
Ice Shelf Flexure
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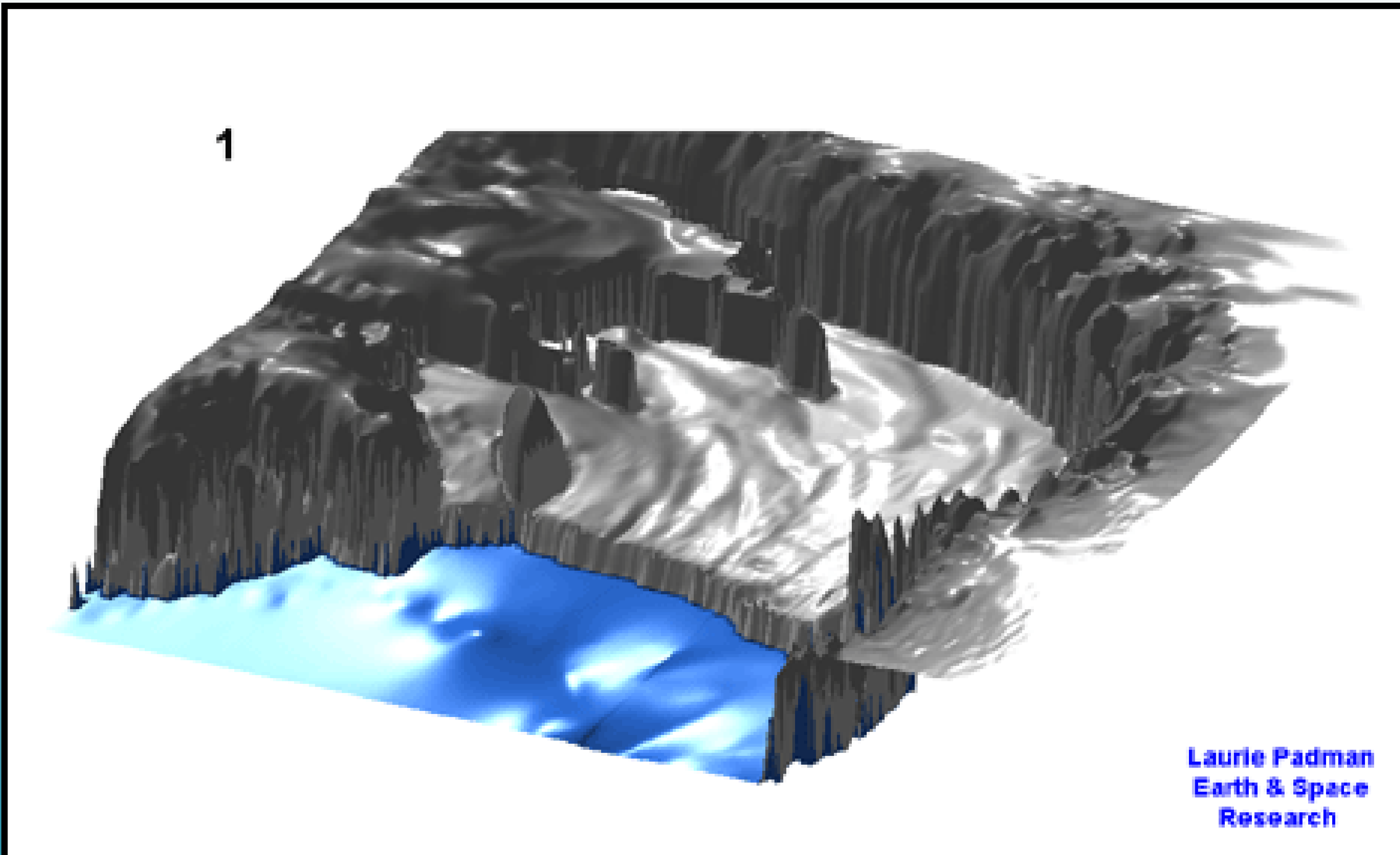
Ice Shelf Hinge Zone

