Fracture Zone Flexure

Wenyuan Fan & Nick Mancinelli

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Outline

Ocean Floor Topography

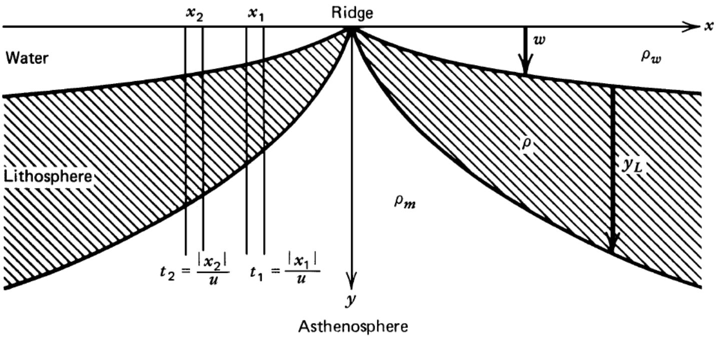

Figure: From Geodynamics Turcotte & Schubert

$$Depth = \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)} \left(\frac{\kappa x}{\pi u_0}\right)^{1/2}$$

Mendocino Fracture Zone

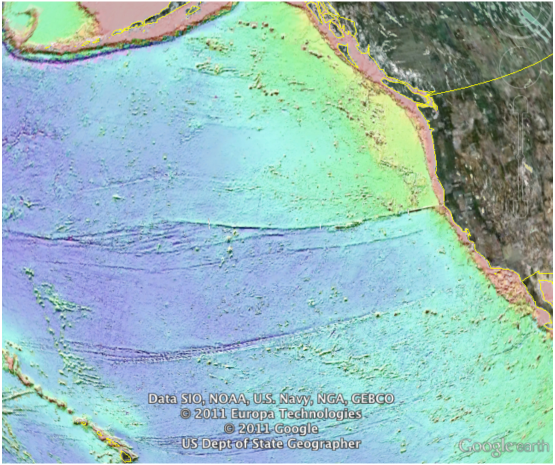

Figure: Mendocino Fracture Zone

Relevent problem in reality

Sandwell and Schubert, 1982

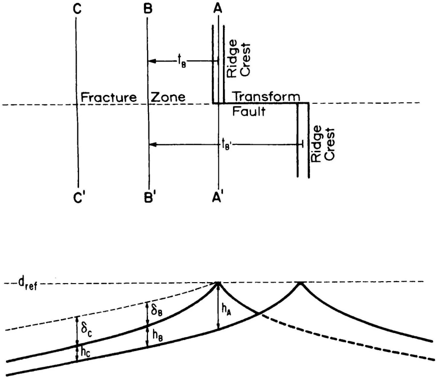

Figure: What brings up the problem?

Depth-Age Relation

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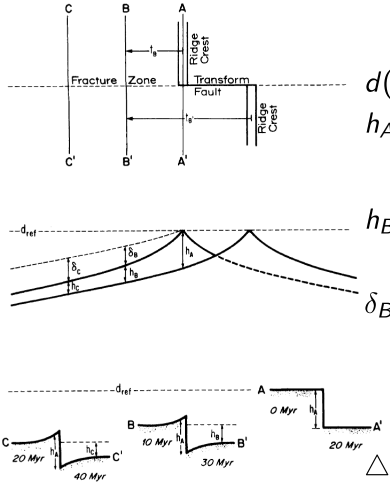
$$d(t) = d_{ref} + \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)}\left(\frac{\kappa t}{\pi}\right)^{1/2}$$

for ages less than about 70 Myr.

