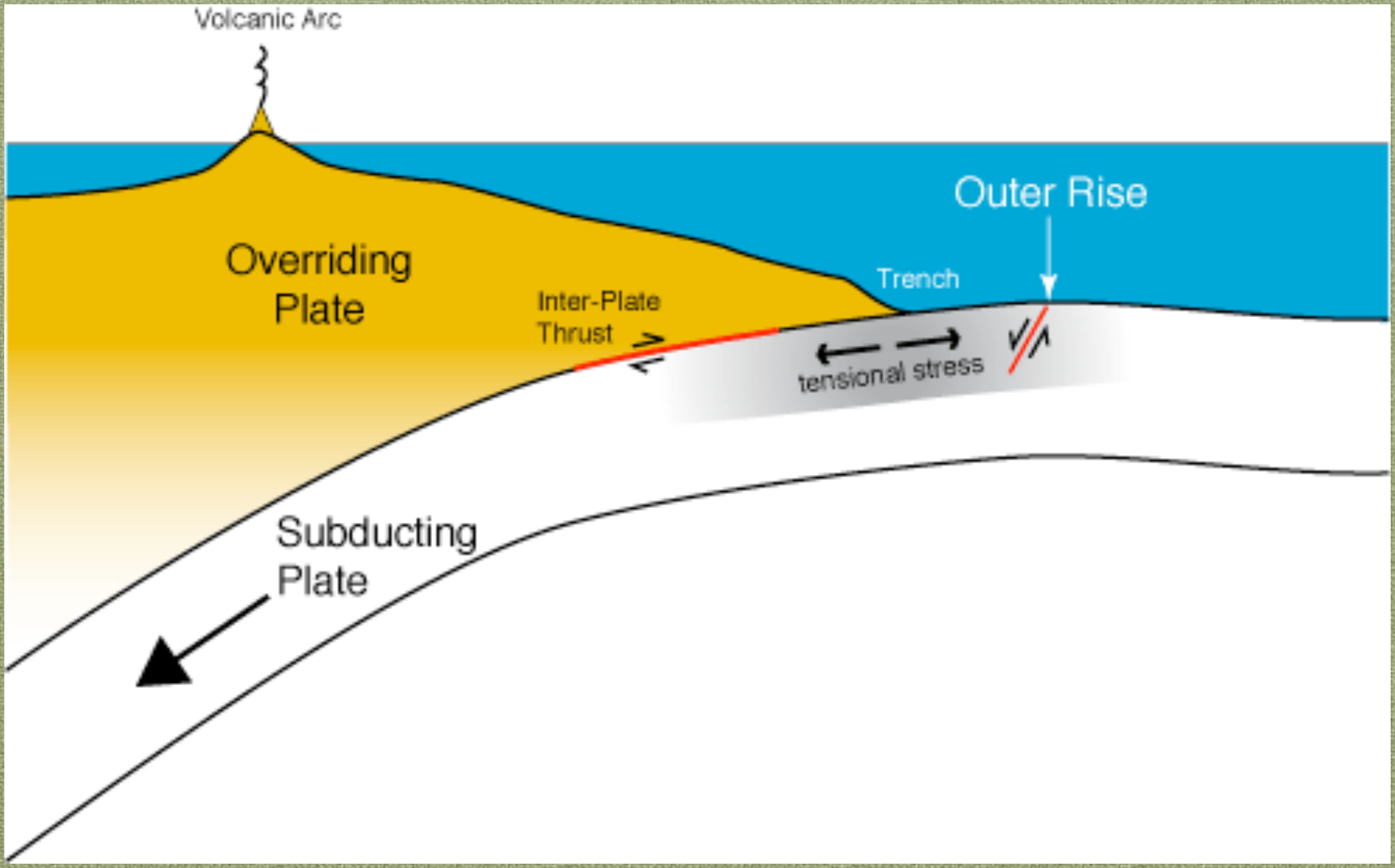
Trench Flexure

Christine Chesley
21 November 2016



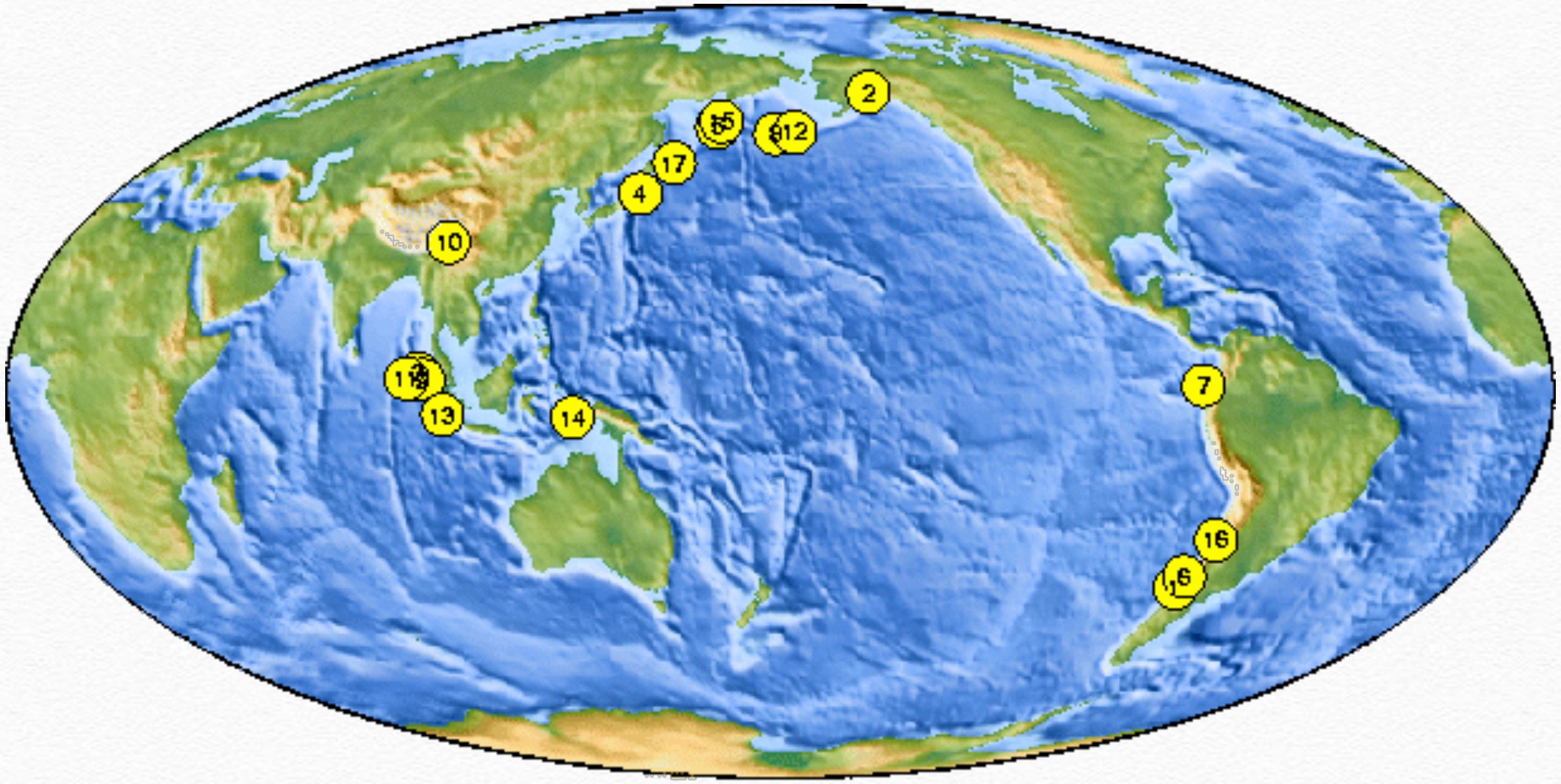
http://walrus.wr.usgs.gov/tsunami/samoa09/

Outline

- 1. Motivation
- 2. Governing Equation
- 3. Key Conclusions of *Caldwell et al.* [1976]
- 4. 6 Trench Flexure Models [*Forsyth,* 1980]

Motivation

- Why we should care?
  - Subduction type EQs are generally the largest and most destructive type

