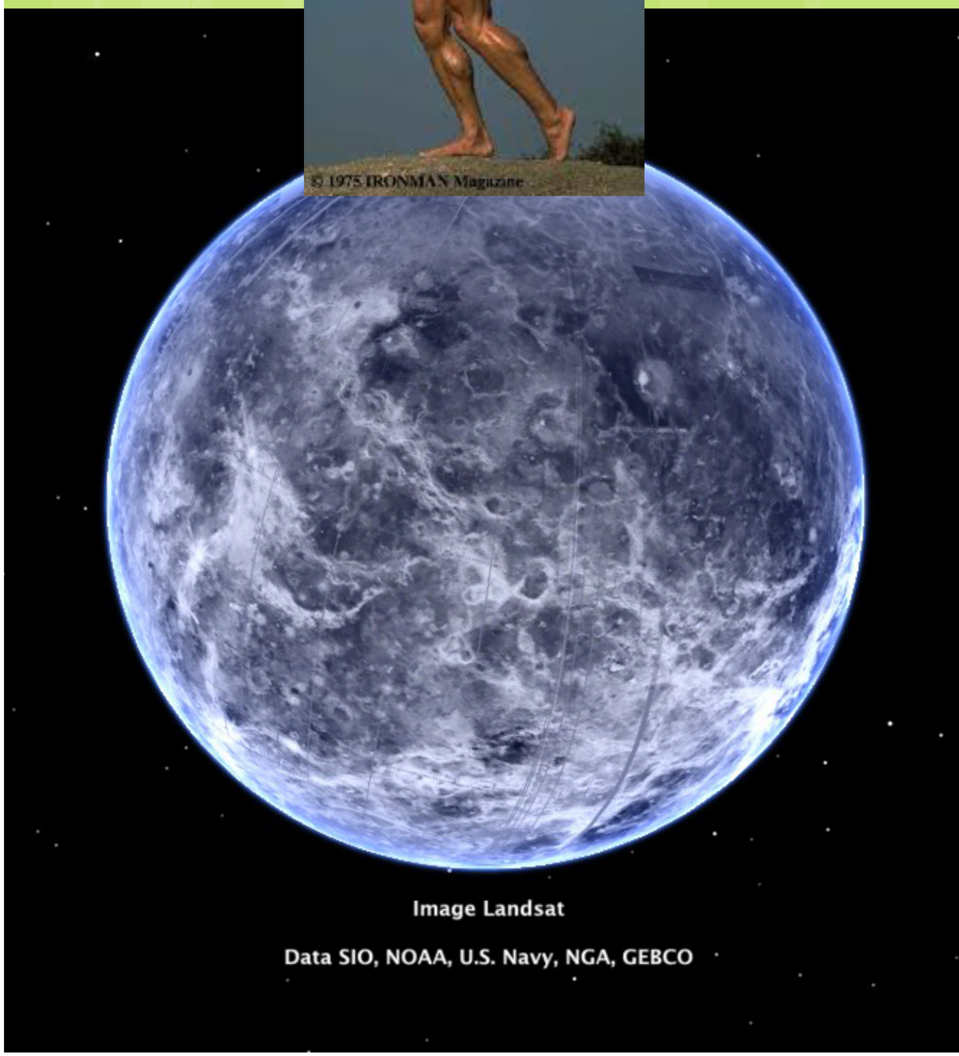


Flexure on Venus

Shelby Jones and
Patrick O'Shea

Objectives

- Discuss flexure on Venus and why it is important
  - Provide a general understanding of flexure on Venus
  - Speak to how flexure is modeled
- Derive equations 2 and 10 in Johnson and Sandwell [1994]
- Explain the findings of this paper with respect to the geothermal gradient

Fun Facts about Venus

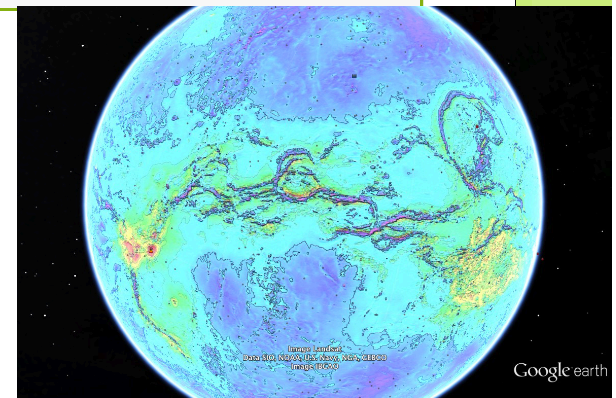
- Venus is very similar to earth
- A day on Venus lasts longer than a year on Venus
  - It takes 243 days for Venus to rotate once, but only 225 days for Venus to orbit the sun
  - And it rotates counterclockwise
- Atmospheric pressure on Venus is 92 times our own
  - Makes it so that there are now small impact craters
- Venus is the hottest planet
  - Average temperature is 492 C, this is because of an atmosphere of 96.5% CO_2

