Frictional Heating During an Earthquake

Te-Yang Yeh
@ Geodynamics HW3 (Group G)

[San Andreas], 2015

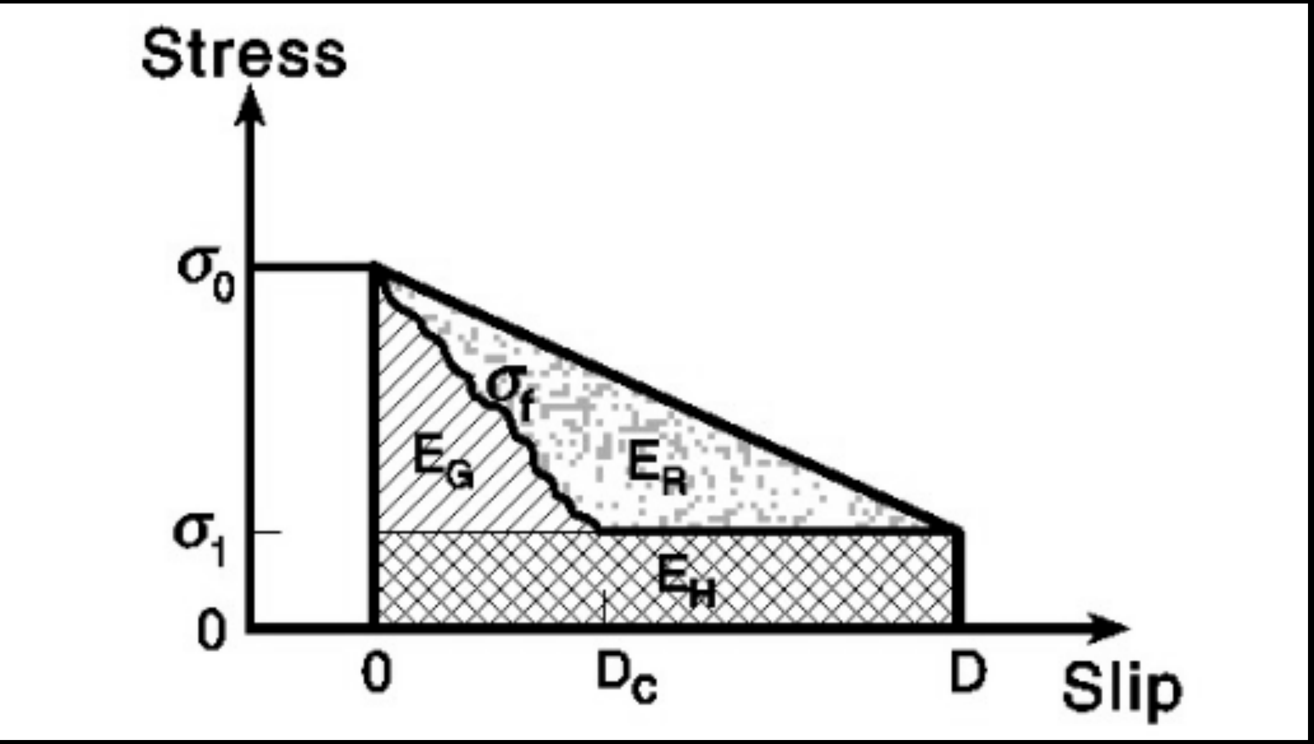
Common misconception

— Some people might think that most of energy generated by an earthquake is released in the form of seismic wave

Energy budget of earthquakes

- Earthquake is viewed as a stress drop process on a fault surface
- Considerable portion of total energy is converted into heat

EG: Fracture energy
ER: Radiated energy
EH: Thermal energy



(Kanamori & Brodsky, 2004)

How does fault slip generate heat?

- Assume that the heat is uniformly distributed within a layer with thickness(w)
- Temperature increase can be calculated by the balancing the work against friction and conductive heat loss to the ambient rocks
- With low thermal conductivity, short time scale, high temperature can be generated.