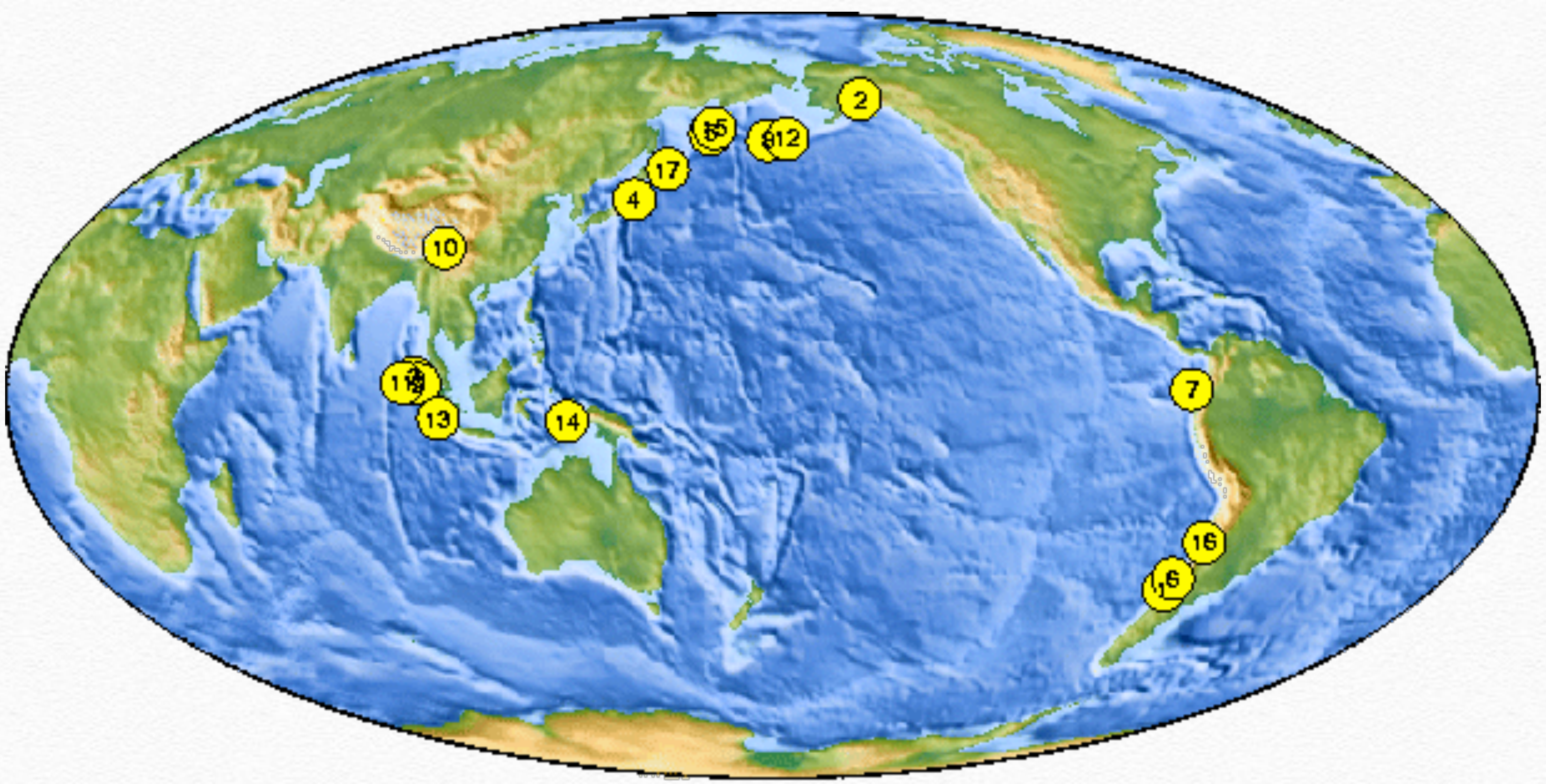


Mw 8.5-9.5 Earthquakes 1900-2012

USGS National Earthquake Information Center

Motivation

- Why we should care?
  - Flexure models can give us insight into stresses and behavior associated with subducting slab



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Governing Equation

- 1D Flexure of an elastic plate

$$D\frac{d^4w}{dx^4} + S\frac{d^2w}{dx^2} + kw = 0$$

D: flexural rigidity
S: horizontal loading
k: hydrostatic restoring force $(\rho_m - \rho_w)g$