Flexure on Venus

- Flexure is important because it helps you understand how thickness and strength of the lithosphere varies spatially and temporally
- The idea that Venus experiences lithospheric flexure is a new idea
  - First identified in the early 90s thanks to Pioneer altimetry data (Solomon & Head 1990)

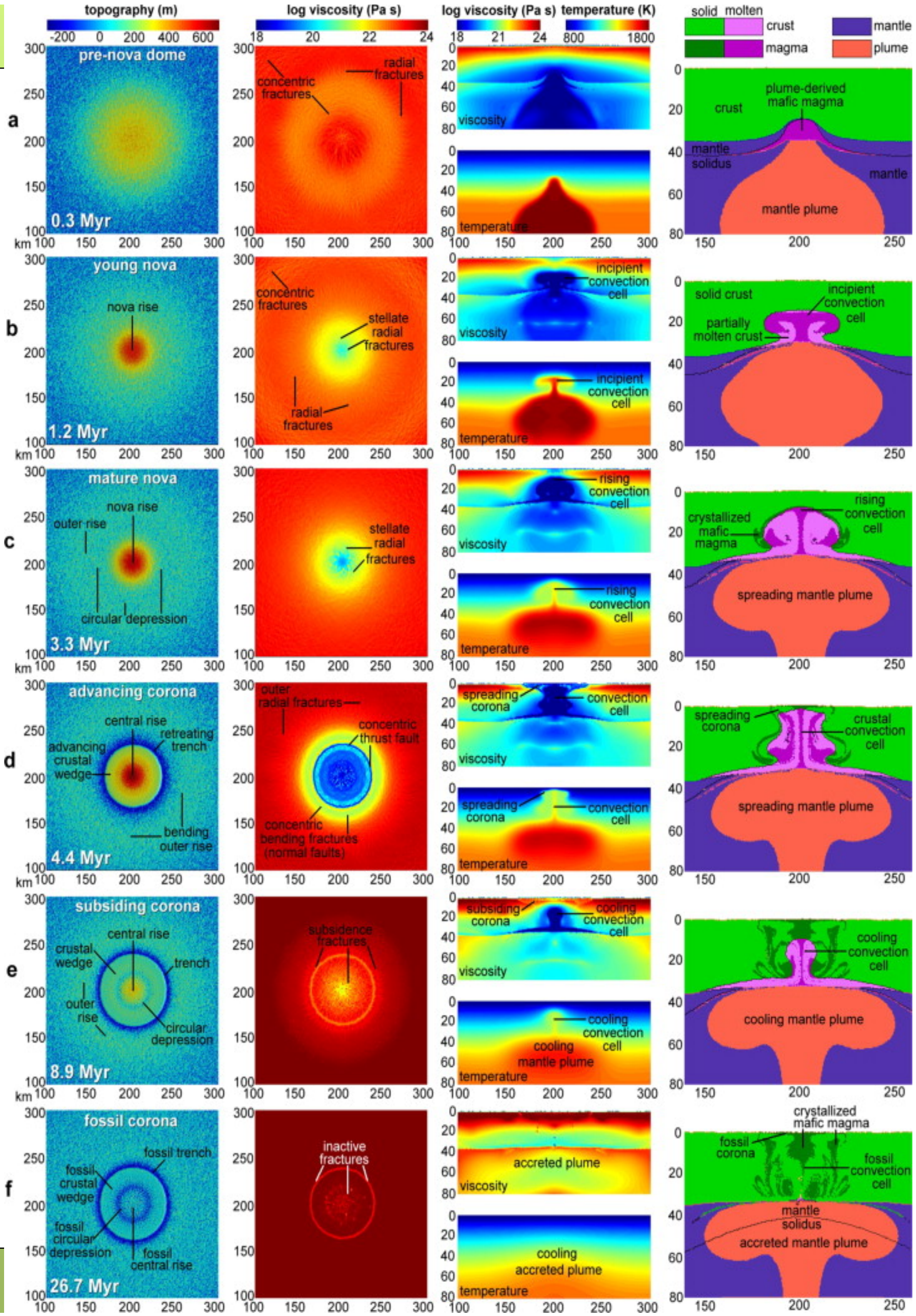
Tectonics on Venus

- Chasmata
  - Linear to arcuate troughs with ridges extending thousands of kilometers
- Tesserae
  - Old highly deformed terrain
  - Evidence for subduction
- Coronae
  - Fractured lithosphere above upwelling or downwelling mantle



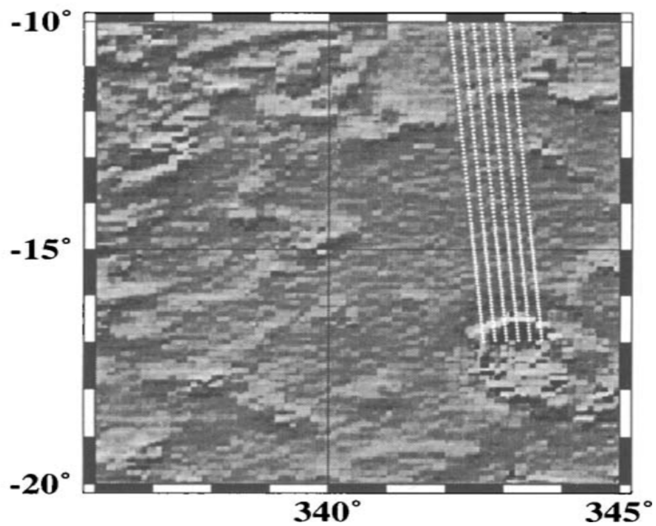
Corona Formation

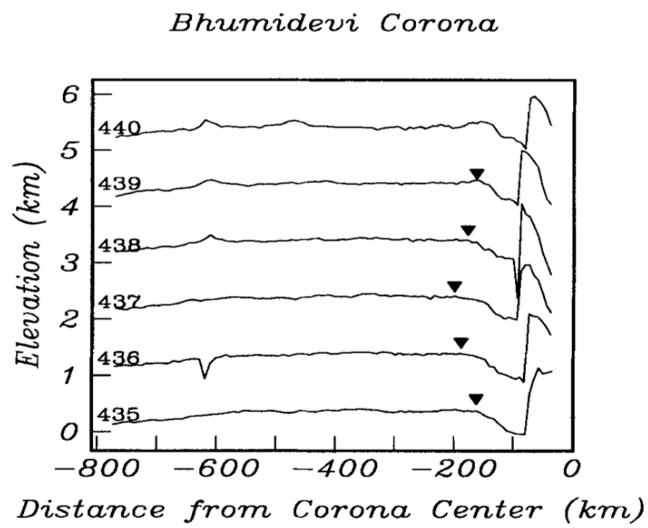
- Coronae are thought to be plume induced
  - Model form Gerya (2014)
- Eventual fracturing of nova causes a
- Hundreds of Coronae are documented on the Venusian surface



Modeling Flexure using Coronae

- Observe lithospheric flexure at coronae edges
  - We use two different thin plate methods to get at the deflection w(x)
    - 2-D Cartesian method
      - Used for features with high planform radii
    - 2-D asymmetric method
      - Used for coronae with elevated outer ring





Derivation of flexure equation

- Differential equation for flexure problems with no in-plane force P

$$D\frac{d^4 w}{dx^4}+\frac{\Delta\rho g}{4}=0$$

- Goal equation

$$w(x)=e^{\frac{-\Delta x}{\alpha}}\left[c_1\cos\left(\frac{\Delta x}{\alpha}\right)+c_2\sin\left(\frac{\Delta x}{\alpha}\right)\right]$$

Derivation of flexure equation cont.

- For simplicity later on

$$\frac{1}{4D}\left[D\frac{d^4w}{dx^4}+\frac{\Delta\rho g}{4}=0\right]$$

- To solve differential equation, determine the characteristic equation

$$\frac{1}{4}r^4+\frac{\Delta\rho g}{4D}=0 \quad + \quad \alpha=\left(\frac{4D}{\Delta\rho g}\right)^{1/4} \quad \Longrightarrow \quad \frac{1}{4}r^4+\frac{1}{\alpha^4}=0$$

Derivation of flexure equation cont.

- Use identities $a^2 + b^2 = (a+bi)(a-bi)$ to find roots
  $a^2 - b^2 = (a+b)(a-b)$

$$r_1 = \frac{-1}{\alpha} + \frac{i}{\alpha} \qquad r_2 = \frac{1}{\alpha} - \frac{i}{\alpha} \qquad r_3 = \frac{-1}{\alpha} - \frac{i}{\alpha} \qquad r_4 = \frac{1}{\alpha} + \frac{i}{\alpha}$$

Derivation of flexure equation cont.

- General solution to homogeneous linear ODE:

$$w(x) = c_1 e^{r_1 x} + c_2 e^{r_2 x} + c_3 e^{r_3 x} + c_4 e^{r_4 x}$$

- Use identities

$$e^{\lambda + \mu t} \Leftrightarrow e^{\lambda t}\cos(\mu t)$$
$$e^{\lambda - \mu t} \Leftrightarrow e^{\lambda t}\sin(\mu t)$$

$$w(x) = c_1 e^{\frac{-x}{\alpha}} \cos\left(\frac{x}{\alpha}\right) + c_2 e^{\frac{x}{\alpha}} \sin\left(\frac{x}{\alpha}\right) + c_3 e^{\frac{-x}{\alpha}} \sin\left(\frac{x}{\alpha}\right) + c_4 e^{\frac{x}{\alpha}} \cos\left(\frac{x}{\alpha}\right)$$

Derivation of flexure equation cont.

- Factor

$$w(x) = e^{\frac{-x}{\alpha}}\left[c_1\cos\left(\frac{x}{\alpha}\right) + c_3\sin\left(\frac{x}{\alpha}\right)\right] + e^{\frac{x}{\alpha}}\left[c_2\cos\left(\frac{x}{\alpha}\right) + c_4\sin\left(\frac{x}{\alpha}\right)\right]$$

- Simplify
  - Symmetry about x=0, so only need w(x≥0)
  - Boundary condition $\lim_{x\to\infty} w(x) = 0$

$$w(x) = e^{\frac{-x}{\alpha}}\left[c_1\cos\left(\frac{x}{\alpha}\right) + c_2\sin\left(\frac{x}{\alpha}\right)\right]$$

Derivation of flexure equation cont.

- Our equation models a flexure where the load is centered at x=0 and the deformation is modeled at x≠0

$$w(x) = e^{\frac{-x}{\alpha}}\left[c_1 \cos\left(\frac{x}{\alpha}\right) + c_2 \sin\left(\frac{x}{\alpha}\right)\right]$$

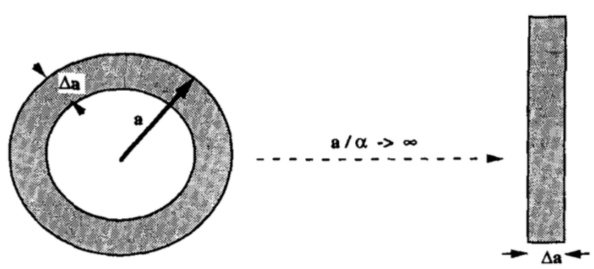
- If we want to model a load at any x, we have to apply a shift

$$w(x) = e^{\frac{-(x-x_0)}{\alpha}}\left[c_1 \cos\left(\frac{x-x_0}{\alpha}\right) + c_2 \sin\left(\frac{x-x_0}{\alpha}\right)\right]$$

Yippie!!

2-D Asymmetric Model: Choosing a Ring Load or a Bar Load

- For coronae with elevated outer rings (most of them), a better approximation for the deflection can be achieved using a 2-D asymmetric model.
- Consider a ring load with outer radius a and ring width Δa
  - As $\Delta a/a$ increases, the ring load better approximates a disk load
  - As $\Delta a/a$ decreases, the ring load better approximates a bar load



(a) Ring Load: *outer radius* = *a*, *width* = Δa

(b) Bar Load: *width* = Δa

Bar Load Approximation

- Flexure due to a bar load for this type of model can be calculated by convolving a bar load geometry with the response due to a line load:
  - w(x)=B(x)*s(x)
  - Where

$$B(x) = \Pi\left(\frac{x}{l}\right)$$

$$s(x) = \frac{V_o}{\Delta\rho g \alpha} e^{\frac{x}{\alpha}} \left[\cos\left(\frac{x}{\alpha}\right) + \sin\left(\frac{x}{\alpha}\right)\right]$$

Obtaining w from a Bar Load, Part I

- Since we are convolving with a boxcar function of height 1, then we can represent the convolution by integrating s(x) with respect to x_0 along the intervals of –l → 0 and 0 → l

$$w(x) = \frac{V_o}{\Delta\rho g\alpha}\left[\int_{-l}^{0} e^{\frac{-x-x_0}{\alpha}}\left(\cos\left(\frac{x-x_0}{\alpha}\right)+\sin\left(\frac{x-x_0}{\alpha}\right)\right)dx_0 \right.$$
$$\left. + \int_{0}^{l} e^{\frac{-x-x_0}{\alpha}}\left(\cos\left(\frac{x-x_0}{\alpha}\right)+\sin\left(\frac{x-x_0}{\alpha}\right)\right)dx_0\right]$$

Obtaining w from a Bar Load, Part II

- Lets make a change of variables to make life easier!

$$x' = x - x_0 \qquad dx_0 = dx'$$

- This change gives us:

$$w(x) = \frac{V_0}{\Delta\rho g \alpha}\left[\int_{x+l}^{x} e^{\frac{-x'}{\alpha}}\left(\cos\left(\frac{x'}{\alpha}\right) + \sin\left(\frac{x'}{\alpha}\right)\right)dx' + \int_{x}^{x-l} e^{\frac{-x'}{\alpha}}\left(\cos\left(\frac{x'}{\alpha}\right) + \sin\left(\frac{x'}{\alpha}\right)\right)dx'\right]$$

$$w(x) = -\int_{x-l}^{x}\int_{x}^{x+l} e^{\frac{-x'}{\alpha}}\left(\cos\left(\frac{x'}{\alpha}\right) + \sin\left(\frac{x'}{\alpha}\right)\right)dx'$$

Obtaining w from a Bar Load, Part III

- Breaking the equation down, the general solution for our integrals is as follows

$$\int e^{-x}(\cos(x) + \sin(x)) = -e^{-x}(\cos(x))$$

- However, if one looks at the integrals we are evaluating, the integration will produce an extra negative sign so that for our case, the integral is equal to

$$e^{-x}(\cos(x))$$

Obtaining w from a Bar Load, Part IV

- Given: $$w(x) = -\int_{x-l}^{x}\int_{x}^{x+l} e^{\frac{-x'}{\alpha}}\left(\cos\left(\frac{x'}{\alpha}\right) + \sin\left(\frac{x'}{\alpha}\right)\right)dx'$$
- And Using the identity: $$-\int e^{-x}(\cos(x) + \sin(x)) = e^{-x}(\cos(x))$$
- We can integrate our equation, evaluate for boundary conditions to get

$$w(x) = \frac{V_0}{\Delta\rho g\alpha}\left(\alpha e^{-\frac{x}{\alpha}}\left(\cos\left(\frac{x}{a}\right)\right) - \alpha e^{-\frac{x+l}{\alpha}}\left(\cos\left(\frac{x+l}{a}\right)\right) + \alpha e^{-\frac{x-l}{\alpha}}\left(\cos\left(\frac{x-l}{a}\right)\right) - \alpha e^{-\frac{x}{\alpha}}\left(\cos\left(\frac{x}{a}\right)\right)\right)$$

- After simplifications, the final formula for w(x) we receive is

$$w(x) = \frac{V_0}{\Delta\rho g}\left(e^{-\frac{x-l}{\alpha}}\left(\cos\left(\frac{x-l}{a}\right)\right) - e^{-\frac{x+l}{\alpha}}\left(\cos\left(\frac{x+l}{a}\right)\right)\right)$$

Brief Look at a Ring Load Model

- Deflection due to a disk load

$$w(r) = \frac{p_0}{\Delta\rho g}\frac{a}{\alpha}\left\{\mathrm{ber}'\left(\frac{a}{\alpha}\right)\mathrm{ker}\left(\frac{r}{\alpha}\right) + \mathrm{bei}'\left(\frac{a}{\alpha}\right)\mathrm{kei}\left(\frac{r}{\alpha}\right)\right\}$$

$$r \geq a$$

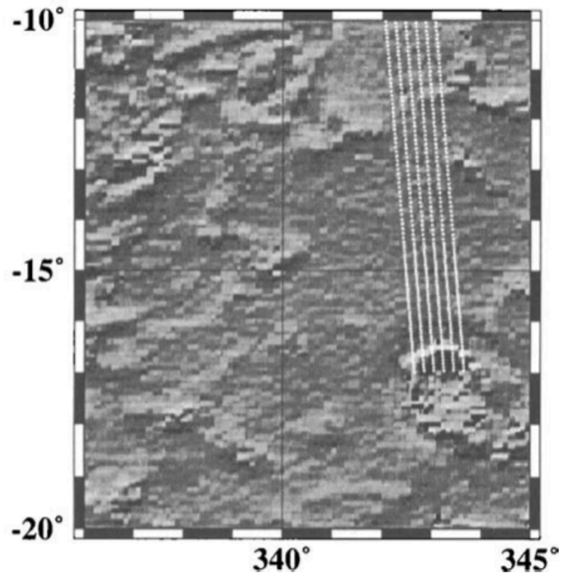
- Deflection due to a ring load

$$w(r) = \frac{p_0}{\Delta\rho g}\left\{c_1\,\mathrm{ker}\left(\frac{r}{\alpha}\right) + c_2\,\mathrm{kei}\left(\frac{r}{\alpha}\right)\right\} \qquad r \geq a$$

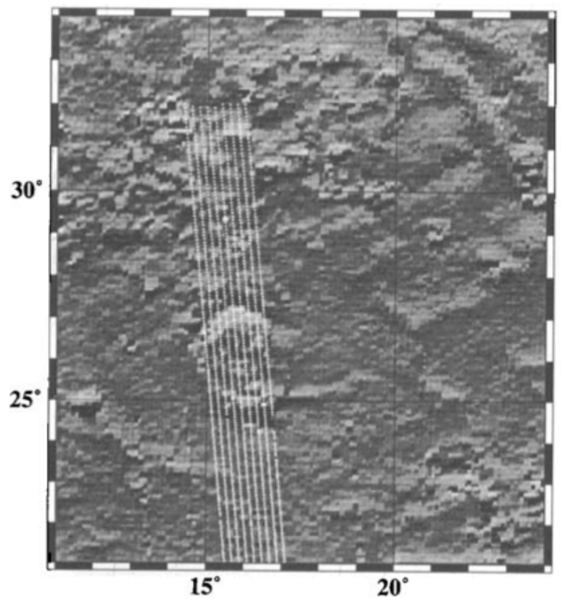
$$c_1 = \left[\left(\frac{a}{\alpha}\right)\mathrm{ber}'\left(\frac{a}{\alpha}\right) - \left(\frac{a-\Delta a}{\alpha}\right)\mathrm{ber}'\left(\frac{a-\Delta a}{\alpha}\right)\right]$$

$$c_2 = \left[\left(\frac{a-\Delta a}{\alpha}\right)\mathrm{bei}'\left(\frac{a-\Delta a}{\alpha}\right) - \left(\frac{a}{\alpha}\right)\mathrm{bei}'\left(\frac{a}{\alpha}\right)\right].$$

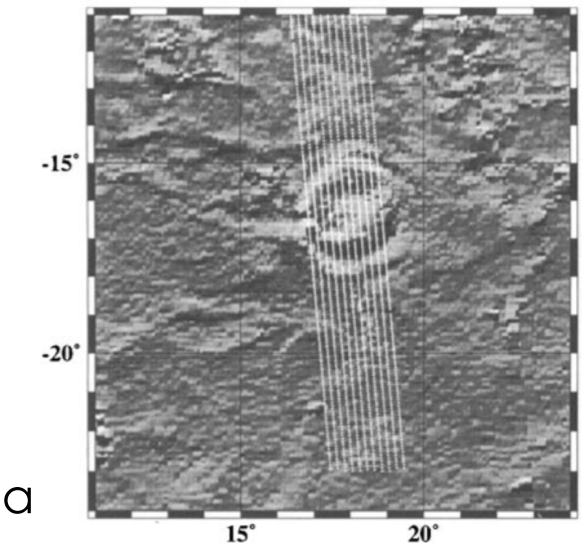
Methods



Bhumidevi Corona

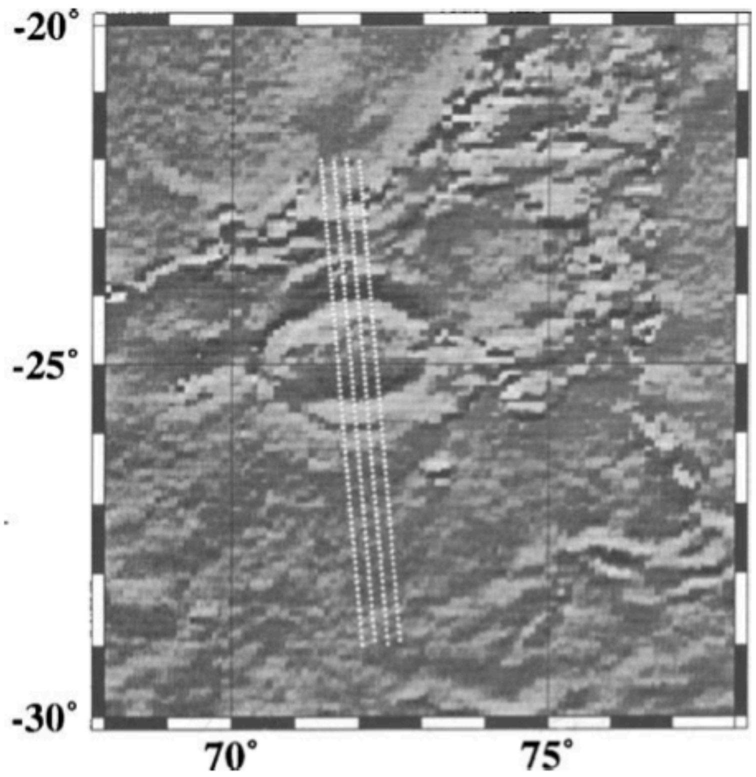


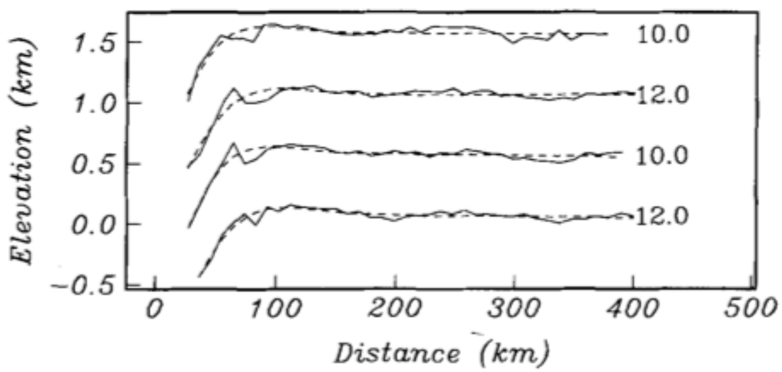
Beyla Corona



Fatua Corona

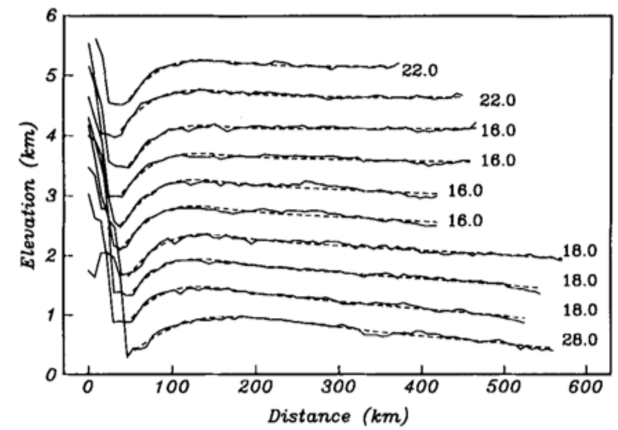
Nishtigri Corona



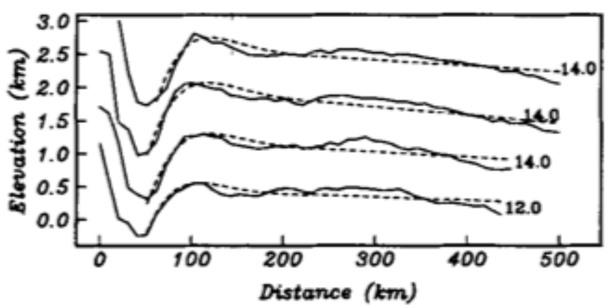


Coronae Profiles

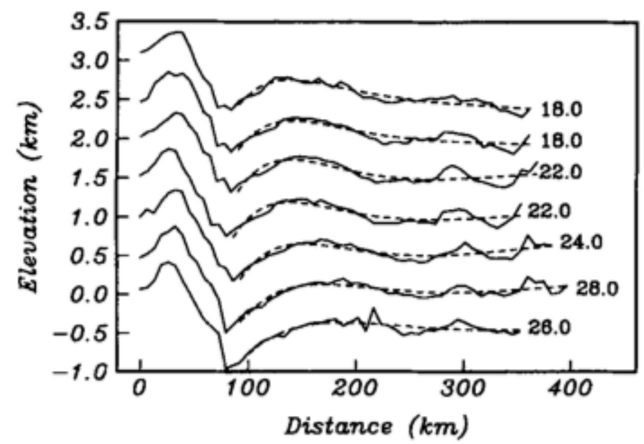
Nightingale Corona



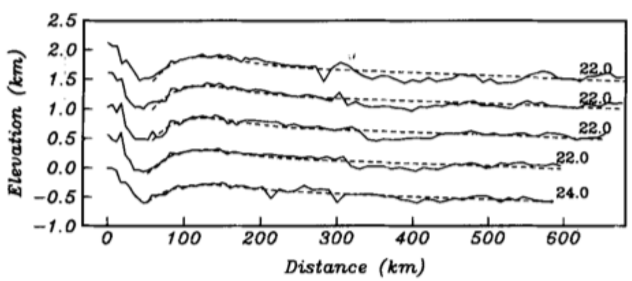
Neyterkob Corona



Demeter Corona (N)

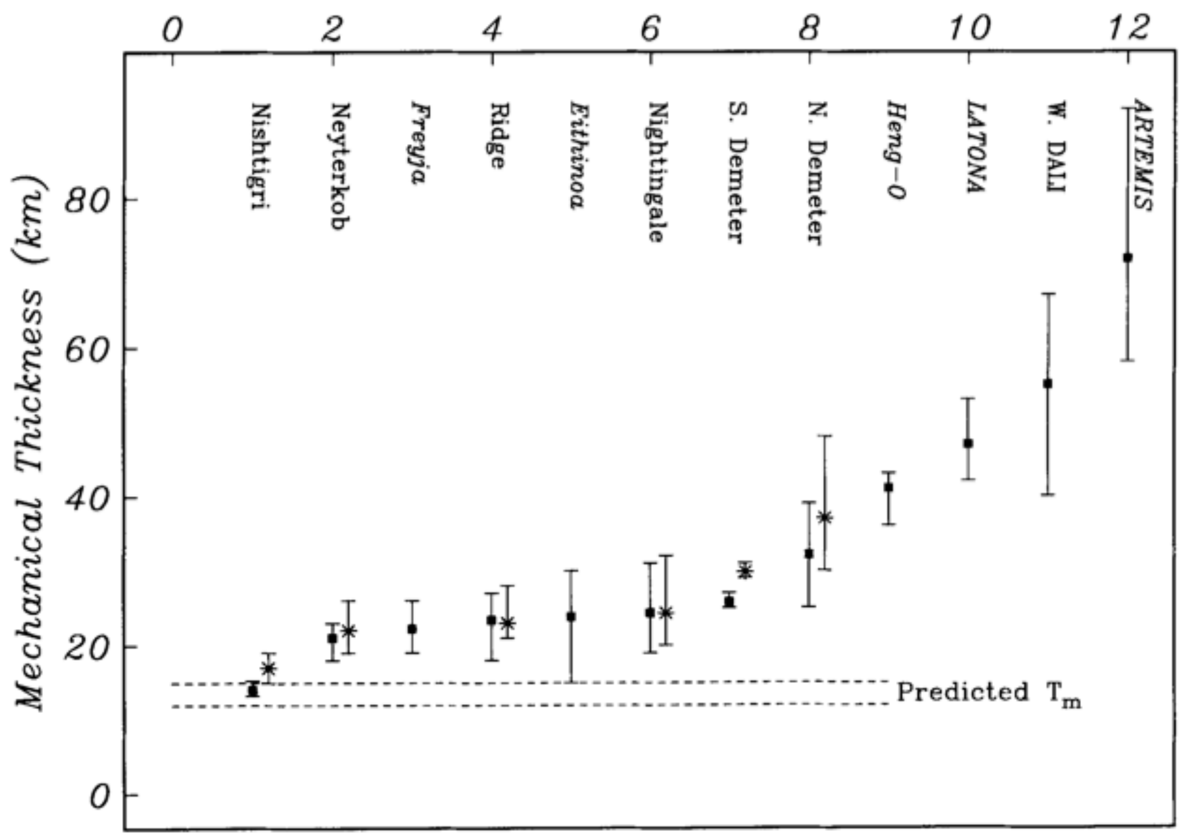


Demeter Corona (S)



Results

Mechanical Thickness



Results

- Effective elastic thickness = 12 – 34 km
- Mechanical thickness = 21 – 37 km
- Thermal gradients = 8 – 14 K km^{-1}
- Heat flow = 26.4 – 46.2 mW m^{-2}

- Rheologically dependent: Dry oceanic lithosphere?

Concluding Remarks

- Relationship to viscous flow
  - Flexures are result of dynamic processes no longer active
  - Coronae maybe in different stages of evolution
  - $T_f = 18$ Myr