Outer Rise Yield Strength

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* Moment-Curvature Formulation
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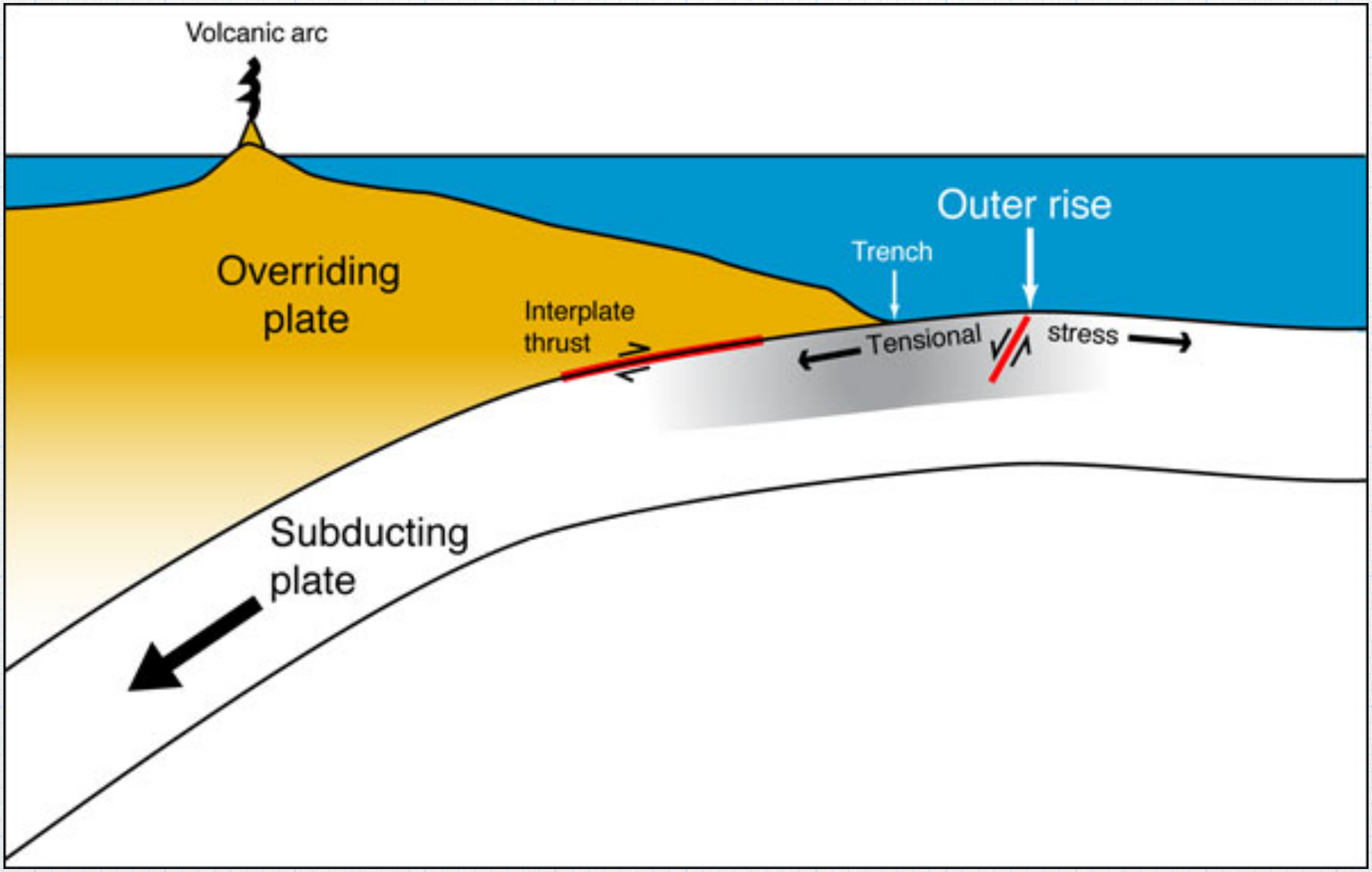
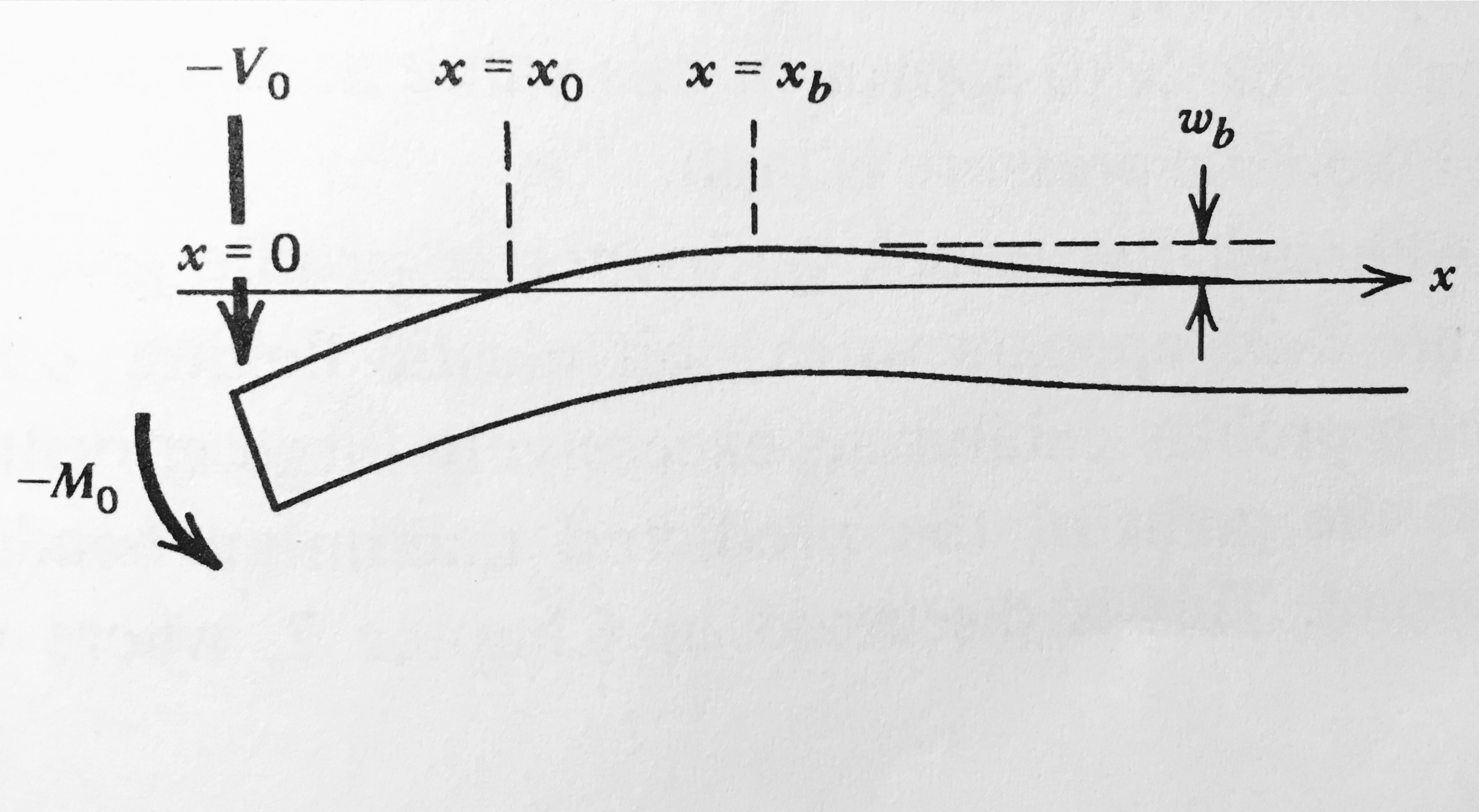
Introduction



