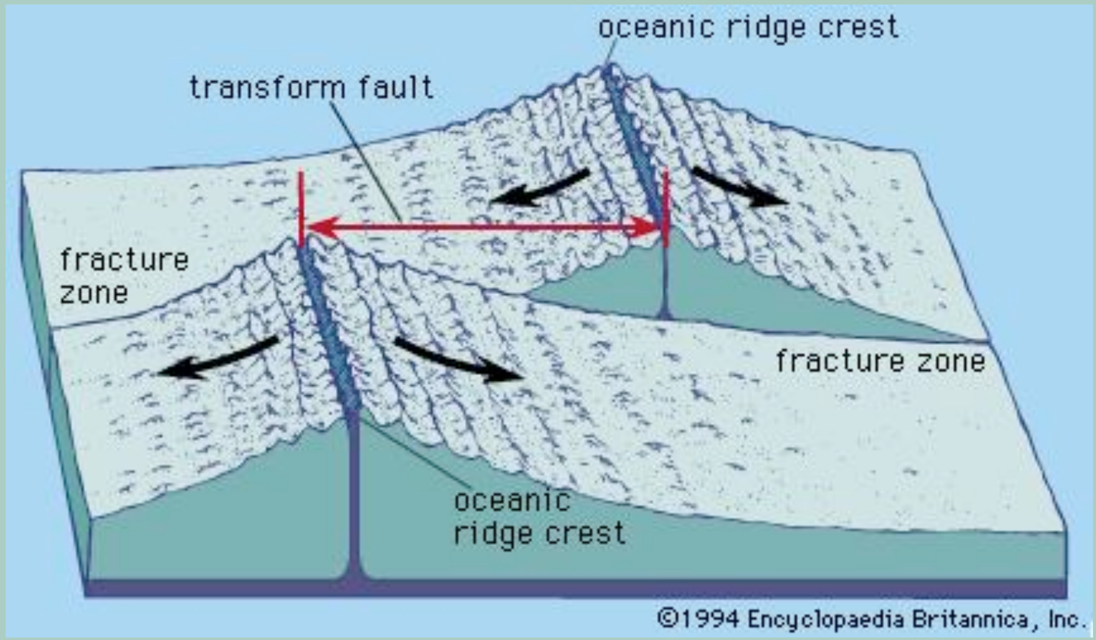
TEMPERATURE EVOLUTION ACROSS AN OCEAN FRACTURE ZONE

Christine Chesley

31 Oct 2016



https://www.britannica.com/science/submarine-fracture-zone

OUTLINE

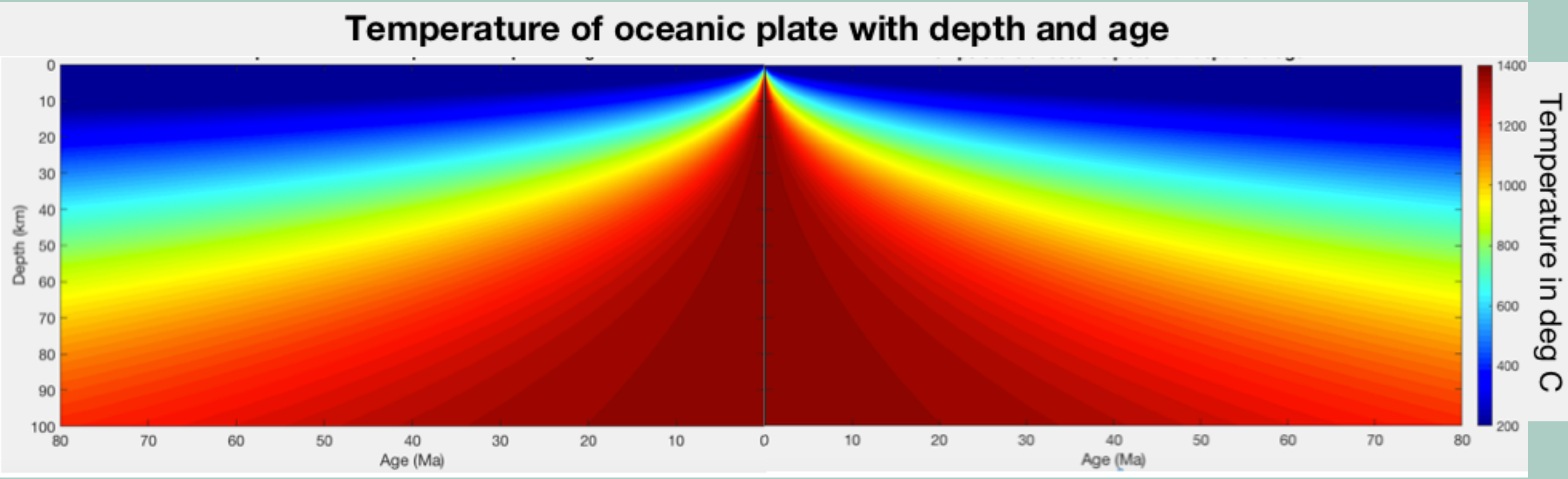
- Posing the Problem
- Equation and BCs
  - Solution
  - Visual Models
- Topography
- Limitations