- Grounded ice sheets are supported by a bed and are steady
- Floating ice shelves are in constant movement due to water movement
- Between them is the ice shelf hinge zone, which is supported by:
  – Hydrostatic pressure from sea below
  – Internal stresses

•Experiences cyclic flexure due to tidal cycle

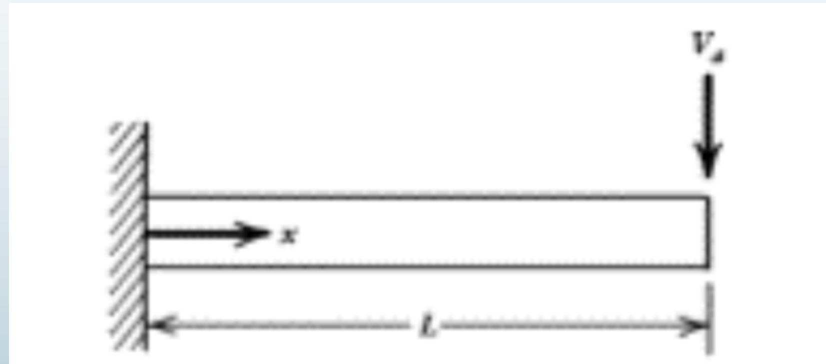
•A large amount of power from the global tidal system is dissipated here

Ice Shelf Flexure



Derivation

- Elastic beam



- D-Flexural Rigidity

$$D \equiv \frac{Eh^3}{12(1-\nu^2)}.$$

- Start with force balance equation

$$\frac{d^2}{dx^2}\left(D(x)\frac{d^2w}{dx^2}\right) + F\frac{d^2w}{dx^2} + \Delta\rho g w = q(x) \qquad (1)$$

flexural resistance + end load + restoring force = vertical load

Parameter	Definition	Value/Unit
$w(x)$	deflection of plate (positive down)	m
$D(x)$	flexural rigidity	N m
h	elastic plate thickness	m
F	end load	N m^{-1}
q	vertical load	N m^{-2}
$\Delta\rho$	density contrast (ρ_m - ρ_w)	
g	acceleration of gravity	9.82 m s^{-2}
E	Young's modulus	6.5 x 10^{10} Pa
ν	Poisson's ratio	0.25

- In our problem, we have no end load so the second term goes away
- Dominant forces during small displacements of a thin elastic beam or sheet are the longitudinal extension and compression that occur above and below a notional neutral surface
- Holdsworth [1969] analysed the problem to show that the problem can be written as follows:
- $D\omega'''' = \rho_{sea} g[A_o(t) - \omega(x)]$

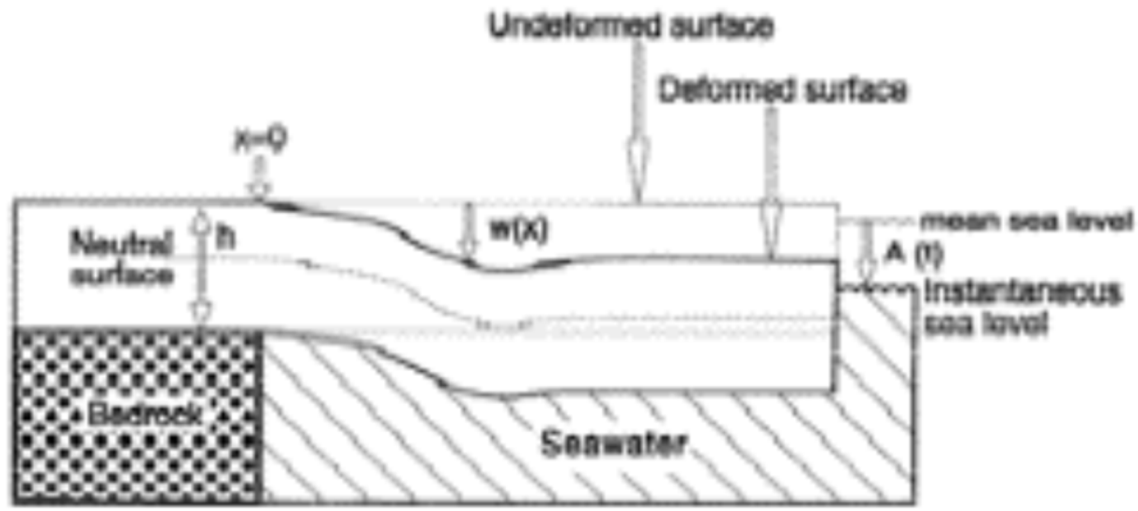


Figure 2. Model of the vertical section through an ice shelf grounding zone. After *Holdsworth* [1969].