Equivalent to work against friction

Frictional stress

$$\Delta T = \frac{Q}{C\rho S w} = \frac{\sigma_{\mathrm{f}} D}{C\rho w}$$

Volumetric specific heat

D: displacement of fault slip
w: thickness of fault zone
S: fault area

(Cardwell et al., 1978; Fialko, 2004)

Formulation of Problem

- Basic assumptions:

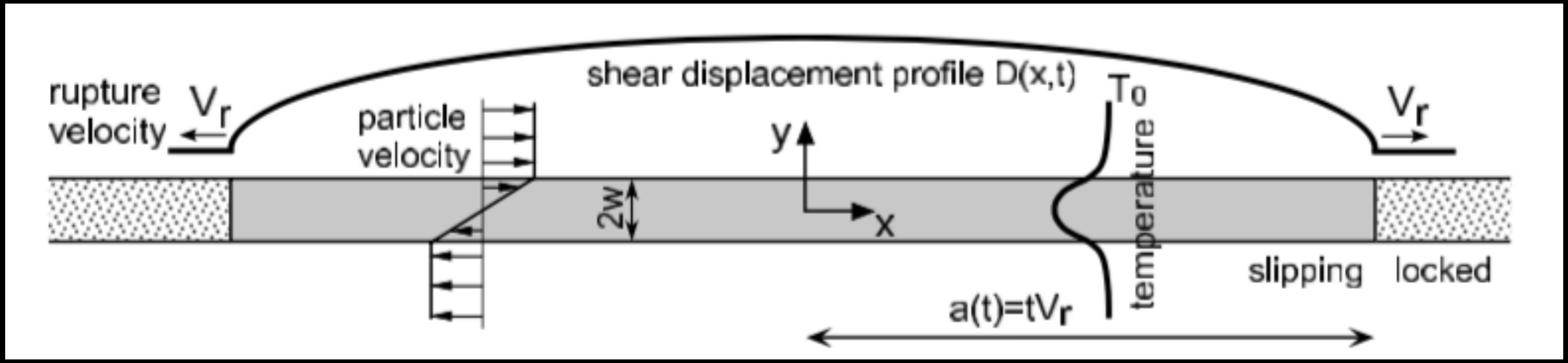
(1) Thickness of fault zone is much smaller than other dimensions of the fault.

(2) Temperature variation across the fault is much higher than temperature variation along the fault.

(3) Heat transfer only occurs in y-direction (1D heat transfer problem)

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho}$$

unsteady-state heat conduction equation with internal heat generation



(Fialko, 2004)

Derivation of temperature profile

Method 1. One could solve the PDE directly $$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \frac{Q}{c\rho}$$

Method 2. A faster way to solve this PDE is to take advantage of the Green's function, which represents point-load response for certain type of source.

The analytical solution of Green's function can be derived by solving

$$\frac{\partial T}{\partial t} = \kappa \frac{\partial^2 T}{\partial y^2} + \delta(y)\delta(t)$$

For the heat generated from fault slip, one could use the Green's function for an infinite plane heat source at y=0 and t=0 is

$$\frac{1}{2\sqrt{\pi \kappa t}} \exp\left(\frac{-y^2}{4\kappa t}\right)$$

(Carslaw and Jaeger, 1959)

Derivation of temperature profile- cont.

The temperature profile T(y,t) will turn out to be the convolution of the Green's function and the source term (Q(y,t)).

Given that 2D convolution take the form

$$f(x,y) * g(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x',y')g(x-x',y-y')dx'dy'$$

We let f(x,y)=Q(y,t), and g(x,y)=Green's function.
The convolution becomes

$$T(y,t) = T_0 + \frac{1}{2\rho c\sqrt{\pi\kappa}} \int_0^t \int_{-\infty}^{\infty} \exp\left[\left(\frac{-(y-y')^2}{4\kappa(t-t')}\right)\right] \frac{Q(y',t')}{(t-t')^{1/2}} dy'dt'$$

(Morse and Feshbach, 1953; Cardwell et al., 1978; Fialko, 2004)

Derivation of temperature profile- cont.

Now, specifying the heat generator Q(y, t)

