Flexure due to seamount loading OR Flexure two ways

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Geodynamics

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Calculation of the Plate Deformation

Governing PDE:

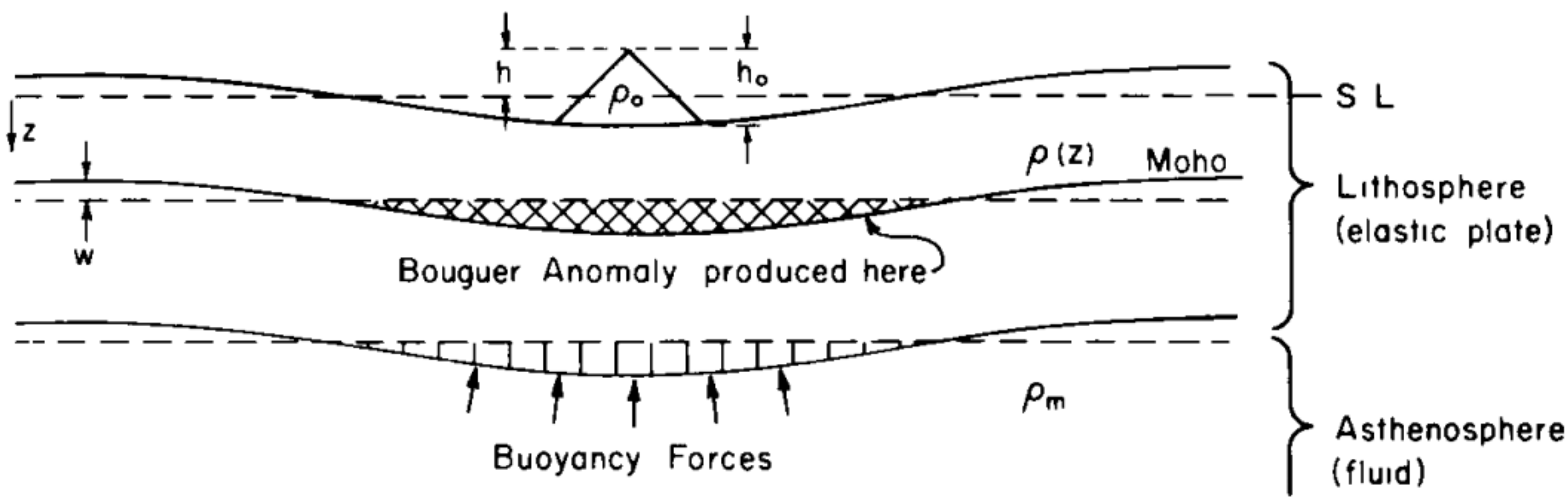
- $D\nabla^4 w(x,y) = p(x,y)$

Flexure

Load

By Definition:

- $p(x,y) = -\rho_o g h_o(x,y) - \rho_m g w(x,y)$
- $h(x,y) = h_o(x,y) + w(x,y)$
- $D = ET^3/12(1-\sigma^2)$



Calculation of the Plate Deformation

2D Fourier Transform 4 times on

$D\nabla^4 w(x,y) = p(x,y)$

$F_4^2[w(x,y)](k_x,k_y) = (2\pi ik)^4 DW(k_x,k_y)$

$= -\rho_o g H_o(k_x,k_y) - \rho_m g W(k_x,k_y)$

Where $W(k_x,k_y) = \int_S w(k_x,k_y) e^{i2\pi \mathbf{k \cdot r}}\, dS$

Calculation of the Plate Deformation

If we want to put a load of some shape (p(x,y))

like a Gaussian, then

$$(2\pi k)^4 D W(\mathbf{k}) + \rho_m g W(\mathbf{k}) = -\rho_o g H_o(\mathbf{k})$$

But if we want to estimate flexure for a measured topography:

$$(2\pi i k)^4 D W(k_x,k_y) = -\rho_o g H_o(k_x,k_y) - \rho_m g W(k_x,k_y)$$

Calculation of the Plate Deformation

$(2\pi ik)^4 DW(\mathbf{k}) = -\rho_o g H_o(\mathbf{k}) - \rho_m g W(\mathbf{k})$

- From initial conditions:

  $H(x,y) = H_o(x,y) + W(x,y)$

  $H_o(x,y) = H(x,y) - W(x,y)$

  $(2\pi k)^4 DW(\mathbf{k}) = -\rho_o g[H(\mathbf{k}) - W(\mathbf{k})] - \rho_m g W(\mathbf{k})$

$$W(\mathbf{k}) = -\left(\frac{\rho_0}{\rho_m - \rho_0}\right)\left(1 + \frac{16\pi^4 k^4 D}{(\rho_m - \rho_0)g}\right)^{-1} H(\mathbf{k})$$

- Expansion:

  $16\pi^4 k^4 DW(\mathbf{k}) = W(\mathbf{k})[\rho_o g - \rho_m g] - \rho_o g[H(\mathbf{k})]$

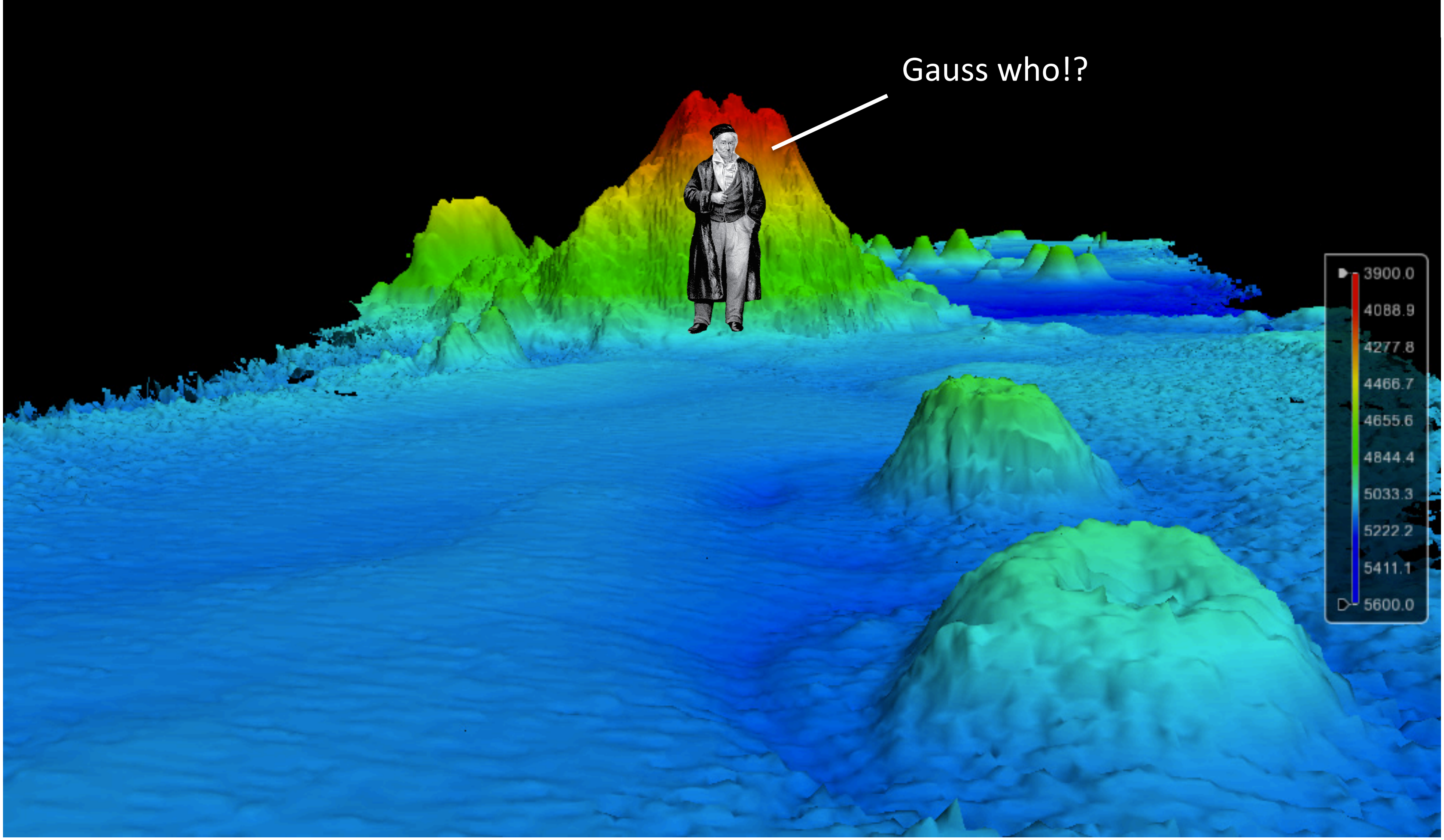
  $W(\mathbf{k})[16\pi^4 k^4 D - g(\rho_o - \rho_m)] = -\rho_o g[H(\mathbf{k})]$