POSING THE PROBLEM

- Temperature profile of oceanic lithosphere depends primarily on depth and…

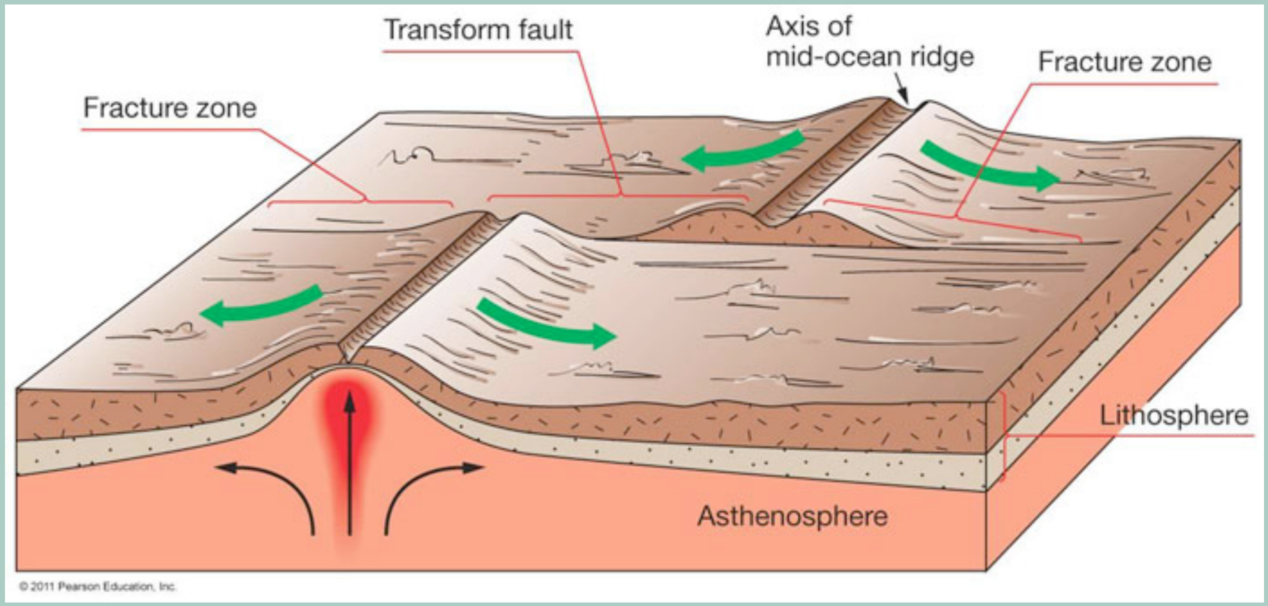
POSING THE PROBLEM

- Temperature profile of oceanic lithosphere depends primarily on depth and…AGE



POSING THE PROBLEM

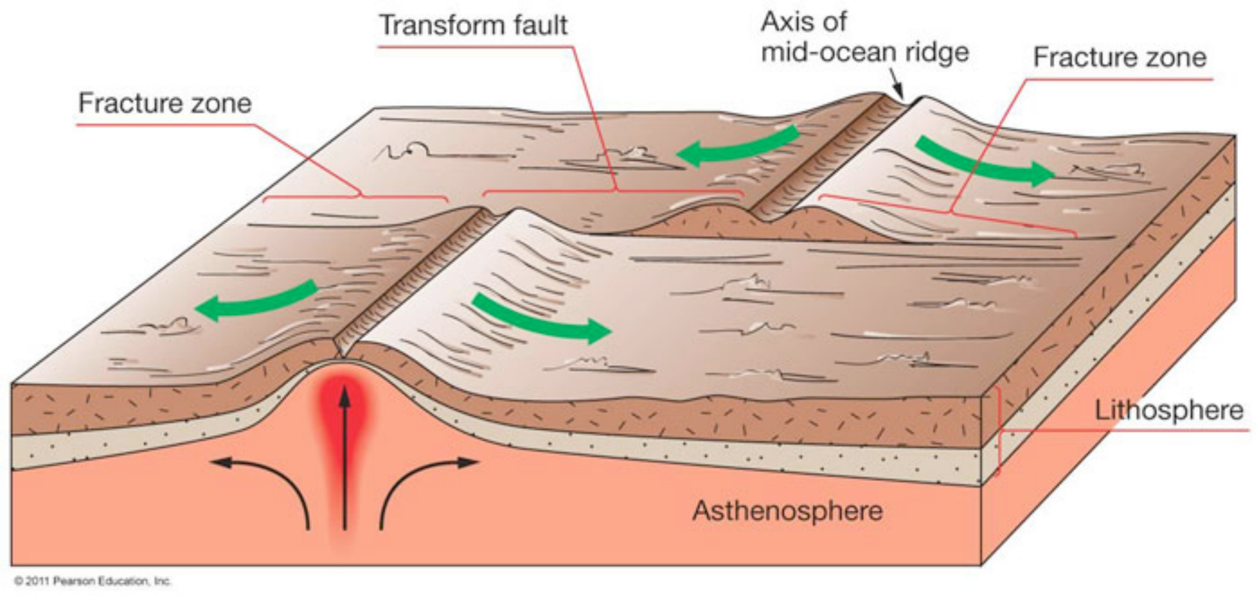
- Recall: Fracture zones
  - Generally aseismic features



http://oceanexplorer.noaa.gov/okeanos/explorations/ex1503/background/edu/purpose.html

POSING THE PROBLEM

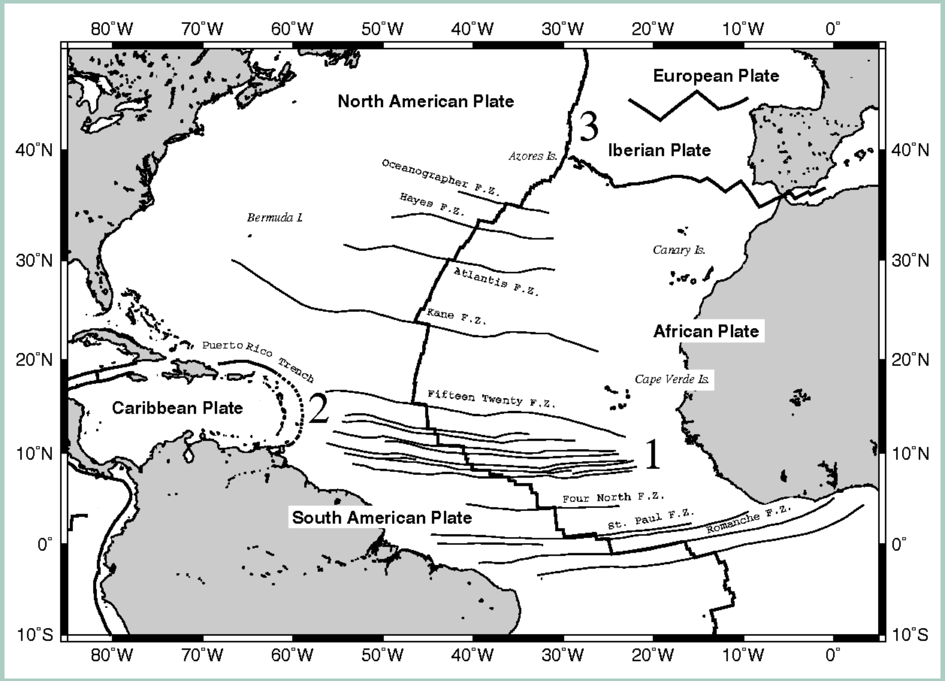
- Recall: Fracture zones
  - Generally aseismic features
  - Plate moves in same direction on either side of scarp