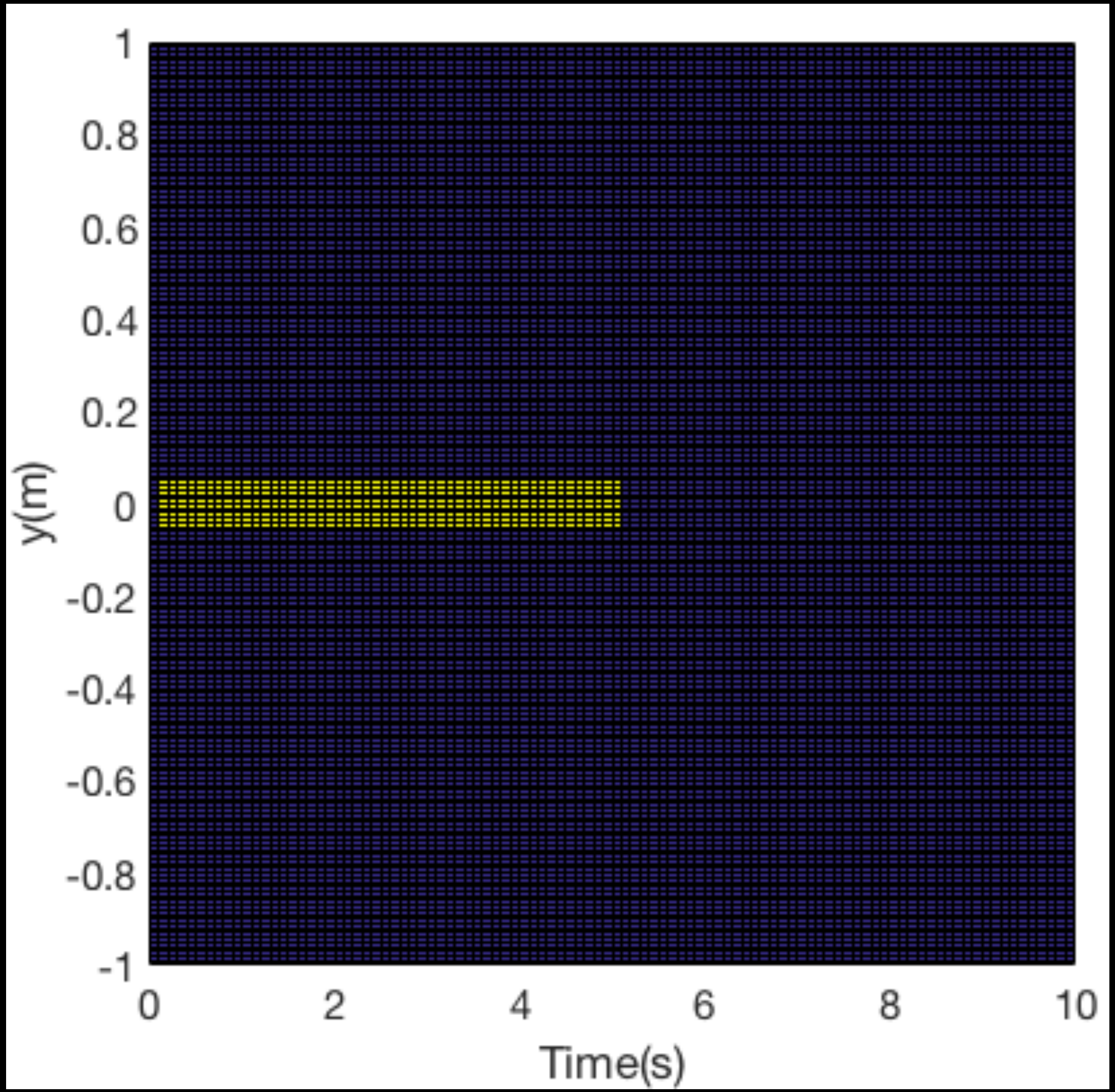
Assuming a layer with homogeneous heat distribution

$$Q(y,t) = \begin{cases} \frac{\sigma_f D}{w\tau}\left[H\left(y+\frac{w}{2}\right) - H\left(y-\frac{w}{2}\right)\right], 0 < t < \tau \\ 0, t < 0 \; or \; t > \tau \end{cases}$$

Rupture duration

(Cardwell et al., 1978)

Substituting this heat source expression back to the convolution….



Derivation of temperature profile- cont.