  $W(\mathbf{k}) = -\rho_o g[H(\mathbf{k})] * [16\pi^4 k^4 D - g(\rho_o - \rho_m)]^{-1}$

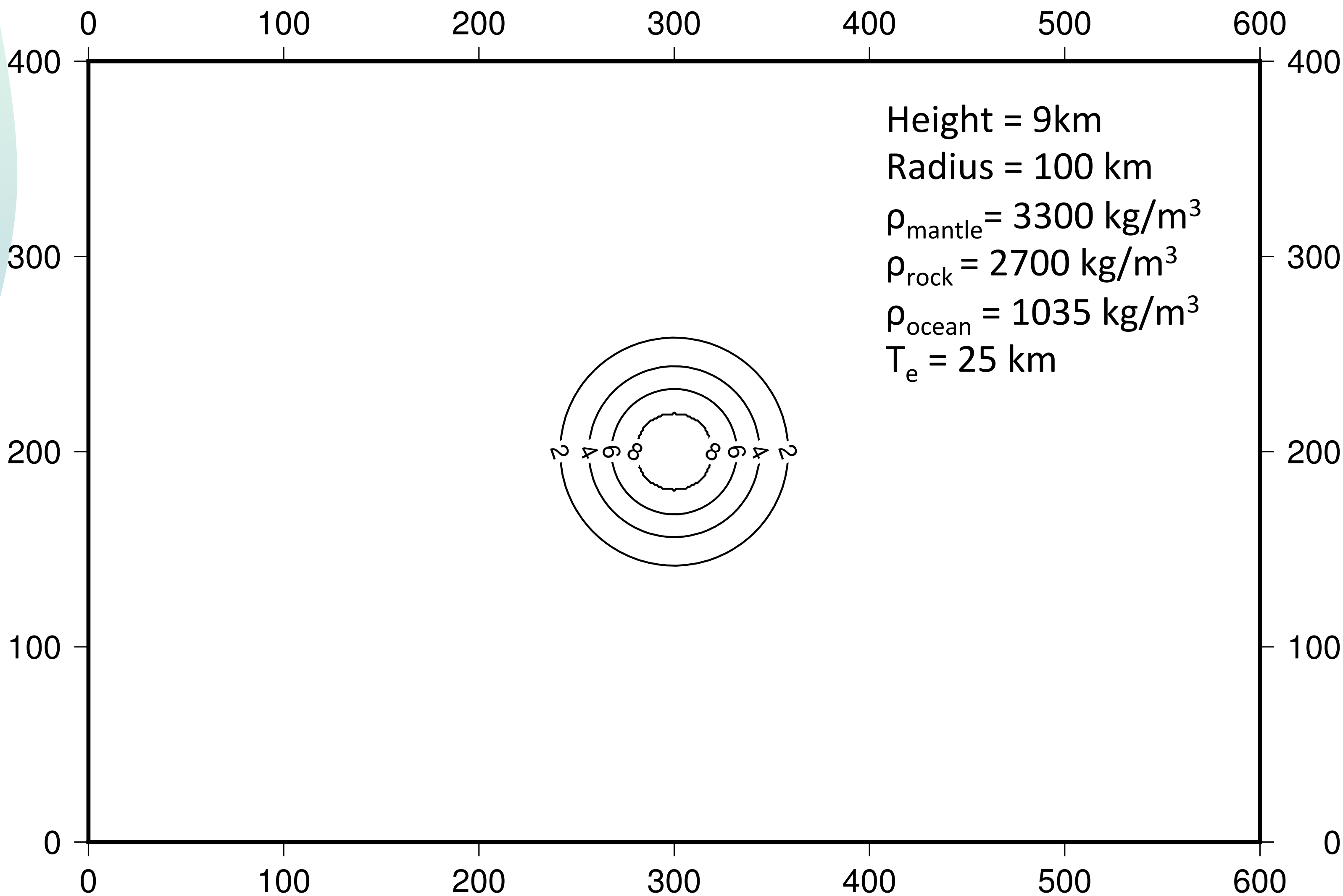
  $W(\mathbf{k}) = -\rho_o H(\mathbf{k})[16\pi^4 k^4 D/g + (\rho_m - \rho_o)]^{-1}$

