Universal Elastic Trench Profile

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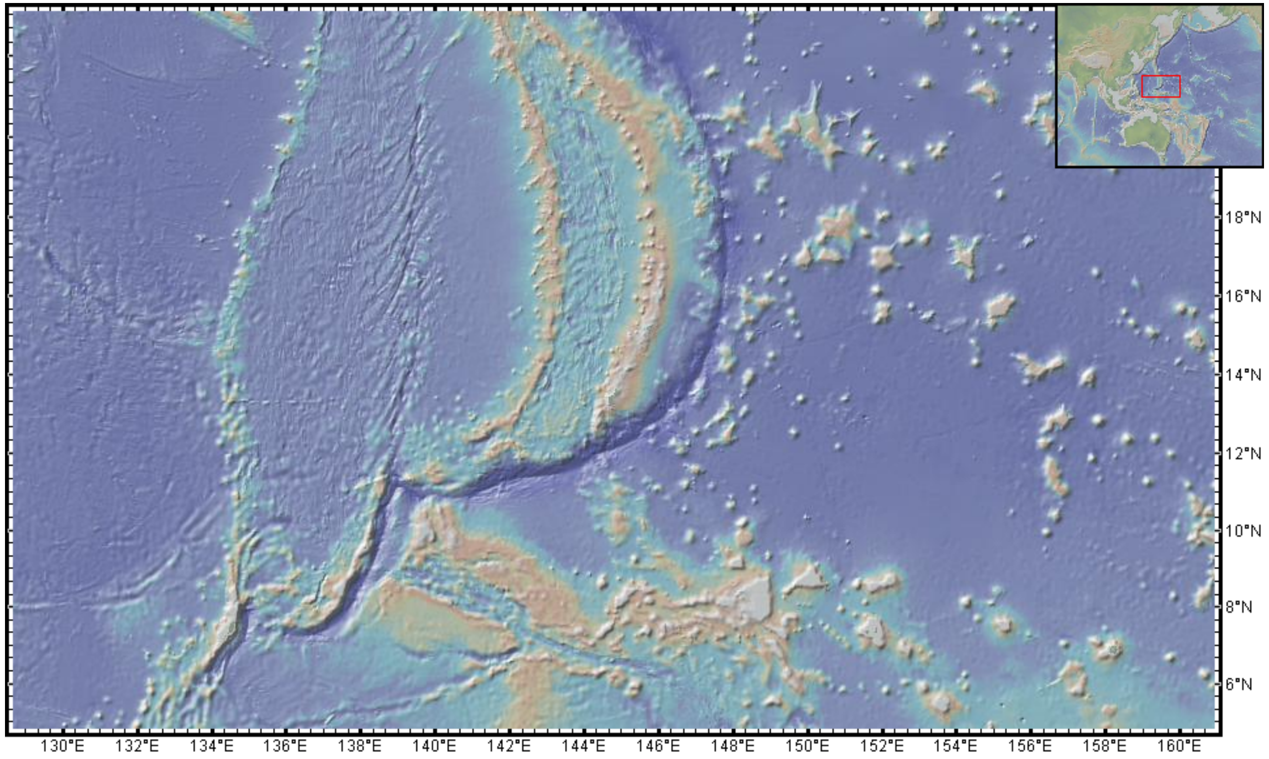
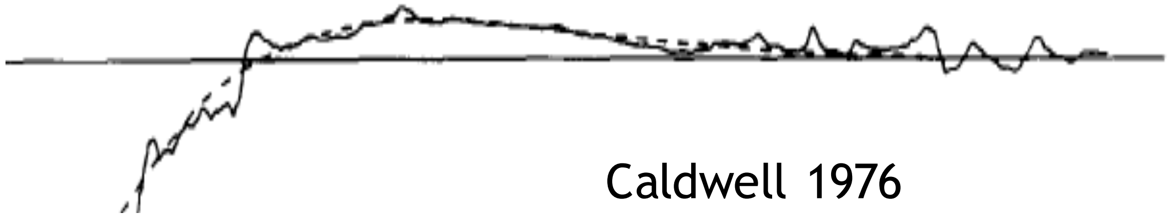
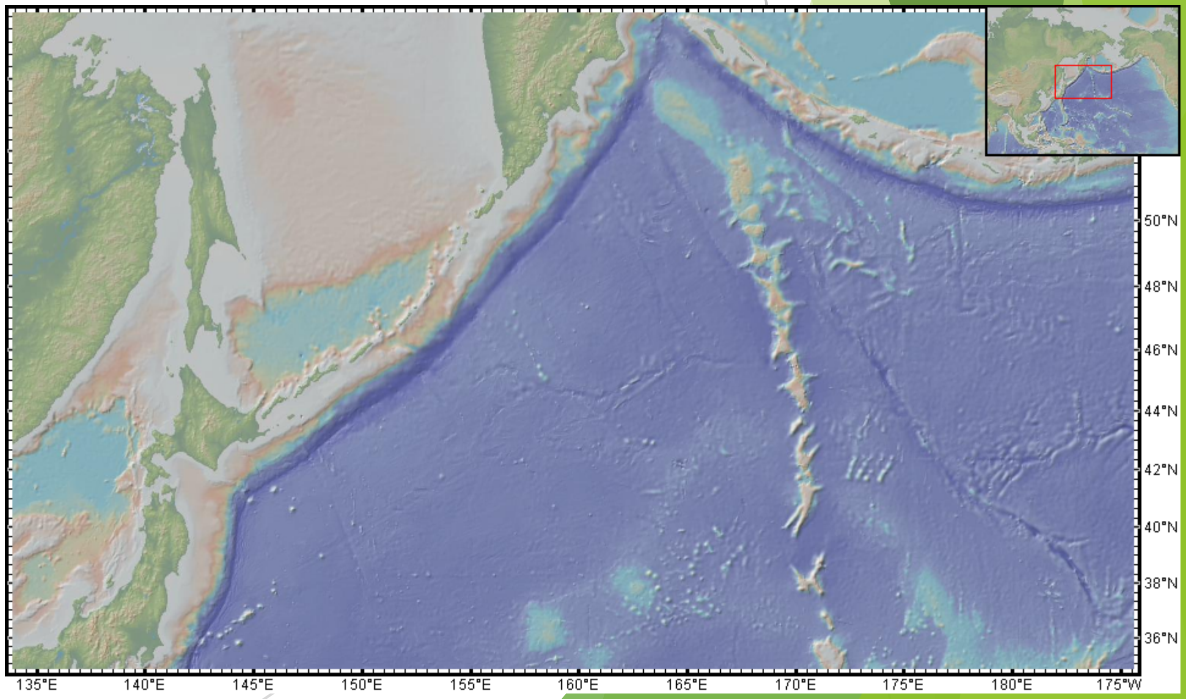
Oceanic Trenches





Caldwell 1976



Vertical Deflection

$$D\frac{d^4w}{dx^4} + S\frac{d^2w}{dx^2} + kw = 0; where\ k = (\rho_m - \rho_w)g$$

General solution assuming s = 0:

$$w(x) = e^{\frac{x}{\alpha}}\left[c_1 \cos\left(\frac{x}{\alpha}\right) + \ c_2 \sin\left(\frac{x}{\alpha}\right)\right] + \ e^{-\frac{x}{\alpha}}\left[c_3 \cos\left(\frac{x}{\alpha}\right) + \ c_4 \sin\left(\frac{x}{\alpha}\right)\right]$$

Vertical Deflection

$$w(x) = e^{\frac{x}{\alpha}}\left[c_1 \cos\left(\frac{x}{\alpha}\right) + c_2 \sin\left(\frac{x}{\alpha}\right)\right] + e^{-\frac{x}{\alpha}}\left[c_3 \cos\left(\frac{x}{\alpha}\right) + c_4 \sin\left(\frac{x}{\alpha}\right)\right]$$

Boundary Condition: $\lim_{x \to \infty} w(x) = 0$

$$e^{-\frac{x}{\alpha}} \to 0$$

$$0 = e^{\frac{x}{\alpha}} c_1 \cos\left(\frac{x}{\alpha}\right) + e^{\frac{x}{\alpha}} c_2 \sin\left(\frac{x}{\alpha}\right)$$

$$e^{\frac{x}{\alpha}} \to \infty$$

$$c_1 = c_2 = 0$$

$$w(x) = e^{-\frac{x}{\alpha}}\left[c_3 \cos\left(\frac{x}{\alpha}\right) + c_4 \sin\left(\frac{x}{\alpha}\right)\right]$$

Vertical Deflection

$$w(x) = e^{-\frac{x}{\alpha}}\left[c_3 \cos\left(\frac{x}{\alpha}\right) + c_4 \sin\left(\frac{x}{\alpha}\right)\right]$$

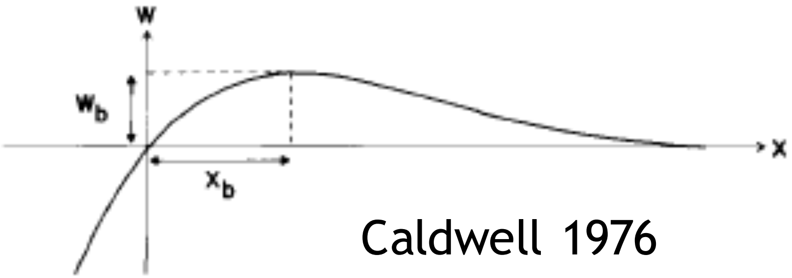
Boundary Condition: $w(0) = 0$

$$0 = 1\left[c_3(1) + c_4(0)\right]$$

$$c_3 = 0$$

$$\boldsymbol{w(x) = Ae^{-\frac{x}{\alpha}}\, sin\left(\frac{x}{\alpha}\right)}; \quad A = c_4$$

Maximum Deflection (w_b)



Caldwell 1976

Take the derivative and set it = 0

$$\frac{dw}{dx} = \frac{Ae^{-\frac{x}{\alpha}}}{\alpha}\left(\cos\left(\frac{x}{\alpha}\right) - \sin\left(\frac{x}{\alpha}\right)\right)$$

$$0 = \frac{Ae^{-\frac{x}{\alpha}}}{\alpha}\left(\cos\left(\frac{x}{\alpha}\right) - \sin\left(\frac{x}{\alpha}\right)\right)$$

$$\sin\left(\frac{x}{\alpha}\right) = \cos\left(\frac{x}{\alpha}\right)$$

$$x_b = \alpha \arctan(1) = \boldsymbol{\frac{\pi}{4}\alpha = x_b}$$

$$w(x_b) = Ae^{-\frac{\pi}{4}} \sin\left(\frac{\pi}{4}\right) = \boldsymbol{\frac{Ae^{-\frac{\pi}{4}}}{\sqrt{2}} = w_b}$$

