Correlation or Causation?
Lake Loading and Paleoseismicity on the Southern San Andreas Fault

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SIO 234: Geodynamics

http://imgs.xkcd.com/comics/correlation.png
Lake Cahuilla and SAFS

- Active tectonic region: San Andreas Fault, San Jacinto Fault, Imperial Fault
- Lake Cahuilla: periodically fills and drains as Colorado River alters course
- Now: lake is naturally drained, but man-made Salton Sea

Luttrell et al., JGR 2007
Timing of major earthquakes and lake draining/filling events appear to be correlated.

Three Possibilities:
- False correlation? (boring!)
- Earthquakes cause draining/filling of the lake? (slightly more interesting…)
- Draining/filling of the lake causes earthquakes? (exciting!)

Luttrell et al., JGR 2007
Draining/filling of the lake causes earthquakes?

• **Coulomb Stress:** \( \sigma_c = \sigma_S + \mu_f \sigma_N \)

• Positive Coulomb stress changes push fault toward failure

• Lake Cahuilla perturbs the effective normal stress in the nearby fault zones in two ways:
  – Direct loading: weight of lake bends lithosphere (flexural effect)
  – Pore pressure effect: fluid percolation to the subsurface reduces effective normal stress
Weight of the water flexes the plate

- Flexural response of the plate to lake:
  - Compression at the bottom of the lake
  - Extension at the rim
- What is the magnitude of this stress?

Luttrell et al., JGR 2007
Let’s do the math

- Model the lithosphere as a thin, 2D elastic plate under a distributed line load (the lake)
- Load is proportional the depth of the lake (max ~ 90m): \( q(x) = \rho_w gh(x) \)
- Solve the flexure equation:

\[
D \frac{d^4 w(x)}{dx^4} + (\rho_m - \rho_w)gw(x) = q(x)
\]
Green’s function solution approach

- Green’s function $G(x,x')$ is the deflection of the plate due to point load at $x = x'$:
- We derived the Green’s function for a point load at $x = 0$ in class (strength $V_0$):

$$D \frac{d^4 G(x)}{dx^4} + (\rho_m - \rho_w) g G(x) = V_0 \delta(x)$$

$$\Rightarrow G(x) = \frac{V_0 \alpha^3}{8D} e^{-|x|/\alpha} \left[ \cos(|x|/\alpha) + \sin(|x|/\alpha) \right]$$

- where $\alpha = \left[ \frac{4D}{((\rho_m - \rho_w)g)} \right]^{1/4} = \lambda_f / 2\pi$
Response = Green’s function convolved with load distribution

- Uniform lake load for $x < 0$: $q(x) = (1 - H(x))$
- What is the plate response for some $x > 0$?
- Convolution of Green’s function and load:

$$W(x) = G(x) * q(x) = \int_{-\infty}^{\infty} G(x - x') q(x') \, dx' = \int_{-\infty}^{\infty} G(x') q(x - x') \, dx'$$

$$W(x) = \int_{-\infty}^{\infty} G(x') (1 - H(x - x')) \, dx'$$

$$= \int_{-\infty}^{x} G(x') \, dx'$$

$$= \int_{-\infty}^{x} \frac{V_0 \alpha^3}{8D} e^{-|x'|/\alpha} \left[ \cos(|x'|/\alpha) + \sin(|x'|/\alpha) \right] \, dx'$$
Let $W_0$ be the deflection of the plate at $x = 0$.

We know that $W(x)$ is antisymmetric about $x = 0$, so once we solve the problem for $x > 0$, we’re done.

Now, for $x > 0$:

$$W(x) = W_0 + \int_0^x \frac{V_0 \alpha^3}{8D} e^{-lx'/\alpha} \left[ \cos(|x'|/\alpha) + \sin(|x'|/\alpha) \right] dx'$$

Let $u = x'/\alpha \rightarrow dx' = \alpha du$

Substitute and integrate by parts:

$$\int e^{-lu} \left[ \cos |u| + \sin |u| \right] du = -e^{-lu} \cos |u|$$

$$\Rightarrow W(x) = W_0 - \frac{V_0 \alpha^4}{8D} \left( e^{-lu} \cos |u| \right) \left. \right|_0^{x/\alpha} = \frac{V_0 \alpha^4}{8D} \left[ 1 - e^{-lx/\alpha} \cos(|x|/\alpha) \right]$$
Putting it all together...

\[ W(x) = W_0 + \frac{V_0 \alpha^4}{8D} \left[ 1 - e^{-lx/\alpha} \cos(|x|/\alpha) \right] \quad \text{for } x \geq 0 \]

- We know the solution is antisymmetric about \( x = 0 \), so if we remove the mean deflection \( (W_0) \), the net deflection of the plate is:

\[ W(x) = \frac{V_0 \alpha^4}{8D} \left[ 1 - e^{-lx/\alpha} \cos(|x|/\alpha) \right] \text{sgn}(x) \]

- Ok, now that we know the deflection of the plate, we can compute the lateral strain \( (\varepsilon_{xx}) \) and stress \( (\sigma_{xx}) \) at some depth \( z \) relative to the center of the plate.
Compute strain $\varepsilon_{xx}(z)$

- Recall that $\varepsilon_{xx}$ is proportional to plate curvature:

$$\varepsilon_{xx} = -z \frac{d^2 W}{dx^2}$$

$$= -z \frac{d^2}{dx^2} \left\{ \frac{V_0 \alpha^4}{8D} \left[ 1 - e^{-lx/\alpha} \cos(|x|/\alpha) \right] \text{sgn}(x) \right\}$$

$$= -z \frac{d}{dx} \left\{ -\frac{V_0 \alpha^3}{8D} e^{-lx/\alpha} \left[ \cos(|x|/\alpha) + \sin(|x|/\alpha) \right] \text{sgn}(x) \right\}$$

$$= -z \frac{V_0 \alpha^2}{4D} \left[ e^{-lx/\alpha} \sin(|x|/\alpha) \right] \text{sgn}(x)$$
Thin plate $\leftrightarrow$ plane stress