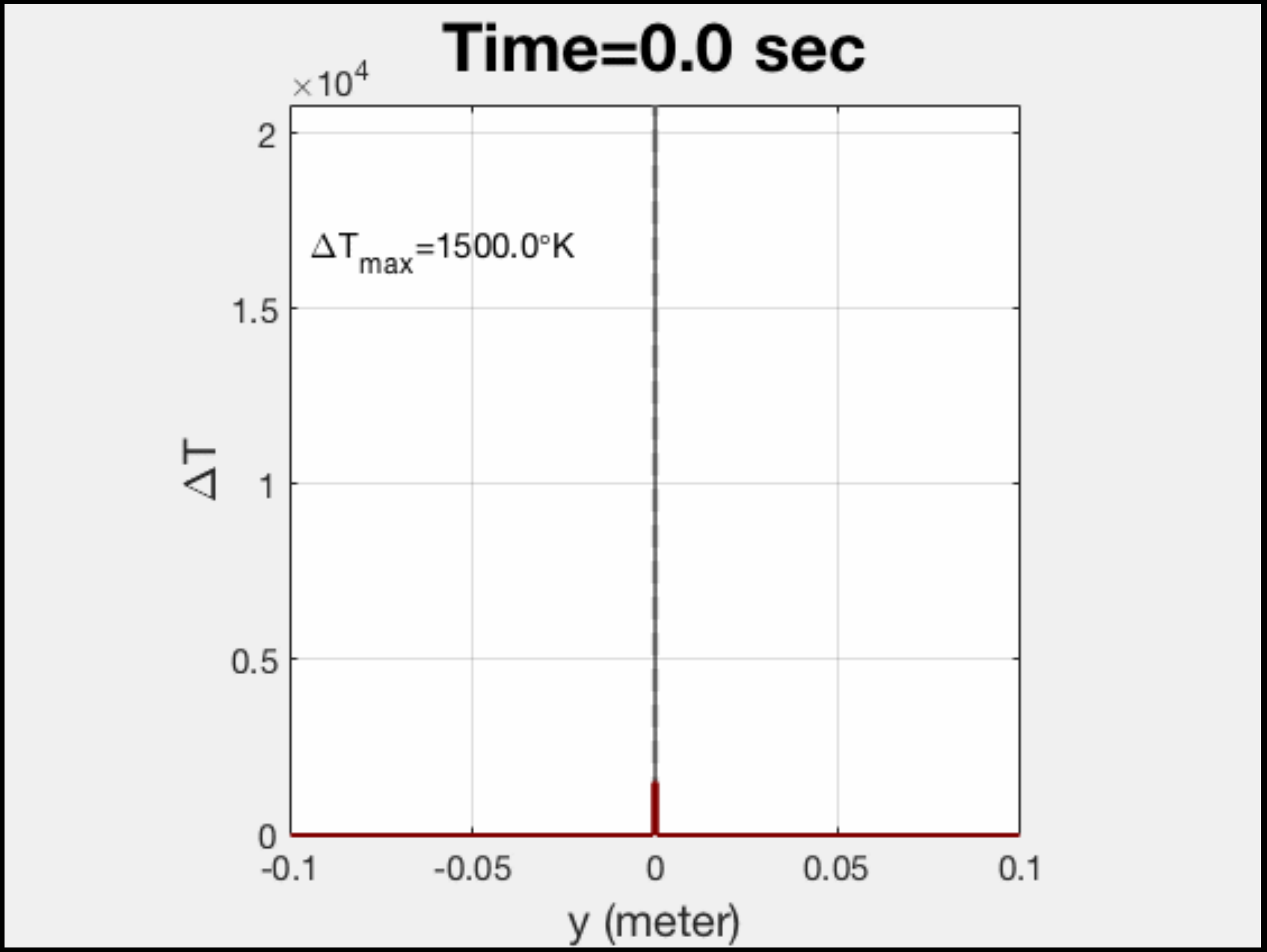
One could derive temperature as a function of y and time

This integration must be evaluated numerically.

$$T(y,t) = \begin{cases} T_0 + \dfrac{\sigma_f D}{2\rho c w \tau}\displaystyle\int_0^{t} \left\{ \mathrm{erf}\left[\dfrac{y + \left(^{w}/_{2}\right)}{\left(4\kappa(t - t')\right)^{1/2}}\right] - \mathrm{erf}\left[\dfrac{y - \left(^{w}/_{2}\right)}{\left(4\kappa(t - t')\right)^{1/2}}\right]\right\} dt', for\ 0 < t < \tau \\ T_0 + \dfrac{\sigma_f D}{2\rho c w \tau}\displaystyle\int_0^{\tau} \left\{ \mathrm{erf}\left[\dfrac{y + \left(^{w}/_{2}\right)}{\left(4\kappa(t - t')\right)^{1/2}}\right] - \mathrm{erf}\left[\dfrac{y - \left(^{w}/_{2}\right)}{\left(4\kappa(t - t')\right)^{1/2}}\right]\right\} dt', for\ t > \tau \end{cases}$$

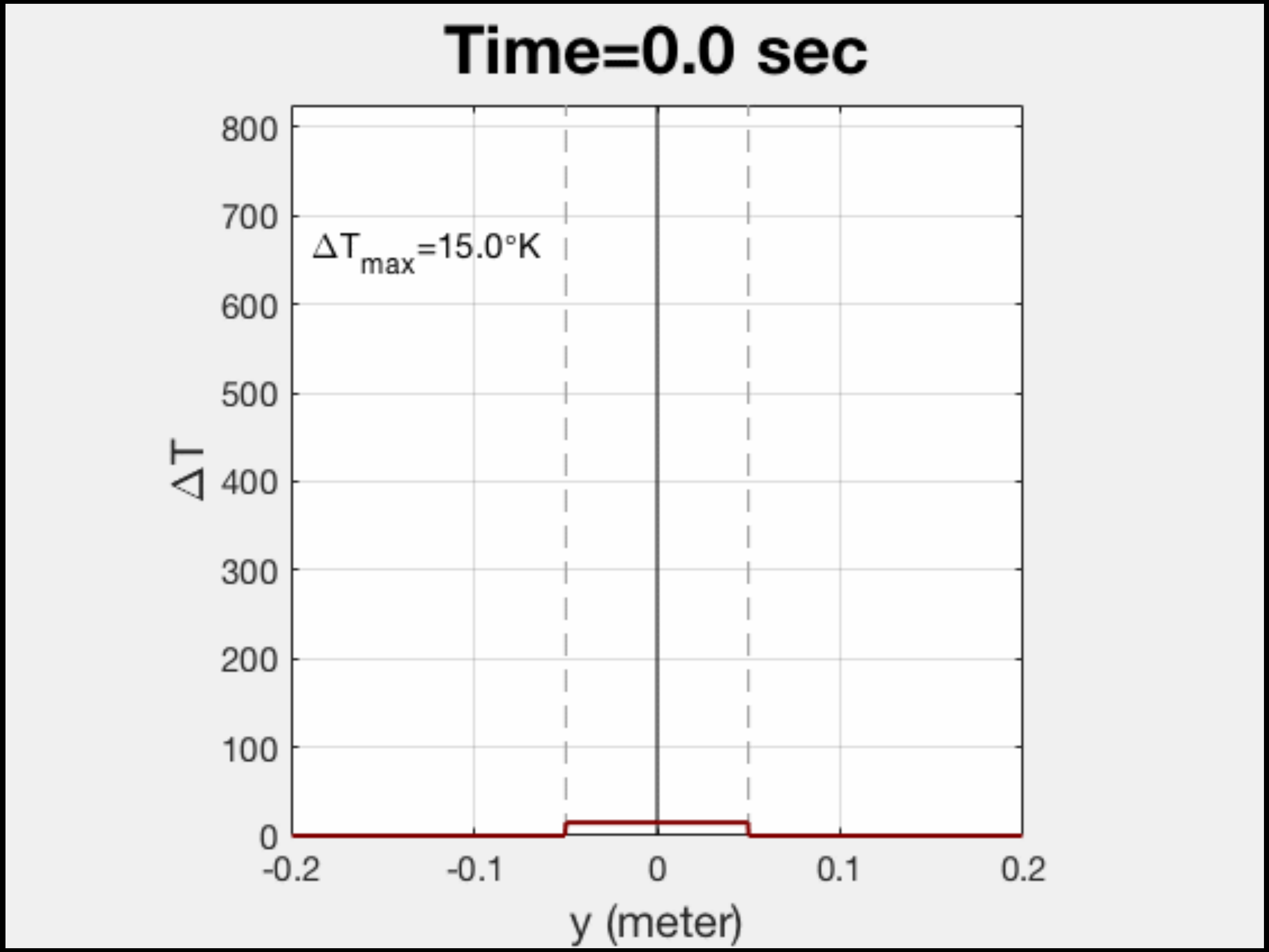
Case 1 - Thin fault

- Thermal diff=1E-6 [m2*s-1]
- Thermal cond=4 [W*m-1*K-1]
- Rupture duration=5s
- Averaged slip (D)=5m
- Frictional stress=60MPa
- Width of fault zone (w)=1mm=**0.001m**