The outer rise is a topographic high, which is a flexural response to the downward deflection of the subducting plate.

Introduction



How to describe the flexure of trench?

Mechanical equilibrium $$\frac{d^2M}{dx^2} - N\frac{d^2w}{dx^2} - \Delta\rho g w = 0$$

horizontal coordinate x

buoyancy force $\Delta\rho g$

plate deflection w

Axial load can be obtained by integrating the stress differences through a vertical cross-section of the plate with thickness H :

$$N = \int_0^H \Delta\sigma \, dz \qquad \Delta\sigma = \sigma_h - \sigma_v$$

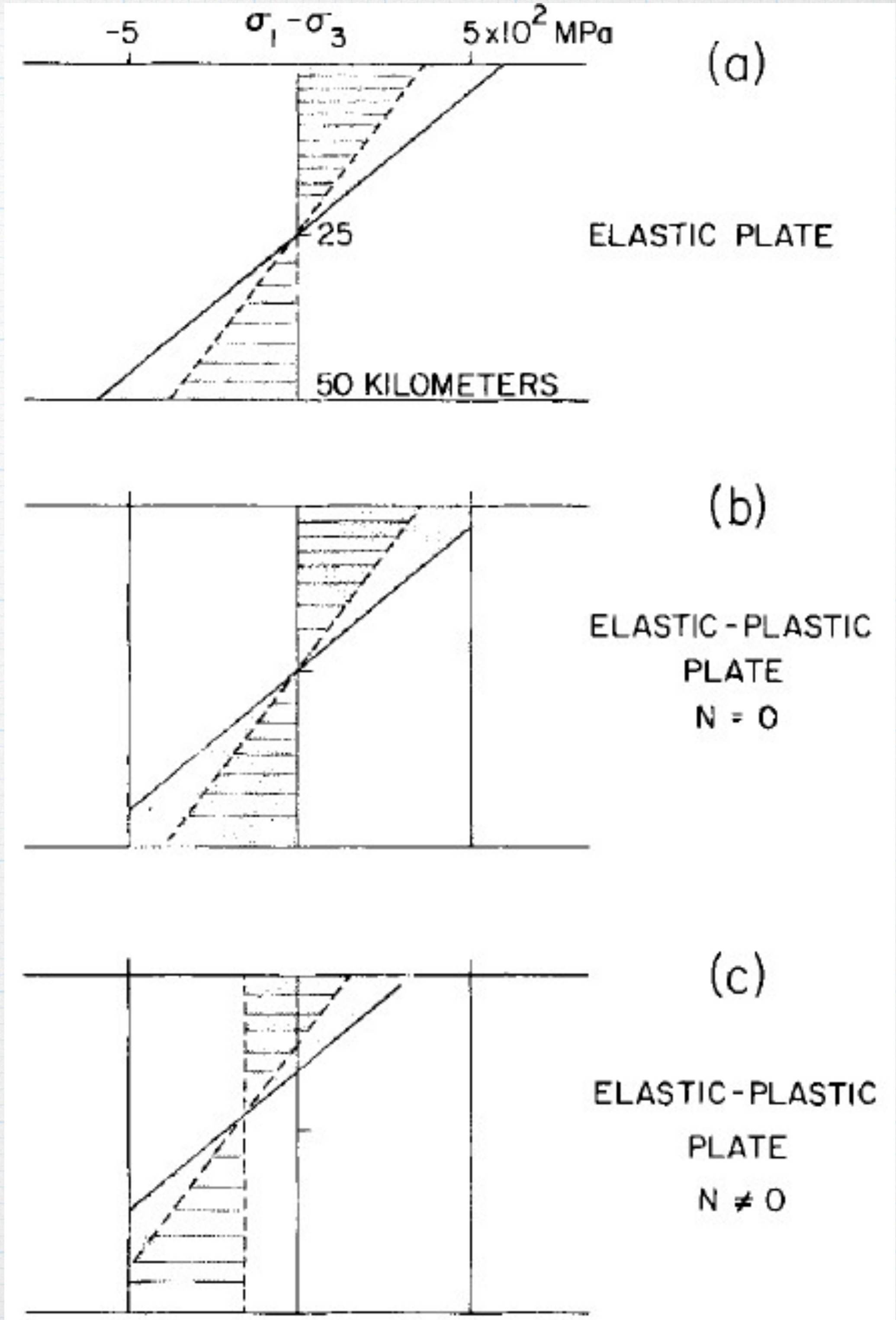
The bending moment is defined by the vertical integral of the fibre stresses σ_f weighted by the distance from the neutral plane of bending at a depth z_n