- Parker and Oldenburg, 1973
- Davis and Lister, 1974
- Oxburgh and Turcotte, 1978
- Parsons and Sclater, 1977

Depth Difference



Sandwell and Schubert, 1982

$$
\begin{aligned}
d(t) &= d_{ref} + \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)}\left(\frac{\kappa t}{\pi}\right)^{1/2} \\
h_A &= d(t_{A'}) - d(0) \\
&= \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)}\left(\frac{\kappa t_{A'}}{\pi}\right)^{1/2} \\
h_B &= d(t_{B'}) - d(t_B) \\
&= \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)}\left(\frac{\kappa}{\pi}\right)^{1/2}(t_{B'}^{1/2} - t_B^{1/2}) \\
\delta_B &= h_A - h_B \\
&= \frac{2\alpha\rho_m(T_m - T_s)}{(\rho_m - \rho_w)}\left(\frac{\kappa}{\pi}\right)^{1/2} \\
&\quad \times(\triangle t^{1/2} + t_B^{1/2} - t_{B'}^{1/2}) \\
\triangle t &= t_{A'} - 0 = t_{B'} - t_B
\end{aligned}
$$

Model

Flexure Equation

$$D\frac{d^4w}{dx^4} + P\frac{d^2w}{dx^2} + g(\rho_m - \rho_w)w = q_a(x)$$

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- $D\frac{d^4w}{dx^4}$ Strength of plate
- $P\frac{d^2w}{dx^2}$ End load
- $g(\rho_m - \rho_w)w$ Restricting force
- $q_a(x)$ Applied load

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Flexure Model

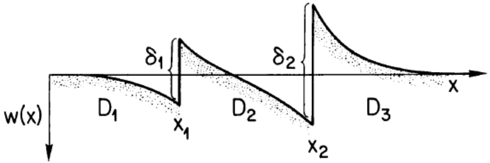

Figure: Model used to calculate the flexural topography

Governing Equation

$$
\begin{array}{ll}
D_1 \frac{d^4 w_1}{dx^4} + g(\rho_m - \rho_w) w_1 = 0 & x < x_1 \\
D_i \frac{d^4 w_i}{dx^4} + g(\rho_m - \rho_w) w_i = 0 & x_{i-1} < x < x_i \\
D_{N+1} \frac{d^4 w_{N+1}}{dx^4} + g(\rho_m - \rho_w) w_{N+1} = 0 & x_N < x
\end{array}
$$

where D_i is the flexural rigidity of ith block.

Physical Constrains(Boundary Conditions)

$$w_i - w_{i+1} = \delta_i$$

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- Finite at infinity far away
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$$w_i - w_{i+1} = \delta_i$$

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Differential Equations

Consider the nth order linear homogeneous differential equation:

$$L[y] = a_n y^{(n)} + a_{n-1} y^{(n-1)} + \cdots + a_1 y = 0$$

where $y = y(x)$. Many years ago, one genius found that $y = e^{rx}$ is a solution of the equation. Then we have:

$$(a_n r^n + a_{n-1} r^{n-1} + \cdots + a_1 r) e^{rx} = 0$$

A polynomial of degree n has n zeros, say we have n roots $r_1, \cdots, r_n$. If $e^{r_1 x}, \cdots, e^{r_n x}$ are linearly independent, the general solution of $L[y] = 0$ is

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$$y = c_1 e^{r_1 x} + c_2 e^{r_2 x} + \cdots + c_n e^{r_n x}$$

Analytical Solution 1

Governing Equation

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4th polynomial

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4th polynomial

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Roots

$$r_{i1} = \left(\frac{g(\rho_m - \rho_w)}{D_i}\right)^{1/4} e^{i\pi/4} \qquad r_{i2} = \left(\frac{g(\rho_m - \rho_w)}{D_i}\right)^{1/4} e^{i3\pi/4}$$

$$r_{i3} = \left(\frac{g(\rho_m - \rho_w)}{D_i}\right)^{1/4} e^{i5\pi/4} \qquad r_{i4} = \left(\frac{g(\rho_m - \rho_w)}{D_i}\right)^{1/4} e^{i7\pi/4}$$

Analytical Solution 2

General Solution

$$w_i(x) = c_{i1}e^{r_1 x} + c_{i2}e^{r_2 x} + c_{i3}e^{r_3 x} + c_{i4}e^{r_4 x}$$