- Boundary conditions
- $\omega=0$ $\omega'=0$ at $x=0$
- Free floating conditions beyond hinge zone
- $\omega=A_o(t)$ at $x=$infinity

- Use Greens function and Fourier transform
- No end load so P=0 $q(x)=\rho_{sea}gA_o(t)\,\delta(x)$
- So $Q(k)=\rho_{sea}gA_o(t)$
- We have then from the initial problem:
- $(2\pi k)^4D\,\omega(k)=\rho_{sea}g[A_o(t)-\omega(k)]$
- $\omega(k)=\rho_{sea}gA_o(t)[(2\pi k)^4D+\rho_{sea}g]^{-1}$

- $\omega(k) = \rho_{sea} g A_o(t)[(2\pi k)^4 D + \rho_{sea} g]^{-1}$
- Let $4\alpha^4 = \rho_{sea} g D^{-1}$
- $\omega(k) = \int \dfrac{\rho_{sea} g A_o(t) \exp[i2\pi kx]}{[(2\pi k)^4 D + \rho_{sea} g]}$
-
- $k' = 2\pi k \quad dk = [2\pi]^{-1} dk'$
- $\omega(k) = \dfrac{\rho_{sea} g A_o(t)}{D} \int \dfrac{\exp[ik'x]}{[k'^4 + 4\alpha^4]} dk'$
-

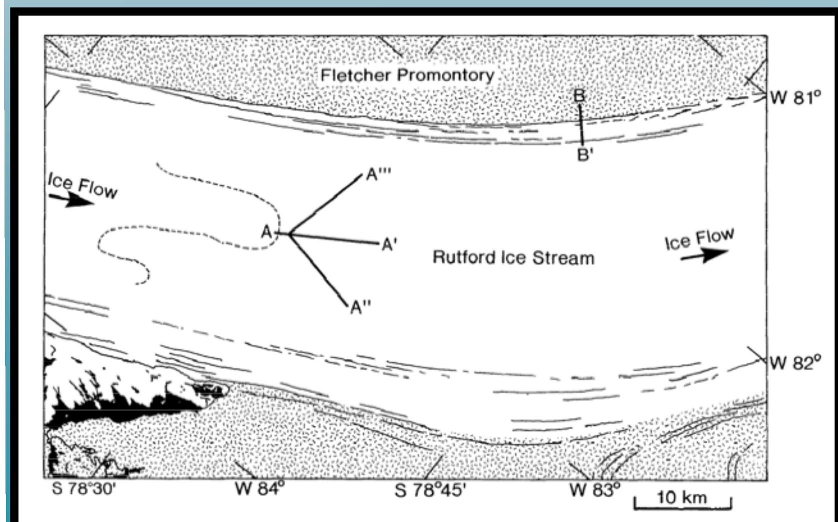
- Use Cauchy's
- $(k^4+4\alpha^4) = (k^2+2i\alpha^2)(k^2-2i\alpha^2) = (k-(1-i)\alpha)\ (k-(-1-i)\alpha)\ (k-(1+i)\alpha)\ (k-(i-1)\alpha)$
- Assume x>0 imk>0
- $\exp(i\theta) = \cos\theta + i\sin\theta$
- Apply boundary conditions
- So $\omega(x) = A_o(t)(1-\exp(-\alpha x)\ [\cos\alpha x + \sin\alpha x])$
- Where $4\alpha^4 = \rho_{sea} g D^{-1}$

