Heat Flow across the San Andreas Fault

Erik Anderson & Brendan Cych

Friction Along Faults Generates Heat

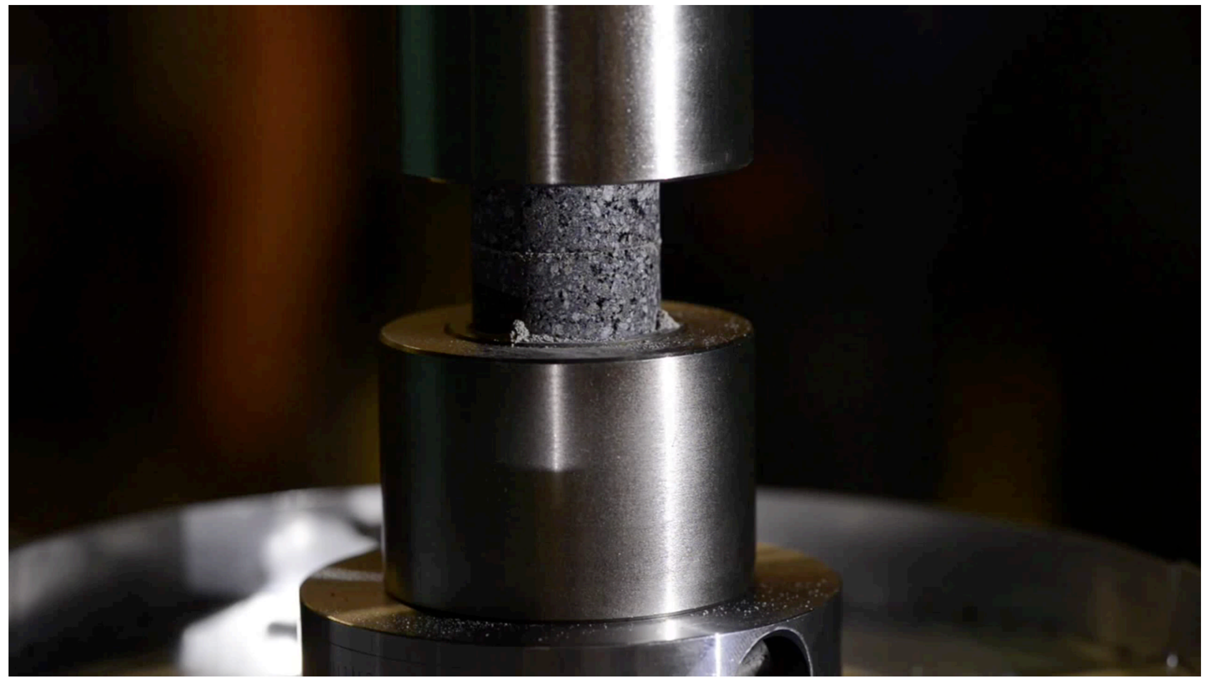
- Energy from slip partitioned into:

  1) Fracture creation

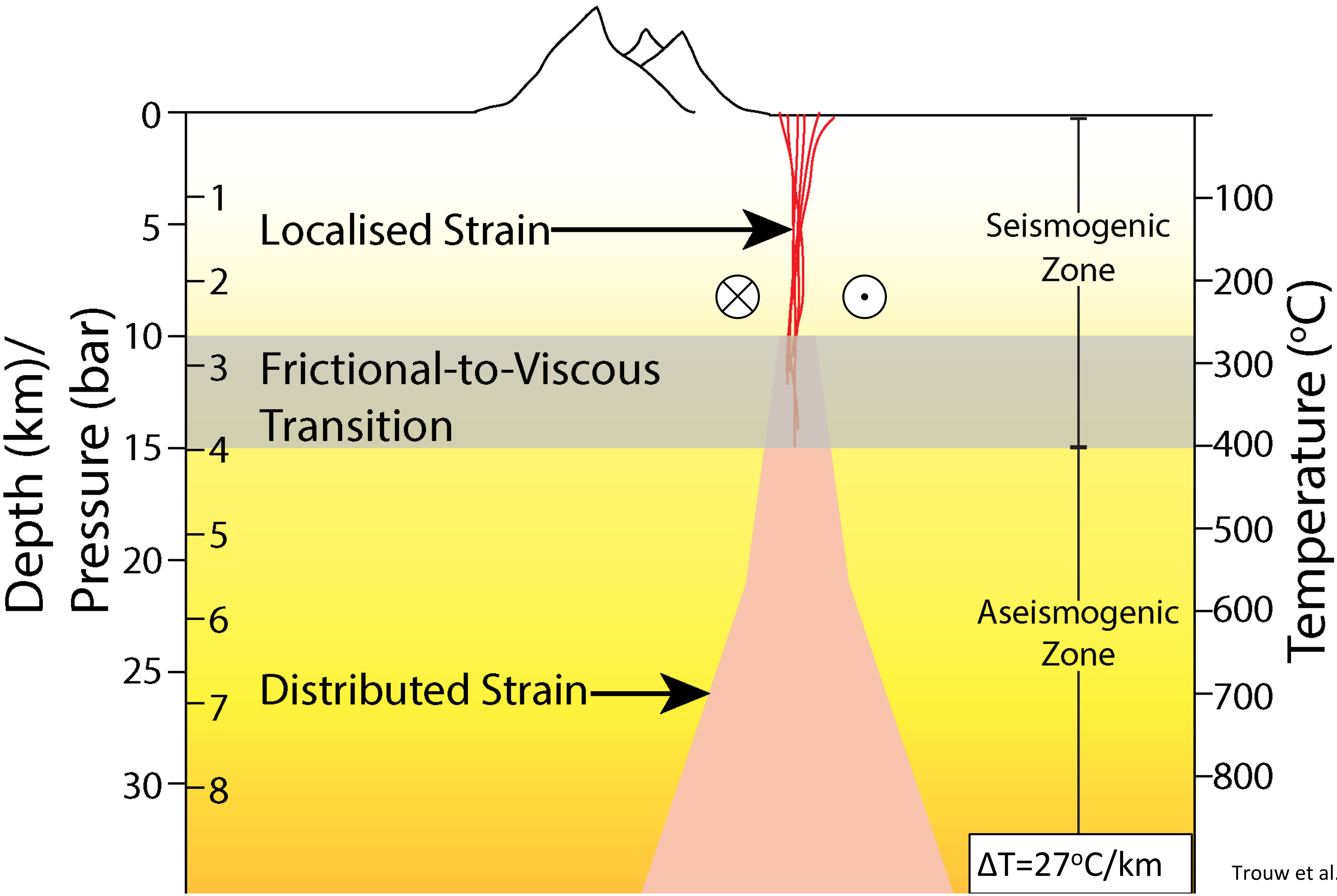
  2) Seismic radiation

  3) Thermal energy

- What component plays the largest role?



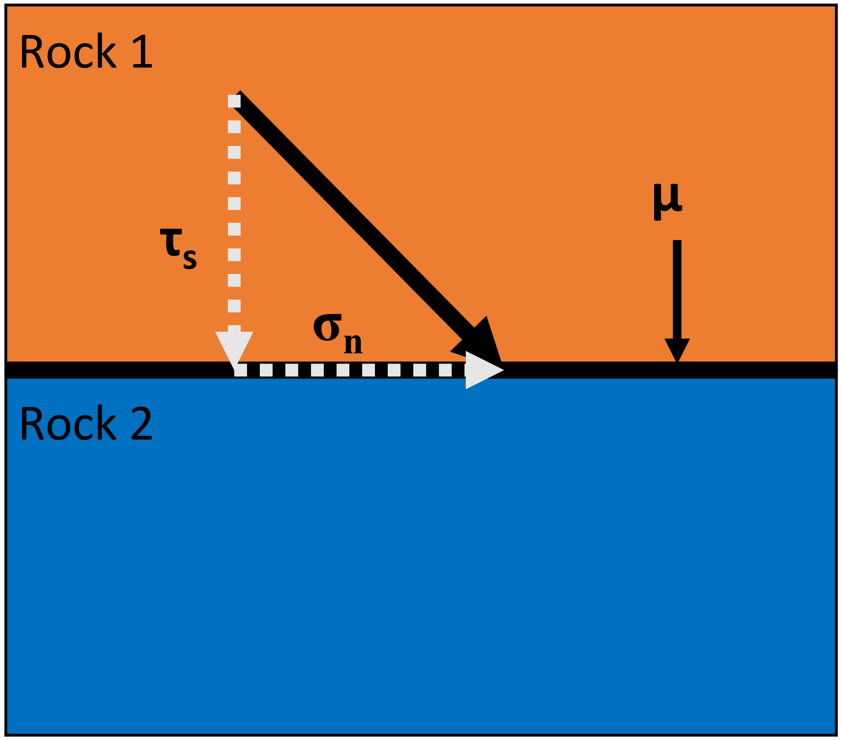
Earthquakes Occur Along Faults in the Upper Crust



Trouw et al., 2010

Radiated Seismic Energy Measurements Underestimate Theory

- $\tau_s = \mu \rho_c z$
  - μ ~0.60
  - ρ_c - 2600 kg/m^3
  - Z – depth (m)
- Average stress drop determined by integrating across depth of seismogenic zone (~12km)
- $\bar{\tau}_s = \frac{1}{D}\int_0^D \mu \rho_c g z$
- $\bar{\tau}_s = \frac{1}{2}\mu \rho_c g D = 92MPa$
- Seismicity observes stress drops of 0.1-10MPa



Setting up the equation and boundary conditions

- Need to consider a line source at z=a to represent heating
- Initial formula for heat flow given by

$$\nabla^2 T = \frac{1}{k} Q(x,z) = \frac{1}{k}\delta(x)\delta(z+a)$$

- Boundary conditions

$$T(x,0) = 0 \qquad \lim_{|z|\to\infty} T(x,z) = 0$$

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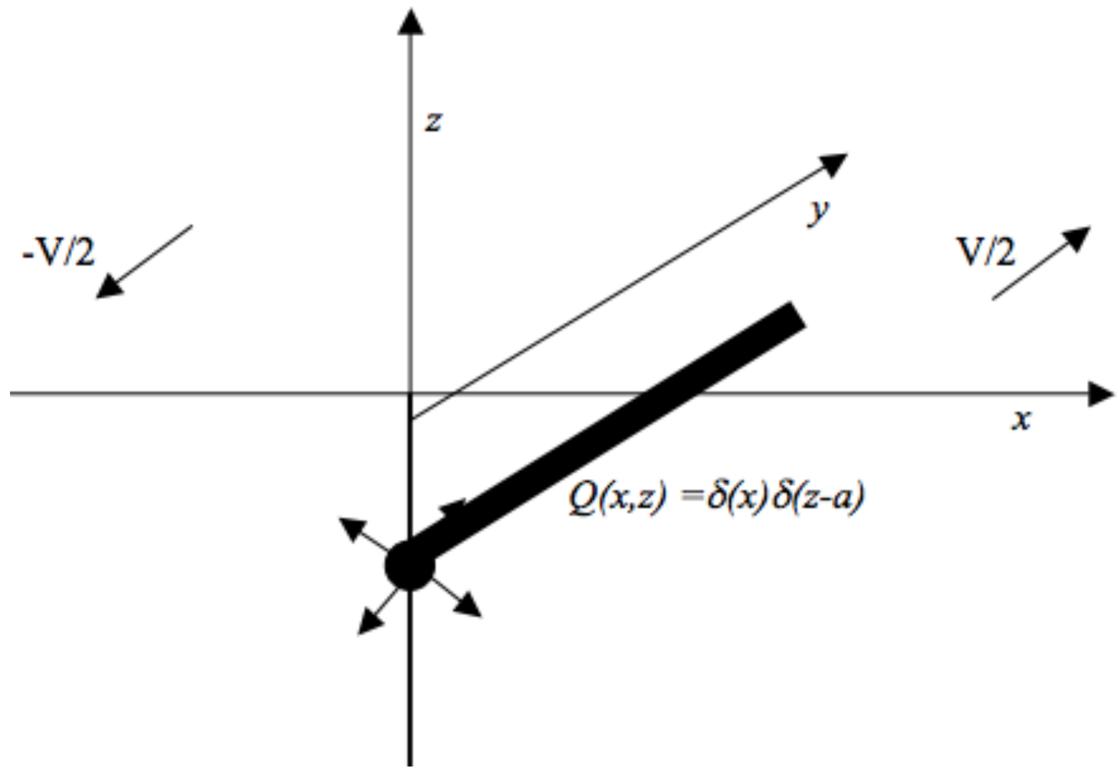
- We will need a heat sink to satisfy these, placed at z=-a