Wavelength of the flexure λ and flexture rigidity are given as:

$$\begin{cases} \lambda = 2\pi\left(\frac{4D}{g(\rho_m - \rho_w)}\right)^{1/4} \\ D = \frac{Eh_e^3}{12(1-\nu^2)} \end{cases}$$

where h_e is the effective elastic thickness as:

$$h_e = 2(\kappa t)^{1/2}\mathrm{erfc}^{-1}\left(\frac{T_m - T_e}{T_m - T_s}\right)$$

Analytical Solution 3

Roots

$$\begin{aligned} r_{i1} &= \frac{2\pi}{\lambda_i}(1+i) & r_{i2} &= \frac{2\pi}{\lambda_i}(-1+i) \\ r_{i3} &= \frac{2\pi}{\lambda_i}(-1-i) & r_{i4} &= \frac{2\pi}{\lambda_i}(1-i) \end{aligned}$$

Analytical Solution 3

Roots

$$r_{i1} = \frac{2\pi}{\lambda_i}(1+i) \qquad r_{i2} = \frac{2\pi}{\lambda_i}(-1+i)$$
$$r_{i3} = \frac{2\pi}{\lambda_i}(-1-i) \qquad r_{i4} = \frac{2\pi}{\lambda_i}(1-i)$$

Flexure Topography

$$\begin{aligned} w_i(x) &= c_{i1}e^{r_1 x} + c_{i2}e^{r_2 x} + c_{i3} + c_{i4}e^{r_4 x} \\ &= e^{-2\pi x/\lambda_i}\left(A_i \sin\frac{2\pi x}{\lambda_i} + B_i \cos\frac{2\pi x}{\lambda_i}\right) \\ &\quad + e^{2\pi x/\lambda_i}\left(C_i \sin\frac{2\pi x}{\lambda_i} + D_i \cos\frac{2\pi x}{\lambda_i}\right) \end{aligned}$$

with help of $e^{i\theta} = \cos\theta + i\sin\theta$. A_i, B_i, C_i, D_i are determined by physical constrains.

2 Blocks situation

Flexure Topography

$$
\begin{aligned}
w_1 &= e^{2\pi x/\lambda_1}\left(C_1 \sin\frac{2\pi x}{\lambda_1} + D_1 \cos\frac{2\pi x}{\lambda_1}\right) \quad x_1 < 0 \\
w_2 &= e^{-2\pi x/\lambda_2}\left(A_2 \sin\frac{2\pi x}{\lambda_2} + B_2 \cos\frac{2\pi x}{\lambda_2}\right) \quad x_2 > 0
\end{aligned}
$$

Based on the four physical constrains, we could solve the four unknown parameter.

Stress and Moment

Moment

$$M_i(x) = -D_i \frac{d^2 w_i(x)}{dx^2}$$

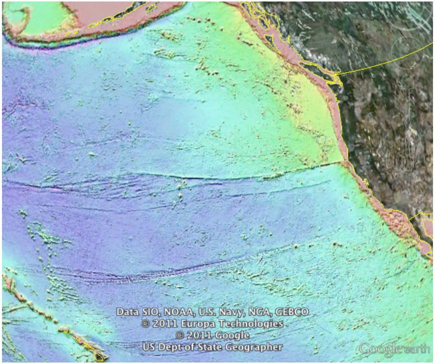
Bending stress

$$\sigma_{xx_i}(x) = \frac{6M_i(x)}{h_e^2} = \frac{-Eh_e}{2(1-\nu^2)} \frac{d^2 w_i(x)}{dx^2}$$