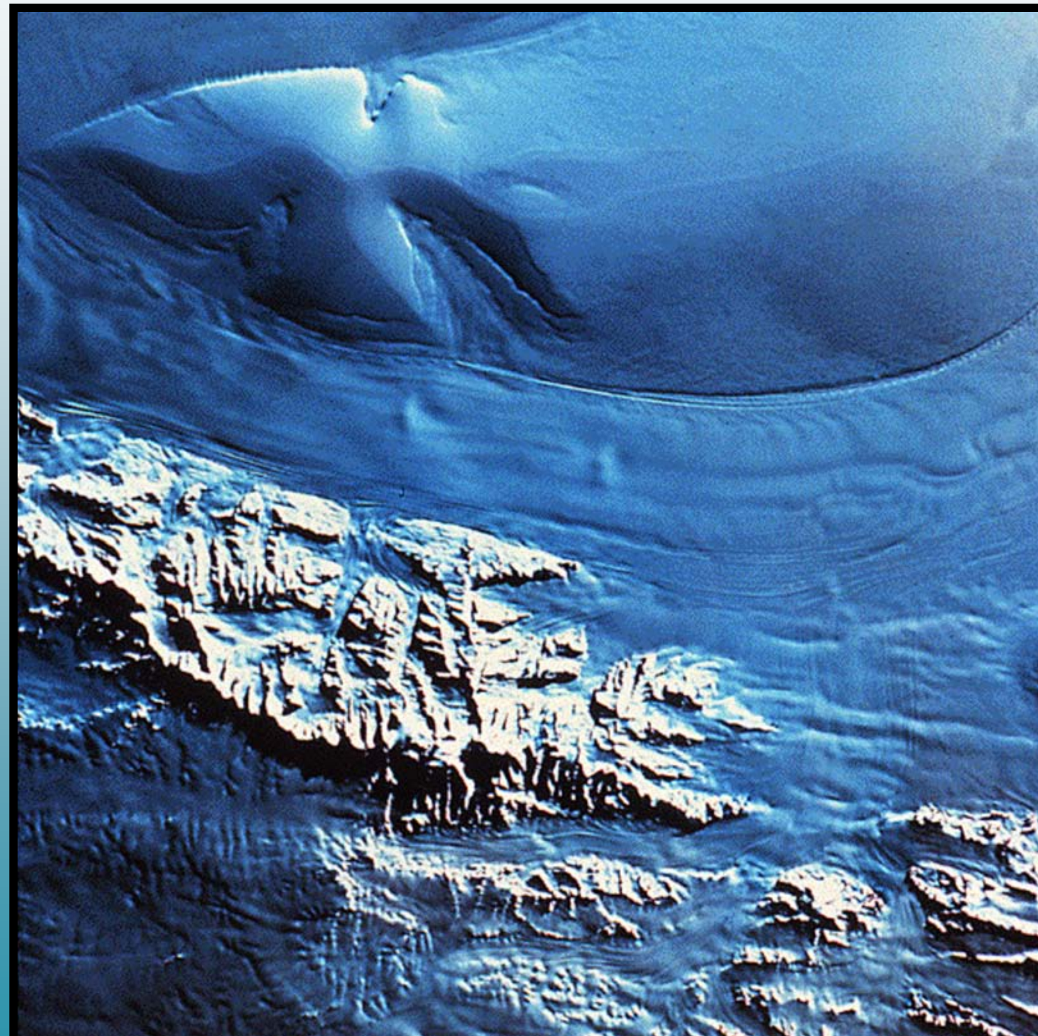
Incorporates both the spatial frequency of the flexure and its decay length

Model Generation

Models were generated for profiles:

- A-A’
- A-A’’
- A-A’’’
- Thickness variation for A-A’