Governing Equation

- 1. Assume the form of the solution

$$w(x) = Ce^{\eta x}$$

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- 2. Characteristic equation is thus

$$D\eta^4 + S\eta^2 + k = 0$$

$$\eta = \pm\sqrt{\frac{-S \pm \sqrt{S^2 - 4kD}}{2D}}$$

Governing Equation

- 3.Rewrite this solution in the form $\eta = \lambda + i\mu$ using algebra

$$\eta = \pm\sqrt{\frac{-S \pm \sqrt{S^2 - 4kD}}{2D}}$$

$$= \pm\sqrt{\frac{-2S \pm 2\sqrt{S^2 - 4kD}}{4D}}$$

$$= \pm\sqrt{\frac{-2S \pm 2\sqrt{S^2 - 4kD} + \sqrt{4kD} - \sqrt{4kD}}{4D}}$$

$$= \pm\sqrt{\frac{\left(-S + \sqrt{4kD}\right) \pm 2\sqrt{S^2 - 4kD} + \left(-S - \sqrt{4kD}\right)}{4D}}$$

Governing Equation

- 3.Rewrite this solution in the form $\eta = \lambda + i\mu$ using algebra

$$= \pm\sqrt{\frac{\left(\sqrt{-S+\sqrt{4kD}} \pm \sqrt{-S-\sqrt{4kD}}\right)^2}{4D}}$$

$$= \pm\sqrt{\frac{\left(\sqrt{-S+\sqrt{4kD}} \pm \sqrt{-S-\sqrt{4kD}}\right)^2}{4D}}$$

$$= \pm\left[\frac{\sqrt{-S+\sqrt{4kD}}}{\sqrt{4D}} \pm \frac{\sqrt{-S-\sqrt{4kD}}}{\sqrt{4D}}\right]$$

$$\eta = \pm\left[\frac{\sqrt{-S+\sqrt{4kD}}}{\sqrt{4D}} \pm \frac{i\sqrt{S+\sqrt{4kD}}}{\sqrt{4D}}\right]$$

Governing Equation

- 4. This gives us a general solution

$$w(x) = e^{\sqrt{\frac{-S+\sqrt{4kD}}{4D}}x}\left(c_1 cos\sqrt{\frac{S+\sqrt{4kD}}{4D}}x + c_2 sin\sqrt{\frac{S+\sqrt{4kD}}{4D}}x\right) + e^{-\sqrt{\frac{-S+\sqrt{4kD}}{4D}}x}\left(c_3 cos\sqrt{\frac{S+\sqrt{4kD}}{4D}}x + c_4 sin\sqrt{\frac{S+\sqrt{4kD}}{4D}}x\right)$$

- 5. Boundary Conditions

Governing Equation

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- 5. Boundary Conditions

$$\lim_{x\to\infty} w(x) = 0 \quad implies \quad c_1 = c_2 = 0$$

$$w(0) = 0 \quad implies \; c_3 = 0$$

Governing Equation

- 4. This gives us a general solution

$$w(x) = e^{\sqrt{\frac{-S+\sqrt{4kD}}{4D}}x}\left(c_1 cos\sqrt{\frac{S+\sqrt{4kD}}{4D}}x + c_2 sin\sqrt{\frac{S+\sqrt{4kD}}{4D}}x\right) + e^{-\sqrt{\frac{-S+\sqrt{4kD}}{4D}}x}\left(c_3 cos\sqrt{\frac{S+\sqrt{4kD}}{4D}}x + c_4 sin\sqrt{\frac{S+\sqrt{4kD}}{4D}}x\right)$$

- 5. Boundary Conditions

$$w(x) = c_4 e^{-\sqrt{\frac{-S+\sqrt{4kD}}{4D}}x} sin\sqrt{\frac{S+\sqrt{4kD}}{4D}}x$$

Governing Equation

- 6. More algebra to mirror *Caldwell et al.* [1976]

$$w(x) = c_4 e^{-\left(\frac{\left(-S+2\sqrt{kD}\right)^2}{4D\cdot 4D}\right)^{\frac{1}{4}} x} \sin\left(\frac{\left(S+2\sqrt{kD}\right)^2}{4D\cdot 4D}\right)^{\frac{1}{4}} x$$

$$w(x) = c_4 e^{-\left(\frac{\left(-S+2\sqrt{kD}\right)^2}{\frac{4D}{k}\cdot 4kD}\right)^{\frac{1}{4}} x} \sin\left(\frac{\left(S+2\sqrt{kD}\right)^2}{\frac{4D}{k}\cdot 4kD}\right)^{\frac{1}{4}} x$$

$$w(x) = c_4 e^{-\left(\frac{\left(-S+2\sqrt{kD}\right)^2}{\frac{4D}{k}\cdot (2\sqrt{kD})^2}\right)^{\frac{1}{4}} x} \sin\left(\frac{\left(S+2\sqrt{kD}\right)^2}{\frac{4D}{k}\cdot (2\sqrt{kD})^2}\right)^{\frac{1}{4}} x$$

Governing Equation

- 6. More algebra to mirror *Caldwell et al.* [1976]

$$w(x) = c_4 e^{-\left(\frac{\left(-\frac{S}{2\sqrt{kD}}+1\right)^2}{\frac{4D}{k}}\right)^{\frac{1}{4}} x} \sin\left(\frac{\left(\frac{S}{2\sqrt{kD}}+1\right)^2}{\frac{4D}{k}}\right)^{\frac{1}{4}} x$$

$$w(x) = c_4 e^{-\frac{x}{(4D/k)^{1/4}}\left(-\frac{S}{2\sqrt{kD}}+1\right)^{1/2}} \sin\left[\frac{x}{(4D/k)^{1/4}}\left(\frac{S}{2\sqrt{kD}}+1\right)^{1/2}\right]$$

$$\epsilon = \frac{S}{2\sqrt{kD}} \qquad \alpha^4 = \frac{4D}{k} \qquad A = c_4$$

Governing Equation

- 6. More algebra to mirror *Caldwell et al.* [1976]

$$w(x) = A\, sin\left(\frac{x}{\alpha}(1+\epsilon)^{1/2}\right) e^{-\frac{x}{\alpha}(1-\epsilon)^{1/2}}$$

Governing Equation

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$$w(x) = A\,sin\left(\frac{x}{\alpha}(1+\epsilon)^{1/2}\right)e^{-\frac{x}{\alpha}(1-\epsilon)^{1/2}}$$

flexural parameter
$(4D/k)^{1/4}$

depends on horizontal loading
$S/(2\sqrt{kD})$

Governing Equation

- x has a maximum at

$$x_b = \frac{\alpha}{(1+\epsilon)^{1/2}} arctan\Big(\frac{1+\epsilon}{1-\epsilon}\Big)^{1/2}$$

- which corresponds to a deflection of

$$w_b = \frac{A(1+\epsilon)^{1/2}}{2^{1/2}} exp\Big[\frac{x_b}{\alpha}(1-\epsilon)^{1/2}\Big]$$

Governing Equation

- Even for an average horizontal stress of 10 kilobars (1 GPa), ϵ is only ~ 0.3

Governing Equation

- Even for an average horizontal stress of 10 kilobars (1 GPa), ϵ is only ~ 0.3

- Letting $\epsilon = 0$, we have

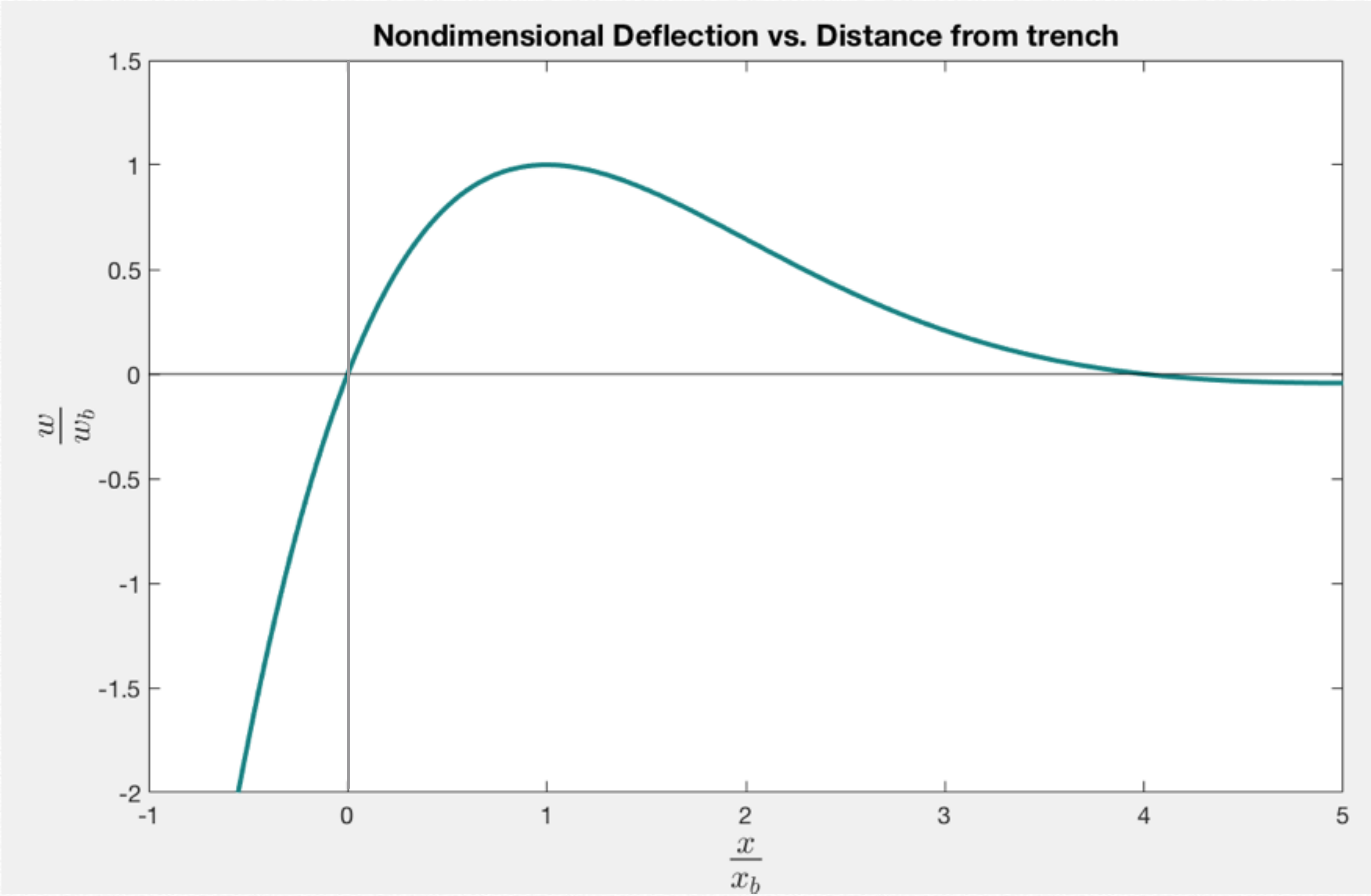
$$x_b = \frac{\pi\alpha}{4}$$

Governing Equation

- It is useful to non-dimensionalize x and w(x) (since A will vary for different profiles)

$$\bar{x} = \frac{x}{x_b} \qquad \bar{w} = \frac{w}{w_b}$$

$$\bar{w} = 2^{1/2} sin\left(\bar{x}\frac{\pi}{4}\right) exp\left[\frac{\pi}{4}(1-\bar{x})\right]$$

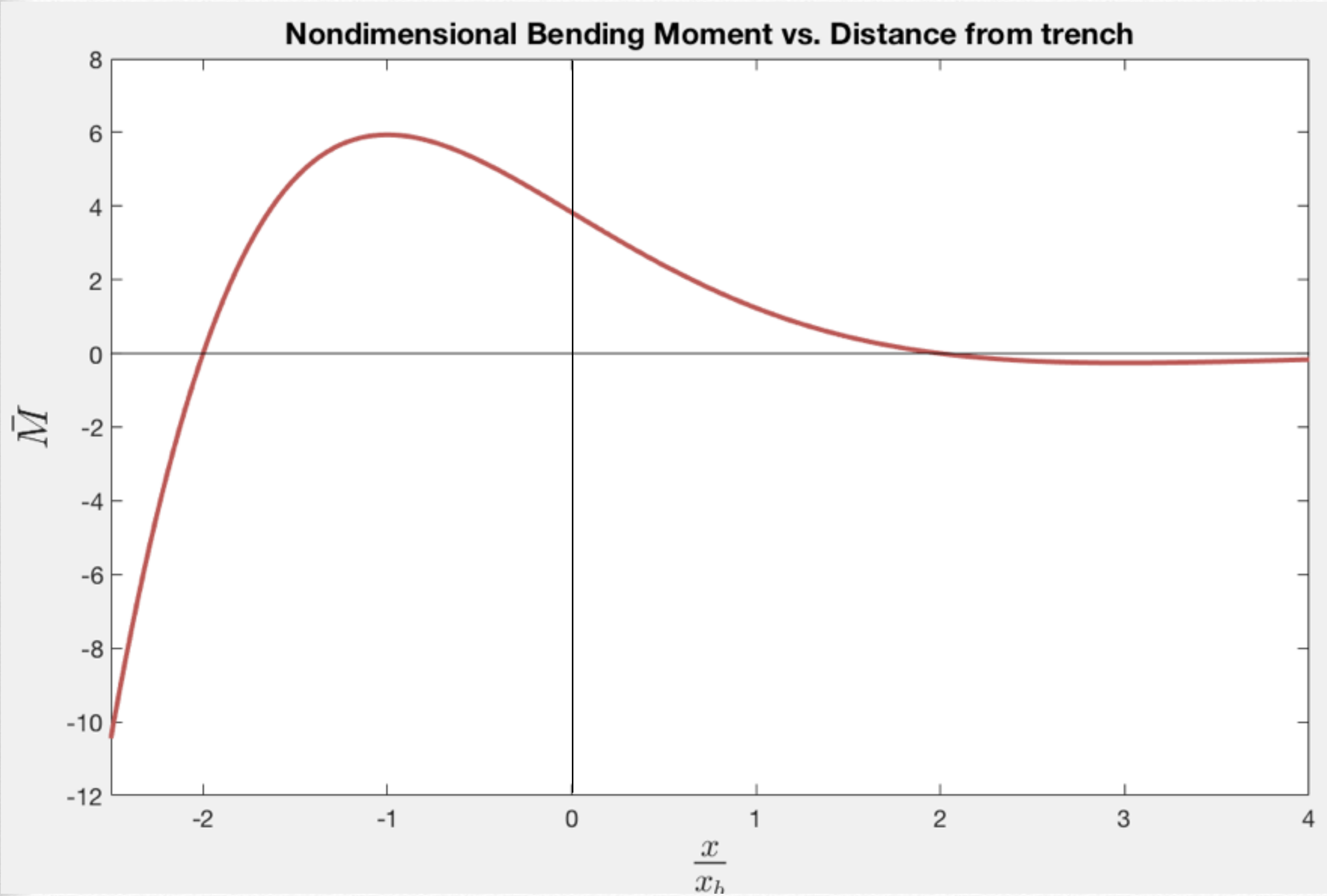

Nondimensional Deflection vs. Distance from trench
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-1
0
1
2
3
4
5
$\frac{w}{w_b}$
$\frac{x}{x_b}$