  $W(\mathbf{k}) = H(k)[-\rho_o/(\rho_m - \rho_o)][16\pi^4 k^4 D/g + 1]^{-1}$

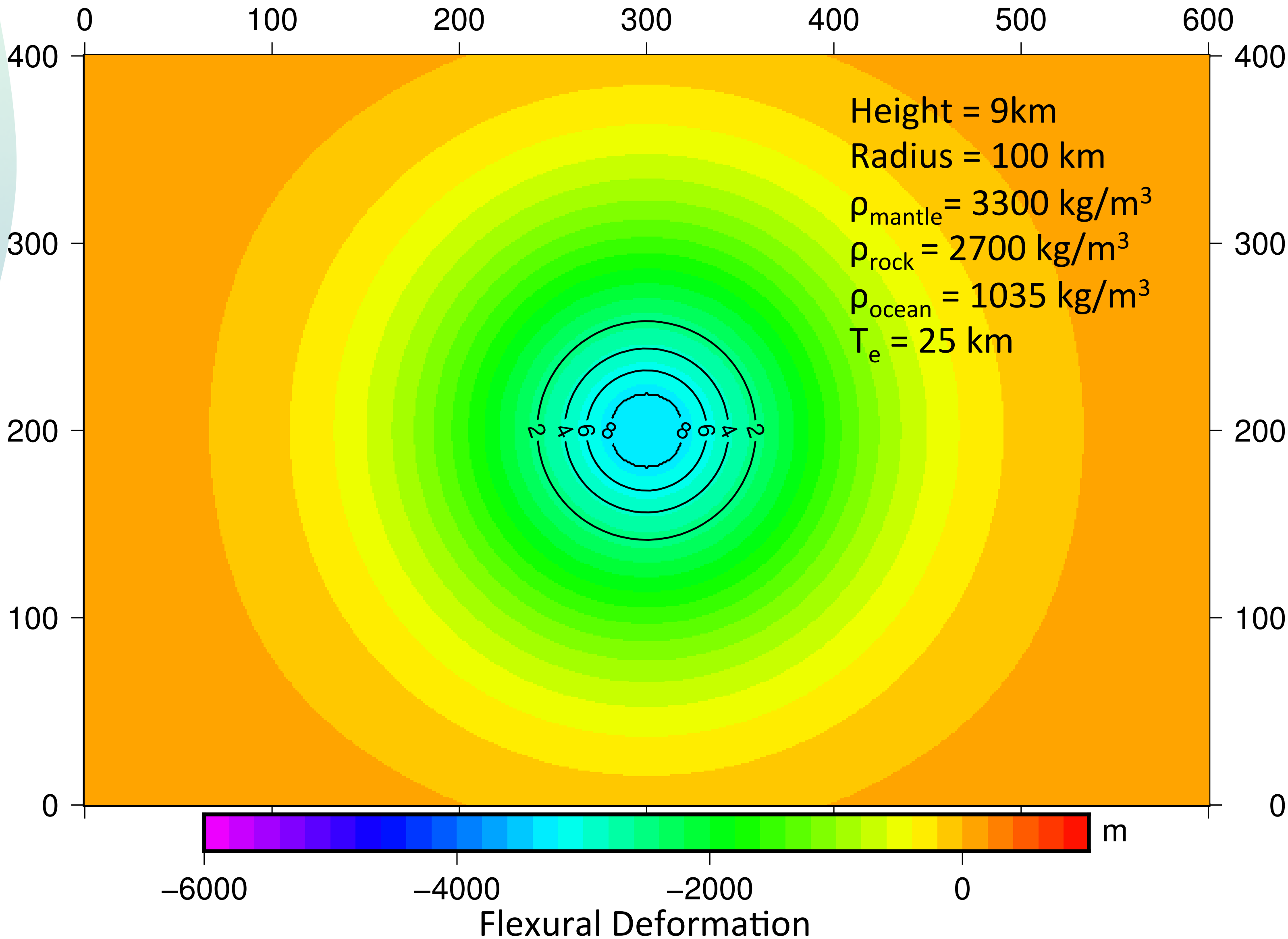
Gaussian Seamount



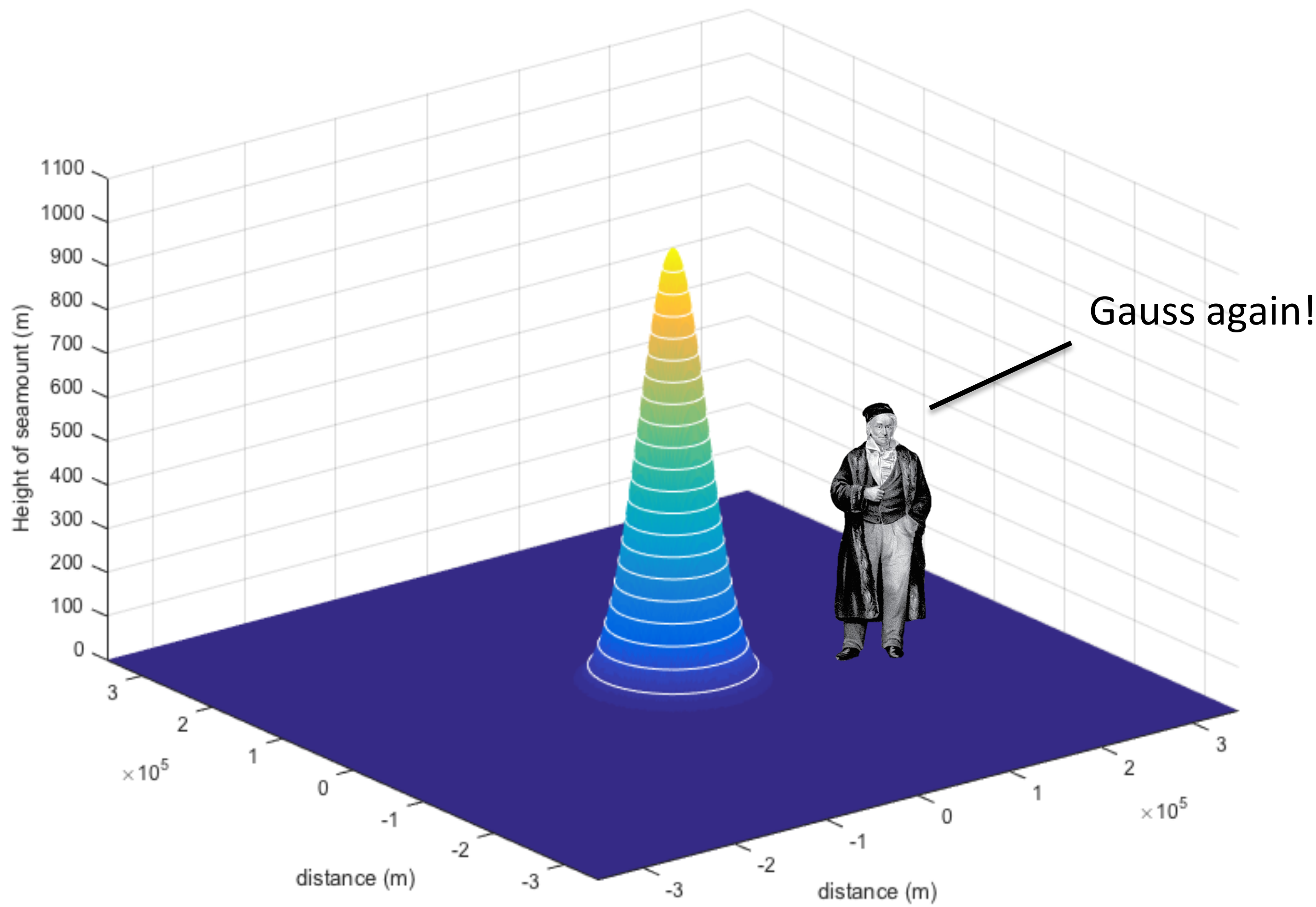