Case 2 - Thick fault

- Thermal diff=1E-6 [m2*s-1]
- Thermal cond=4 [W*m-1*K-1]
- Rupture duration=5s
- Averaged slip (D)=5m
- Frictional stress=60MPa
- Width of fault zone (w)=10cm=**0.1m**

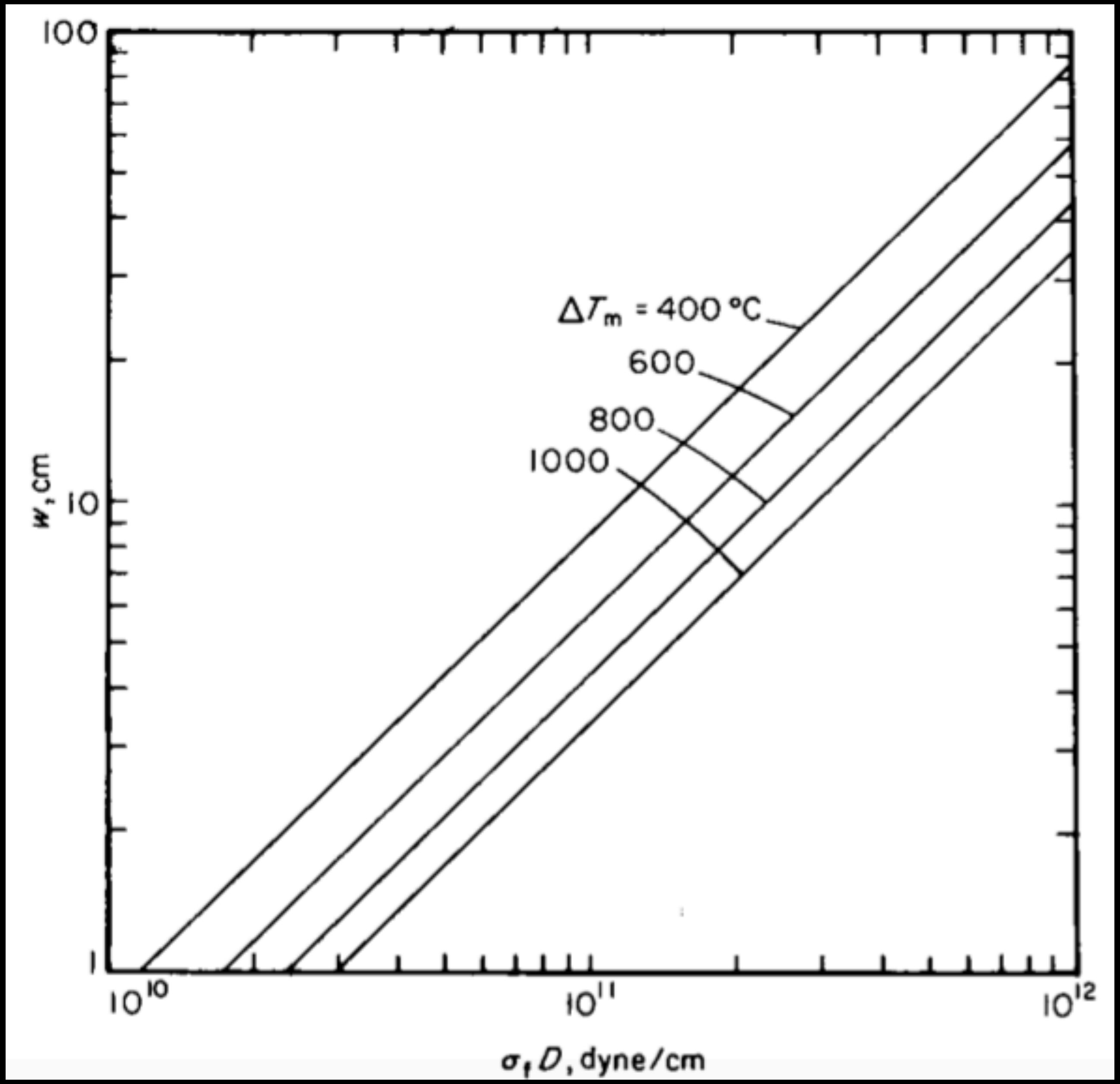


Maximum temperature increase

- From the derived solution, one could also estimate the maximum temperature increase at y=0

$$\Delta T = \frac{\sigma_f D}{\rho c w} \text{ --> } w = \frac{\sigma_f D}{\rho c \Delta T}$$