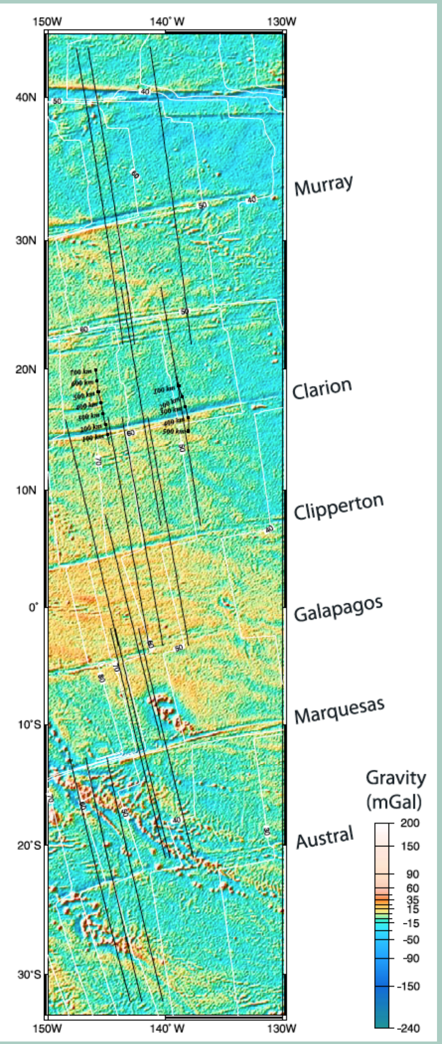
http://oceanexplorer.noaa.gov/okeanos/explorations/ex1503/background/edu/purpose.html

POSING THE PROBLEM

- Fracture zones—Age, and therefore Temperature, offset



FZs in the Atlantic Ocean



FZs in east and central Pacific Ocean [*Hall and Gurnis*, 2005]

(http://crack.seismo.unr.edu/ftp/pub/louie/class/plate/images_predict.HTML)

EQUATIONS AND BC's

- 2D Time Varying Heat Equation

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial x^2} = \frac{1}{\kappa}\frac{dT}{dt}$$

EQUATIONS AND BC's

- 2D Time Varying Heat Equation

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial x^2} = \frac{1}{\kappa}\frac{dT}{dt}$$

- Boundary Conditions (Sandwell and Schubert, 1982)

$$T(x,z,t_0) = \begin{cases} T_m & x < 0 \\ T_m \operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t_0}}\right) & x > 0 \end{cases}$$

$$T(x,0,t) = 0 \qquad T(x,\infty,t) = T_m$$

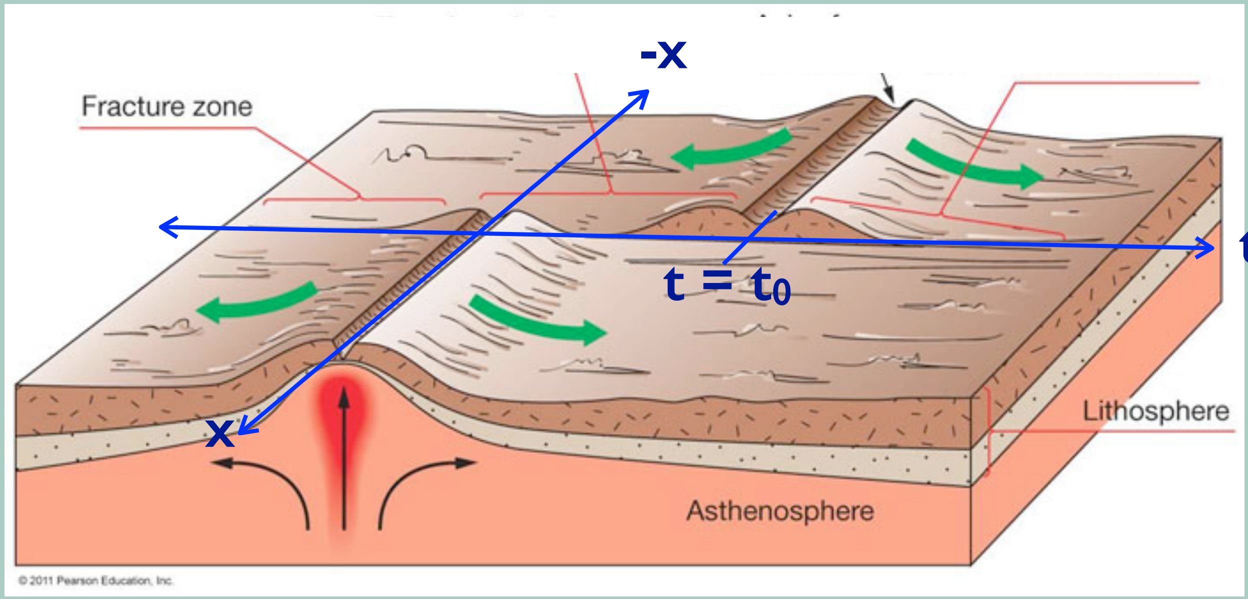
EQUATIONS AND BC's

- Boundary Conditions (Sandwell and Schubert, 1982)

$$T(x,z,t_0) = \begin{cases} T_m & x < 0 \\ T_m \operatorname{erf}\left(\dfrac{z}{2\sqrt{\kappa t_0}}\right) & x > 0 \end{cases}$$

$$T(x,0,t) = 0$$

$$T(x,\infty,t) = T_m$$



EQUATIONS AND BC's

- 2D Time Varying Heat Equation

$$\frac{\partial^2 T}{\partial z^2} + \frac{\partial^2 T}{\partial x^2} = \frac{1}{\kappa}\frac{dT}{dt}$$