Solving the differential equation for dT/dz

$$\nabla^2 T = \frac{1}{k} Q(x,z) = \frac{1}{k}\delta(x)\delta(z+a)$$

- We already solved our differential equation for heat flow in class. (Notes on Fourier transforms).
- Take the Fourier transform of both sides, using the derivative property on the LHS and definition of the delta function on RHS
- Take the inverse transform in the Z direction, using the Cauchy residue theorem to make this easier.
- Take the inverse transform in the x direction, using the derivative property with respect to z to get dT/dz

Solving the differential equation for dT/dz

- Note that our solution is different to in the notes, as we have a conduction term

$$\frac{\partial T(x,z)}{\partial Z} = -\frac{1}{2\pi k}\frac{z+a}{(x^2+(z+a)^2)}$$

- We need to add the line sink to the equation so we obtain

$$\frac{\partial T(x,z)}{\partial Z} = -\frac{1}{2\pi k}\left\{\frac{z+a}{(x^2+(z+a)^2)} - \frac{z-a}{(x^2+(z-a)^2)}\right\}$$

Finding the surface heat conduction using Fourier's Law

- As we have included conduction in our equation, we can solve for the surface heat flow using Fourier's law of thermal conduction

$$q = -k\frac{dT}{dz}$$

$$q(x,z) = \frac{1}{2\pi}\left\{\frac{z+a}{(x^2+(z+a)^2)} - \frac{z-a}{(x^2+(z-a)^2)}\right\}$$

Obtaining the Green's Function

- For our surface heat flow, we solve for z=0

$$q(x,0) = \frac{1}{\pi}\left\{\frac{a}{x^2 + a^2}\right\}$$

This gives us a Green's function which we can then convolve with an arbitrary source.

$$G = \frac{1}{\pi}\left\{\frac{z}{x^2 + z^2}\right\}$$

Convolving with the Green's Function

- Our heat source is not a line source at depth, it's a plane, where the heat flow at depth z is given by

$$q(z) = u\tau(z)$$

- We can then convolve this with our Green's Function to get our surface heat flow for this source.

$$q(x) = \frac{u}{\pi} \int_0^D \left\{ \frac{z\,\tau(z)}{x^2 + z^2} \right\} dz$$

Convolving with the Green's Function

- We can use our initial formula for the normal stress, given by $\tau(z) = \mu\rho_c g z$
- We can then plug this into our equation for the surface heat flow.

$$q(x) = \frac{\mu\rho_c g u}{\pi} \int_0^D \frac{z^2}{x^2 + z^2} dz$$

Solving this integral

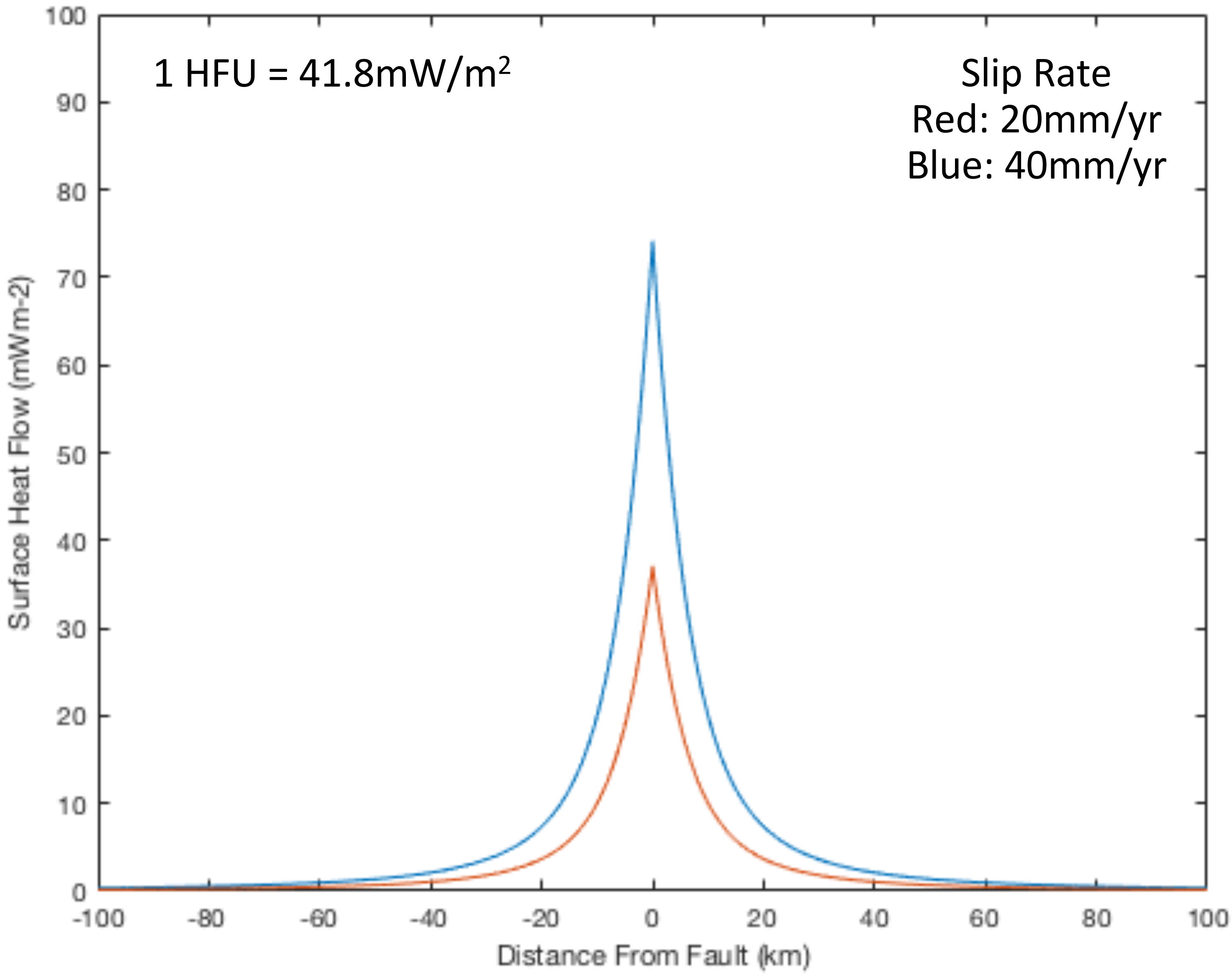
We can rearrange the equation in the integral

$$\frac{z^2}{x^2+z^2} = \frac{z^2+x^2-x^2}{x^2+z^2}$$

$$= 1 - \frac{x^2}{x^2+z^2}$$

$$q(x) = \frac{\mu \rho_c g u}{\pi} \int_0^D 1 - \frac{x^2}{x^2+z^2} \, dz$$

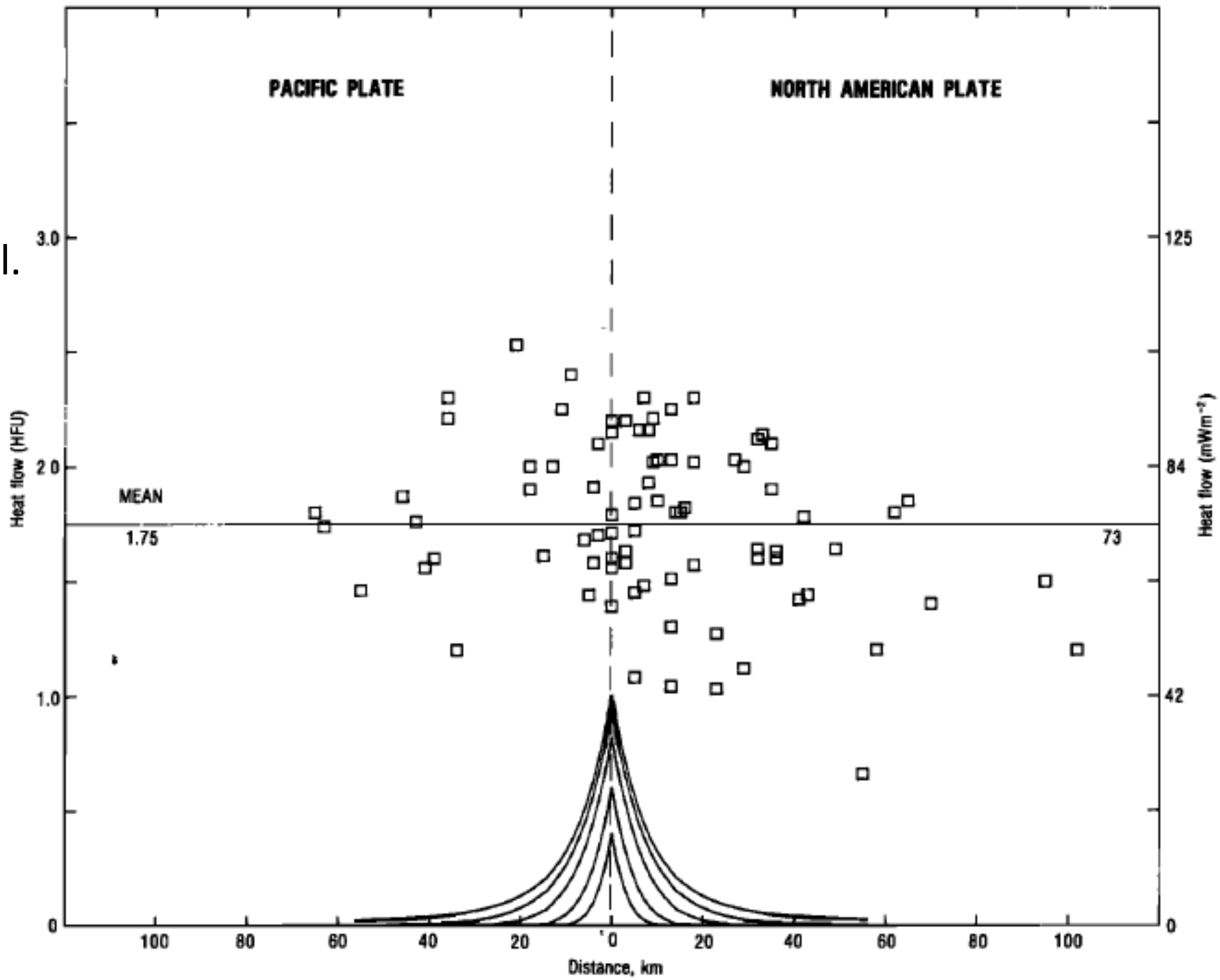
$$= \frac{\mu \rho_c g u}{\pi} \left\{ D - x \tan^{-1} \frac{D}{x} \right\}$$



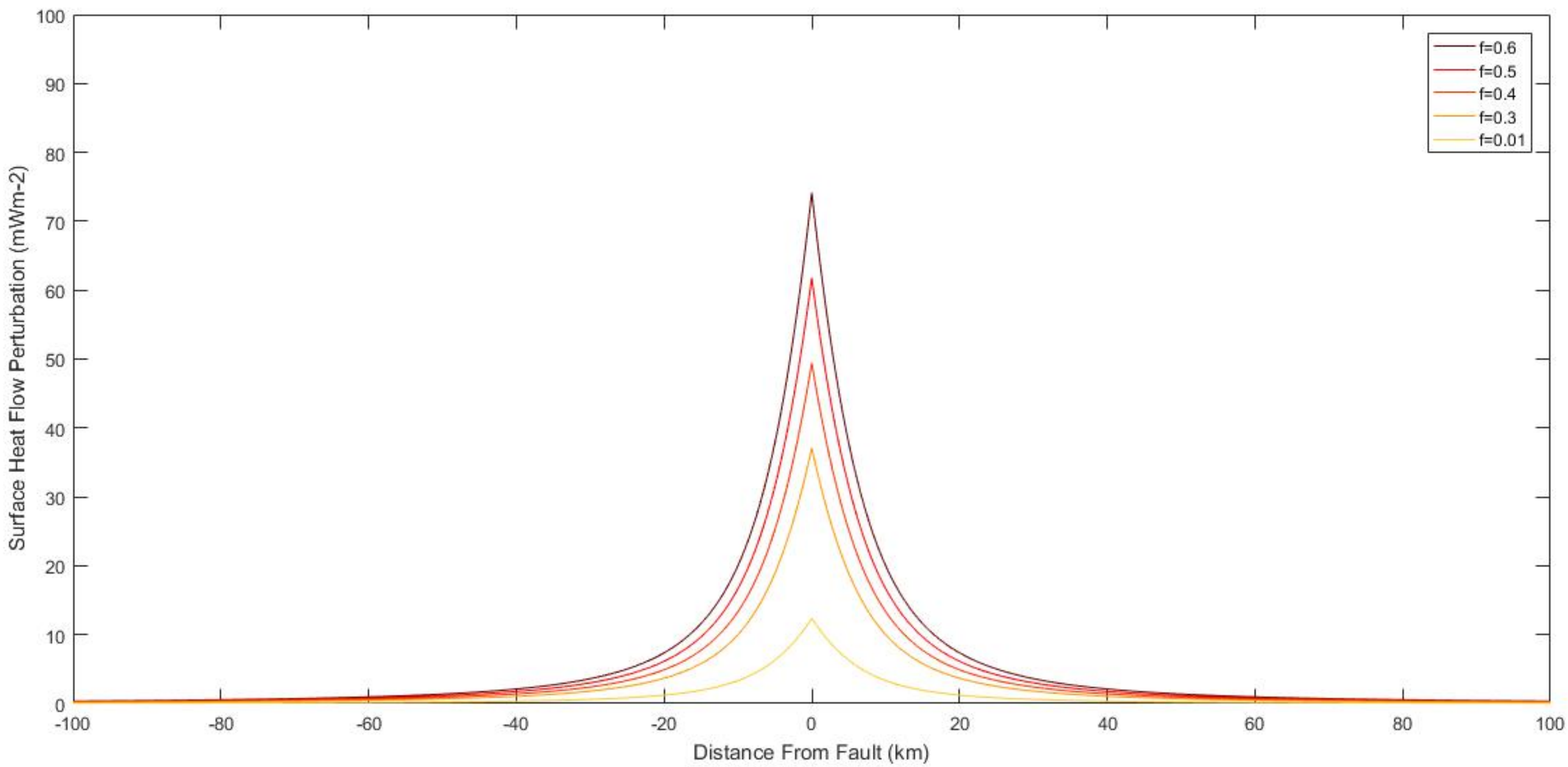
1 HFU = 41.8mW/m2
Slip Rate
Red: 20mm/yr
Blue: 40mm/yr
Surface Heat Flow (mWm-2)
Distance From Fault (km)
0
10
20
30
40
50
60
70
80
90
100
-100
-80
-60
-40
-20
0
20
40
60
80
100

Comparing Model with Surface Observations of Heat Flow

- Lachenbruch and Sass (1980) observe no perturbed surface heat flow measurements predicted by their model.



Changing Frictional Coefficient Cannot Produce Heat Flow Anomaly



Accounting for Hydrothermal Circulation

$$q(x) = \frac{\mu(\rho_c - \rho_w)gu}{\pi} \int_d^D 1 - \frac{x^2}{x^2 + z^2} dz$$

$$= \frac{\mu(\rho_c - \rho_w)gu}{\pi} \left\{(D - d) + (x \tan^{-1}\frac{d}{x} - x \tan^{-1}\frac{D}{x})\right\}$$

Hydrothermal Circulation Broadens Heat Anomaly Profile