Influence of Seasonal Temperature Variations on Surface Temperature of a glacier

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Contents

Introduction

- The top 15~20m of a glacier are subject to seasonal variations of temperature.
- Heat flow (heat diffusivity, conduction) is dominant in the top layer.
- Similar to the ‘classical’ Wine cellar problem.

<u>Objective</u>

- Derive the formula of a temperature profile and changes with time *T(z,t)*.
- Discuss the seasonal variations in heat flow

Derivation

Starting from the Fourier law of heat diffusion

$$\frac{\partial T}{\partial t} = \alpha_T \frac{\partial^2 T}{\partial z^2}$$

where $\alpha_T = \frac{k_t}{\rho c}$ is the thermal diffusivity

BC: a cyclic variation of temperature at the surface (z=0)

$$T(0,t) = \Delta T \sin(2\pi f t)$$

Fourier transform in time domain

$$\frac{\partial T}{\partial t} = \alpha_T \frac{\partial^2 T}{\partial z^2}$$

Left: $$F\left[\frac{\partial T}{\partial t}\right] = (2\pi i f)\tilde{T}(z,f)$$ *(derivative property of FT)*

Right: $$F\left[\alpha_T \frac{\partial^2 T}{\partial z^2}\right] = \alpha_T \frac{\partial^2}{\partial z^2}\tilde{T}(z,f)$$ *(linearity of derivation and integration operator)*

Rewriting the PDE

$$(2\pi i f)\tilde{T}(z,f) = \alpha_T \frac{\partial^2}{\partial z^2}\tilde{T}(z,f)$$

Re-order the ODE

$$\frac{\partial^2}{\partial z^2}\tilde{T}(z,f) - \frac{(2\pi i f)}{\alpha_T}\tilde{T}(z,f) = 0$$

A general solution of the 2nd order ODC is

$$\tilde{T}(z,f) = C_1 \exp(ikz) + C_2 \exp(-ikz)$$

where

$$k = \sqrt{\frac{2\pi i f}{\alpha_T}} = A(1+i)$$

Attenuation depth ~
decay rate in terms of depth

$$A = \sqrt{\frac{\pi f}{\alpha_T}}$$

KEY: $\sqrt{i} = \dfrac{1+i}{\sqrt{2}}$

Back to BC

$$T(0,t) = \Delta T \sin(2\pi f t) \quad \Rightarrow \quad \tilde{T}(0,f) = \frac{\Delta T}{2i}\left[\delta(f - f_0) - \delta(f + f_0)\right]$$

$$T(\infty,t) = 0 \quad \Rightarrow \quad \tilde{T}(\infty,f) = 0$$

$$C_2 = \frac{\Delta T}{2i}\left[\delta(f - f_0) - \delta(f + f_0)\right]$$

Now, the full solution can be written as

$$\tilde{T}(z,f) = \frac{\Delta T}{2i}\left[\delta(f - f_0) - \delta(f + f_0)\right]\exp[-A(1+i)z]$$