- Boundary Conditions (Sandwell and Schubert, 1982)

$$T(x,z,t_0) = T_m - T_m \mathrm{H(x)erfc}\left(\frac{z}{2\sqrt{\kappa t_0}}\right)$$

$$T(x,0,t) = 0 \qquad T(x,\infty,t) = T_m$$

SOLUTION

- 1. Define dimensionless variable $\theta = \dfrac{T}{T_m}$

SOLUTION

- 1. Define dimensionless variable $\theta = \frac{T}{T_m}$

$$\frac{\partial^2\theta}{\partial z^2} + \frac{\partial^2\theta}{\partial x^2} = \frac{1}{\kappa}\frac{d\theta}{dt}$$

$$\theta(x, z, t_0) = 1 - \mathrm{H}(\mathrm{x})\mathrm{erfc}\left(\frac{z}{2\sqrt{\kappa t_0}}\right)$$

$$\theta(x, 0, t) = 0$$

$$\theta(x, \infty, t) = 1$$

SOLUTION

- 1. Define dimensionless variable $\theta = \frac{T}{T_m}$

$$\frac{\partial^2\theta}{\partial z^2} + \frac{\partial^2\theta}{\partial x^2} = \frac{1}{\kappa}\frac{d\theta}{dt}$$

$$\theta(x, z, t_0) = \left(1 - H(x)\right)\left(2H(z) - 1\right) + H(x)\operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t_0}}\right)$$

Method of Images
(à la Lindsey and Kanitsch)

$$\theta(x, 0, t) = 0 \qquad \theta(x, \infty, t) = 1$$

SOLUTION

- 2. Fourier Transform wrt x and z (note derivative property)

$$-4\pi^2\kappa(k_x^2 + k_z^2)\times\theta(k_x, k_z, t) = \frac{d\theta(k_x, k_z, t)}{dt}$$

SOLUTION

- 2. Fourier Transform wrt x and z (note derivative property)

$$-4\pi^2\kappa(k_x^2 + k_z^2)\times\theta(k_x, k_z, t) = \frac{d\theta(k_x, k_z, t)}{dt}$$

- 3. Integrate wrt t

$$\theta(k_x, k_z, t) = \theta(k_x, k_z, t_0)e^{-4\pi^2\kappa(k_x^2+k_z^2)t}$$

SOLUTION

- 2. Fourier Transform wrt x and z (note derivative property)

$$-4\pi^2\kappa(k_x^2 + k_z^2)\times\theta(k_x, k_z, t) = \frac{d\theta(k_x, k_z, t)}{dt}$$

- 3. Integrate wrt t (the Green's function in the k-domain)

$$\theta(k_x, k_z, t) = \theta(k_x, k_z, t_0)e^{-4\pi^2\kappa(k_x^2+k_z^2)t}$$

SOLUTION

- 4. Fourier transform $\theta(x,z,t_0)$ boundary condition

SOLUTION

- 4. Fourier transform $\theta(x,z,t_0)$ boundary condition
- 5. Multiply this by Green's function in k-domain

SOLUTION

- 4. Fourier transform $\theta(x,z,t_0)$ boundary condition
- 5. Multiply this by Green's function in k-domain
- 6. Inverse Fourier transform in k_x and k_z
  - it helps to know that $$\Im\left[erf\left(\frac{z}{2\sqrt{\kappa t}}\right)\right] = \frac{1}{i\pi k_z} e^{-4\pi^2 k_z^2 \kappa t} = -\Im\left[erfc\left(\frac{z}{2\sqrt{\kappa t}}\right)\right]$$

SOLUTION

- 4. Fourier transform $\theta(x,z,t_0)$ boundary condition
- 5. Multiply this by Green's function in k-domain
- 6. Inverse Fourier transform in k_x and k_z
  - it helps to know that $\Im\left[erf\left(\frac{z}{2\sqrt{\kappa t}}\right)\right] = \frac{1}{i\pi k_z} e^{-4\pi^2 k_z^2 \kappa t} = -\Im\left[erfc\left(\frac{z}{2\sqrt{\kappa t}}\right)\right]$
- 7. Solution!

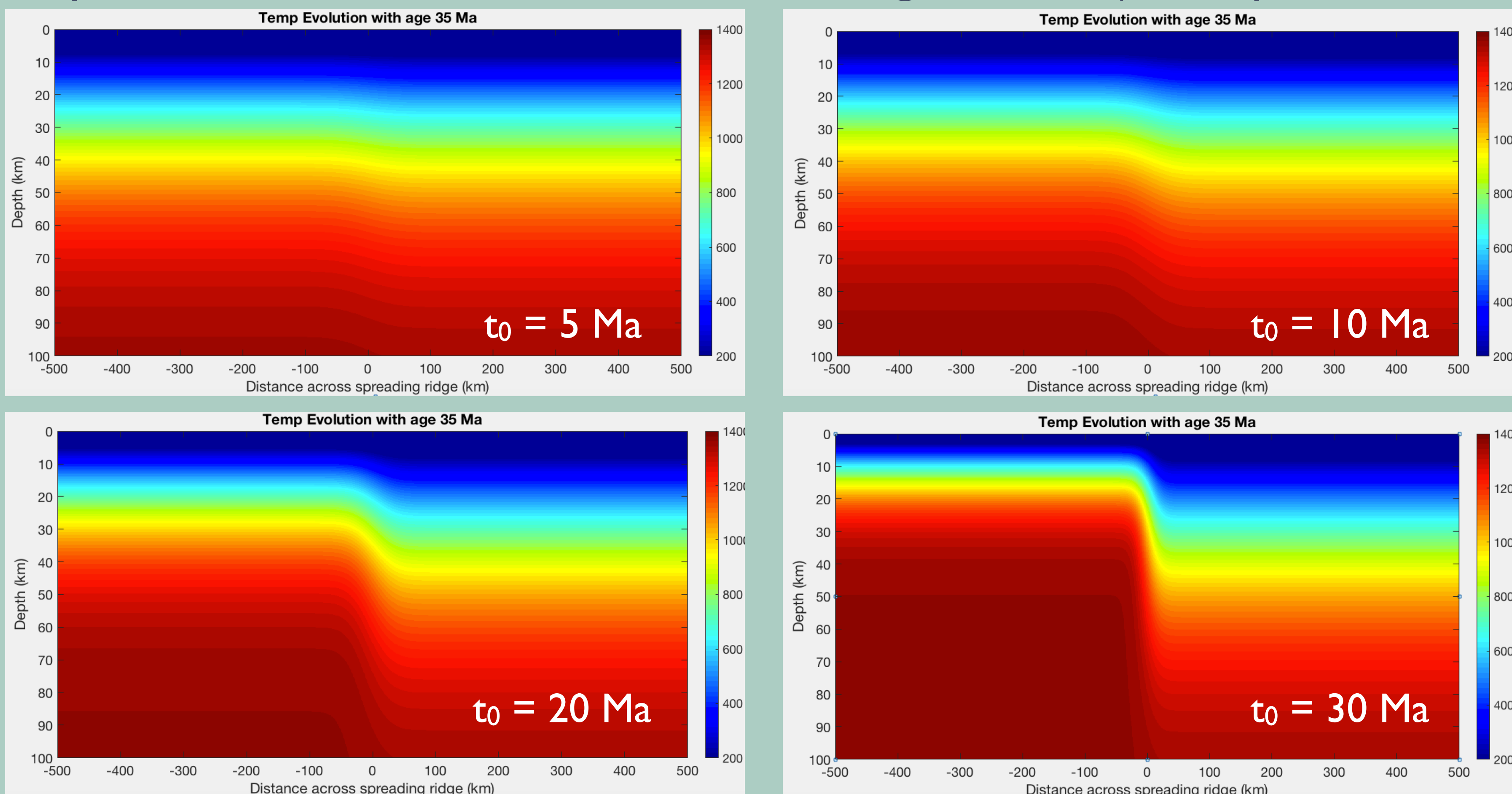
$$T(x,z,t_0) = \frac{T_m}{2}\left(\operatorname{erfc}\left(\frac{x}{2\sqrt{\kappa(t-t_0)}}\right)\operatorname{erf}\left(\frac{z}{2\sqrt{\kappa(t-t_0)}}\right) + \operatorname{erfc}\left(\frac{-x}{2\sqrt{\kappa(t-t_0)}}\right)\operatorname{erf}\left(\frac{z}{2\sqrt{\kappa t}}\right)\right)$$

SOLUTION

- MATLAB can verify this for us if we ask nicely

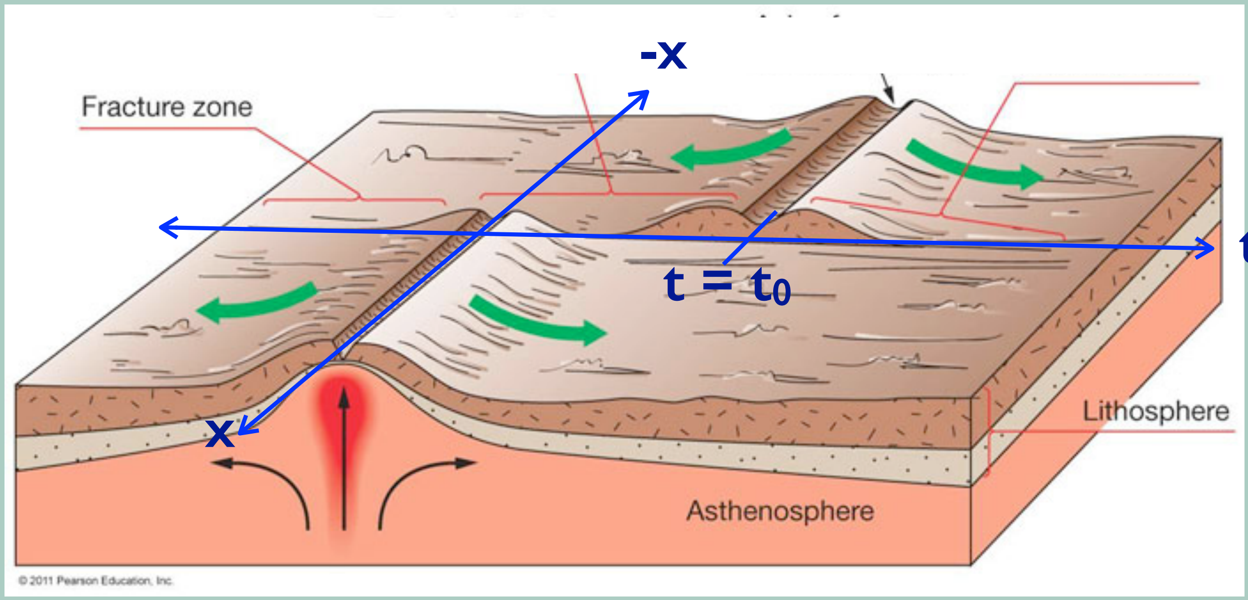
VISUAL MODELS

- Depth vs. Distance from FZ for different age offsets (older plate = 35 Ma)



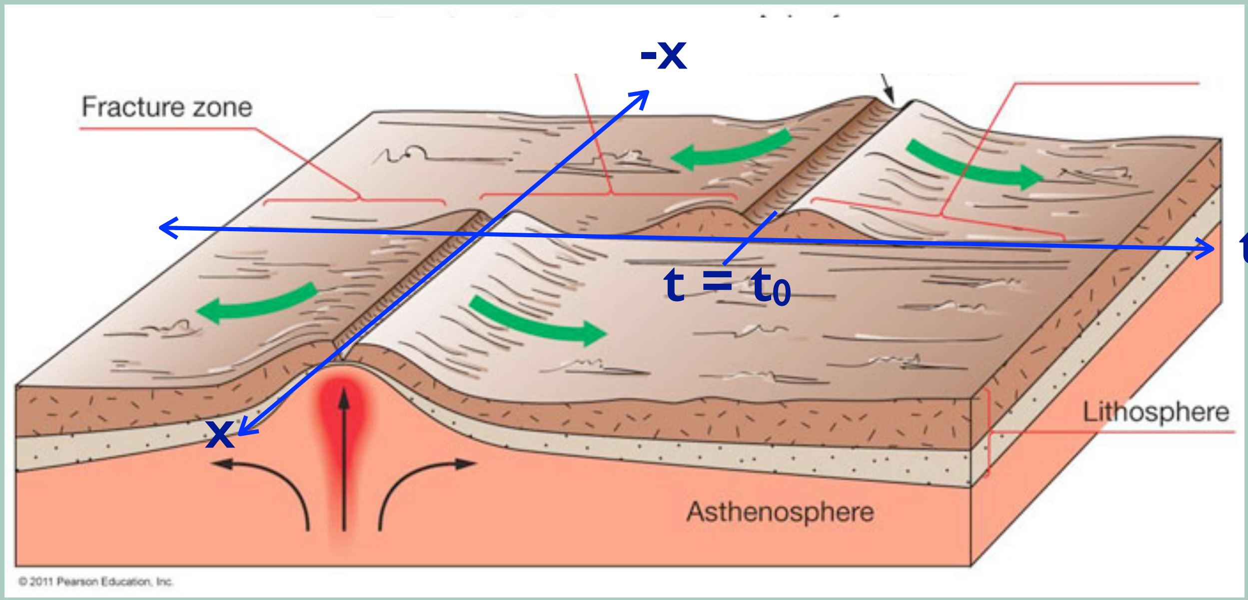
VISUAL MODELS

- **Depth vs. Distance with increasing age—video**
- Depth vs. Age Moving laterally across FZ—video
- Lateral Distance vs. Age—video



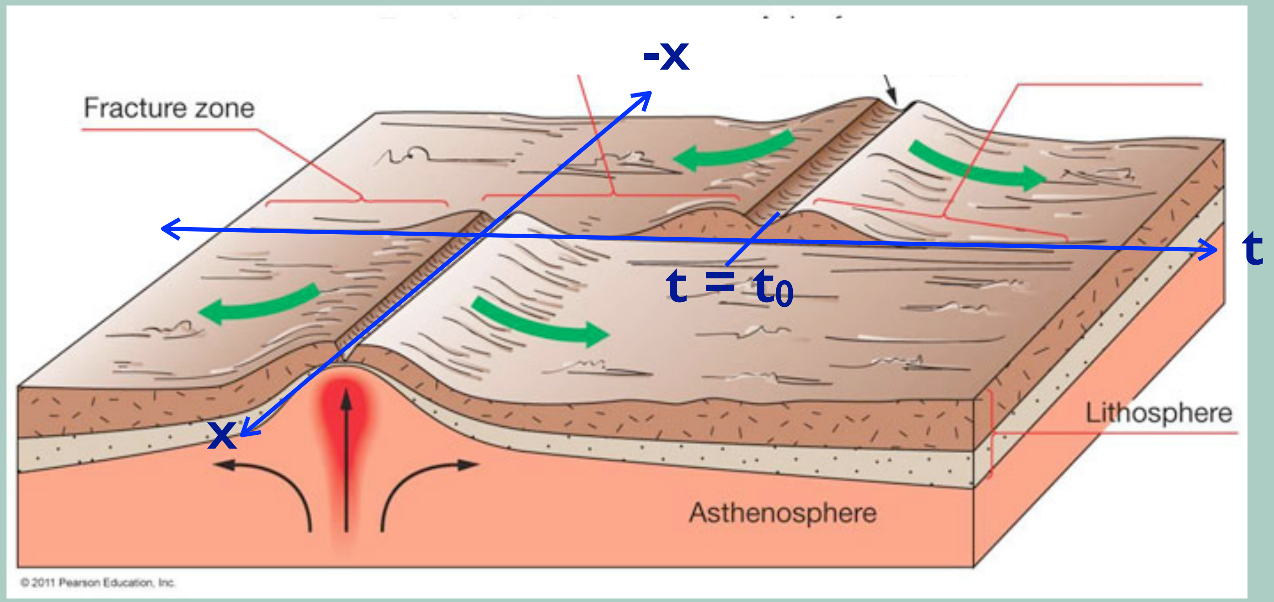
VISUAL MODELS

- Depth vs. Distance with increasing age—video
- **Depth vs. Age Moving laterally across FZ—video**
- Lateral Distance vs. Age—video



VISUAL MODELS

- Depth vs. Distance with increasing age—video
- Depth vs. Age Moving laterally across FZ—video
- **Lateral Distance vs. Age—video**



TOPOGRAPHY

- Assume Isostasy, then, similar to in class, we need to evaluate

$$d(t) = d_{ref} - \frac{\alpha\rho_m}{\rho_m - \rho_w}\int_0^\infty (T(x,z,t) - T_m)dz$$

- which can be rewritten as

$$d(t) = d_{ref} + \frac{\alpha\rho_m T_m}{2(\rho_m - \rho_w)}\int_0^\infty \left(\operatorname{erfc}\left(\frac{x}{2\sqrt{\kappa(t-t_0)}}\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{\kappa(t-t_0)}}\right) + \operatorname{erfc}\left(\frac{-x}{2\sqrt{\kappa(t-t_0)}}\right)\operatorname{erfc}\left(\frac{z}{2\sqrt{\kappa t}}\right)\right)dz$$