Gaussian Seamounts in GMT



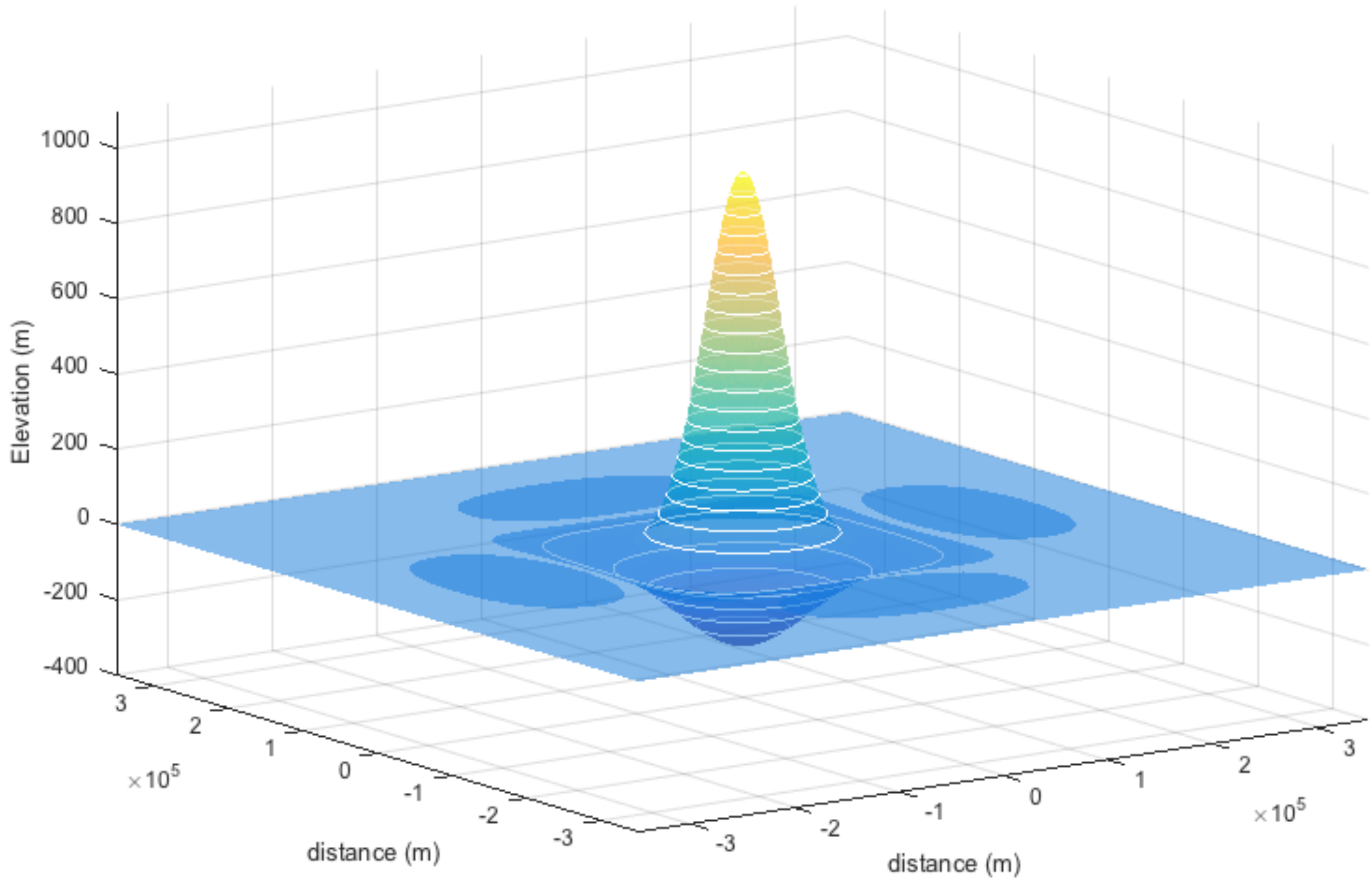
Gaussian Seamounts in GMT



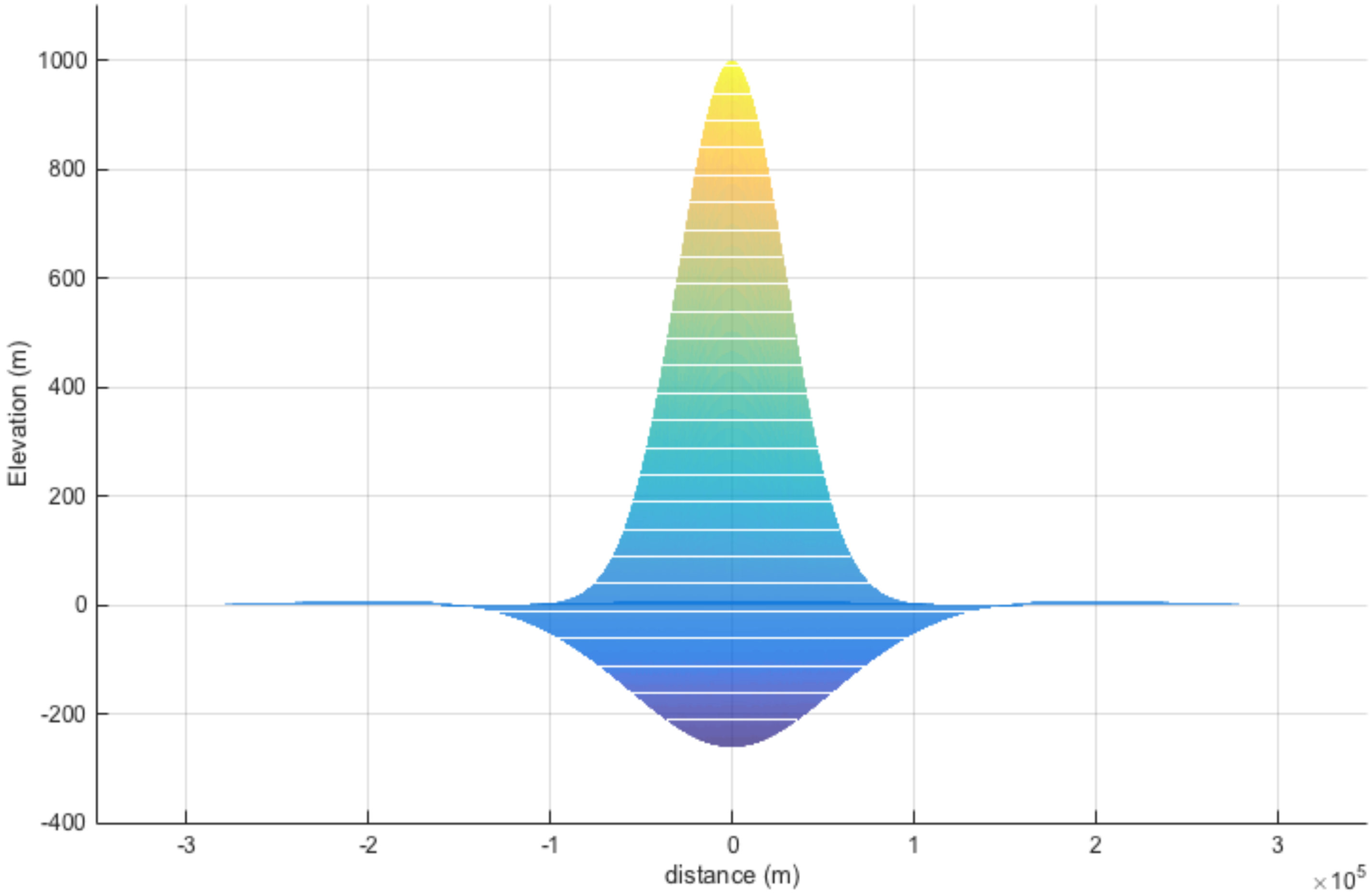