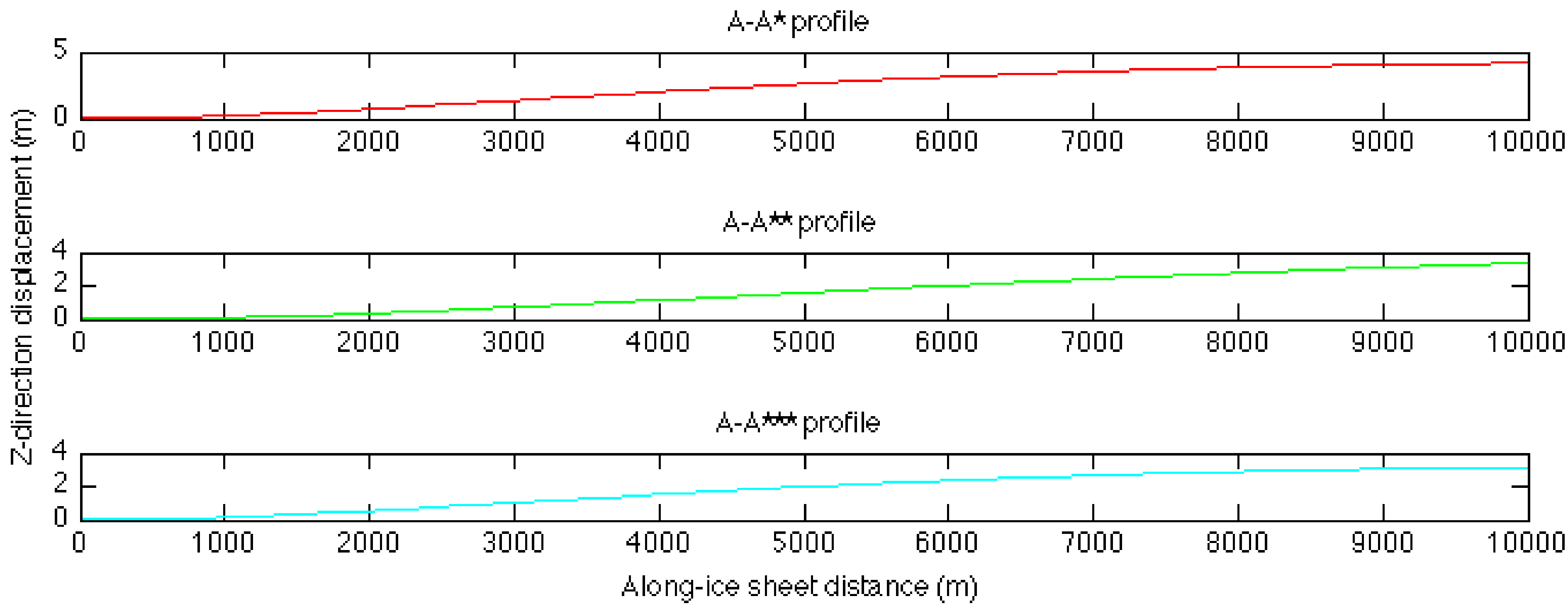


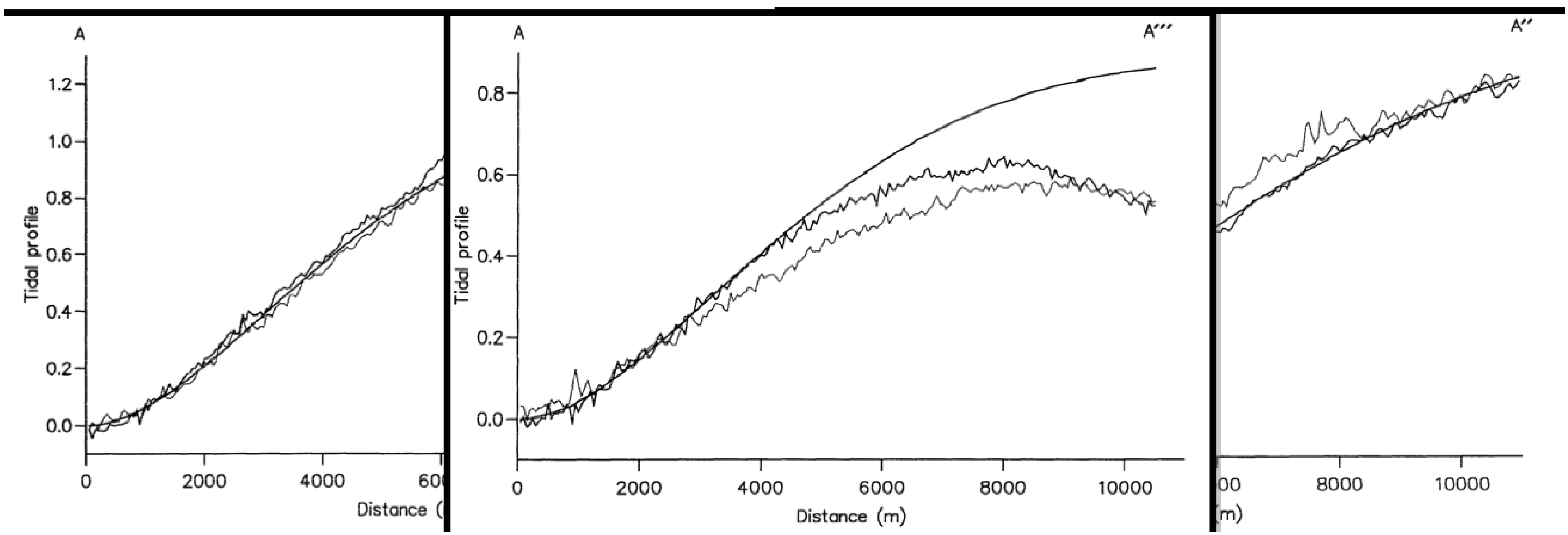


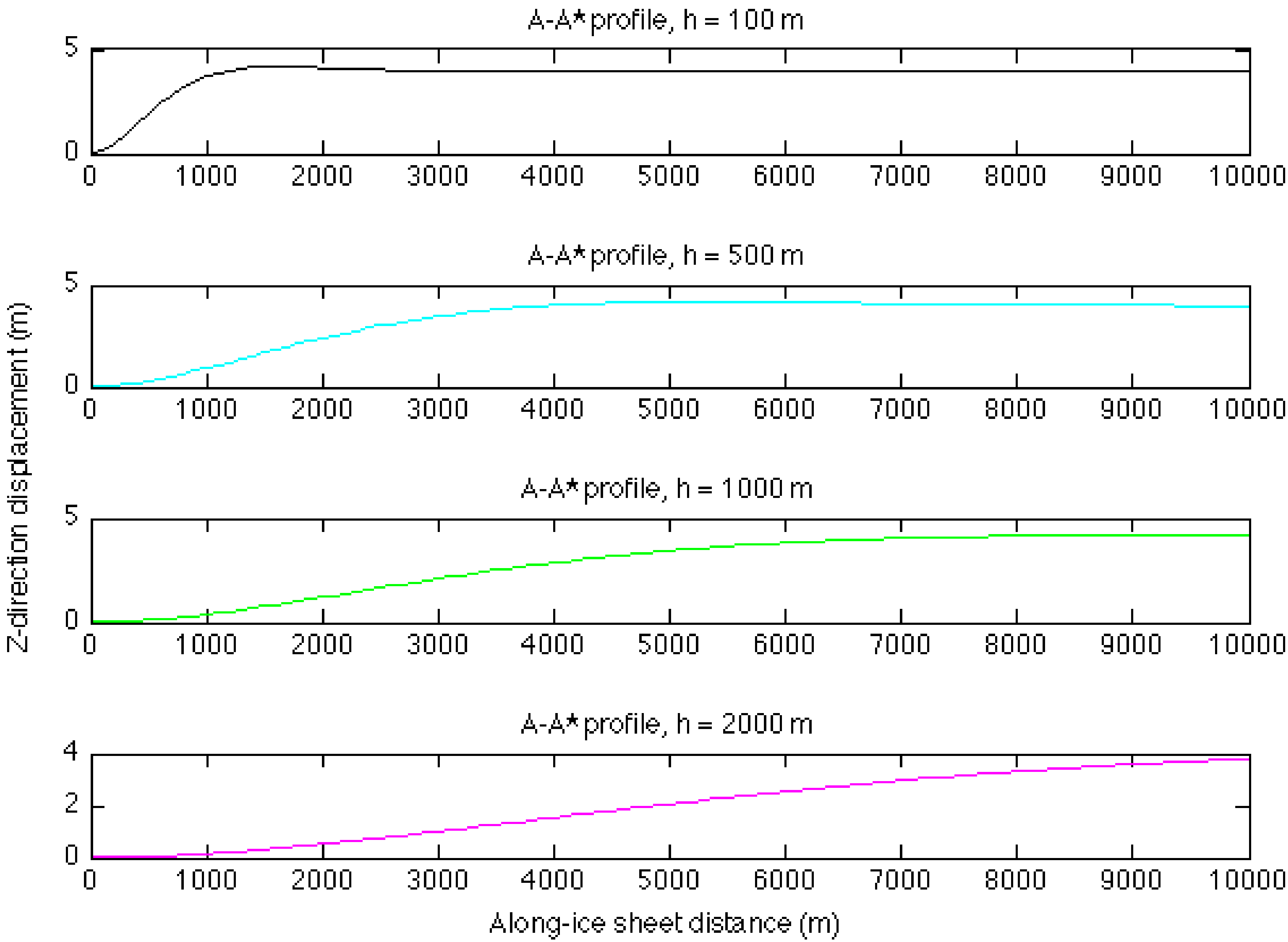
NASA Landsat Image

Model Code

```
X = 0:100:10000;                    %Defines x-grid
h1 = 100; h2 = 500; h3 = 1000; h4 = 2000;   %Selection of ice stream thicknesses (m)
A01 = 4.1;                          %Tidal influence beyond the hinge zone (m), value from Vaughan
A02 = 4;
A03 = 3.1;

%Beta parameter calculations
beta1 = 2.5*(10^-4);
beta2 = 1.7*(10^-4);
beta3 = 2.5*(10^-4);

%Calculation loop for displacement along x-grid
for i = 1:101
   w1(i) = A01*(1-(exp(-1*beta1*X(i)))*(cos(beta1*X(i))+sin(beta1*X(i))));
   w2(i) = A02*(1-(exp(-1*beta2*X(i)))*(cos(beta2*X(i))+sin(beta2*X(i))));