Shear stress(Average)

$$\sigma_{xz_i}(x) = -\frac{-D_i}{h_e} \frac{d^3 w_i(x)}{dx^3}$$

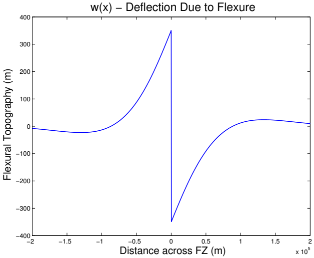
Time to Deal with Real World



Model Assumptions

- FZ scarp fossilized (no slip on plane)
- No lateral heat conduction

Model Results



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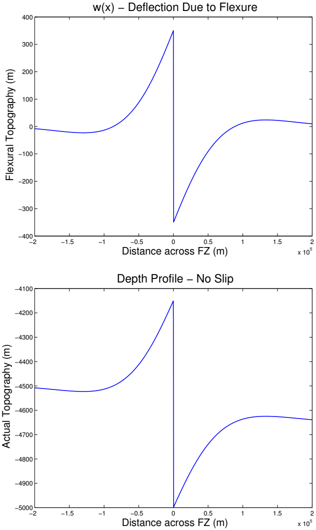

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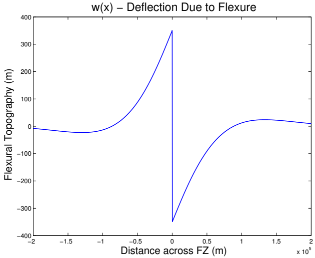

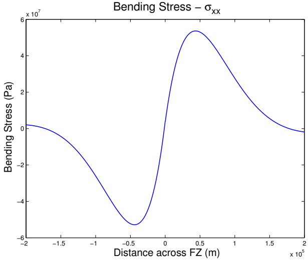

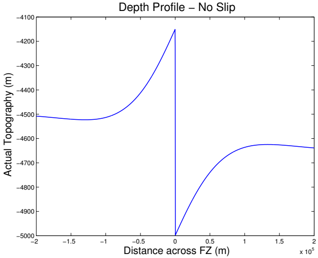

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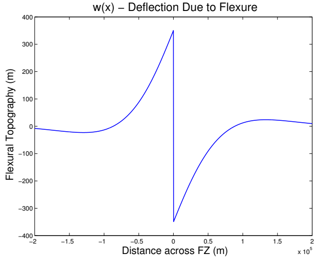
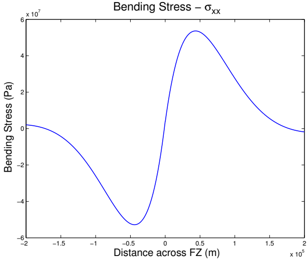
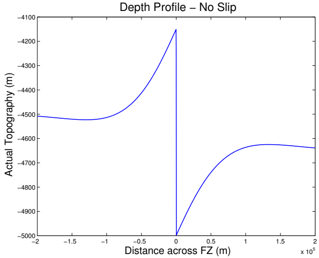
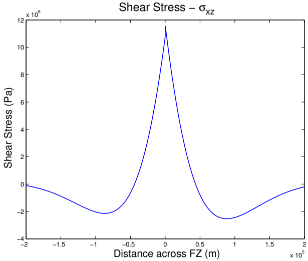


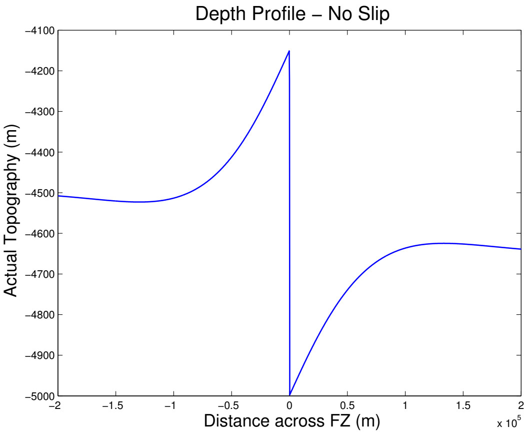






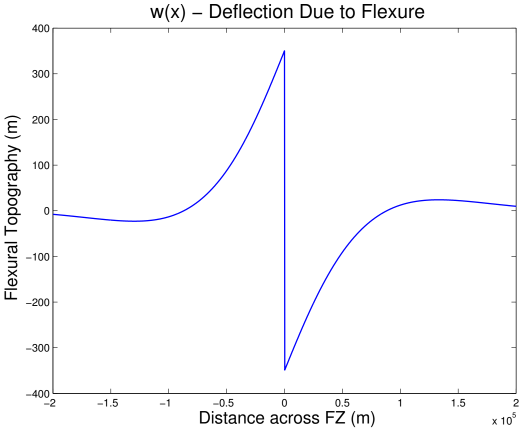
Topography

Crustal ages: 46 Myr $x < 0$ and 54 Myr $x > 0$



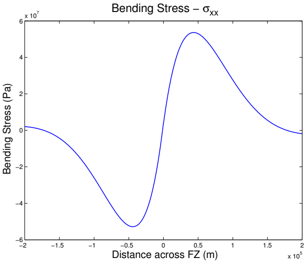
Deflection Due to Flexture

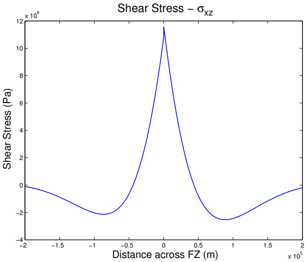
Flexural wavelengths: $342km(x < 0)$ and $358km(x > 0)$



Deflection Due to Flexture

Flexural rigidities: $4.99 \times 10^{10}\ Pa \times km^4$ and $6.00 \times 10^{10}\ Pa \times km^4$
Effective elastic thicknesses: $20.5\ km$ and $21.8\ km$

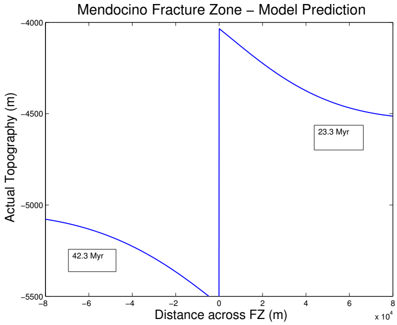


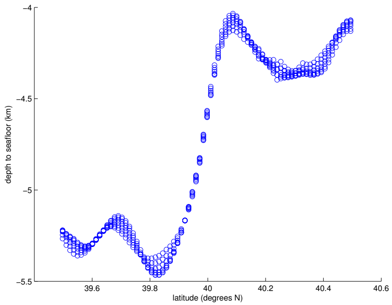


Model VS. Reality

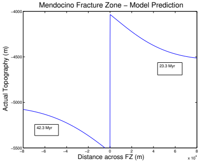
Age of block

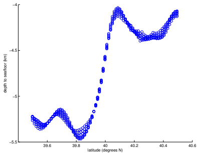
Age of Left Block 42.3 Myr **&** Age of Right Block 23.3 Myr

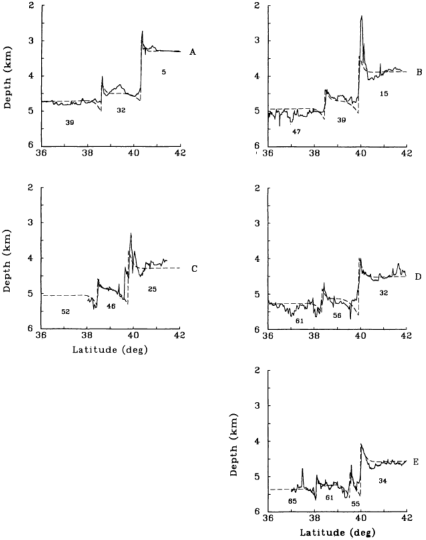




Model VS Reality 2







Comparisons between theoretical and actual bathymetric profiles for Mendocino-Pioneer FZ Pair (Sandwell and Schubert):

- Model fits multiple FZs fairly well
- Asymmetric flexure
- Elastic coupling (apparent tilting)

Conclusion

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5. Maximum bending stress (100 MPa) much less than stresses encountered in subduction zones.
6. In the case of multiple fracture zones, there is elastic coupling when two adjacent FZs are spaced less than a flexural wavelength apart. This is true for the Mendocino-Pioneer FZ pair.

Questions?