Matlab from first principles



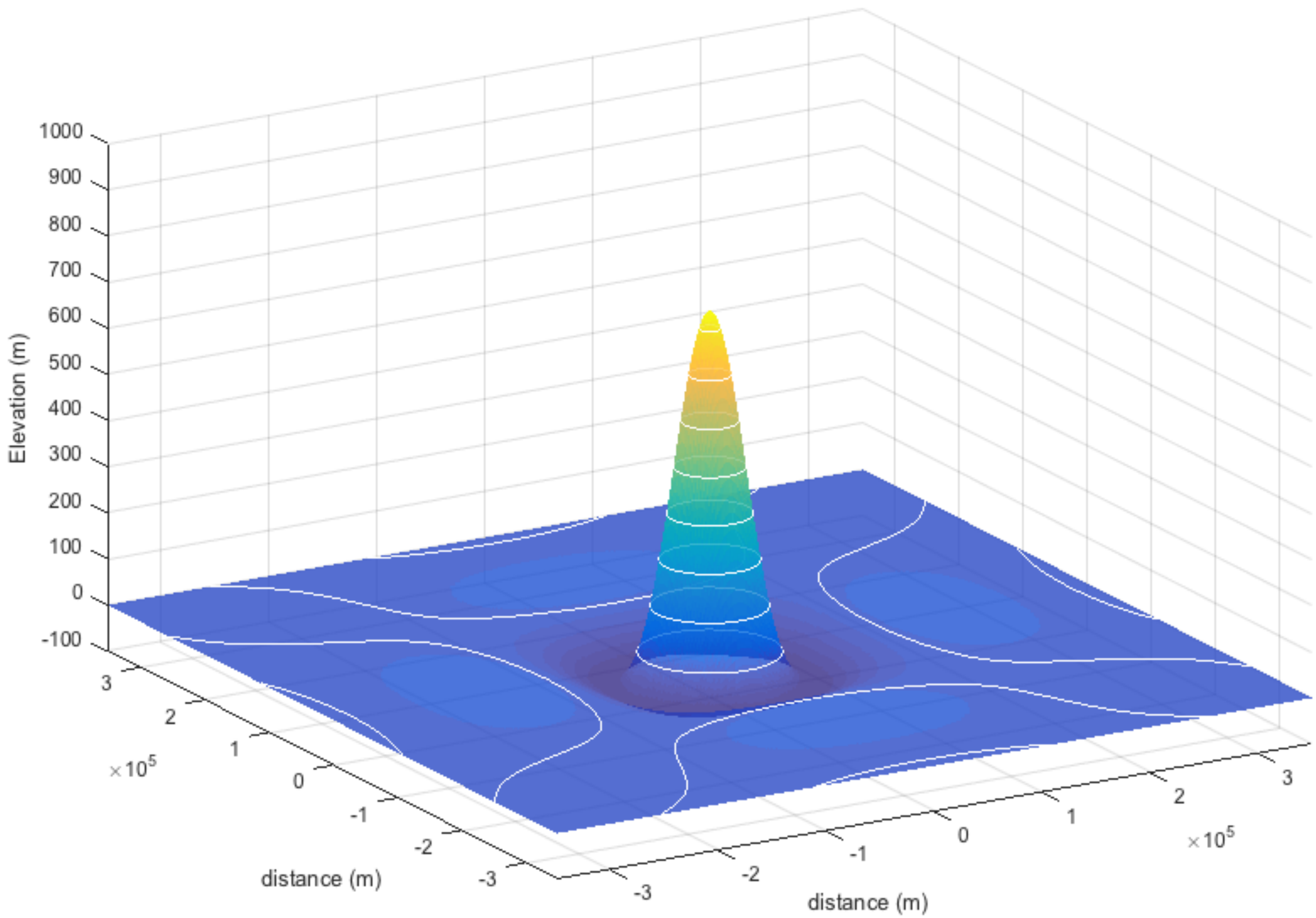
Flexure under load



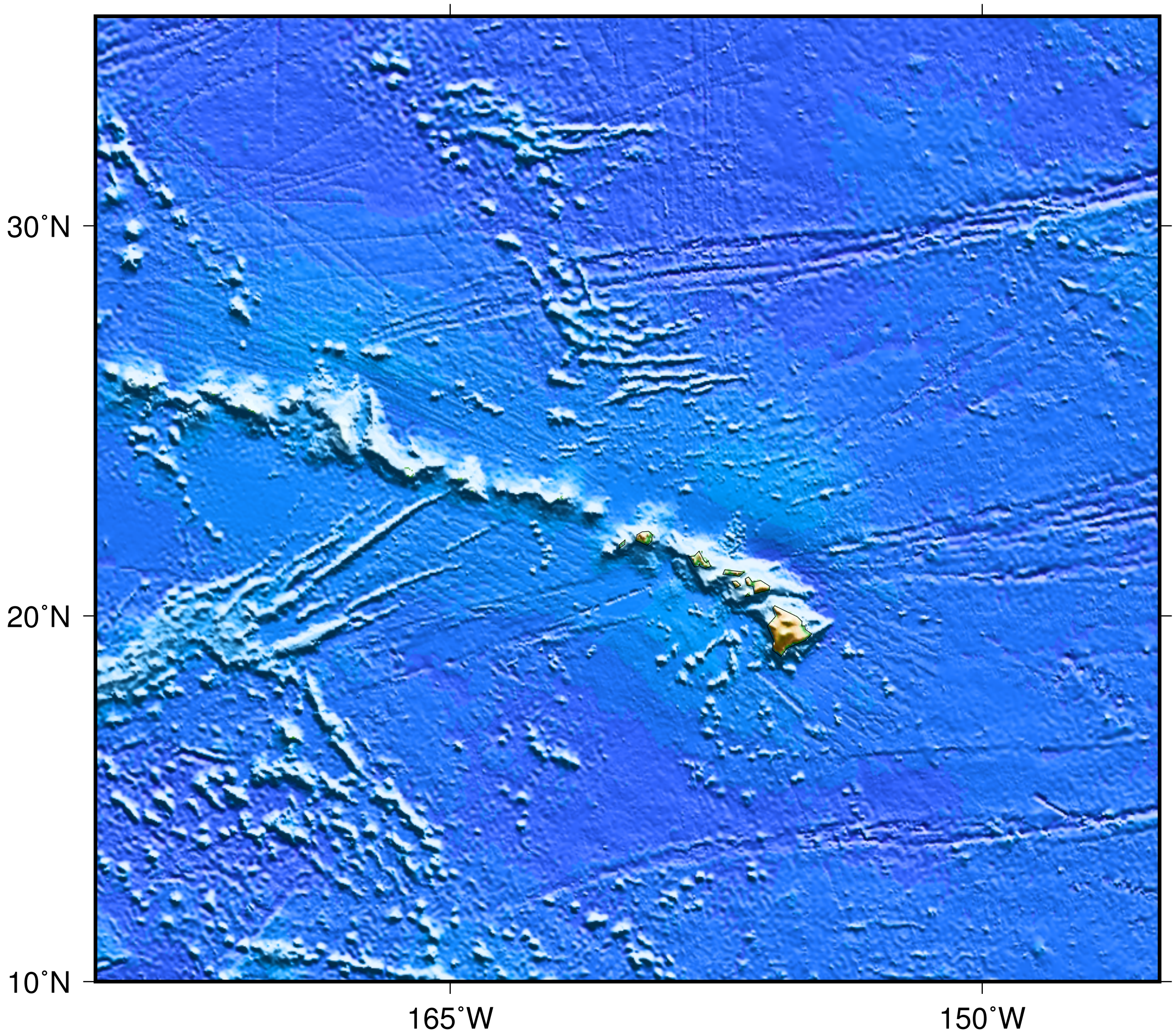
Flexure under load part II – The one tower