   w3(i) = A03*(1-(exp(-1*beta3*X(i)))*(cos(beta3*X(i))+sin(beta3*X(i))));

end

figure(1); clf;
subplot(3,1,1), plot(X,w1,'r'), title('A-A* profile'), xlabel('Along-ice sheet distance (m)'), ylabel('Z-direction
displacement (m)');
subplot(3,1,2), plot(X,w2,'g'), title('A-A** profile'), xlabel('Along-ice sheet distance (m)'), ylabel('Z-direction
displacement (m)');
subplot(3,1,3), plot(X,w3,'c'), title('A-A*** profile'), xlabel('Along-ice sheet distance (m)'), ylabel('Z-direction
displacement (m)');
```


A-A* profile
A-A** profile
A-A*** profile
Z-direction displacement (m)
Along-ice sheet distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000


A
A'''
A''
Tidal profile
Distance (m)
0.0
0.2
0.4
0.6
0.8
1.0
1.2
0
2000
4000
6000
8000
10000



A-A* profile, h = 100 m
A-A* profile, h = 500 m
A-A* profile, h = 1000 m
A-A* profile, h = 2000 m
Z-direction displacement (m)
Along-ice sheet distance (m)
0
1000
2000
3000
4000
5000
6000
7000
8000
9000
10000
5
4
2

Findings

- A new method for obtaining tidal profiles for ice shelf hinge zones
- Model based on elastic beam theory
  - Explains data from previous studies
- Average elastic modulus that fits nearly all the sites: E=0.88 +- 0.35 Gpa
- Model can be used to predict a characteristic based of the others like the Bulk Modulus if thickness of the ice sheet is known

Schlaten Glacier, Austria