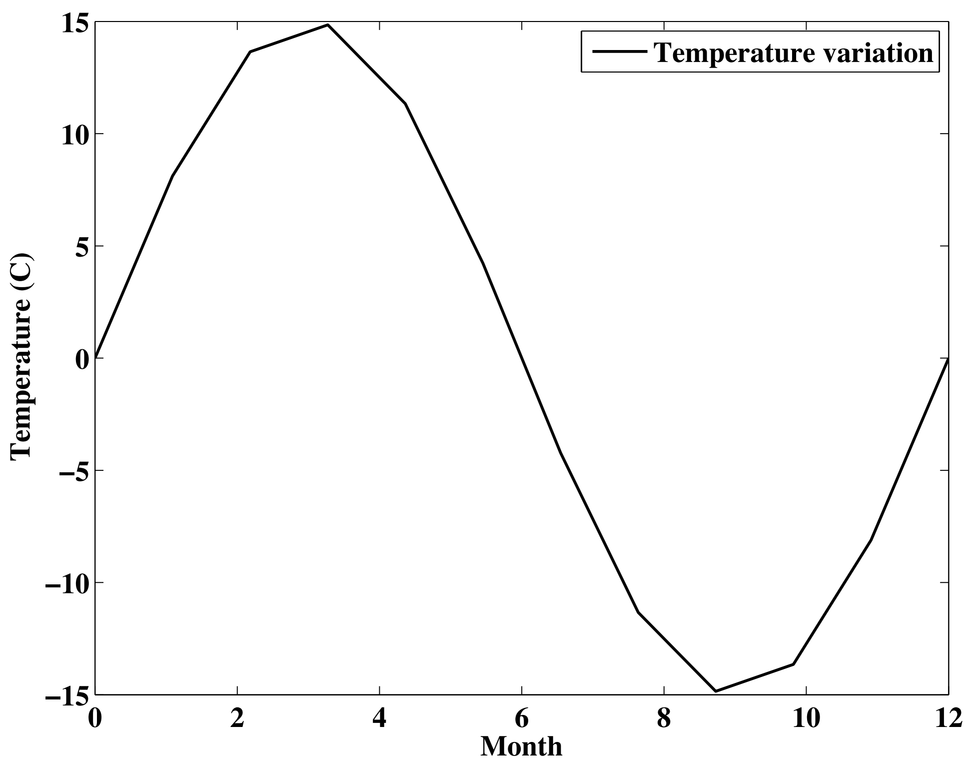
Inverse FT in frequency domain

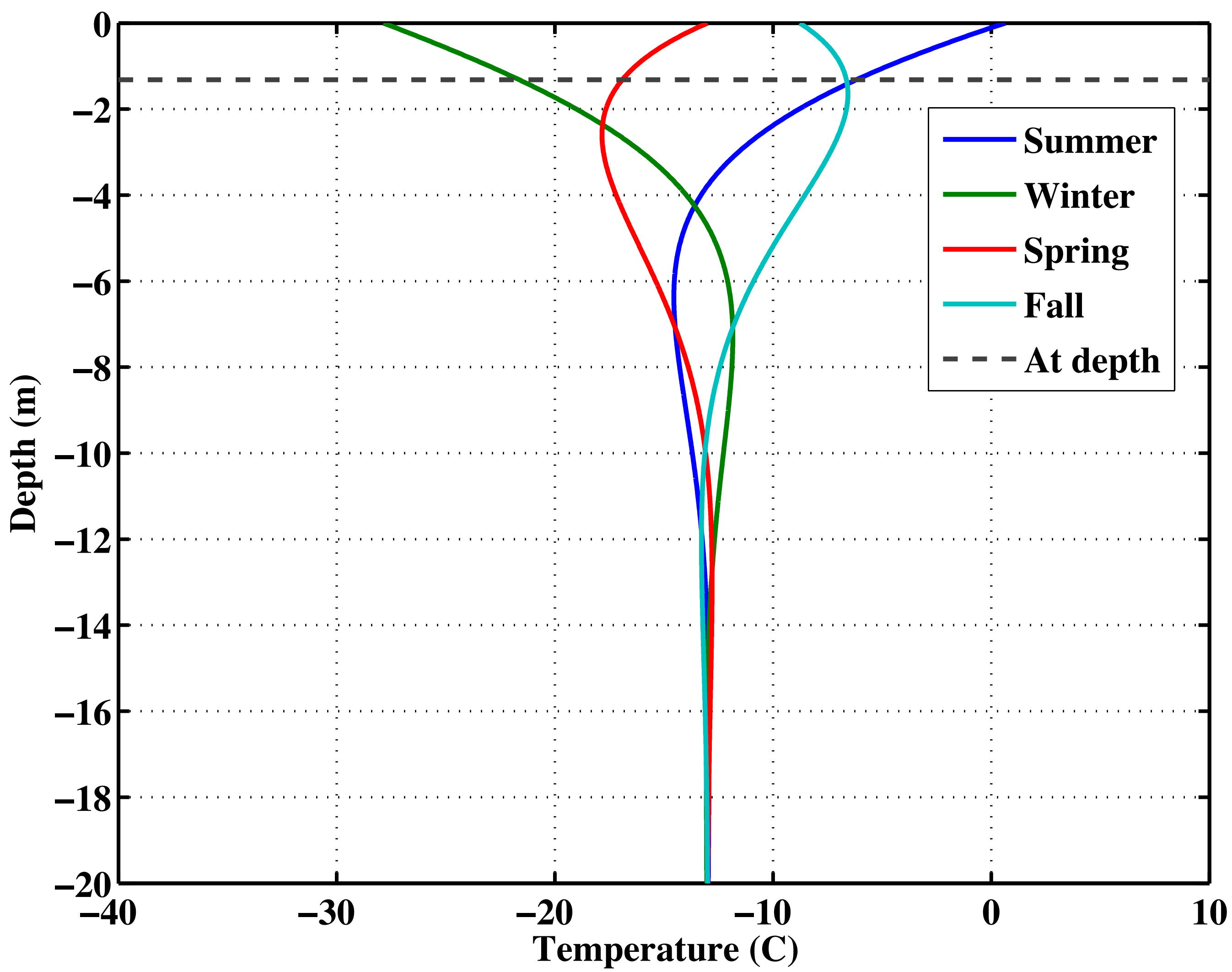
$$T(z,f) = \int_{-\infty}^{\infty} \frac{\Delta T}{2i}\left[\delta(f - f_0) - \delta(f + f_0)\right]\exp[-A(1+i)z]\exp(2\pi i f t)df$$

Result #1

$$T(z,f) = \Delta T \exp(-zA)\sin(2\pi f t - zA)$$

Thermal diffusivity = $1.09 \times 10^{-6} m^2 s^{-1}$
Perturb amp = 30 C



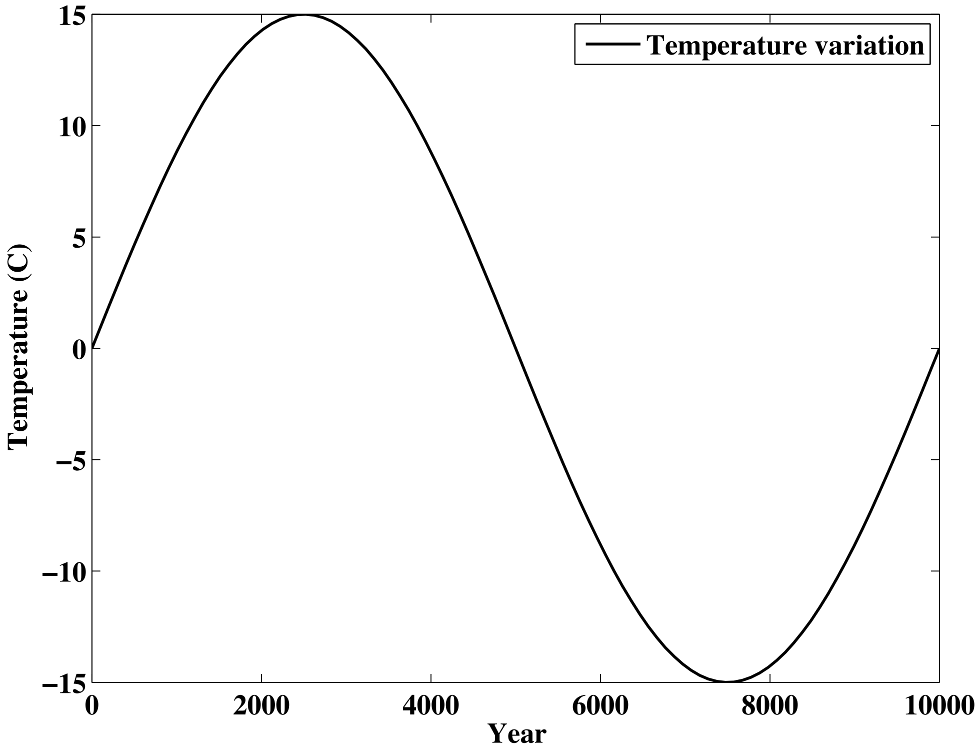


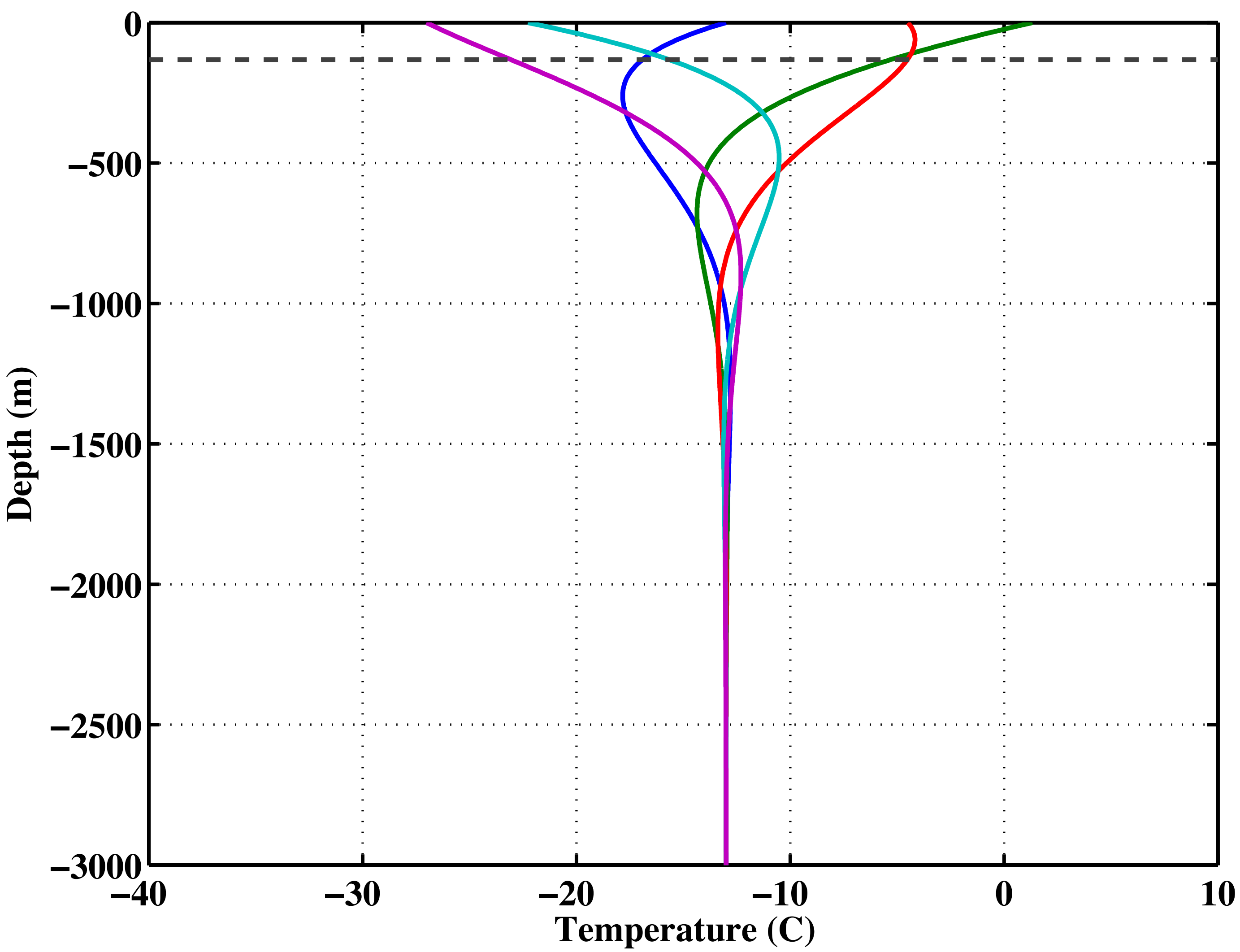
Result #2

$$T(z,f) = \Delta T \exp(-zA)\sin(2\pi ft - zA)$$

Thermal diffusivity = $1.09 \times 10^{-6} m^2 s^{-1}$
Perturb amp = 30 C

Boundary condition
10,000-year period





Conclusion

- The amplitude of the temp variation decrease with the exponential term, varies with depth
- the higher the freq, the more rapid attenuation with depth.
- The maximum heat flux is 1/8 of the period ahead of the maximum temperature.
- Long term temperature changes (~10,000 yrs) penetrates much deeper into the ice.
- Limitations
    - Surface layer is not homogeneous (conductivity and diffusivity spatially varies
    - The surface boundary is not a perfect sine function.
    - The ice is moving and thus, head advection must be considered
    - Melt water will have a great impact