$$M = \int_0^H \sigma_f (z - z_n) dz$$

$$\sigma_f = \Delta\sigma = \sigma_h - \sigma_v$$

reach maxima at surface and base

$$z = T_e$$



For a thin elastic plate

$$M(x) = -DK(x)$$

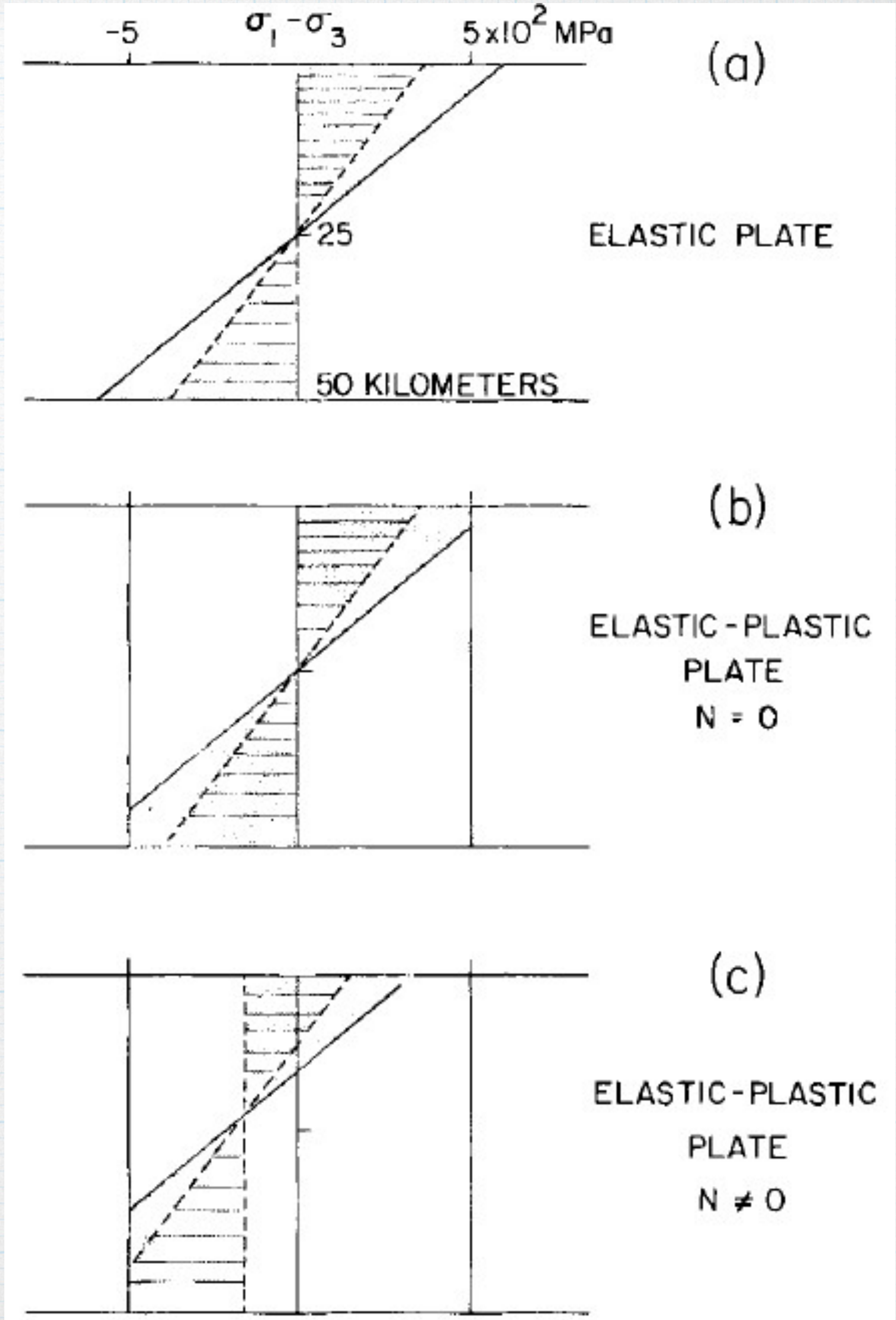
flexural rigidity

$$D = ET_e^{\,3} \big/ 12(1 - \nu^2)$$

curvature of the plate

$$K = \frac{d^2w}{dx^2}$$

the more sharply bent, the thinner, if no finite yield strength

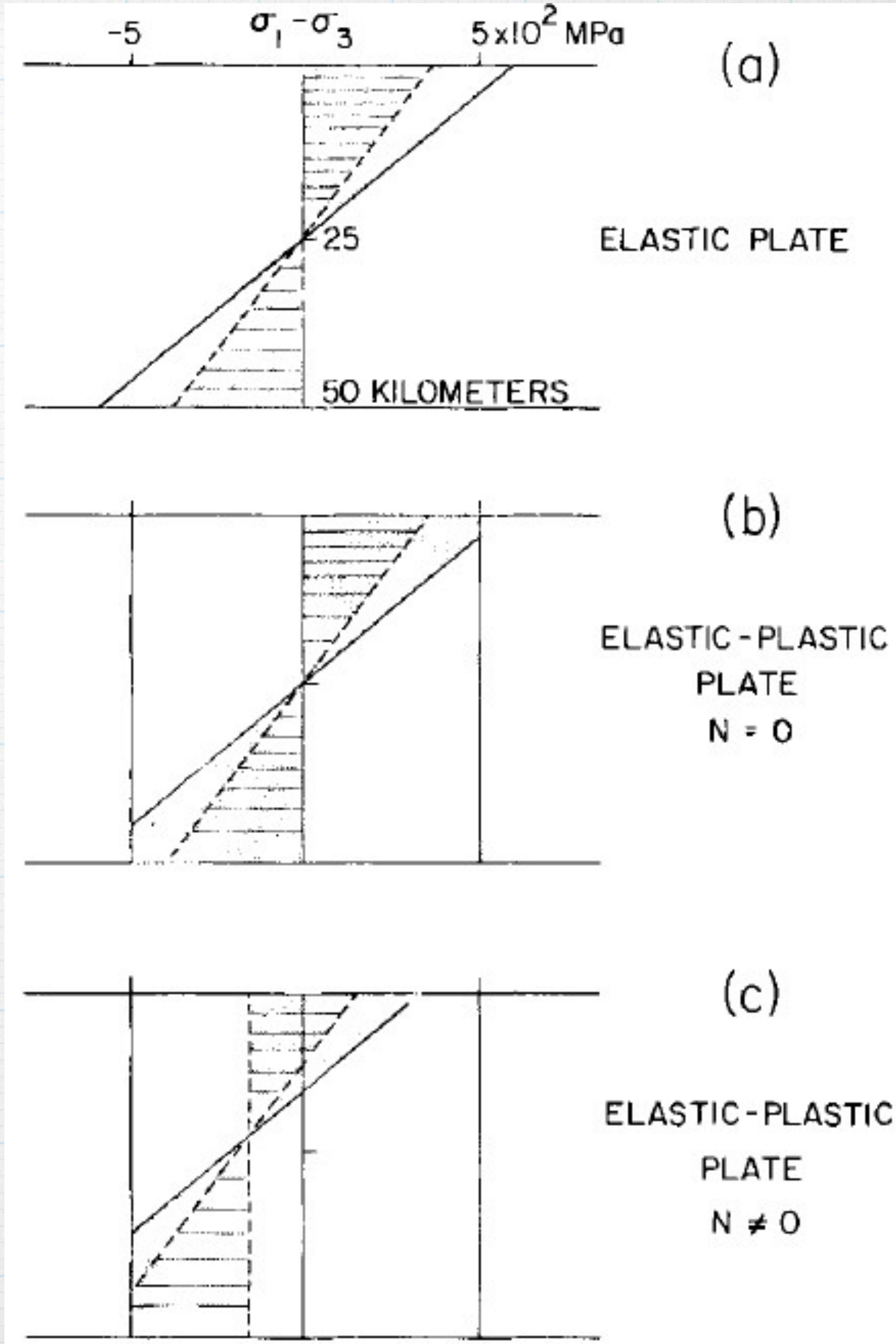


Real earth materials do have a finite strength.

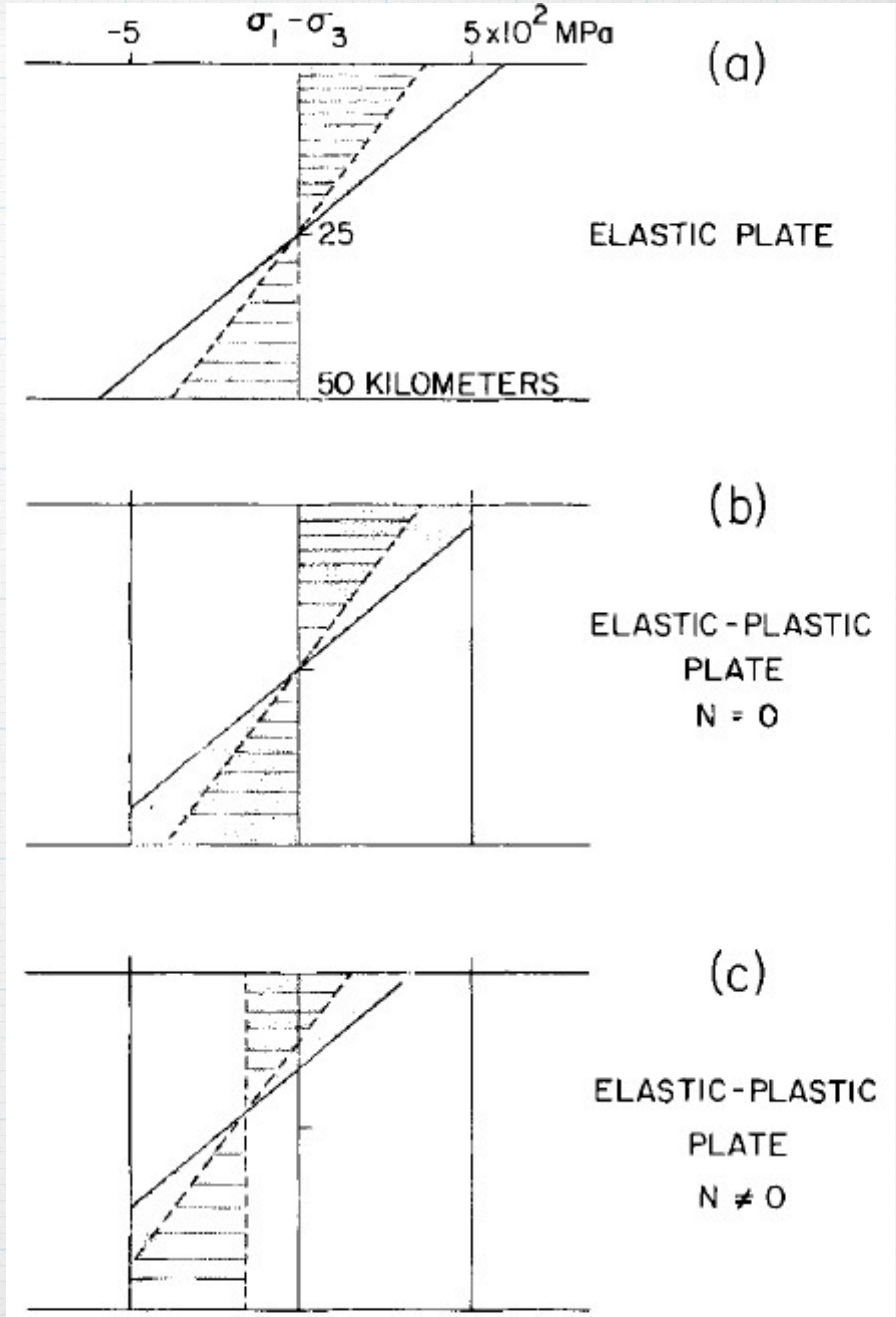
Stress difference are linearly proportional to distance from the neutral axis in the elastic plate.

The plate behaves elastically up to the yield stress, at which point the plate fails. Additional strain causes no increases in stress.

Axial loading forces can cause an apparent plate thinning.



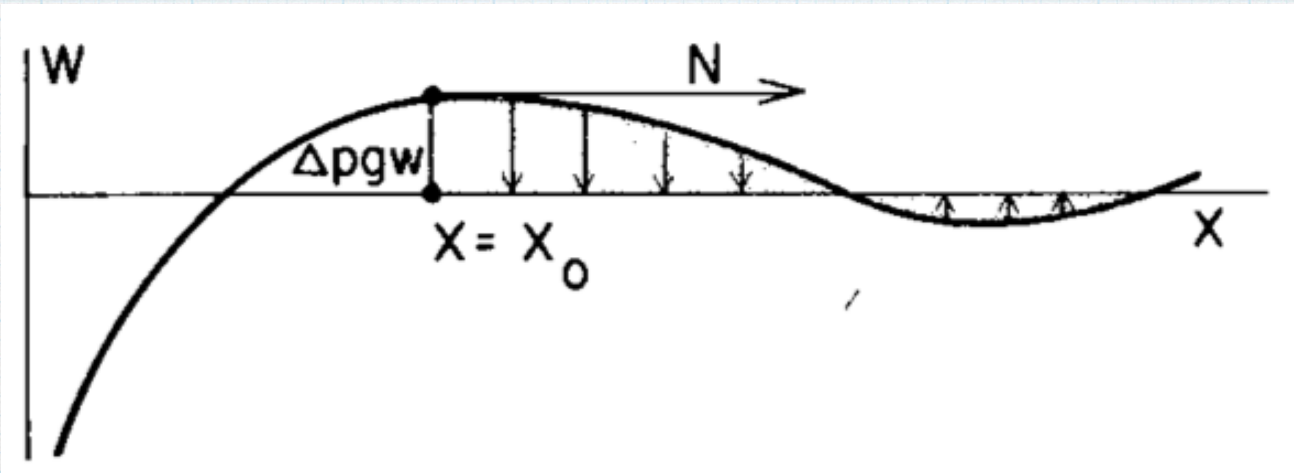
* Consider the finite yield strength of the lithosphere is important
* Since using elastic theory at a location in which the stresses are high enough that plastic behavior is occurring can result in a large underestimate of plate thickness



Moment-Curvature Formulation

Mechanical equilibrium $$\frac{d^2M}{dx^2} - N\frac{d^2w}{dx^2} - \Delta\rho g w = 0$$

Integrating twice $$M(x_0) = \int_{x_0}^{\infty} \Delta\rho g w(x)(x - x_0)dx + Nw(x_0)$$



measures the moment at point x_0 regardless of rheological assumptions.

$$M(x_0)=\int_{x_0}^{\infty}\Delta\rho g w(x)(x-x_0)dx+Nw(x_0)$$

To make the eq. only depend on observable quantities, choose x_0 so that $w(x_0)=0$

To analyze the observed data, we have to choose a curve to fit the date. The curvature from bending a thin elastic beam is acceptable.

$$w(x)=A\exp(-x/\alpha)\sin(x/\alpha)$$

A and α are related to the height w_b of the outer rise and the distance x_b from the first zero crossing to w_b

$$w_b=A\exp(-\pi/4)/\sqrt{2}$$

$$x_b=\pi\alpha/4$$

Let $x_0 = 0$

$$
\begin{aligned}
M(x=0) &= \int_0^{\infty} \Delta\rho g w(x) x dx \\
&= \Delta\rho g A\alpha^2 \int_0^{\infty} x e^{-x} \sin x dx \\
&= \Delta\rho g A\alpha^2 [-\frac{1}{2} e^{-x} [x \sin x + (x+1)\cos x]]|_0^{\infty} \\
&= \frac{1}{2}\Delta\rho g A\alpha^2 \\
&= \Delta\rho g w_b (4x_b/\pi)^2 exp(\pi/4)/\sqrt(2)
\end{aligned}
$$

$$K(x=0) = d^2w/dx^2$$

$$dw/dx = \frac{A}{\alpha} e^{-\frac{x}{\alpha}} (\cos\frac{x}{\alpha} - \sin\frac{x}{\alpha})$$

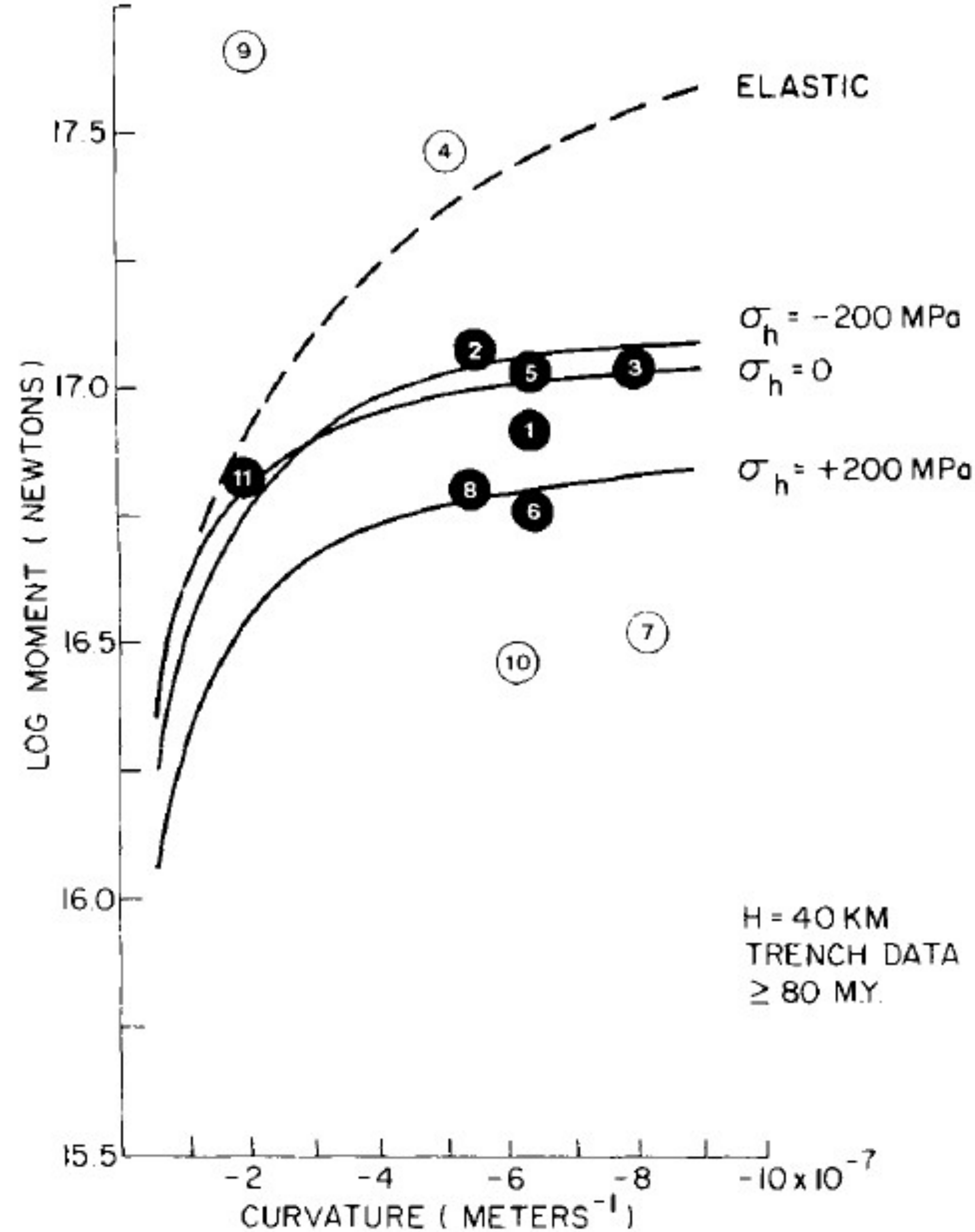
$$
\begin{aligned}
d^2w/dx^2 &= \frac{2A}{\alpha^2} e^{-\frac{x}{\alpha}} \cos\frac{x}{\alpha} \\
&= -\sqrt(2)\pi^2 w_b exp(\pi/4)/(8x_b^2)
\end{aligned}
$$

No.	Location	Profile	Age (Myr)	Velocity ($mm\,yr^{-1}$)	Strain rate $\times 10^{-16}\,s^{-1}$	W_b m	X_b km	Curvature $\times 10^{-7}\,m^{-1}$	Moment $\times 10^{16}$ N	Source
1	Marianas	Scan 5	>165	73	1.4	500	55	6.3	8.6	Caldwell *et al.* (1976)
2	Marianas		>165	73	1.2	550	63	5.4	12	Carey & Dubois (1981)
3	Bonin	Bent 2–2	130	87	2.1	640	55	8.0	11	Jones *et al.* (1978)
4	Bonin	Japanyon 4	130	87	1.5	820	78	5.1	29	Jones *et al.* (1978)
5	Bonin	Hunt 1–4	130	87	1.6	570	59	6.3	11	Jones *et al.* (1978)
6	Bonin	Bent 1–3	130	87	1.6	420	49	6.5	5.8	Jones *et al.* (1978)
7	Bonin	Antipode 3	130	87	1.9	350	40	8.2	3.3	Jones *et al.* (1978)
8	Bonin	Hunt 3 Aries 7	130	87	1.4	400	53	5.5	6.4	Caldwell *et al.* (1976)
9	Japan	Bent 1–1	130	87	0.6	620	115	1.8	47	Jones *et al.* (1978)
10	Kuril	Zetes 2	100	83	1.4	280	42	6.1	2.8	Caldwell *et al.* (1976)
11	Kermadec	Geo 318	100	83	0.5	240	71	1.8	6.9	Carey & Dubois (1981)
12	Aleutian	Seamap 13–4	55	72	1.0	350	53	4.8	5.6	Caldwell *et al.* (1976)
13	Middle America	Iguana 4–2	20	84	0.8	129	36	3.9	0.92	Jones *et al.* (1978)
14	Middle America	Iguana 2	20	84	0.8	106	34	3.5	0.70	Jones *et al.* (1978)

The points are observed data.

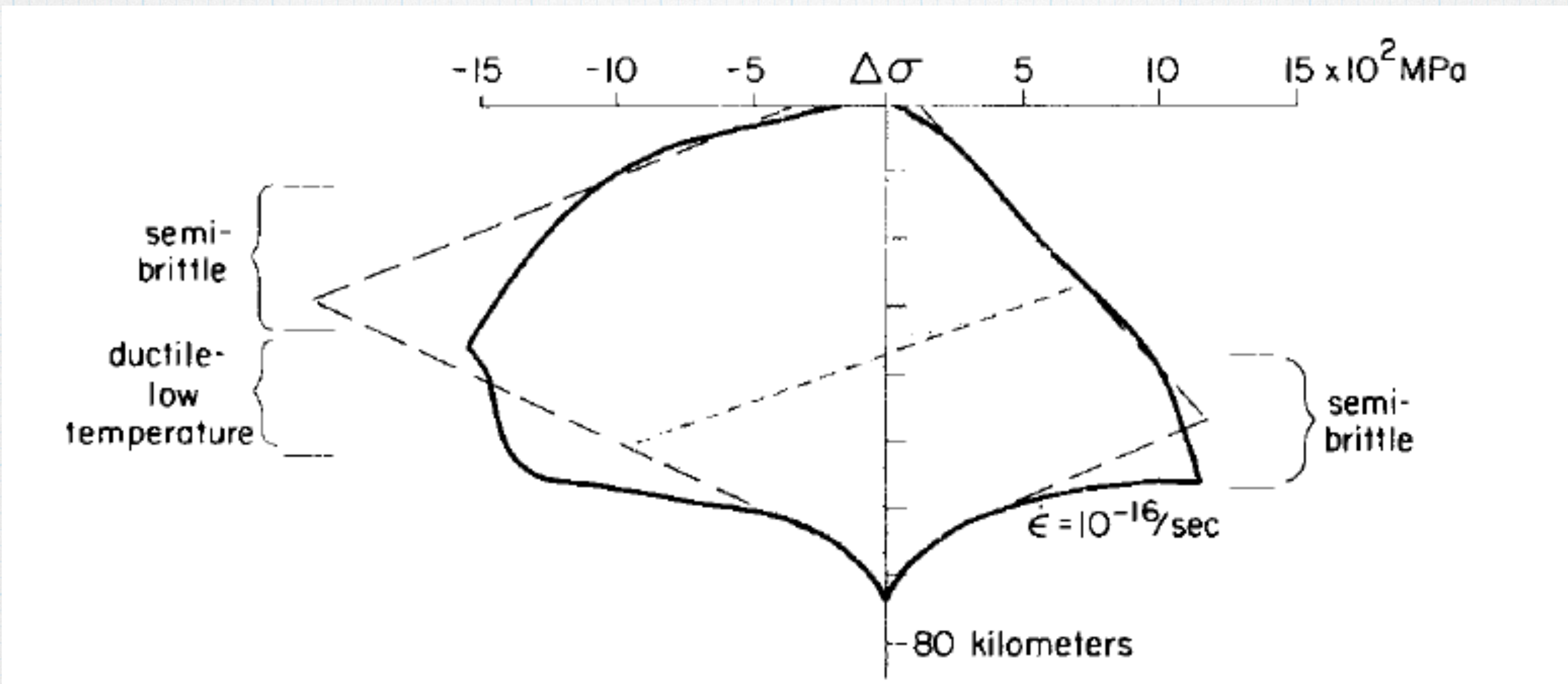
The line are theoretical moment/curvature

the dash line is elastic situation



For curvature of the order of $7\times10^{-7}m^{-1}$

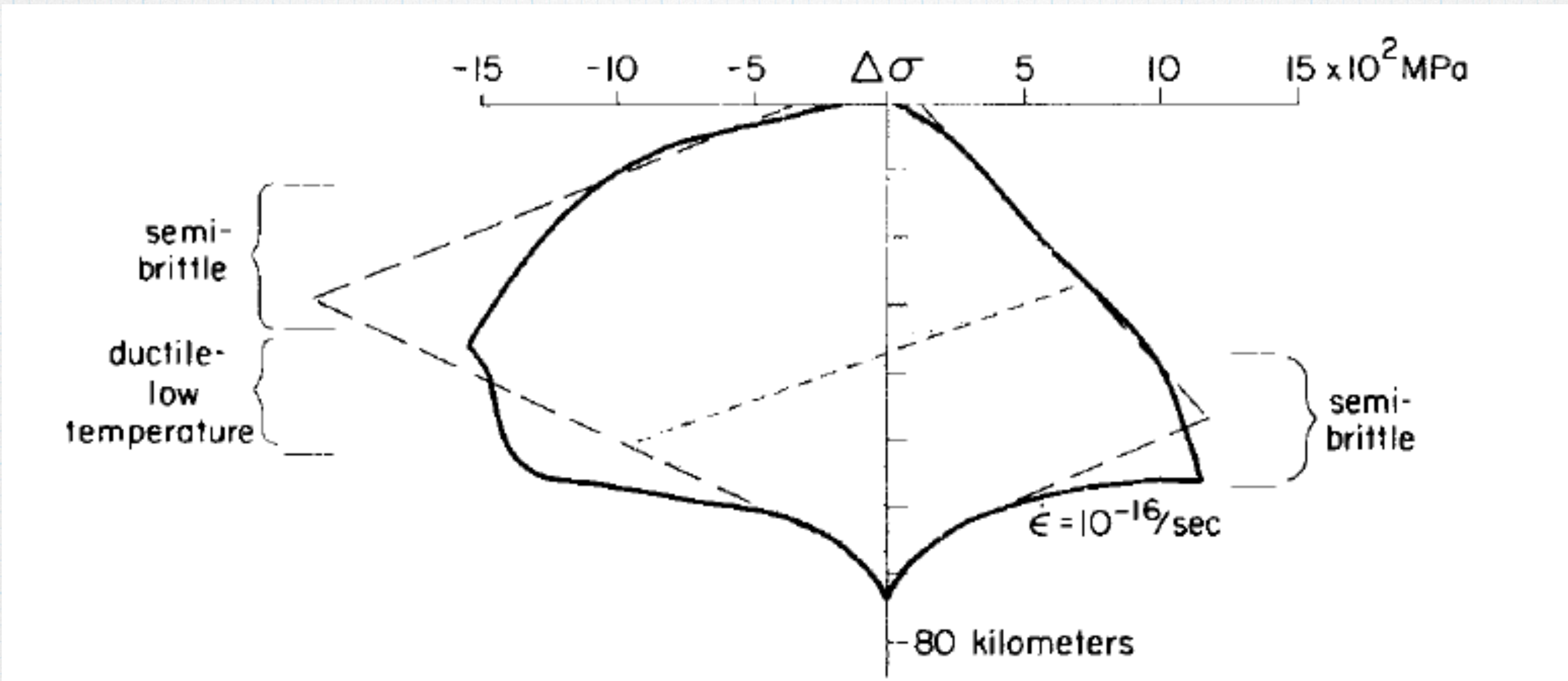
The yield envelope produces a moment of $4\times10^{17}N$ as the dash line shows, which is clearly too large to explain most of the data.



Explanation:

Decreasing the depth of the base of the yield envelope by increasing the geothermal gradient or reducing Q.

(The higher temperature, the less elastic rocks will be)



Conclusion

* The plate behaves elastically up to the yield stress, at which point the plate fails
* It is important to consider the finite yield strength of the lithosphere when modeling flexure at subduction zones
* The bending moment can be measured from the topography and it must be very large

Thank you!

Questions?