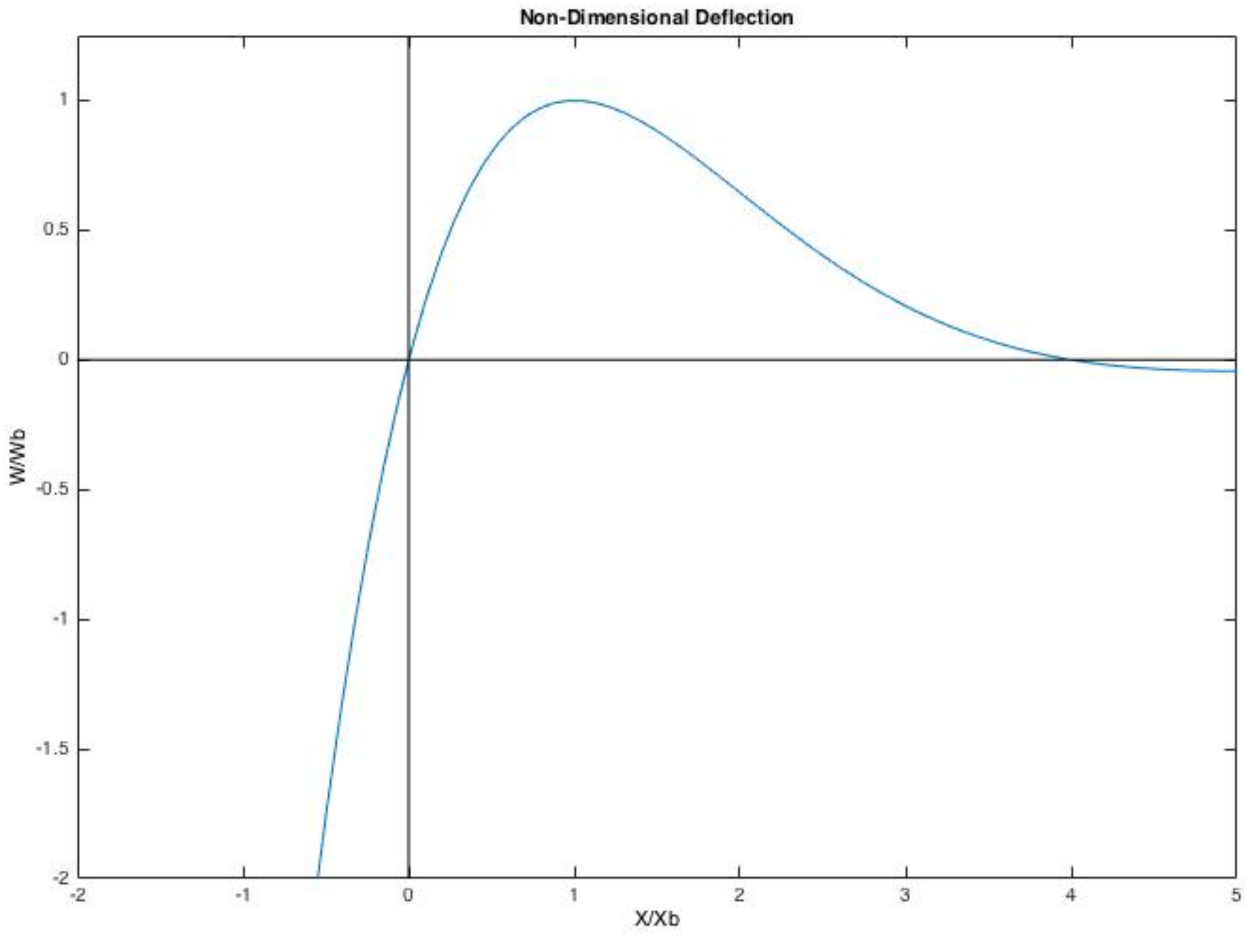
Non-Dimensional Deflection $(\overline{w})$

Putting things in non-dimensional terms

$$\bar{x} = \frac{x}{x_b} \text{ therefore } x = \frac{\pi\alpha}{4}\,\bar{x}$$

$$\bar{w} = \frac{w}{w_b} = \frac{Ae^{-\frac{x}{\alpha}}\,\sin\left(\frac{x}{\alpha}\right)}{\frac{Ae^{-\frac{\pi}{4}}}{\sqrt{2}}}$$

$$\bar{w} = \sqrt{2}\, e^{\frac{\pi}{4}(1-\ \bar{x})}\,\sin\left(\frac{\pi}{4}\bar{x}\right)$$



Bending Moment: $\bar{M} = \frac{\sqrt{2}\,\pi^2}{8} e^{\frac{\pi}{4}(1-\ \bar{x})} \cos\left(\frac{\pi}{4}\bar{x}\right)$

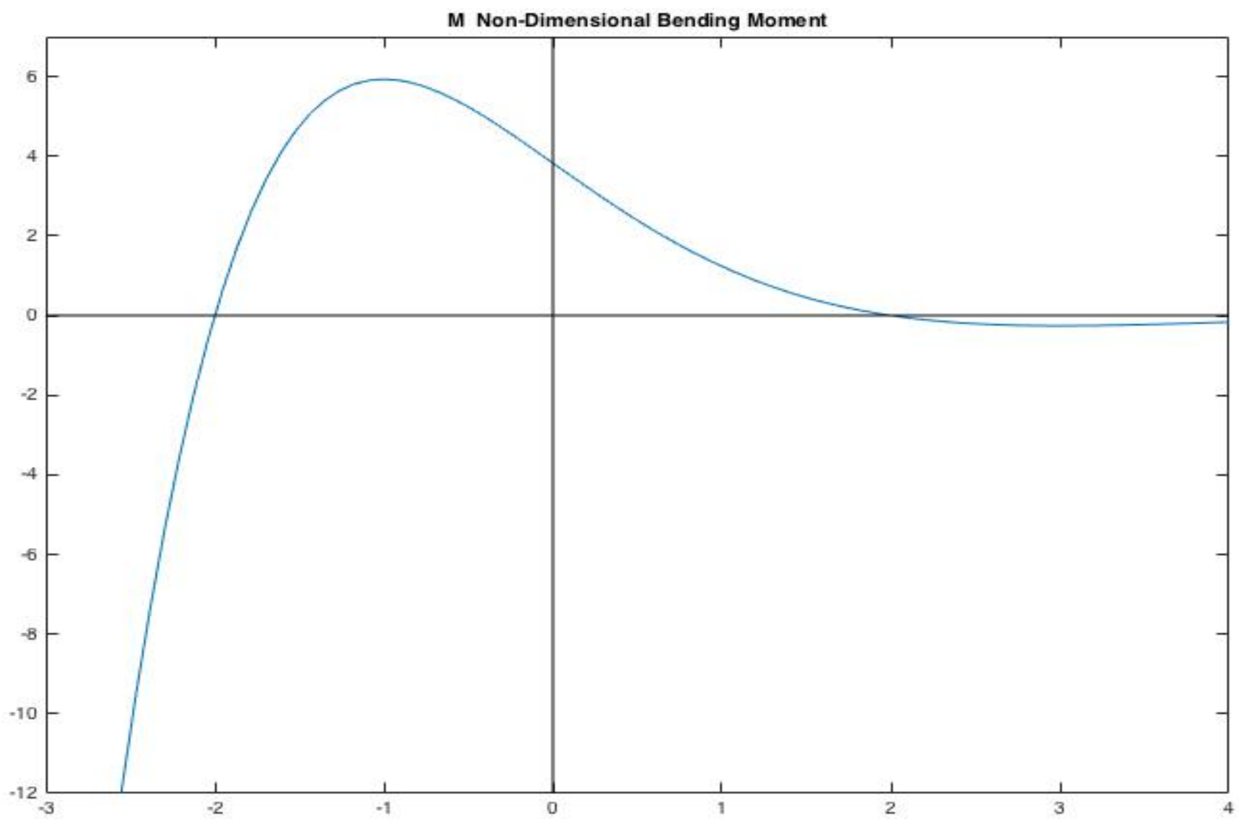
$$M = D\frac{d^2w}{dx^2}$$

$$\bar{M} = \frac{M{x_b}^2}{D\,w_b} = \frac{\frac{d^2w}{dx^2}{x_b}^2}{w_b}$$

$$\frac{dw}{dx} = \frac{Ae^{-\frac{x}{\alpha}}}{\alpha}\left(\cos\left(\frac{x}{\alpha}\right) - \sin\left(\frac{x}{\alpha}\right)\right)$$

$$\frac{d^2w}{dx^2} = -\frac{2Ae^{-\frac{x}{\alpha}}}{\alpha^2}\cos\left(\frac{x}{\alpha}\right)$$

$$\bar{M} = \frac{-\frac{2Ae^{-\frac{x}{\alpha}}}{\alpha^2}\cos\left(\frac{x}{\alpha}\right)\left(\frac{\pi}{4}\alpha\right)^2}{\frac{Ae^{-\frac{\pi}{4}}}{\sqrt{2}}} = \frac{\sqrt{2}\,\pi^2}{8} e^{\frac{\pi}{4}(1-\ \bar{x})} \cos\left(\frac{\pi}{4}\bar{x}\right)$$



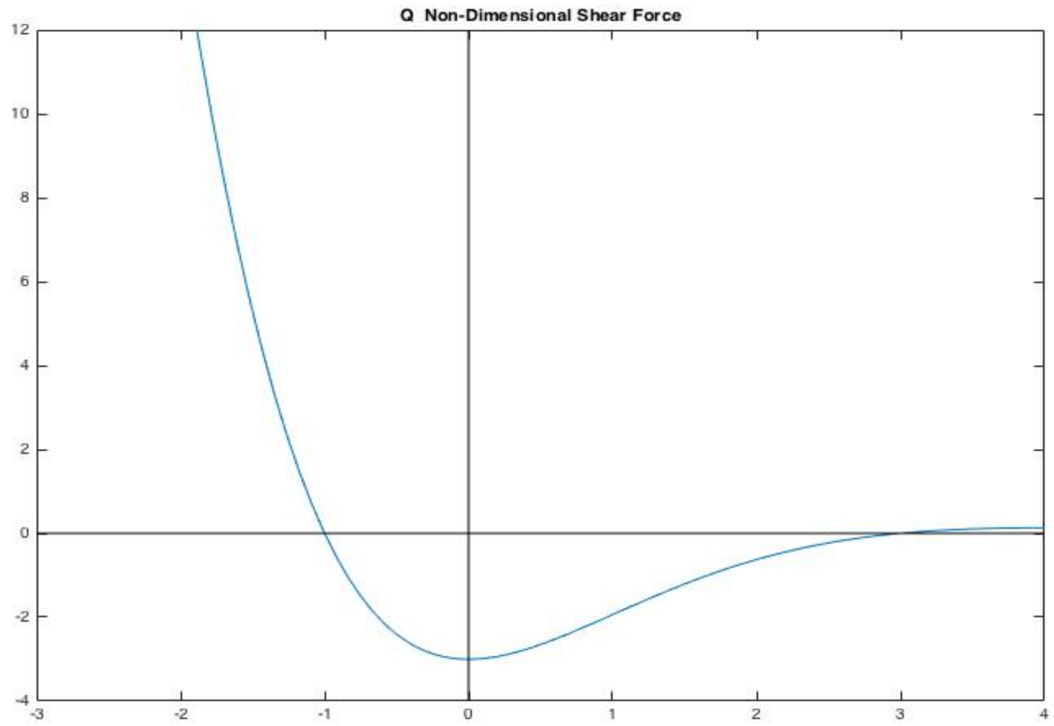
Shear Force: $\bar{Q} = -\frac{\sqrt{2}\,\pi^3}{32} e^{\frac{\pi}{4}(1-\bar{x})}\left(\cos\left(\frac{\pi}{4}\bar{x}\right) + \sin\left(\frac{\pi}{4}\bar{x}\right)\right)$

$$Q = \frac{dM}{dx} = D\frac{d^3w}{dx^3}$$

$$\bar{Q} = \frac{Q{x_b}^3}{D\,w_b} = \frac{\frac{d^3w}{dx^3}{x_b}^3}{w_b}$$

$$\frac{d^2w}{dx^2} = -\frac{2Ae^{-\frac{x}{\alpha}}}{\alpha^2}\cos\left(\frac{x}{\alpha}\right)$$

$$\frac{d^3w}{dx^3} = -\frac{2Ae^{-\frac{x}{\alpha}}}{\alpha^3}\left(\cos\left(\frac{x}{\alpha}\right) + \sin\left(\frac{x}{\alpha}\right)\right)$$



$$\bar{Q} = \frac{-\frac{2Ae^{-\frac{x}{\alpha}}}{\alpha^3}\left(\cos\left(\frac{x}{\alpha}\right)+\sin\left(\frac{x}{\alpha}\right)\right)\left(\frac{\pi}{4}\alpha\right)^3}{\frac{Ae^{-\frac{\pi}{4}}}{\sqrt{2}}} = -\frac{\sqrt{2}\,\pi^3}{32} e^{\frac{\pi}{4}(1-\bar{x})}\left(\cos\left(\frac{\pi}{4}\bar{x}\right) + \sin\left(\frac{\pi}{4}\bar{x}\right)\right)$$

If S ≠ 0

$$\varepsilon = \frac{S}{2(kD)^{\frac{1}{2}}}$$

$$w = A\ \sin(\frac{x}{\alpha}(1+\ \varepsilon)^{\frac{1}{2}})\, e^{-\frac{x}{\alpha}(1-\ \varepsilon)^{\frac{1}{2}}}$$

$$x_b = \frac{\propto}{(1+\ \varepsilon)^{\frac{1}{2}}}\tan^{-1}(\frac{1+\ \varepsilon}{1-\ \varepsilon})^{\frac{1}{2}}$$

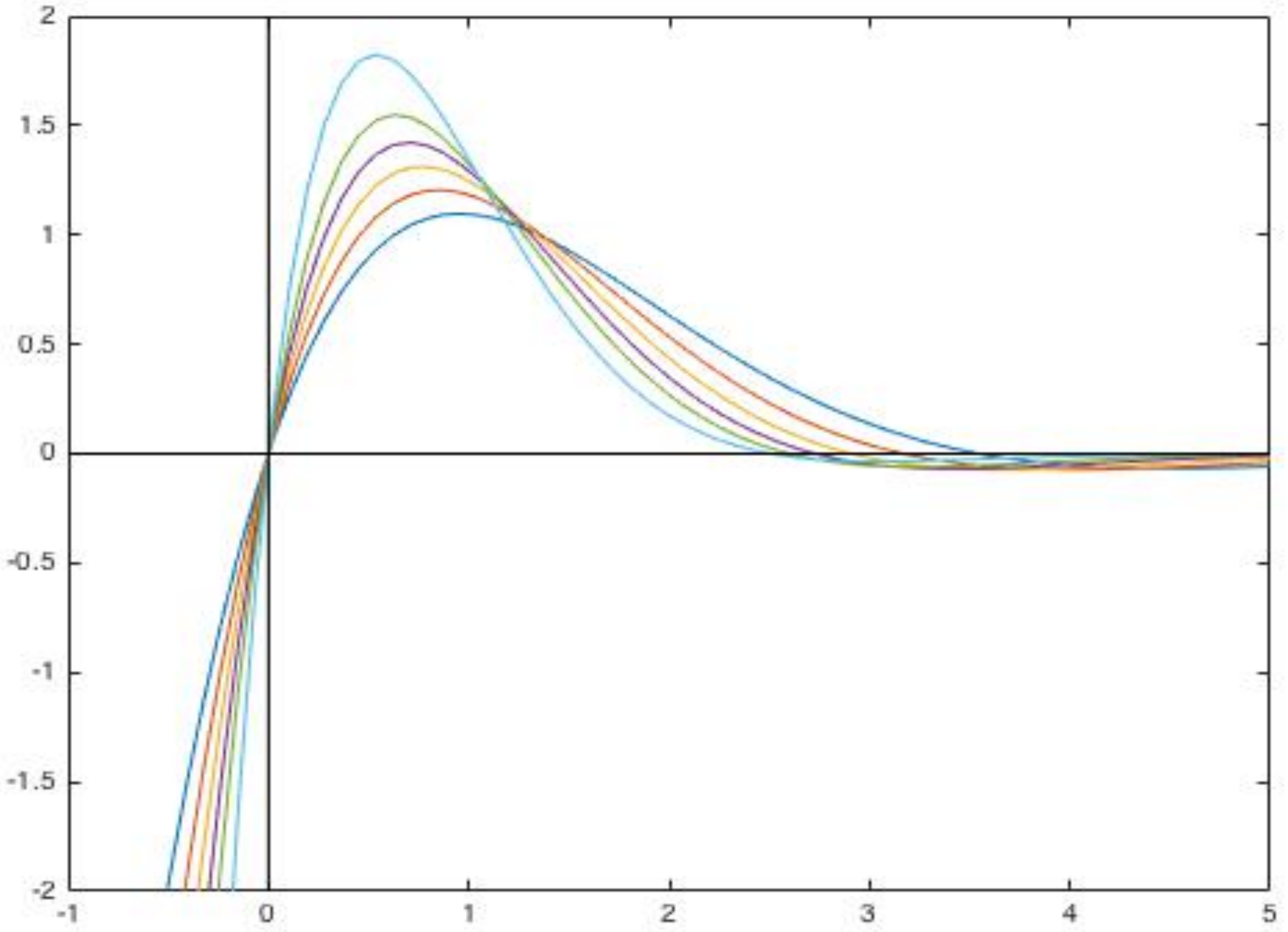
$$w_b = \frac{A(1+\varepsilon)^{\frac{1}{2}}}{2^{\frac{1}{2}}}e^{-\frac{x_b}{\propto}(1-\ \varepsilon)^{\frac{1}{2}}}$$

Assuming S = 0

$$w(x) = Ae^{-\frac{x}{\alpha}}\ \sin\left(\frac{x}{\alpha}\right)$$

$$x_b = \frac{\pi}{4}\alpha$$

$$w_b = \ sin\frac{Ae^{-\frac{\pi}{4}}}{\sqrt{2}}$$


2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5

Comparison: Theoretical Deflection Curve and Observed Bathymetric Profiles

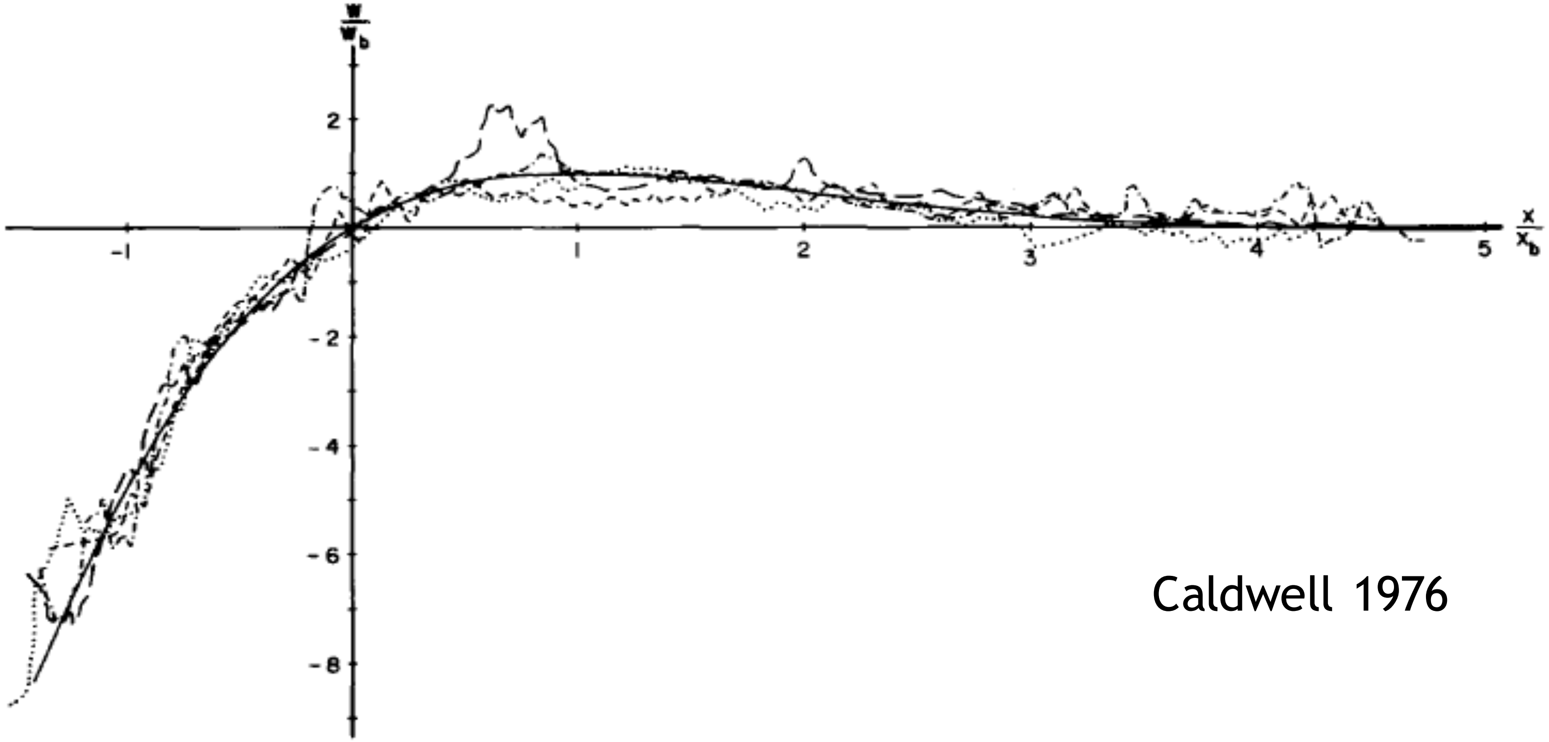


Fig. 6. The solid line is the universal deflection curve and the broken lines are the corrected and normalized bathymetric profiles. — — — is the Mariana profile (data from Scan 5 cruise and [19]), — · — · is the Bonin profile (Hunt 3 and Aries 7 cruises), · · · · · is the Kuril profile (Zetes 2 cruise and [18]), and - - - - - is the central Aleutian profile (Seamap 13 cruise and [17]).

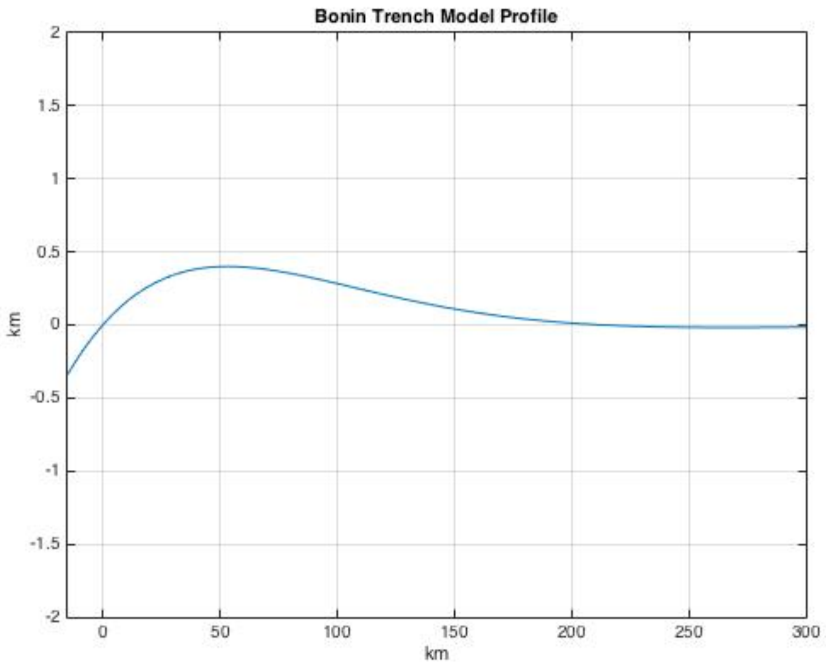
Caldwell 1976

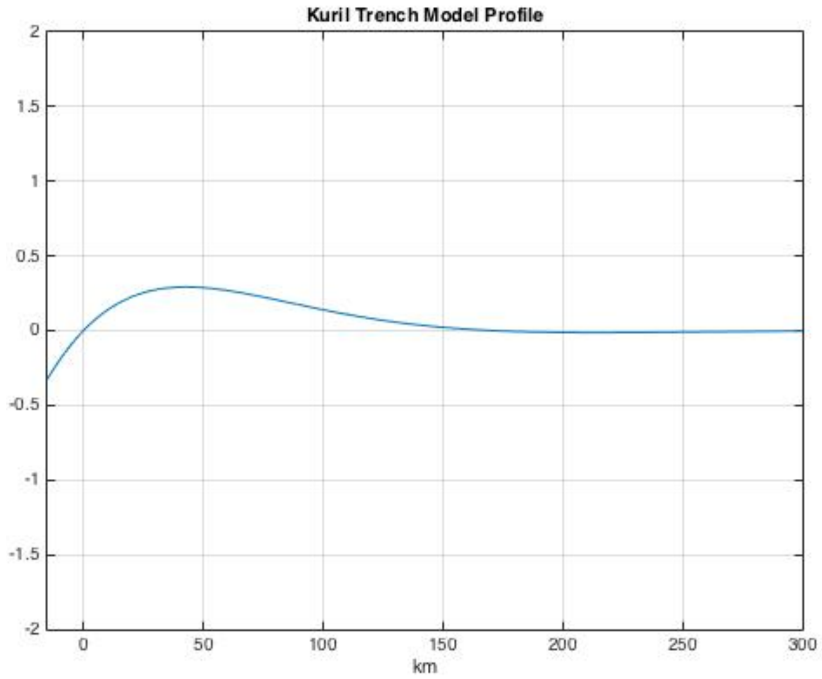
TABLE 1

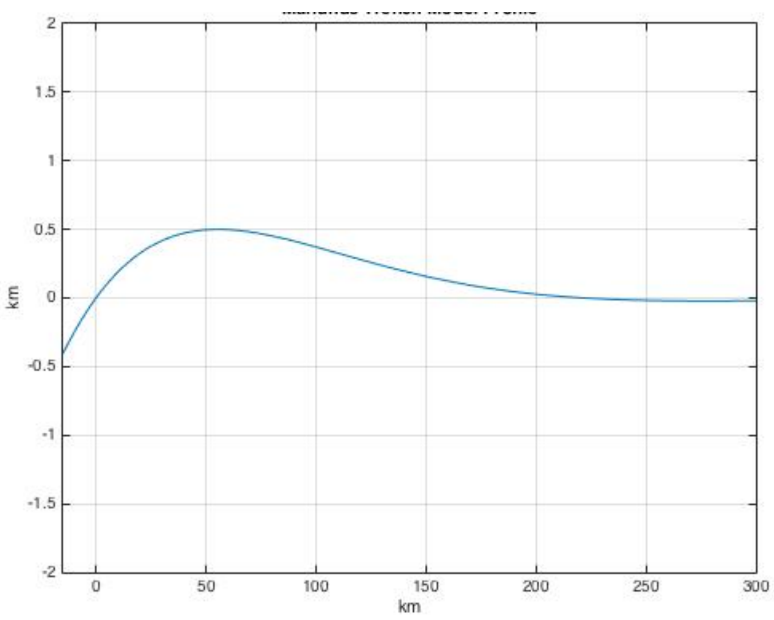
Trench parameters

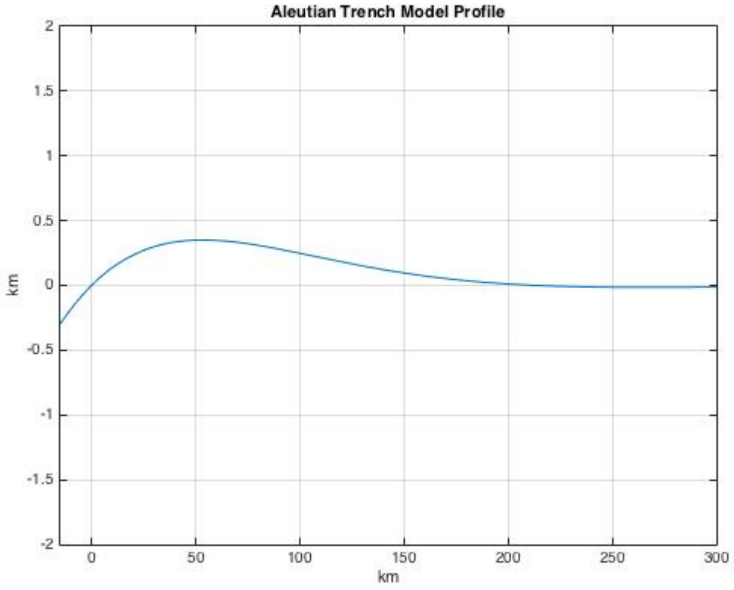
Trench name	Mariana	Bonin	Aleutian	Kuril
x_b (km)	55	53	53	42
w_b (km)	0.50	0.40	0.35	0.28
Flexural rigidity (dyne cm)	1.4×10^{30}	1.2×10^{30}	1.2×10^{30}	0.5×10^{30}
Lithospheric thickness (km)	29	28	28	20
Trench axis distance from point of zero deflection (km)	71	70	58	68
Maximum bending stress (kbar)	9.2	7.6	6.6	6.2
Point of maximum bending stress (in km) seaward of trench axis	20	20	8	30
Shear force at trench axis (dyne/cm)	1.0×10^{15}	9.8×10^{14}	3.1×10^{14}	1.4×10^{15}
Bending moment at trench axis (dyne)	1.3×10^{22}	9.5×10^{21}	8.9×10^{21}	2.8×10^{21}
Shear force at point where bending moment equals zero (dyne/cm)	8.0×10^{15}	5.3×10^{15}	4.6×10^{15}	3.2×10^{15}
Point of zero bending moment (in km) landward of trench axis	50	40	50	20

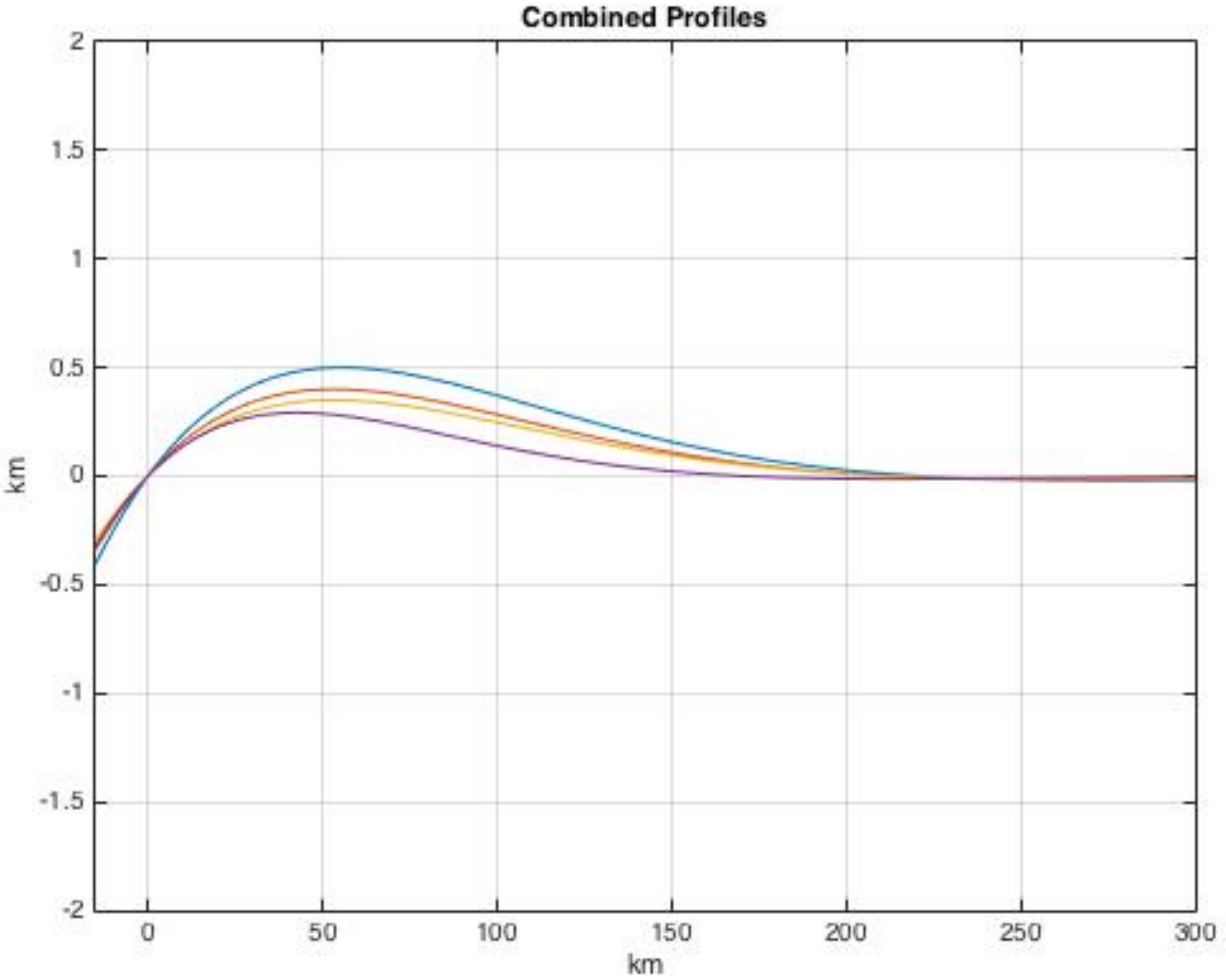
Caldwell 1976


Bonin Trench Model Profile
km
km


Kuril Trench Model Profile
km


km
km


Aleutian Trench Model Profile
km
km


Combined Profiles
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
km
0
50
100
150
200
250
300
km

Conclusions

- The model deflection profile is based on vertical forces and bending moments and assumes zero horizontal forces.
- The profiles produced by a universal thin elastic plate model approximate the shape and amplitude of the observed profiles across several trenches
- Conclusions
  1. Lithosphere behaves elastically at a trench
  2. Horizontal forces are negligible

Limitations

- Model profiles deviate from observed profiles with younger plates

- Focal mechanisms indicate horizontal forces are acting on the descending plate

- X_b and W_b weren't measured directly; determined by finding the combination that best fit the observed profiles

- Investigation omits misfit values

References

- Caldwell, J.G. & Turcotte, D.L. (1976). On the Applicability of a Universal Elastic Trench Profile. *Earth and Planetary Science Letters*, 31.

- Turcotte, D., & Schubert, G. (2010). Elasticity and Flexure. *Geodynamics* (Third ed.). Cambridge University Press.