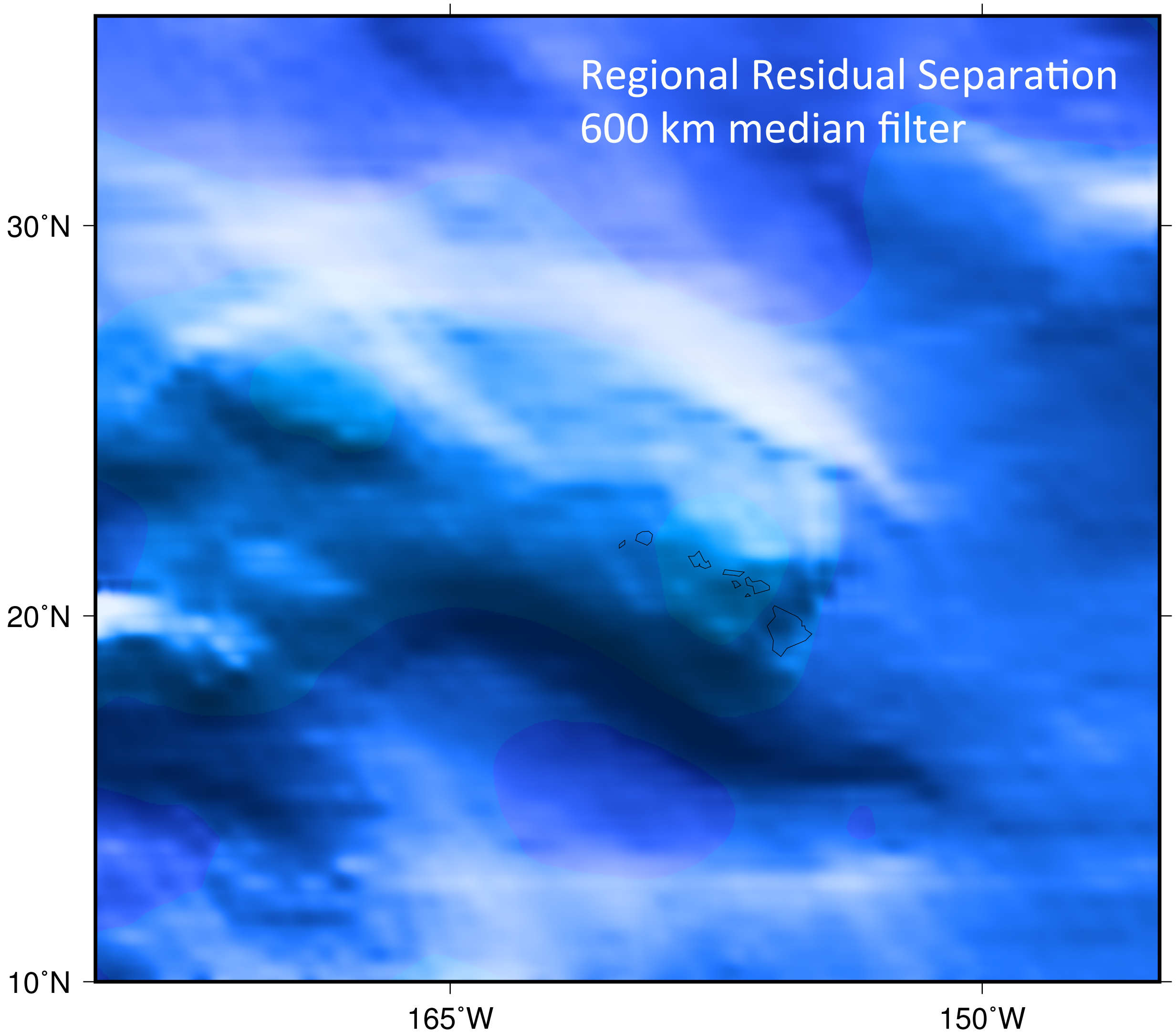
Overall elevation



Hawaiian Flexure



Hawaiian Flexure



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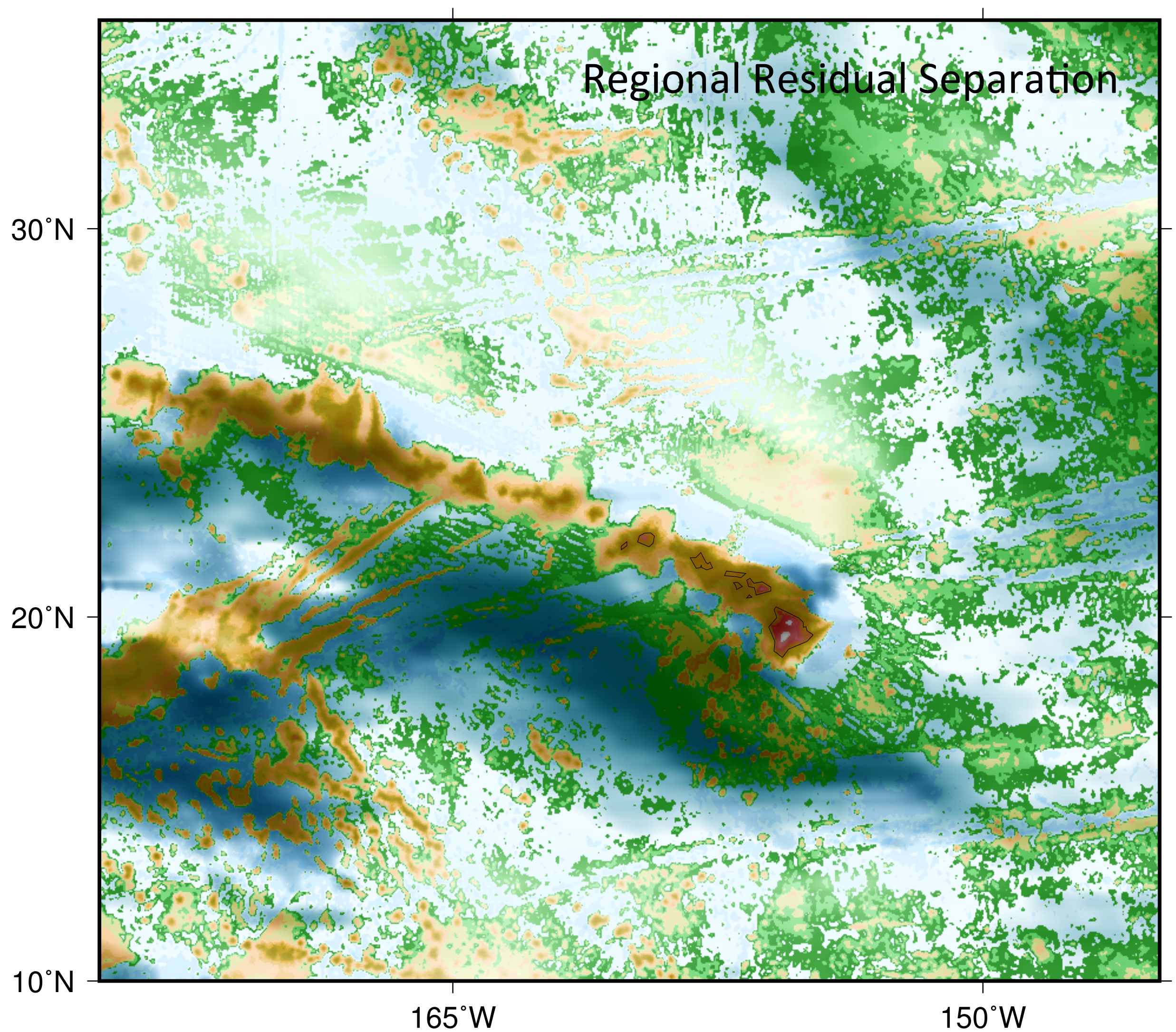

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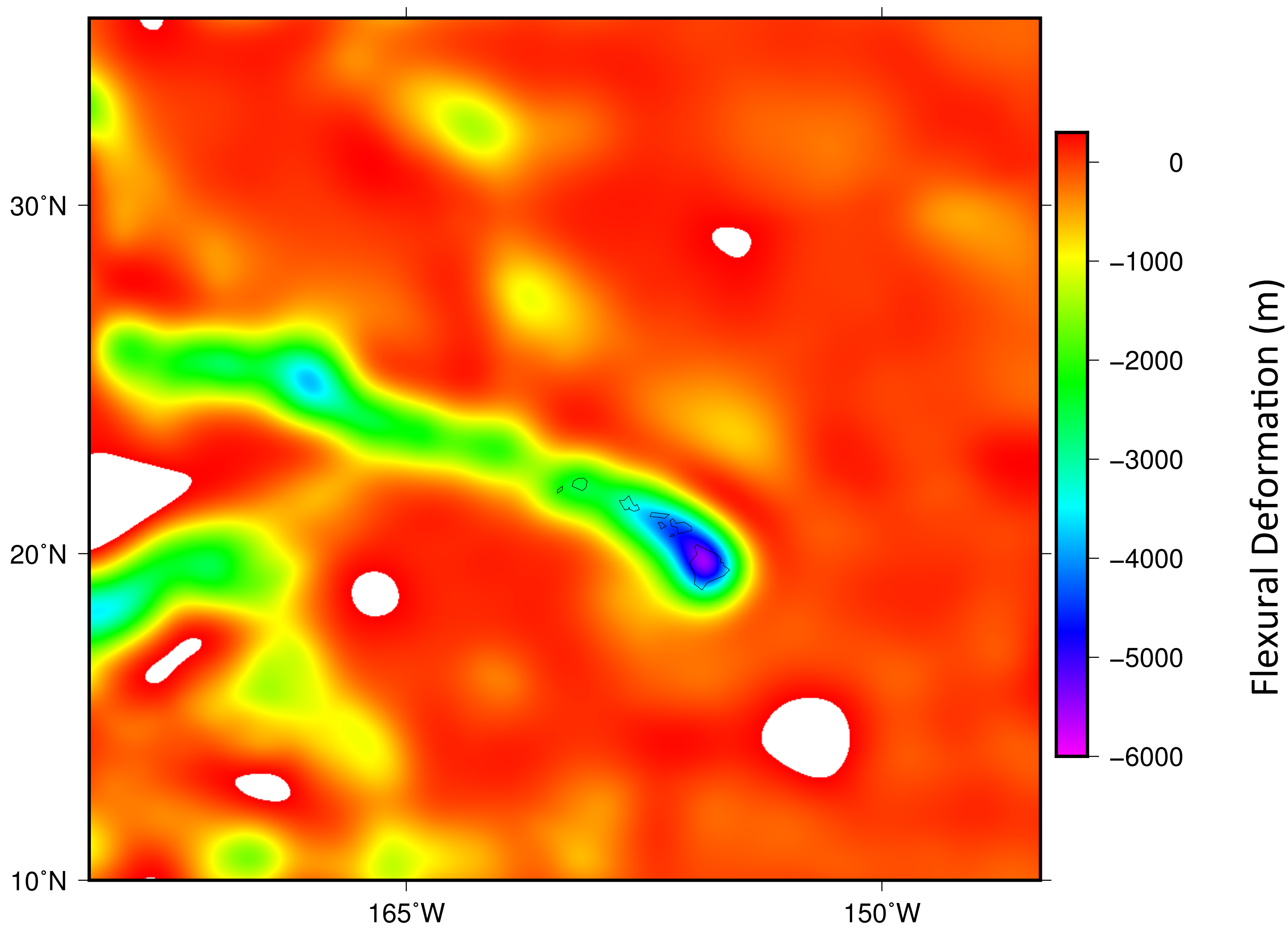
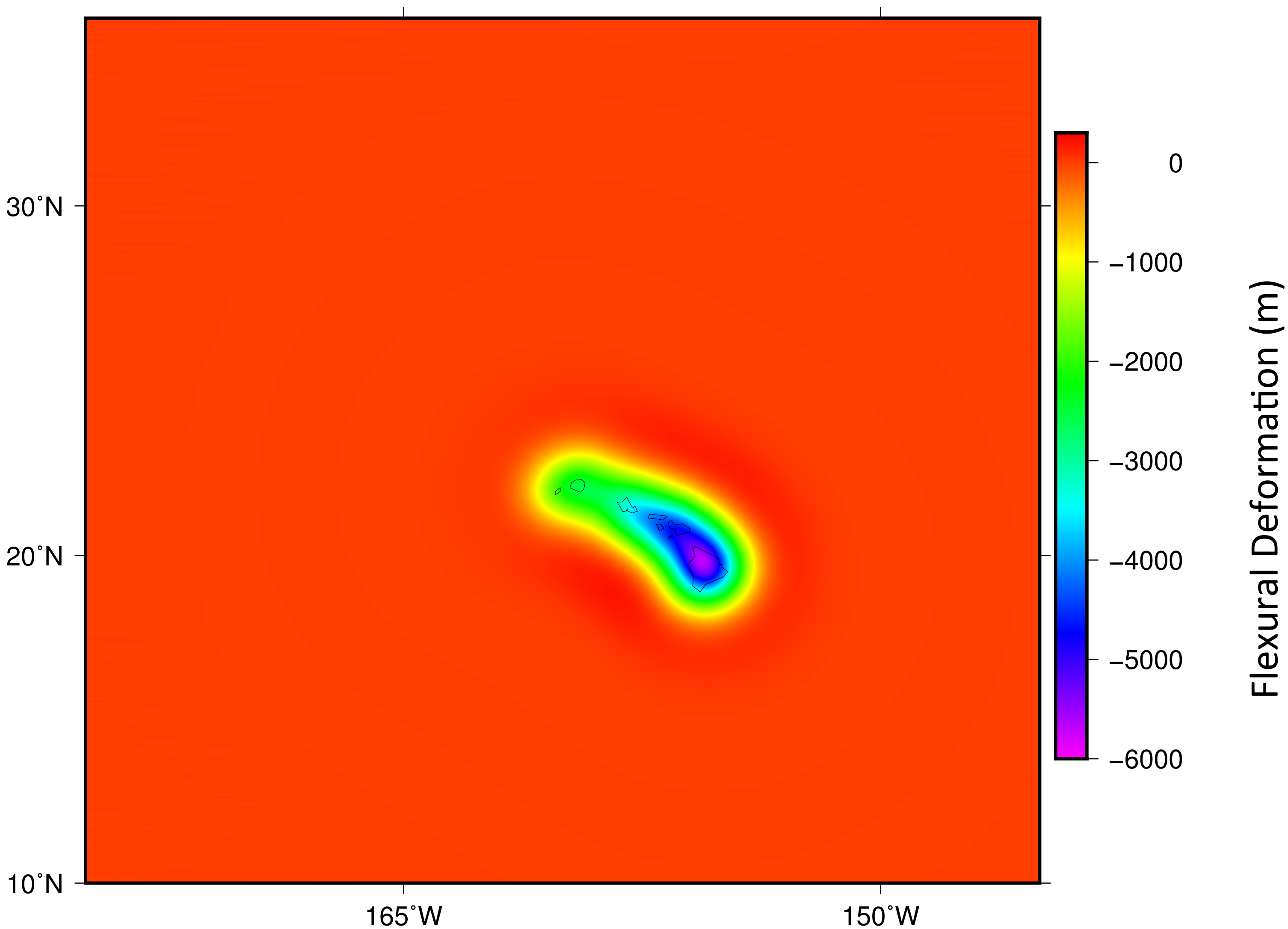
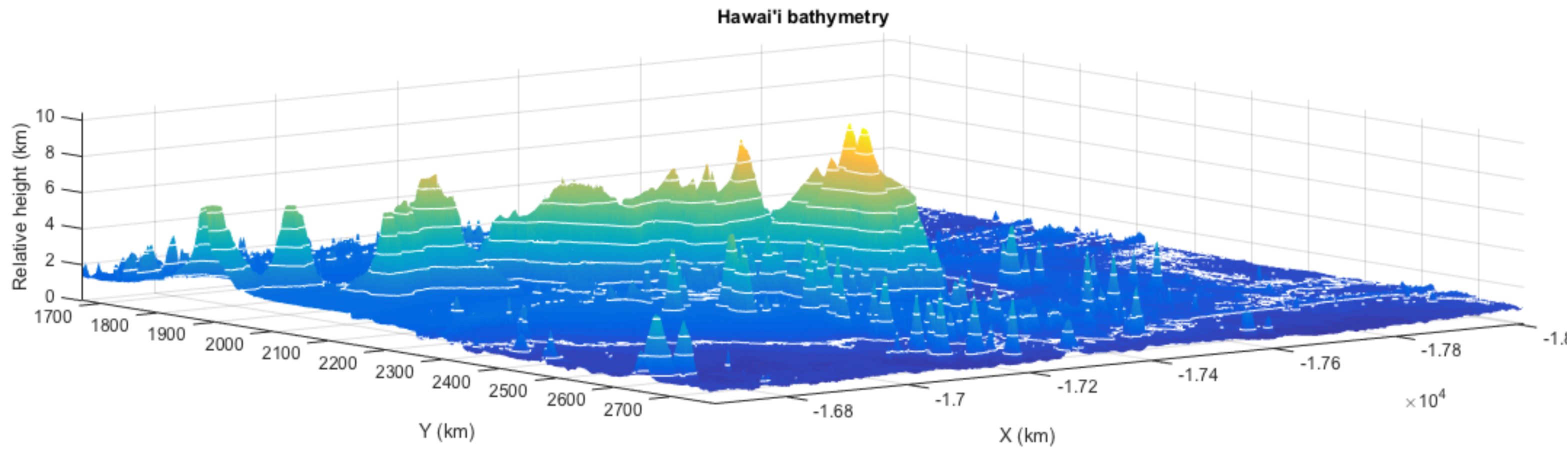
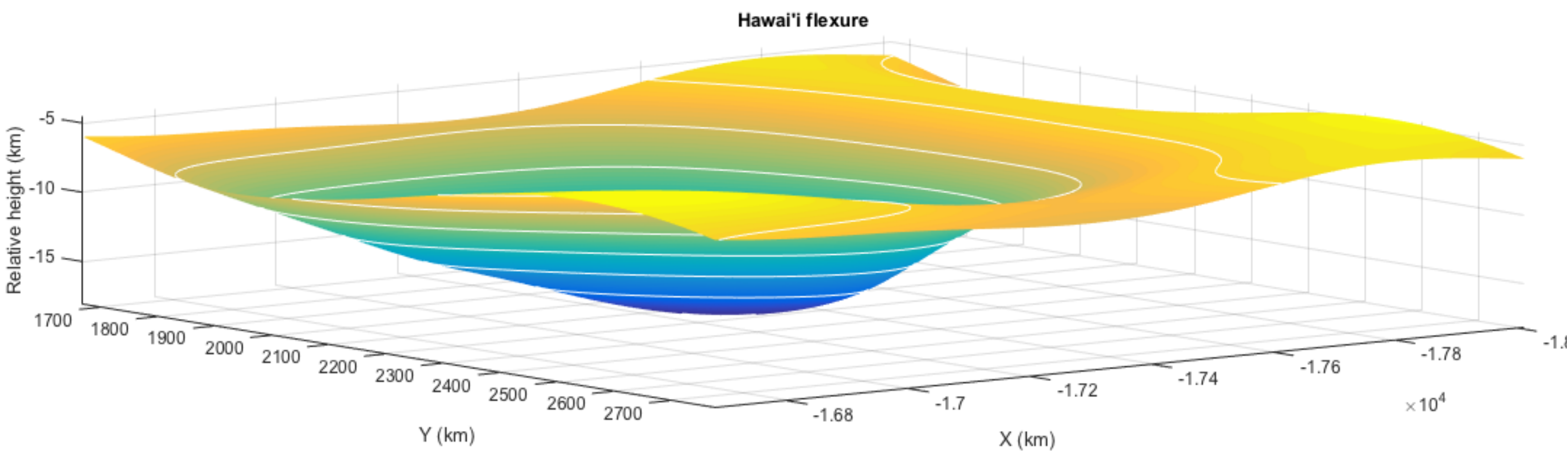


Hawaiian Flexure



Matlab from first principles





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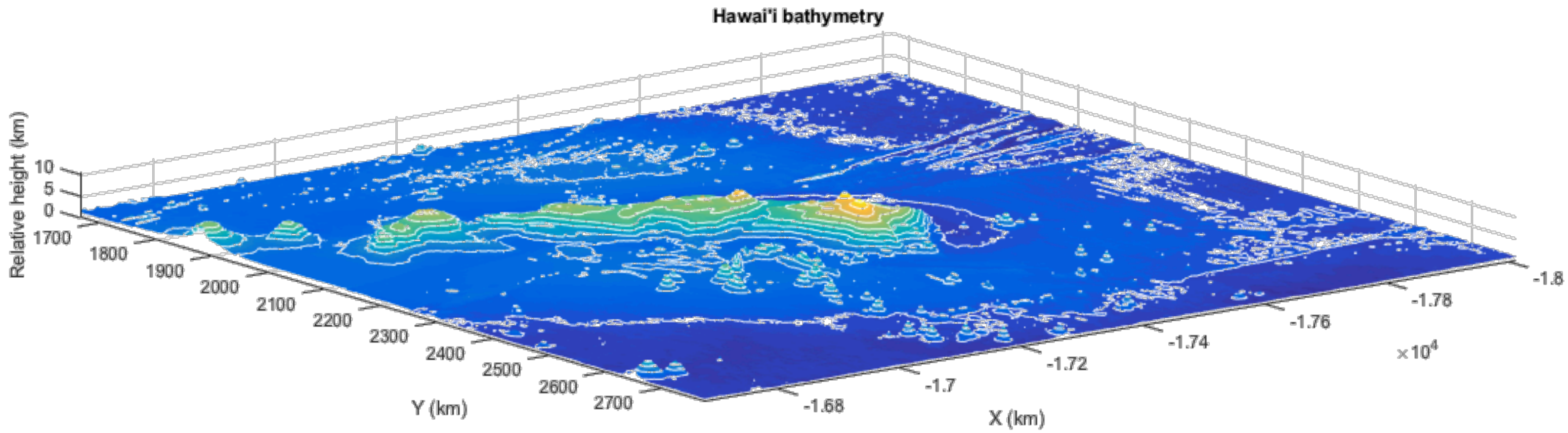

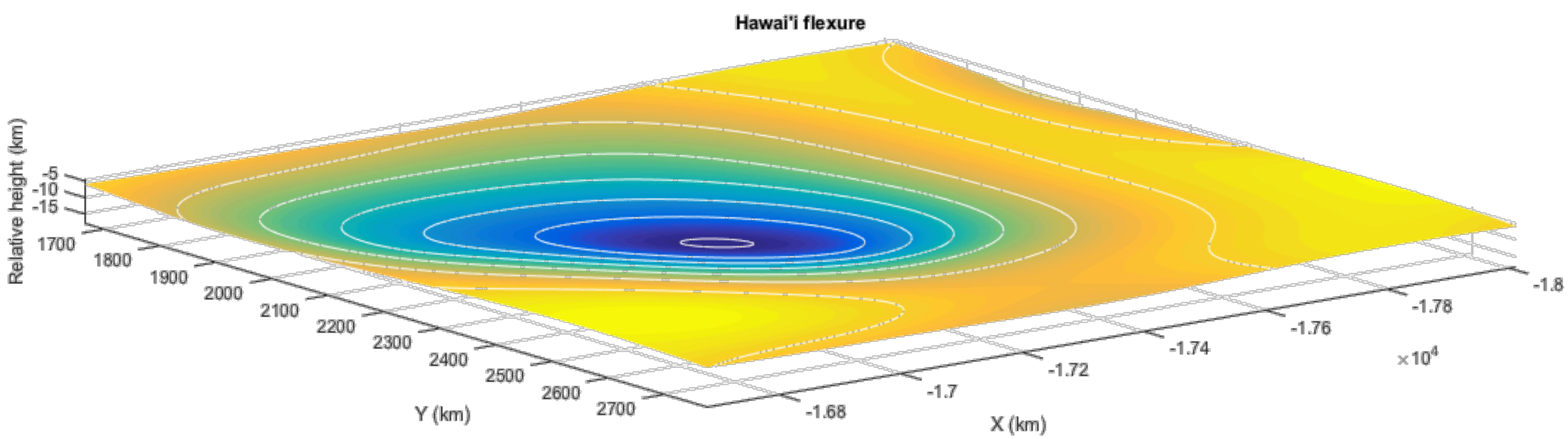

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