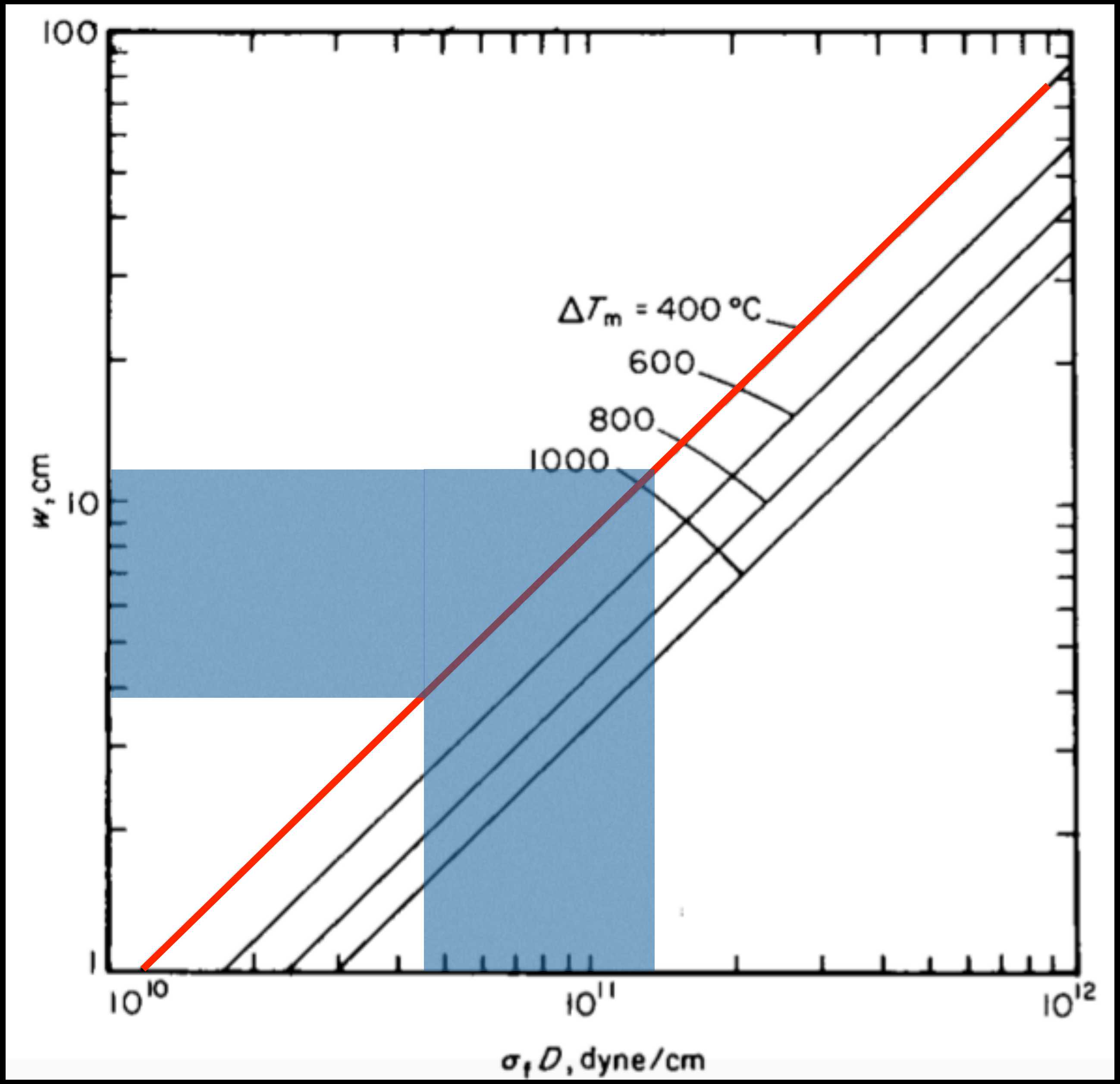
- Normally, averaged slip on the fault (D) increases with earthquake magnitude Mw



(Cardwell et al., 1978)

Partial melting zone

- It is reasonable to assume that ambient rock temperature before faulting is about 400 deg C
- Rock experiment suggests that melting of glassy rocks should should have taken place at about 800 deg C (Wallace, 1976)
- The result shows that the partial melting should be expected on faults zones 1-10 cm wide during moderately large earthquakes.



(Cardwell et al., 1978)

Reference

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