Tidal flexure at ice shelf margins

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Ice shelf margin

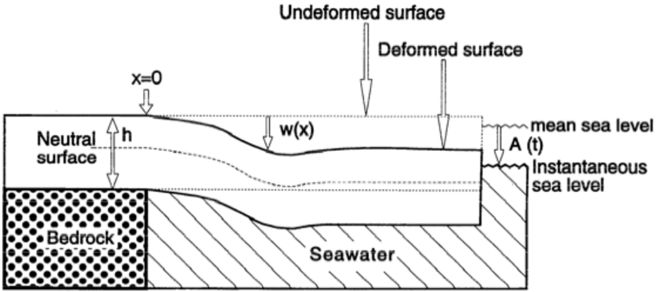


Figure 2. Model of the vertical section through an ice shelf grounding zone. After *Holdsworth* [1969].

Typical values:

- Tidal amplitude $\approx 3 - 5$ m
- Width of hinge zone $\approx 1 - 10$ km

Hinge zones

Grounded ice sheet: supported, unaffected by tides

Floating ice shelf: constant motion (with tides)

Hinge zone:

- hydrostatic pressure & internal stresses
- cyclic flexure
- differential movement

Tidal flexure implications

- Power dissipated from global tides into ice hinge zones
  $\rightarrow$ rheology governing flexure
- Ice shelf flow influenced by repeated straining in hinge zones
  $\rightarrow$ mechanism & magnitude of flexure
- InSAR measurements of ice sheet velocities
  $\rightarrow$ model and remove tidal displacements in the hinge zone
- Ice shelf evolution linked to sea tides
  $\rightarrow$ determine limits of tidal range

Elastic beam model

Robin [1958]:
Positions of surface cracks and regions of maximum stress coincide

Holdsworth [1977]:
Extend model to include rheology (value of E based on laboratory tests)
— dominant flow mechanism poorly understood

Vaughan [1995]:
Derives E from site data; develops widely applicable flexure model

Ice rheology

Glacier ice deforms by many processes — different ones dominate at different points in time

Want to understand dominant flow mechanism over various timescales/stress regimes

Difficult!

Ice properties

- Wide range of values for Young's modulus, E (0.83 – 10.0 GPa)
- Poisson's ratio $\nu \approx 0.3$
- Small strains (10^{-6} to 10^{-5}) $\implies$ elastic problem

Elastic beam model

Tidal flexure

$$D\frac{\partial^4 w}{\partial x^4} = \rho_{sea} g \left[A_0(t) - w(x)\right] \quad (1)$$

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

x axis - horizontal and orthogonal to the grounding line

$A_0(t)$ - level of the ice shelf if it were floating in isostatic equilibrium

$w(x)$ - vertical displacement of the ice sheet surface

ρ_{sea} - density of sea water

g - gravitational acceleration

D - flexural rigidity

h - thickness of ice shelf

Elastic beam model

Boundary conditions

- Grounding line, $x = 0$: $w(0) = \frac{\partial w(0)}{\partial x} = 0$
- Free-floating, $x = \infty$: $w = A_0(t)$

Steps to solution

1. Find particular, $w_p(x)$, and homogeneous solutions, $w_h(x)$
2. Write general form of the solution $w(x) = w_p(x) + w_h(x)$
3. Apply boundary conditions $\implies$ solution to model problem

Particular solution

Rewrite Eq. 1:

$$D\frac{\partial^4 w}{\partial x^4} + \rho_{sea} g w(x) = \rho_{sea} g A_0(t)$$

Since the forcing function is constant in x, we guess that $w_p(x)$ is a constant function C:

$$w_p(x) = C \implies \frac{\partial^4 w_p}{\partial x^4} = 0$$

Thus, Eq. 1 simplifies to

$$\rho_{sea} g C = \rho_{sea} g A_0(t) \implies C = A_0(t)$$

Hence,

$$\boxed{w_p(x) = A_0(t)}$$

Homogenous solution

Roots of the characteristic equation $r^4 + \frac{\rho_{sea} g}{D} = 0$:

$$r = \sqrt[4]{\frac{\rho_{sea} g}{D}} e^{i\theta} = \sqrt[4]{\frac{\rho_{sea} g}{D}} \left[\cos(\theta) + i \sin(\theta)\right], \quad \theta = \pm\frac{\pi}{4}, \pm\frac{3\pi}{4}.$$

The four roots are of the form $r = a + ib$, where

$$a = b = \pm\frac{1}{\sqrt{2}} \sqrt[4]{\frac{\rho_{sea} g}{D}}$$

Hence, the general form of the homogenous solution is

$$\boxed{w_h(x) = e^{\beta x}\left[c_1 \cos \beta x + c_2 \sin \beta x\right] + e^{-\beta x}\left[c_3 \cos \beta x + c_4 \sin \beta x\right]}$$

with spatial wavenumber

$$\beta = \frac{1}{\sqrt{2}} \sqrt[4]{\frac{\rho_{sea} g}{D}} = \sqrt[4]{\frac{3\rho_{sea} g(1-\nu^2)}{Eh^3}}$$

General solution

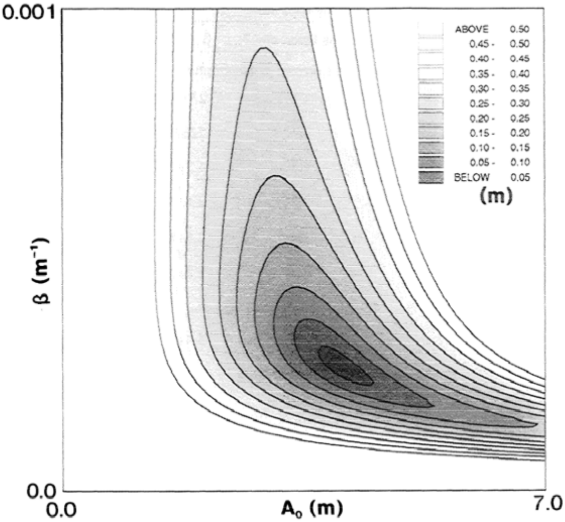
$$w(x) = A_0(t) + e^{\beta x}\left[c_1 \cos \beta x + c_2 \sin \beta x\right] + e^{-\beta x}\left[c_3 \cos \beta x + c_4 \sin \beta x\right]$$

- Far-field condition $w(x) = A_0(t) \implies w_h(x) = 0 \implies \boxed{c_1 = c_2 = 0}$
- Grounding line $w(0) = 0 \implies 0 = A_0(t) + c_3 \implies \boxed{c_3 = -A_0(t)}$
- Grounding line $\frac{\partial w(0)}{\partial x} = 0 \implies 0 = -\beta\left[c_3 - c_4\right] \implies \boxed{c_4 = -A_0(t)}$

Tidal deflection

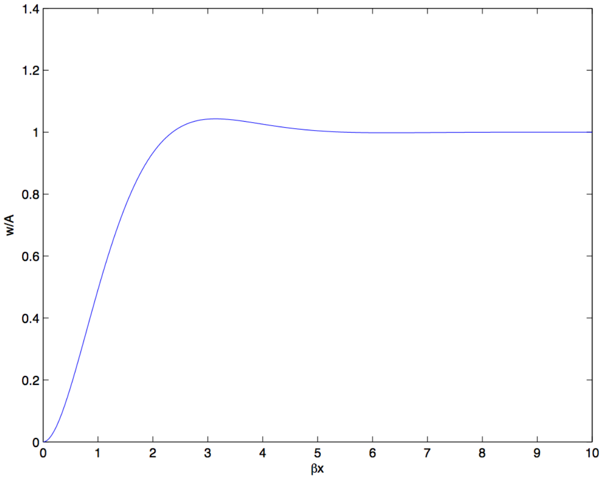
$$w(x) = A_0(t)\left[1 - e^{-\beta x}(\cos \beta x + \sin \beta x)\right]$$

Parameter fitting

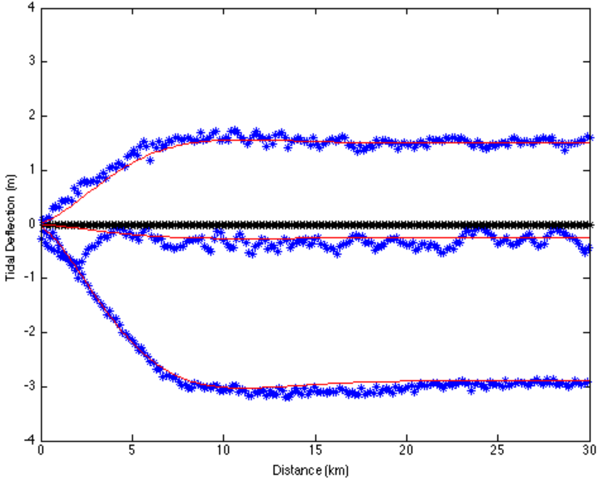


Vaughan, D. G., Tidal flexure at ice shelf margins, *Journal of Geophysical Research*, *100*, 6213 - 6224, 1995.

Tidal profile



Ronne Ice Shelf



Fricker, H. A., and L. Padman, Ice shelf grounding zone structure from ICESat laser altimetry, *Geophysical Research Letters*, *33*, L15502, 2006.

Summary

Vaughan [1995]: finds mean wavenumber and amplitude values $\rightarrow$ widely applicable elastic model

Fricker and Padman [2006]: reproduced tidal profile plots and compared against model results

- $\beta \approx 0.27$ km^{-1}
- $A_0(t) \approx$ -3 – 1.5 m

References

Fricker, H. A., and L. Padman, Ice shelf grounding zone structure from ICESat laser altimetry, *Geophysical Research Letters*, *33*, L15502, 2006.

Holdsworth, G., Flexure of a floating ice tongue, *Journal of Glaciology*, *8*, 385 - 397, 1969.

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