- For a plate of thickness $H$:  \[ D = \frac{EH^3}{12(1 - \nu^2)} \]
- And in plane stress ($\sigma_{zz} = 0$):  \[ \sigma_{xx} = \frac{E}{(1 - \nu^2)} \varepsilon_{xx} \]
- So at a depth $-z_0$ (relative to center of plate):

\[
\sigma_{xx} = \frac{E}{(1 - \nu^2)} \left\{ z_0 \frac{V_0 \alpha^2}{4D} \left[ e^{-\frac{|x|}{\alpha}} \sin(|x|/\alpha) \right] \text{sgn}(x) \right\}
\]

\[
\sigma_{xx} = \frac{12D}{H^3} \left\{ z_0 \frac{V_0 \alpha^2}{4D} \left[ e^{-\frac{|x|}{\alpha}} \sin(|x|/\alpha) \right] \text{sgn}(x) \right\}
\]

\[
\sigma_{xx} = \frac{3V_0 \alpha^2}{H^2} \left( \frac{z_0}{H} \right) \left[ e^{-\frac{|x|}{\alpha}} \sin(|x|/\alpha) \right] \text{sgn}(x)
\]

- Compression under lake, extension outside its rim
2D Thin Plate Model Results

- Load: \( q(x) = \rho_w g h(x) \)
  
  \[ \text{for } x < 0 \text{ and } h = 3.3m \]

- \( H = 30 \text{ km} \)
- \( E = 70 \text{ GPa}, \nu = 0.25 \)
Strength of the pore pressure effect at depth depends on the permeability of the subsurface. Two limiting cases:

- “Full percolation” limit (high permeability): hydrostatic increase in pore pressure at depth $\Rightarrow \gamma = 1$
- “No percolation” limit (low permeability): no increase in pore pressure at depth $\Rightarrow \gamma = 0$

\[
\Delta \sigma_C = \mu_f \Delta \sigma_N = \mu_f \Delta P = \mu_f \gamma \rho_w gh
\]
Two important time scales...

1. Although we treated it as an elastostatic problem (thin plate), the flexural response is not instantaneous
   – Plate has a finite thickness
   – Time scale depends on the viscosity of asthenosphere
2. Pore pressure also does not propagate instantaneously
   – It diffuses (like heat conduction)

• The flexural timescale (1) is much slower than the diffusion timescale (2), so we can model the pore pressure effect as instantaneous (relative to the flexural effect)
Stress response to rise in lake level

- Coulomb stress in crust beneath the lake
- Pore pressure increase \(\rightarrow\) Instantaneous increase in Coulomb stress
- Flexural compression of lithosphere \(\rightarrow\) Long-term decrease in Coulomb stress over relaxation time scale

Luttrell et al., JGR 2007
Overview of Lake Cahuilla study of *Luttrell et al.* (JGR 2007)

- Topographic record gives lake height $h(x,y,t)$
- Convolve lake load with the point load’s Green’s function to compute flexural response (Fourier domain computation)
- Study temporal and spatial variations in Coulomb stress at 5km depth for two lithospheric models ($H = 25\text{km}, \tau = 70\text{yr}$ and $H = 35\text{km}$ and $\tau = 30\text{yr}$)
- Two limiting cases: no percolation of pore pressure to seismogenic depth ($\Upsilon = 0$) and full percolation ($\Upsilon = 1$)
Stress changes over one loading cycle

- Lake appears at $t = 0$, disappears at $t = 10\tau_m$
- Bombay Beach and Indio on SSAF
  - One inside lake, one outside of it
- Pore pressure effect influences only sites in Lake
- Rebound after lake disappears $\rightarrow$ increase in Coulomb stress

Luttrell et al., JGR 2007
Lake loading stresses and Earthquakes: 600 AD to Present

- Coulomb stress increases after both lake rises and lake falls
- Faster rebound for low $\tau \rightarrow$ amplifies stresses
- Magnitude of stresses depend on permeability and location

Luttrell et al., JGR 2007
Recap of Key Findings of *Luttrell et al.*’s Lake Cahuilla Study

• Lake cycle causes time-dependent and spatially variable changes in Coulomb stress

• Positive Coulomb stress changes right after lake level rise (pore pressure effect) or after fall (extensional stress in rebound)

• Maximum Coulomb stress changes near SAF: 0.2-0.6 MPa, depending on fluid permeability
  – This is an order of magnitude less than integrated tectonic stresses, but large enough to push near-critical faults to failure

• Plate thickness controls wavelength of deformation, stress perturbations amplify for short relaxation times
The 2008 $M_w$ 7.9 Wenchuan Earthquake: A Modern Analog?

Ge et al., GRL, 2009
CALVIN AND HOBBES

THE MORE YOU KNOW, THE HARDER IT IS TO TAKE DECISIVE ACTION.

ONCE YOU BECOME INFORMED, YOU START SEEING COMPLEXITIES AND SHADES OF GRAY.

YOU REALIZE THAT NOTHING IS AS CLEAR AND SIMPLE AS IT FIRST APPEARS. ULTIMATELY, KNOWLEDGE IS PARALYZING.

BEING A MAN OF ACTION, I CAN'T AFFORD TO TAKE THAT RISK.

YOU'RE IGNORANT BUT AT LEAST YOU ACT ON IT.
References:
