TIDAL FLEXURE OF ICE SHELF MARGIN

Ishita Pal

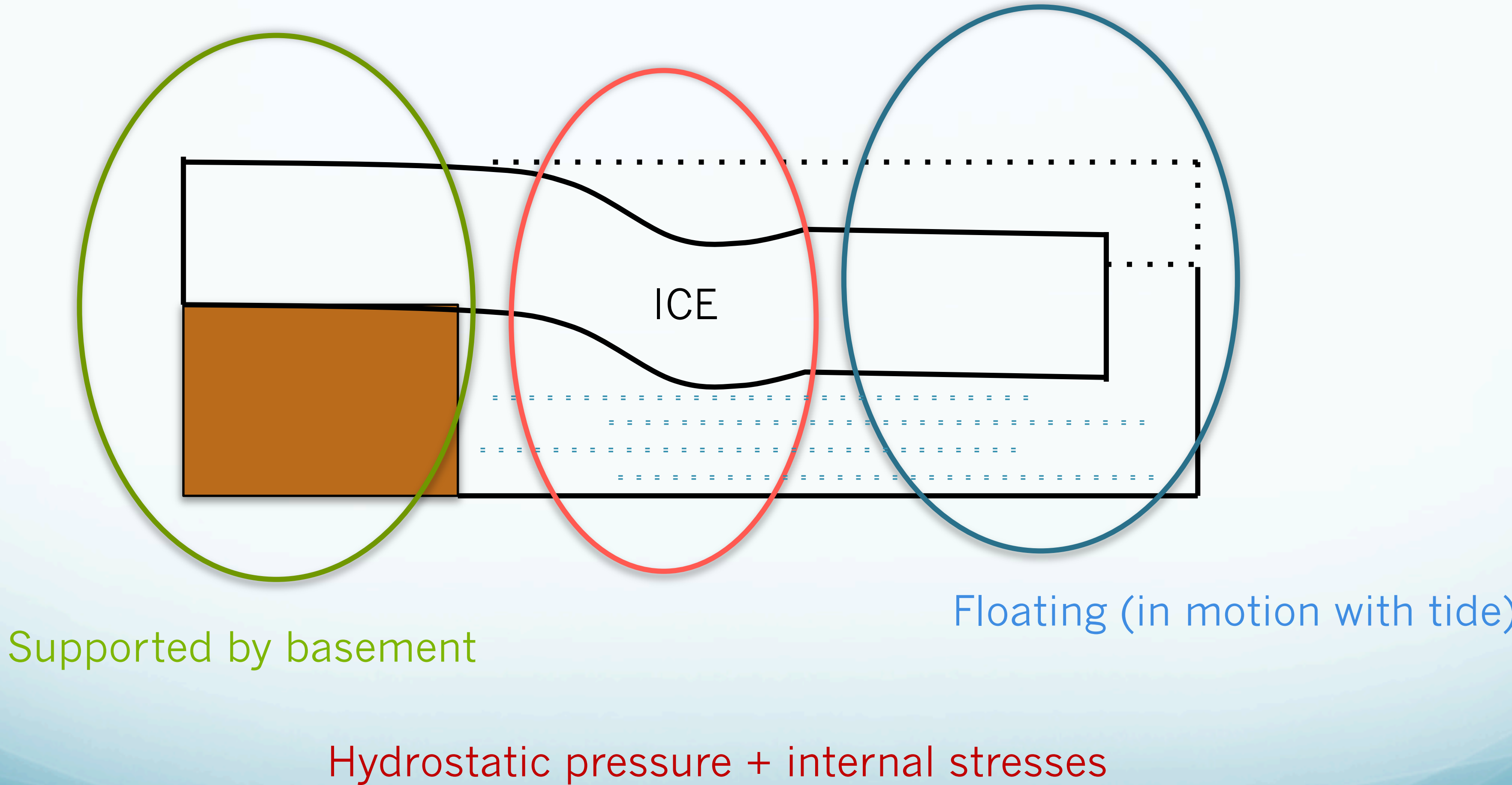
HW6

Introduction to Geodynamics

2016

BUT WHY?

- *Power dissipation*
- *Glacier fabrics affecting flow*
- *Ice shelf evolution*
- *Change in flexure indicates global warming*


ICE
Supported by basement
Floating (in motion with tide)
Hydrostatic pressure + internal stresses

Kinematic GPS technique → Tidal displacement profile

PART 1

- Elastic beam theory
- Derivation of elastic modulus

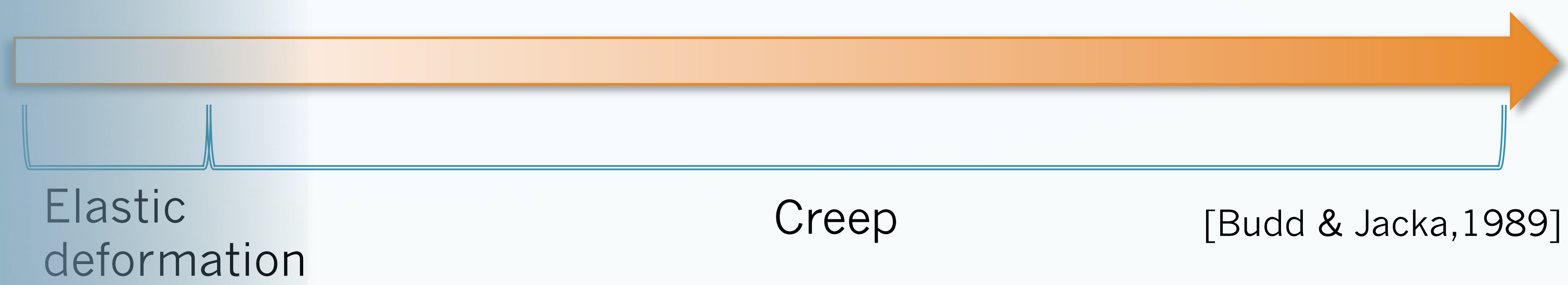
30 km wide, 150 km long, 2000 km thick

PART 2

- Application of the model to other data to find the elastic modulus

After Vaughan,1995

Tidal cycle



[Budd & Jacka,1989]

[Sinha,1978]

Effective Elastic Modulus= Truly elastic + viscoelastic response

Depends on :

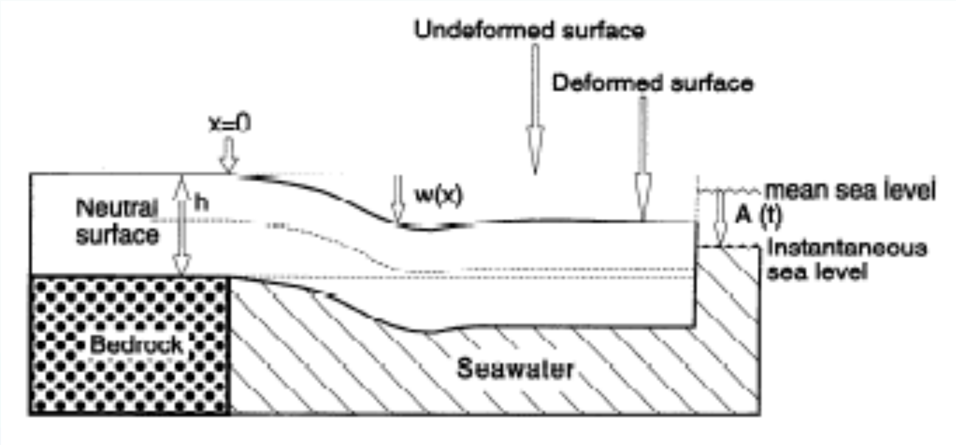
- Load time and Temperature

Differential equation for deformation of elastic beam:

$$D\frac{\partial w^4}{\partial x^4} = \rho g[Ao(t) - w(x)]$$

$D = \frac{Eh^3}{12(1-v^2)}$ = flexural rigidity

h=thickness of ice shelf
v= Poisson's ratio
E=elastic modulus
ρ = density of sea water
g= acceleration due to gravity of earth
w(x)= vertical displacement of ice sheet surface
Ao(t)= tidal displacement beyond influence of hinge zone



After Vaughan,1995

$$D\frac{\partial w^4}{\partial x^4} = \rho g[Ao(t) - w(x)]$$

$$\frac{D}{\rho g}\frac{\partial w^4}{\partial x^4} + w(x) = Ao(t) \qquad \text{..........(i)}$$

Boundary conditions:

$$w = 0, \frac{\partial w}{\partial x} = 0 \; at \; x = 0$$

$$w = Ao(t) at \; x = \infty$$

Assuming:

$$\frac{D}{\rho g}\frac{\partial w^4}{\partial x^4} + w(x) = 0$$

&

$$w(x) = Ce^{mx}$$

Equ. (i) becomes:

$$\frac{D}{\rho g}\frac{\partial Ce^{mx}}{\partial x^4} + Ce^{mx} = 0$$

$$m = \sqrt{i}\;\left(\frac{\rho g}{D}\right)^{1/4}$$

$$w(x) = Ce^{\sqrt{i}\;\left(\frac{\rho g}{D}\right)^{1/4} x}$$

Now, let's assume: $w(x)=C_2$ ....(a)

$C_2=Ao(t)$ [from (i)]

Boundary conditions:

$$w = 0, \frac{\partial w}{\partial x} = 0 \; at \; x = 0$$

$$w = Ao(t) at \; x = \infty$$

Also, from previous assumption:

$$w(x) = Ce^{\sqrt{i}\; (\frac{\rho g}{D})^{1/4} x}$$ ....(b)

Using the boundary conditions we get C= -Ao(t)

Adding up (a) and (b):

$$w(x) = Ao(t)[1 - e^{\sqrt{i}\; (\frac{\rho g}{D})^{\frac{1}{4}} x}]$$

$$w(x) = Ao(t)[1 - e^{(i+1)\; (\frac{\rho g}{D})^{\frac{1}{4}} x}]$$

$$w(x) = Ao(t)[1 - e^{\beta x}. e^{i\beta x}$$

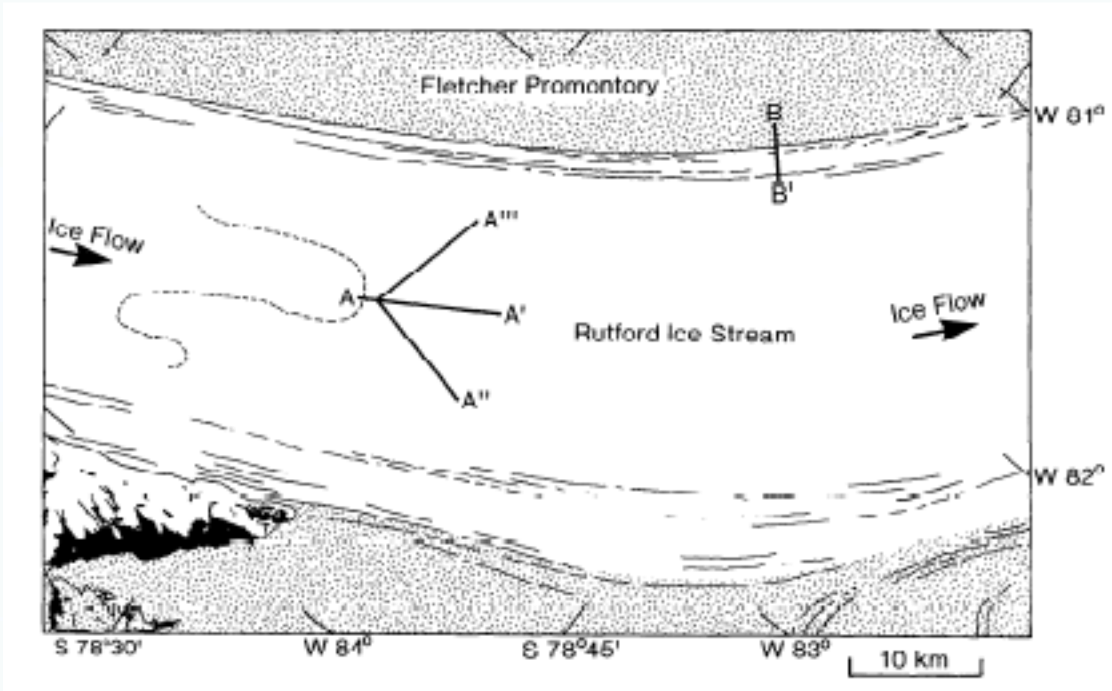
$$w(x) = Ao(t)[1 - e^{\beta x}(cos\beta x + isin\beta x)]$$

$$\sqrt{i} = \frac{1}{\sqrt{2}} + \frac{i}{\sqrt{2}}$$

$$\beta = (\frac{\rho g}{4D})^{\frac{1}{4}}$$

$$\beta^4 = 3\rho g \frac{1 - v^2}{Eh^3}$$

$$w(x) = Ao(t)[1 - e^{\beta x}(cos\beta x + sin\beta x)]$$



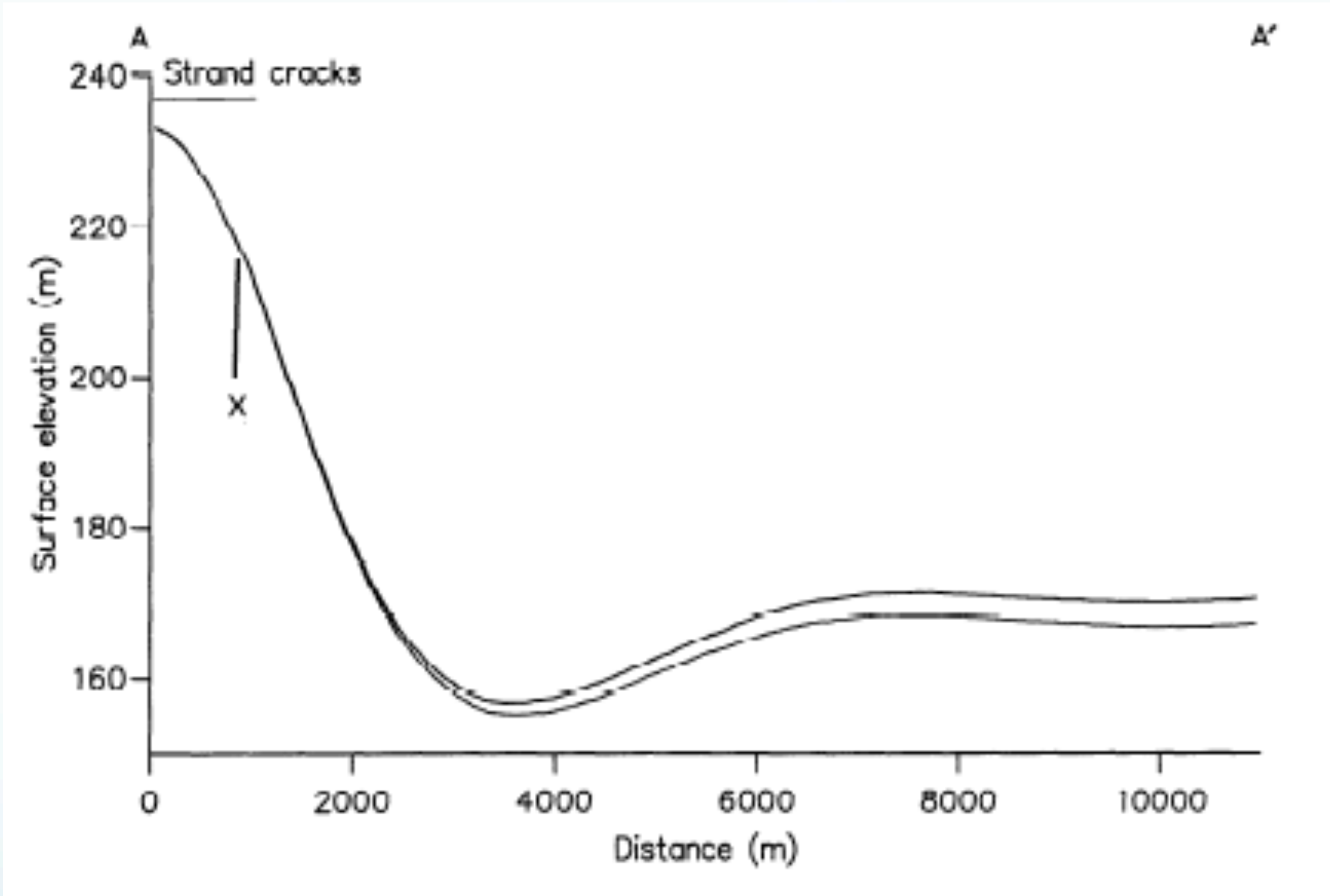
After Vaughan,1995

Tidal correction

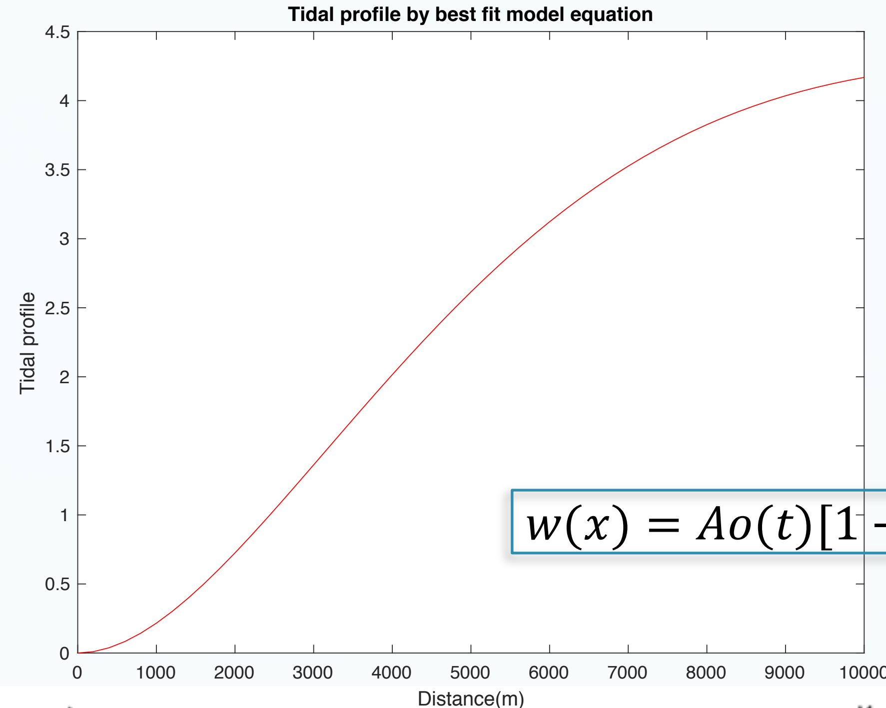
$$d = \frac{e - e'}{p - p'}$$

=Tidal profile

e,p= elevation, predicted tidal height at 1st visit
e',p'= elevation, predicted tidal height at 2nd visit



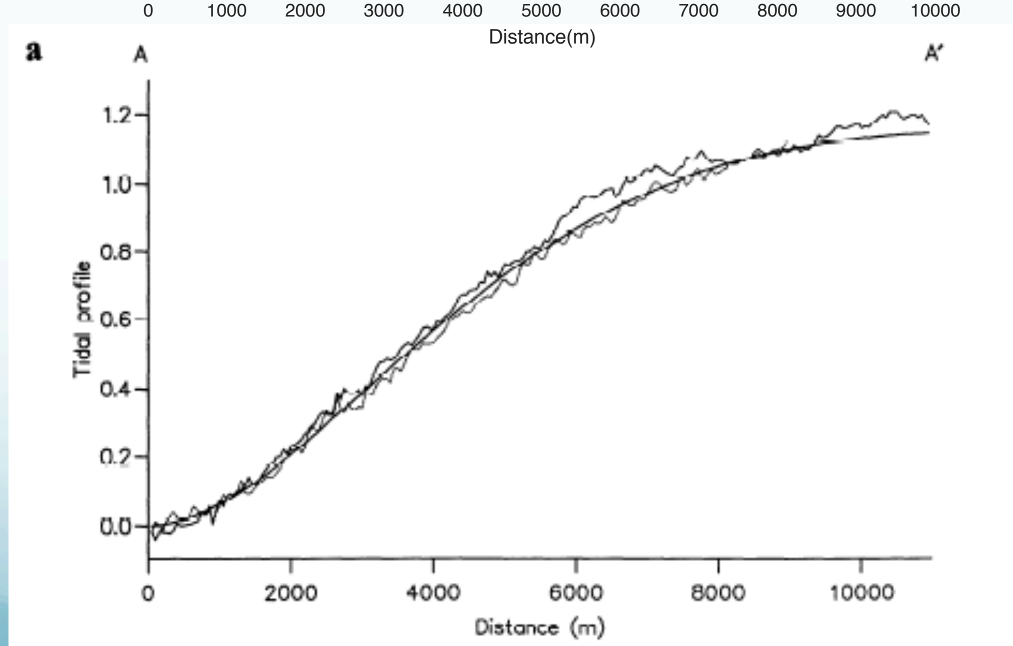
After Vaughan,1995



Ao(t)= 4.1 m

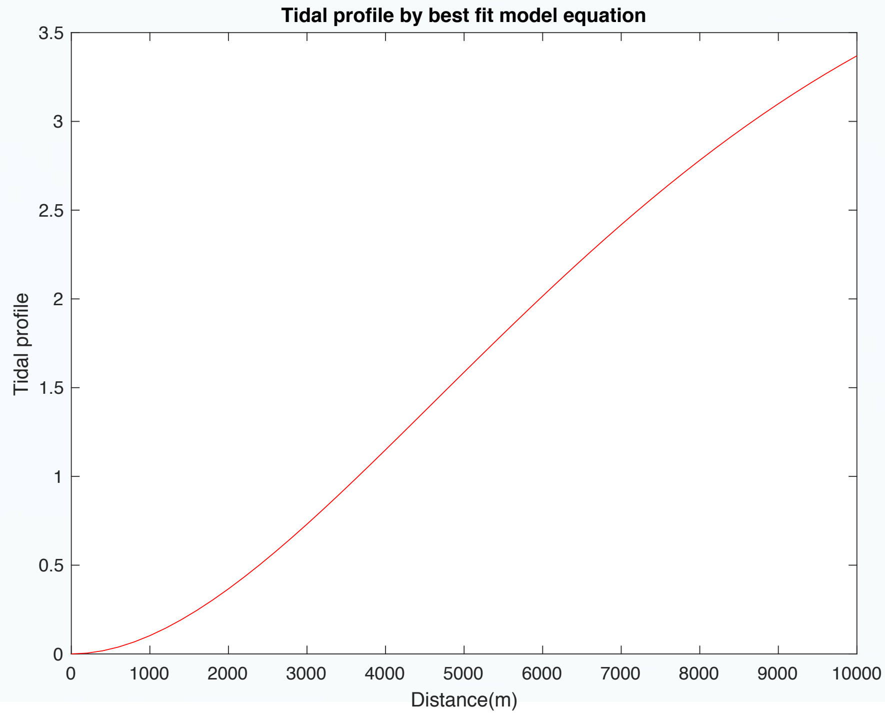
β = 2.5* 10^-4

$$w(x) = Ao(t)[1 - e^{\beta x}(cos\beta x + sin\beta x)]$$

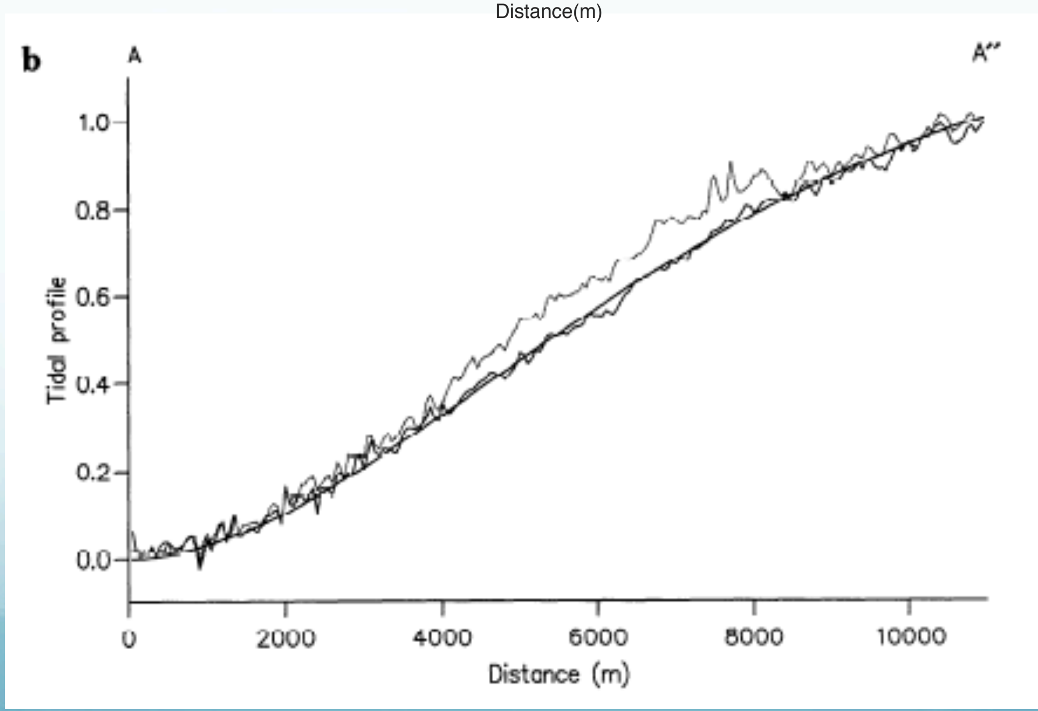


$$d = \frac{e - e'}{p - p'}$$

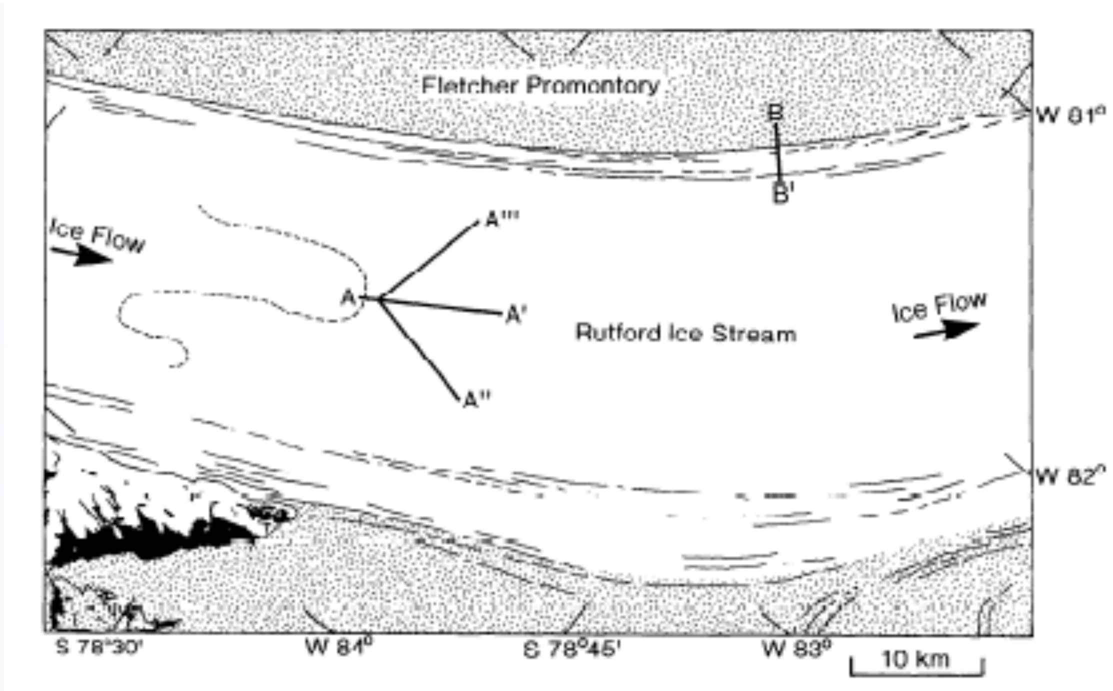
After Vaughan,1995

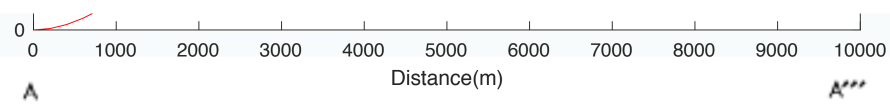


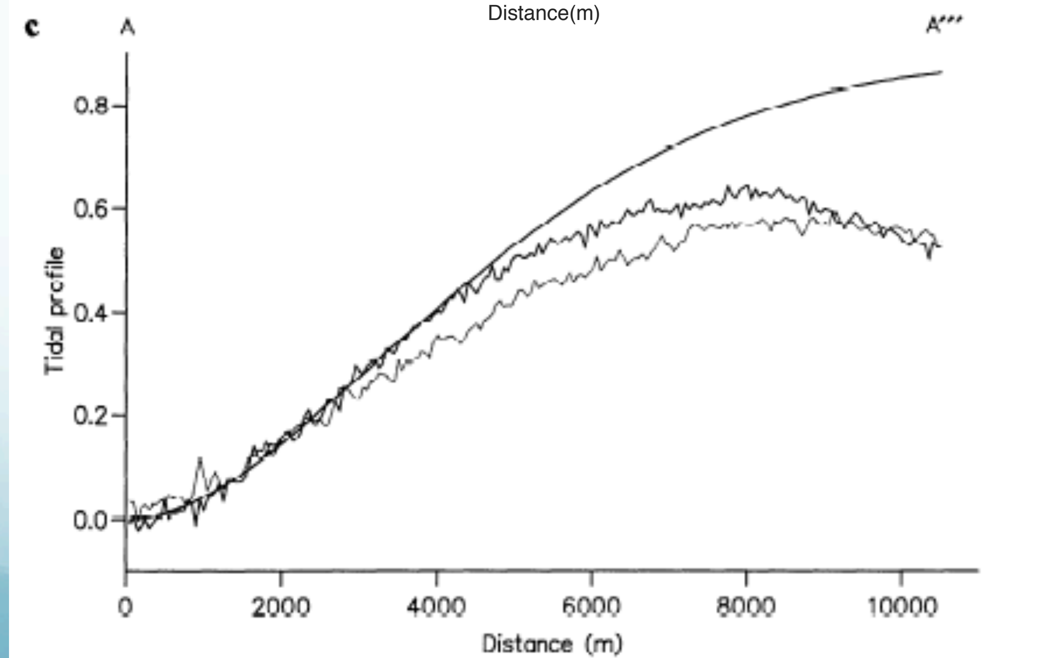
Ao(t)= 4.0 m
β = 1.7* 10^-4



After Vaughan,1995







Ao(t)= 3.1 m

β = 2.5* 10^-4

After Vaughan, 1995

Ao(t)= (3.9 ± 0.2) m

β = (2.43 ± 0.43) * 10^-4

h= 1550 m

v= 0.33

$$\beta^4 = 3\rho g \frac{1 - v^2}{E h^3}$$

E=(0.57- 2.4) GPa

Location	Thickness (h) m
Doake Ice Rumples, Antarctica	1000
Bach Ice Shelf, Antarctica	250
Jakobshavns Glacier, Greenland	450
Ekstrom Ice Shelf, Antarctica	150-200
Maudheim Ice Shelf, Antarctica	....

Fit the model line with the observed data

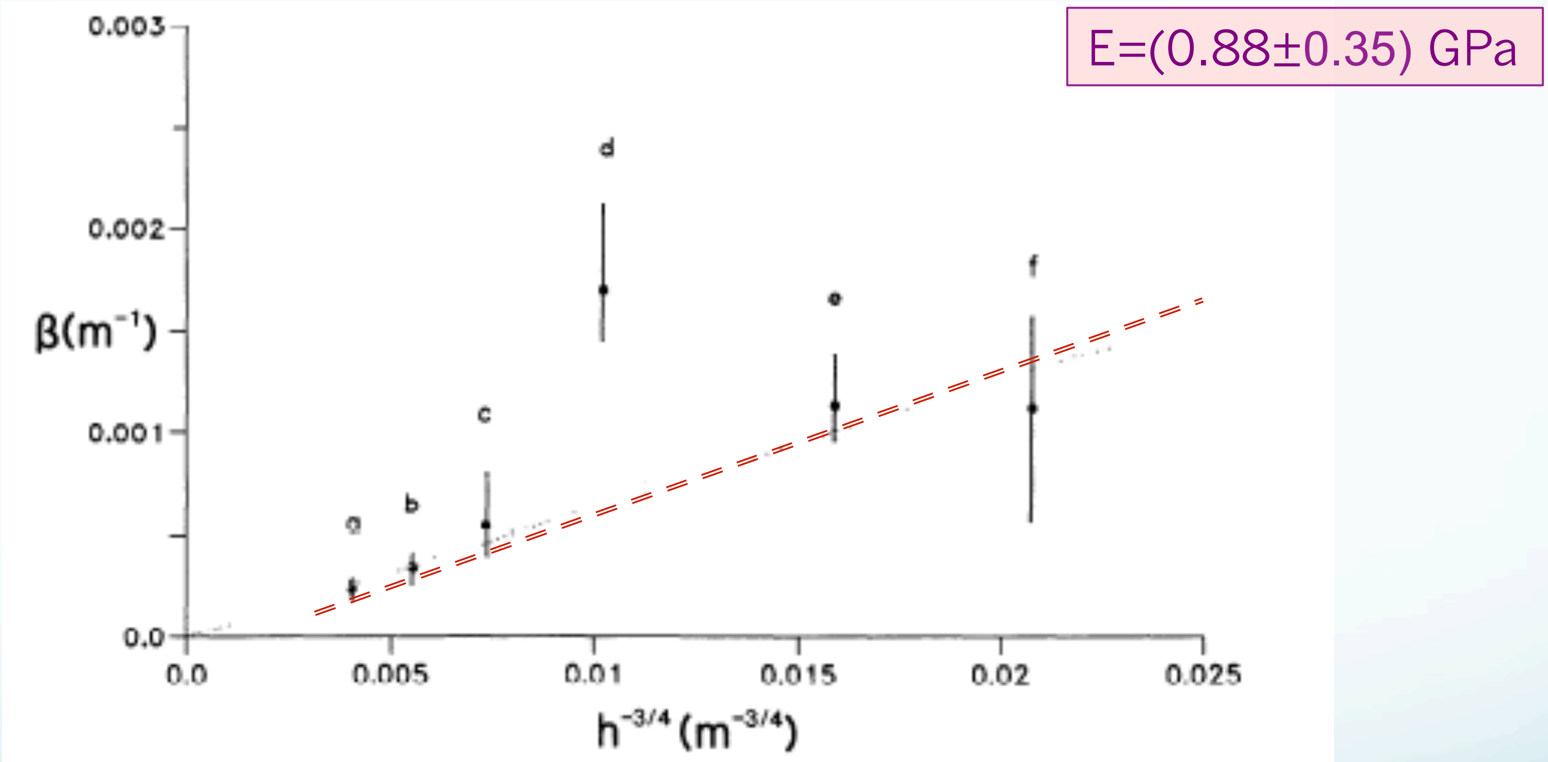
↓

Infer Ao & β values from the graph

↓

Calculate E

$$\beta^4 = 3\rho g \frac{1 - v^2}{E h^3}$$



After Vaughan,1995

a= Rutford Ice Streaming Groundline
b= Doake Ice Rumples
c= Bach Ice Shelf
d= Jakobshavns Glacier
e= Ekstorm Ice Shelf
f=Maudheim Ice Shelf

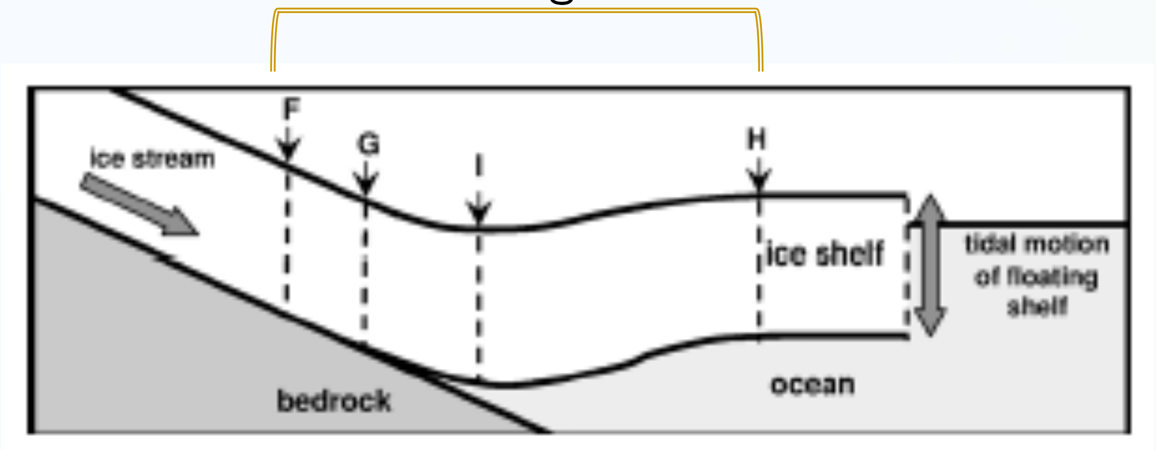
CONCLUSIONS:

- Average Elastic Modulus of 0.88 ± 0.35 GPa
- No evidence of variation of E with temperature
- No evidence of variation of E with structural style
- Tidal flexing can be considered in isolation from Ice Flow

LIMITATIONS:

- Not considered geometry of the bed rock
- Uniform tidal correction

Grounding zone



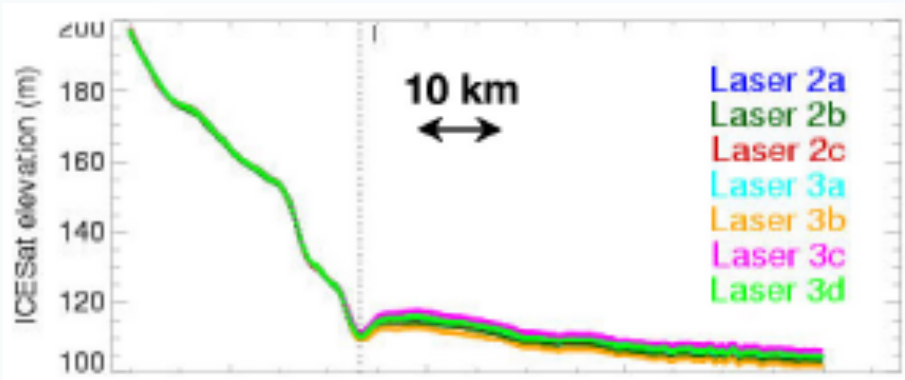
F=limit of ice flexure from tidal movement

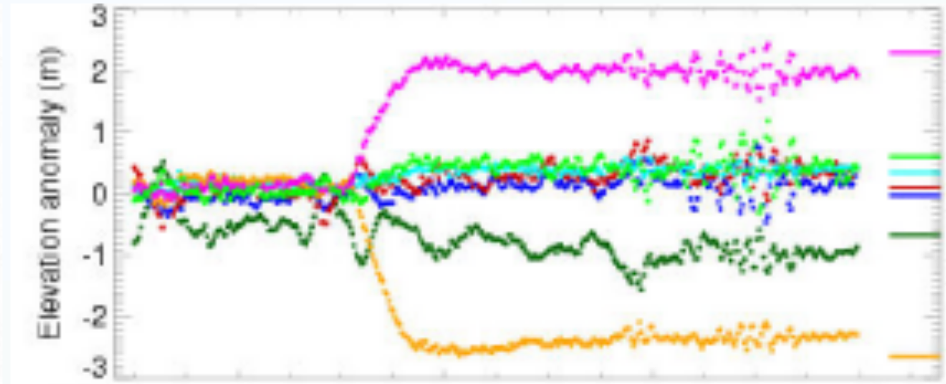
G=limit of ice floatation

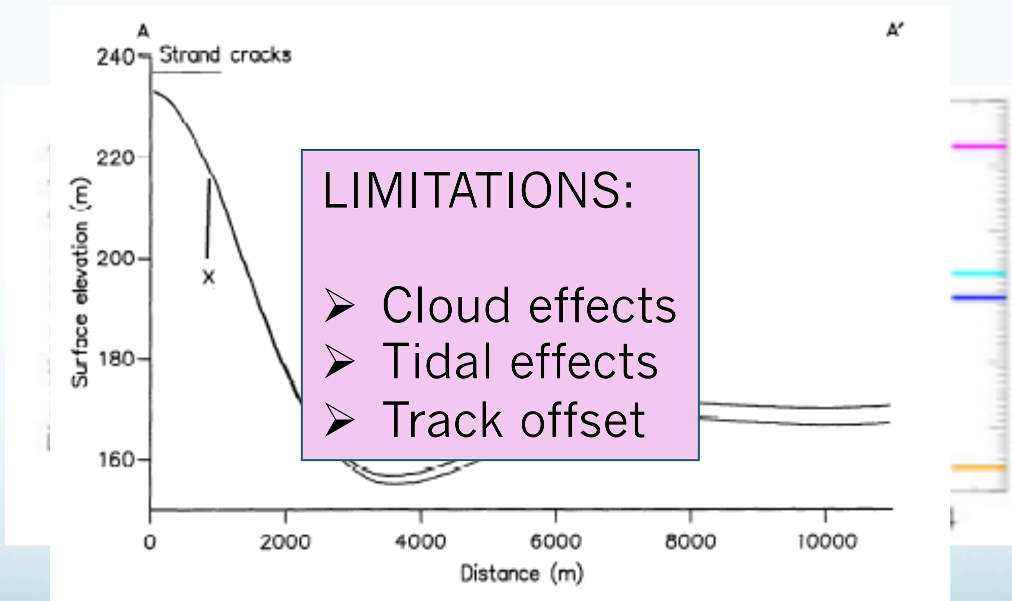
I=inflection point

H=seaward limit of ice fexure

[After Fricker and Padman, 2006]







LIMITATIONS:

- Cloud effects
- Tidal effects
- Track offset

After Fricker and Padman, 2006

After Vaughan,1995

REFERENCES

- Vaughan D.G., *Tidal flexure at ice shelf margins,* Journal of Geophysical Research, Vol 100, Pages 6213-6224, 1994
- Fricker H.A. and Padman L, *Ice shelf grounding zone structure* from ICESat laser altimetry, Geophysical Research Letters,Vol 33, L15502,2006
- Chuter S.J. and Bamber J.L., *Antarctic Ice shelf thickness from CryoSat-2 radar altimetry*, Geophysical Research Letters, Vol 42, Pages 10721-10729, 2015

THANK YOU