TOPOGRAPHY

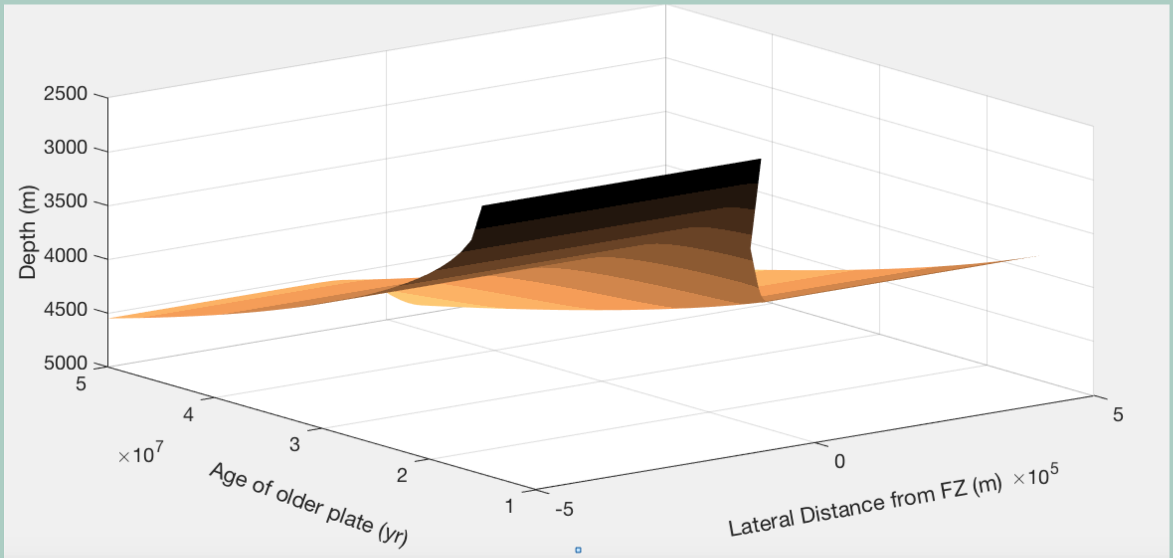
- And because

$$\int_0^\infty erfc(y)dy = \frac{1}{\sqrt{\pi}}$$

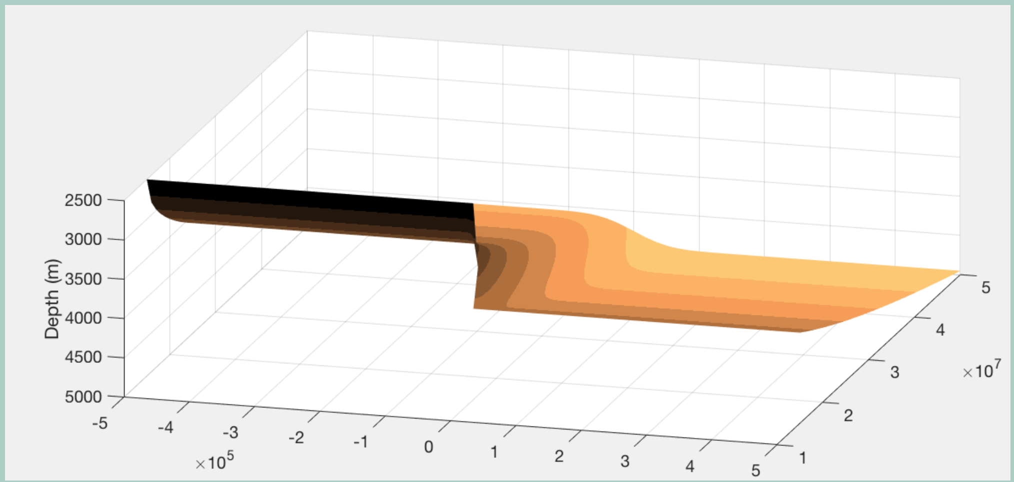
- this is equivalent to

$$d(t) = d_{ref} + \frac{\alpha\rho_m T_m}{(\rho_m - \rho_w)}\left[\sqrt{\frac{\kappa(t-t_0)}{\pi}}\,erfc\left(\frac{x}{2\sqrt{\kappa(t-t_0)}}\right) + \sqrt{\frac{\kappa t}{\pi}}\,erfc\left(\frac{-x}{2\sqrt{\kappa(t-t_0)}}\right)\right]$$

TOPOGRAPHY



Depth (m)
2500
3000
3500
4000
4500
5000
5
4
3
2
1
$\times 10^7$
Age of older plate (yr)
-5
0
5
Lateral Distance from FZ (m)
$\times 10^5$



Depth (m)
2500
3000
3500
4000
4500
5000
-5
-4
-3
-2
-1
0
1
2
3
4
5
$\times 10^5$
1
2
3
4
5
$\times 10^7$

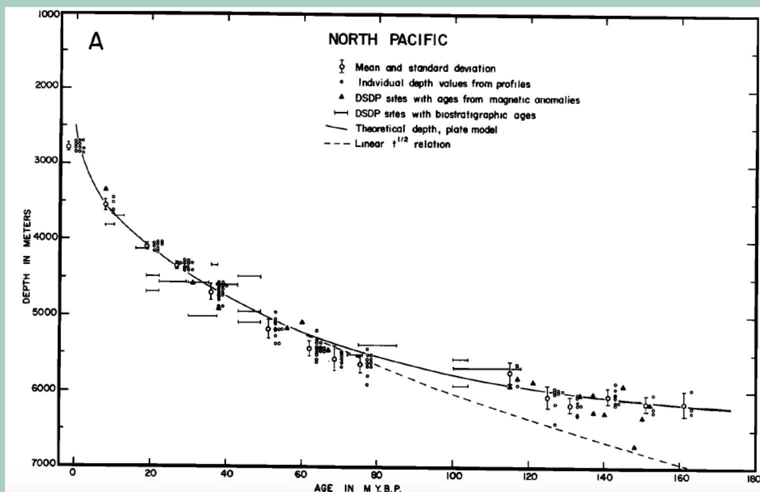
LIMITATIONS

- 1. We have assumed half-space cooling

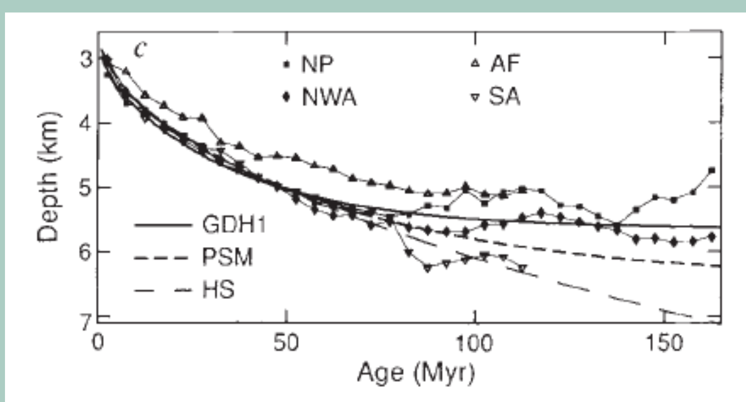
LIMITATIONS

- 1. We have assumed half-space cooling
  - Assumption is good at describing **young** (< 70/80 Ma) oceanic lithosphere, **not old**

