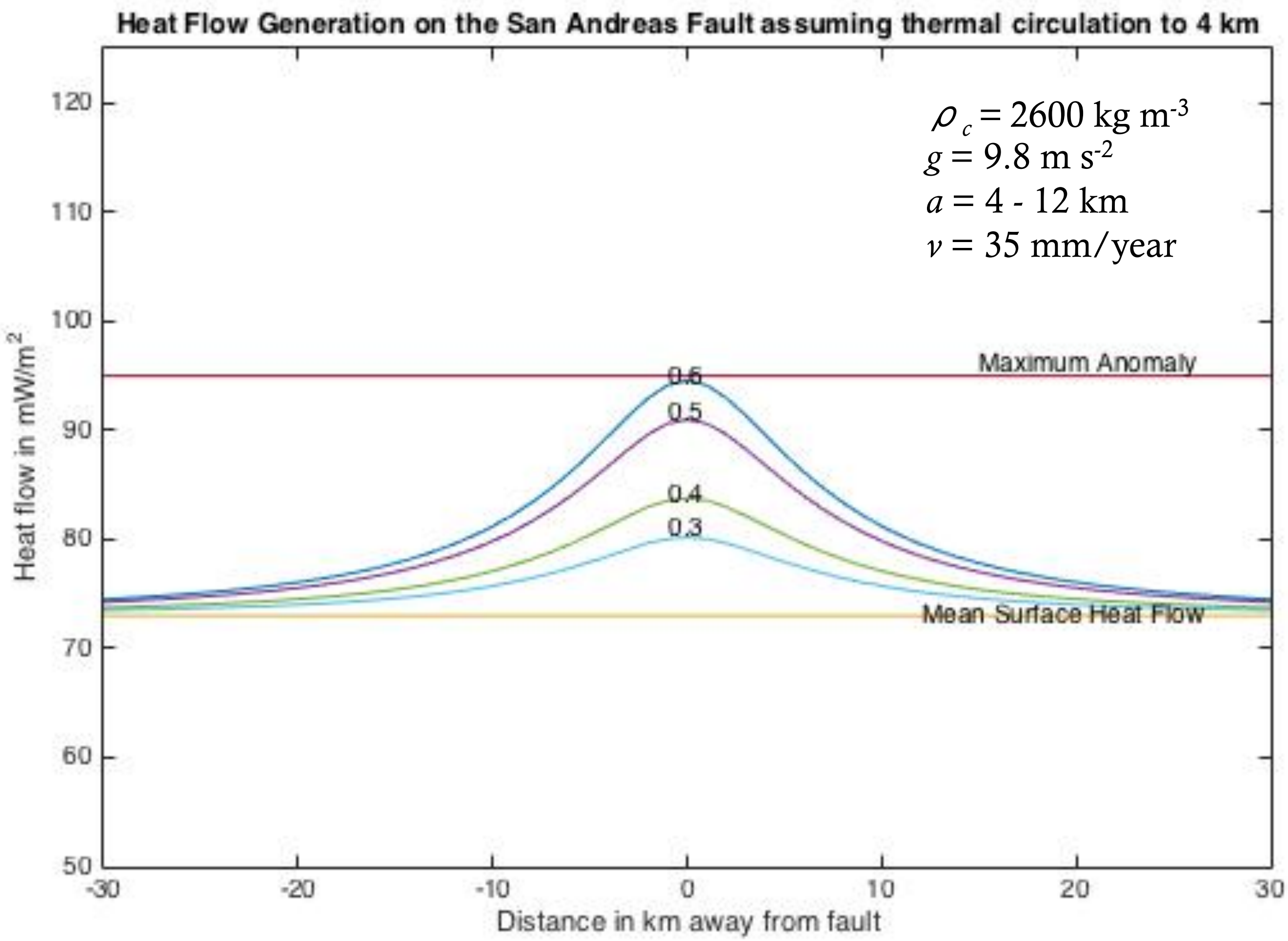
# Expected Heat Flow Anomaly



# Expected Heat Flow Anomaly (including pore fluid pressure)



# Expected Heat Flow Anomaly (including hydrothermal circulation)



# Discussion

- No evidence for significant hydrothermal circulation along the San Andreas (Lachenbruch & Sass 1980, Turcotte et al 1980)
- Error may be associated with coefficient of static friction
  - Evidence for talc found in serpentinite could account for low $\mu$ for creeping portions of SAF (Moore & Rymer, 2007)
- Still a large debate as to whether the San Andreas Fault is a strong or weak fault (Saffer et al 2003)

# References


Fulton, Patrick M., Robert N. Harris, Demian M. Saffer, and Emily E. Brodsky. "Does Hydrologic Circulation Mask Frictional Heat on Faults after Large Earthquakes?" Journal of Geophysical Research 115.B9 (2010): n, pag Print.

Lachenbruch, Arthur H., and J. H. Sass. "Heat Flow and Energetics of the San Andreas Fault Zone." Journal of Geophysical Research 85.B11 (1980): 6185-222. Print.

Moore, Diane E., and Michael J. Rymer. "Talc-bearing Serpentinite and the Creeping Section of the San Andreas Fault." *Nature* 448.7155 (2007): 795-97. Print.

Saffer, Demian M. "Topographically Driven Groundwater Flow and the San Andreas Heat Flow Paradox Revisited." Journal of Geophysical Research 108.B5 (2003): n, pag Print.

Sandwell, D. T., "Fourier Transform Methods in Geophysics." topex.ucsd.edu/geodynamics/01Fourier.pdf (2013) Accessed September 30, 2015

Turcotte, D. L., P. H. Tag, and R. F. Cooper. "A Steady State Model for the Distribution of Stress and Temperature on the San Andreas Fault." Journal of Geophysical Research 85.B11 (1980): 6224-230. Print.

Turcotte, D. L., & Schubert, G. (2002). *Geodynamics* Cambridge University Press, Cambridge. Print.