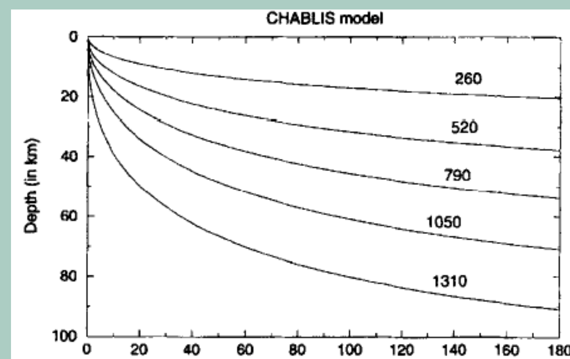
LIMITATIONS



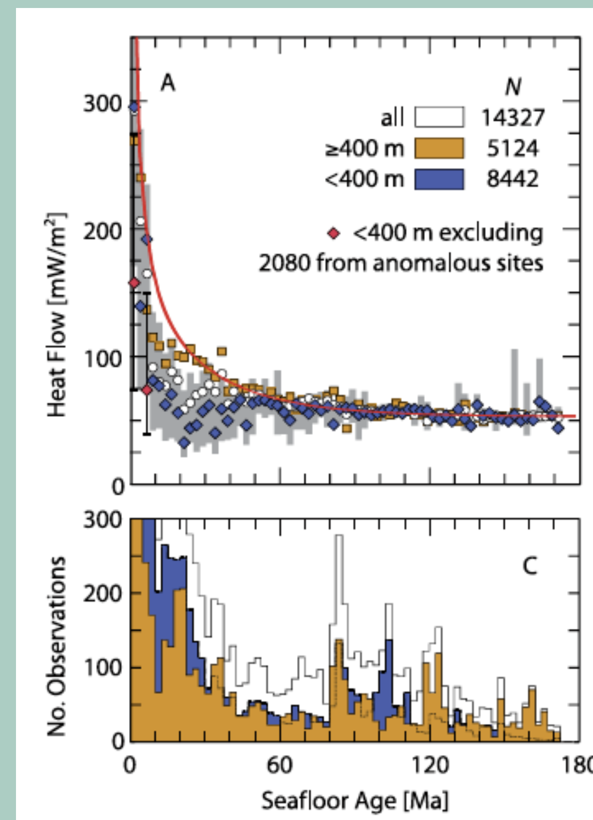
Parsons and Sclater, 1977



Stein and Stein, 1992



Doin and Fleitout, 1996



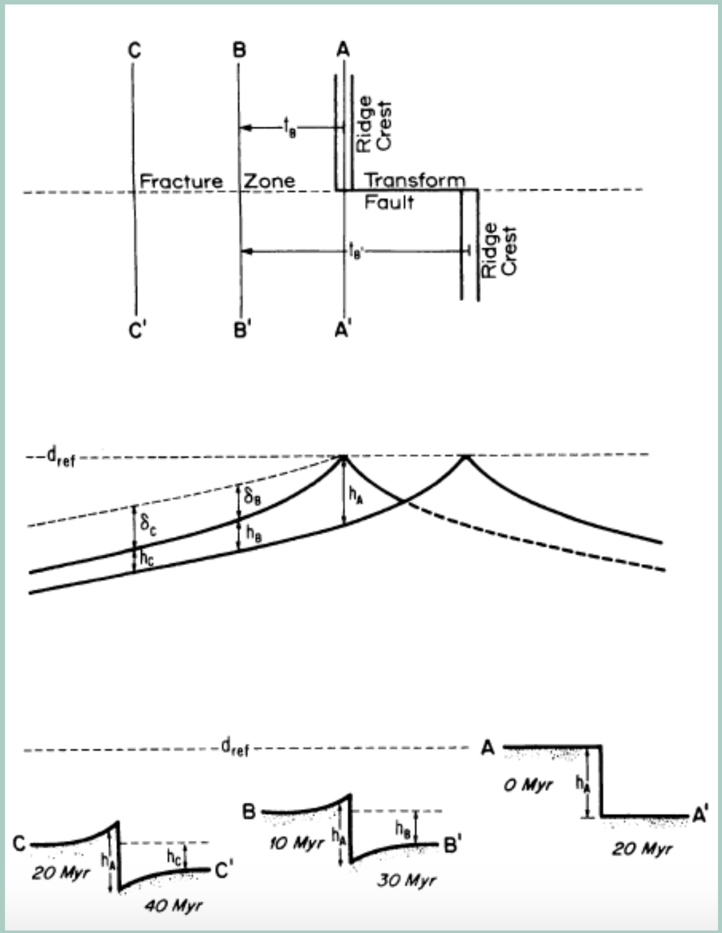
Hasterok, 2013

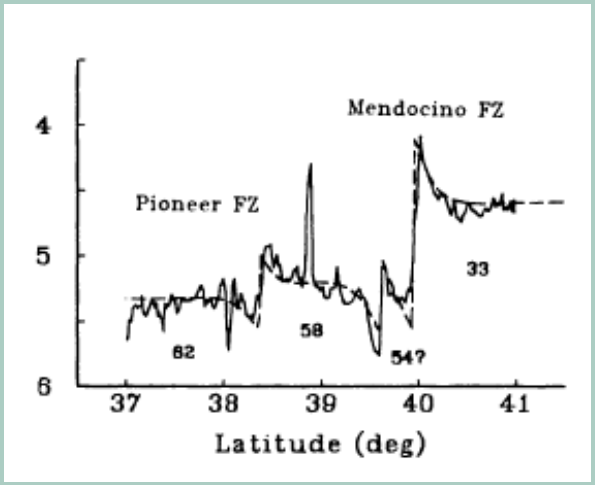
LIMITATIONS

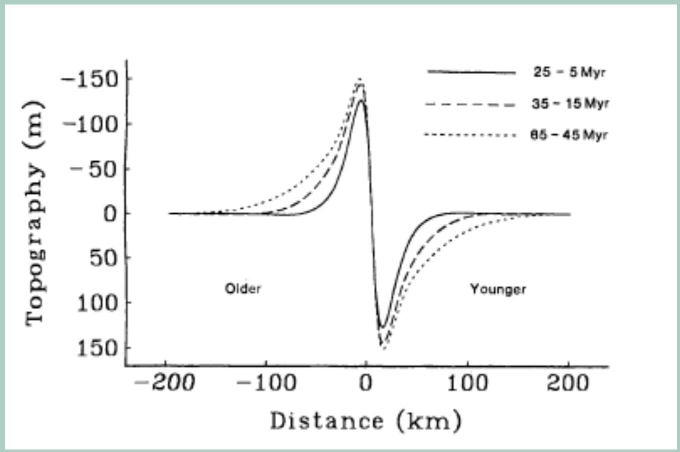
- 2. We have not taken FLEXURE into consideration

LIMITATIONS

Sandwell and Schubert [1982]







REFERENCES

- Doin, M. P., & Fleitout, L. (1996). Thermal evolution of the oceanic lithosphere: an alternative view. EPSL, 142(1), 121-136.
- Hall, C. E., & Gurnis, M. (2005). Strength of fracture zones from their bathymetric and gravitational evolution. J. Geophys. Res.: Solid Earth, 110(B1).
- Hasterok, D. (2013). Global patterns and vigor of ventilated hydrothermal circulation through young seafloor. EPSL, 380, 12-20.
- Parsons, B., & Sclater, J. G. (1977). An analysis of the variation of ocean floor bathymetry and heat flow with age. J. Geophys. Res., 82(5), 803-827.
- Sandwell, D., and G. Schubert (1982), Lithospheric flexure at fracture zones, J. Geophys. Res., 87(B6), 4657–4667
- Stein, C.A., & Stein, S. (1992). A model for the global variation in oceanic depth and heat flow with lithospheric age. Nature 359, 123-129.

VISUAL MODELS

- 15 Ma age offset