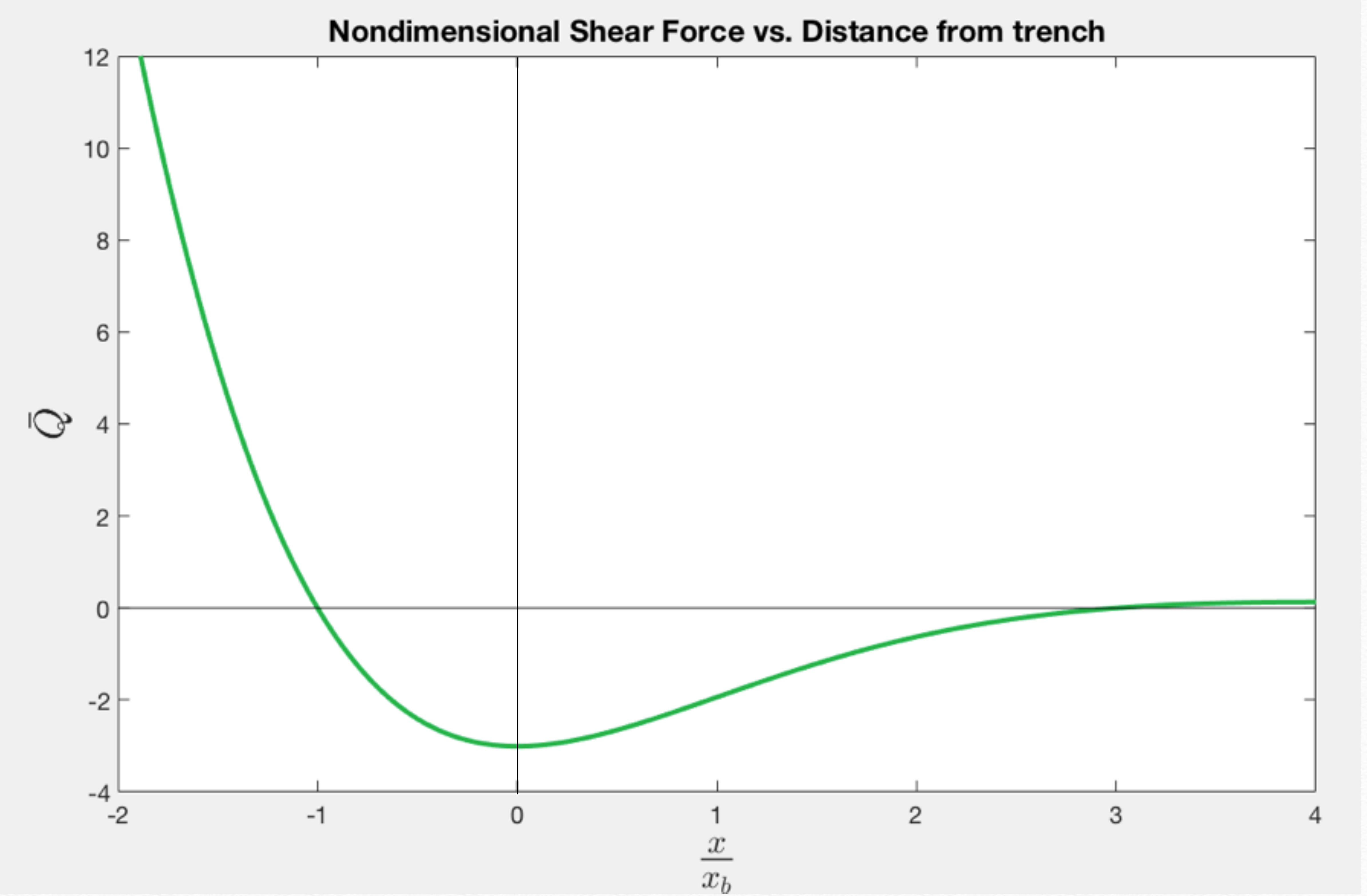
Governing Equation

- We may similarly non-dimensionalize the moment, M, and shear force, Q

$$\bar{M} = \frac{M {x_b}^2}{D w_b}$$
$$= \frac{2^{1/2}\pi^2}{8} cos\left(\frac{\pi\bar{x}}{4}\right) exp\left[\frac{\pi}{4}(1-\bar{x})\right]$$

$$\bar{Q} = \frac{Q {x_b}^3}{D w_b}$$
$$= \frac{2^{1/2}\pi^3}{32}\left[cos\left(\frac{\pi\bar{x}}{4}\right) + sin\left(\frac{\pi\bar{x}}{4}\right)\right] exp\left[\frac{\pi}{4}(1-\bar{x})\right]$$


Nondimensional Bending Moment vs. Distance from trench
8
6
4
2
0
-2
-4
-6
-8
-10
-12
$\bar{M}$
-2
-1
0
1
2
3
4
$\frac{x}{x_b}$



Nondimensional Shear Force vs. Distance from trench
$\bar{Q}$
$\frac{x}{x_b}$
12
10
8
6
4
2
0
-2
-4
-2
-1
0
1
2
3
4

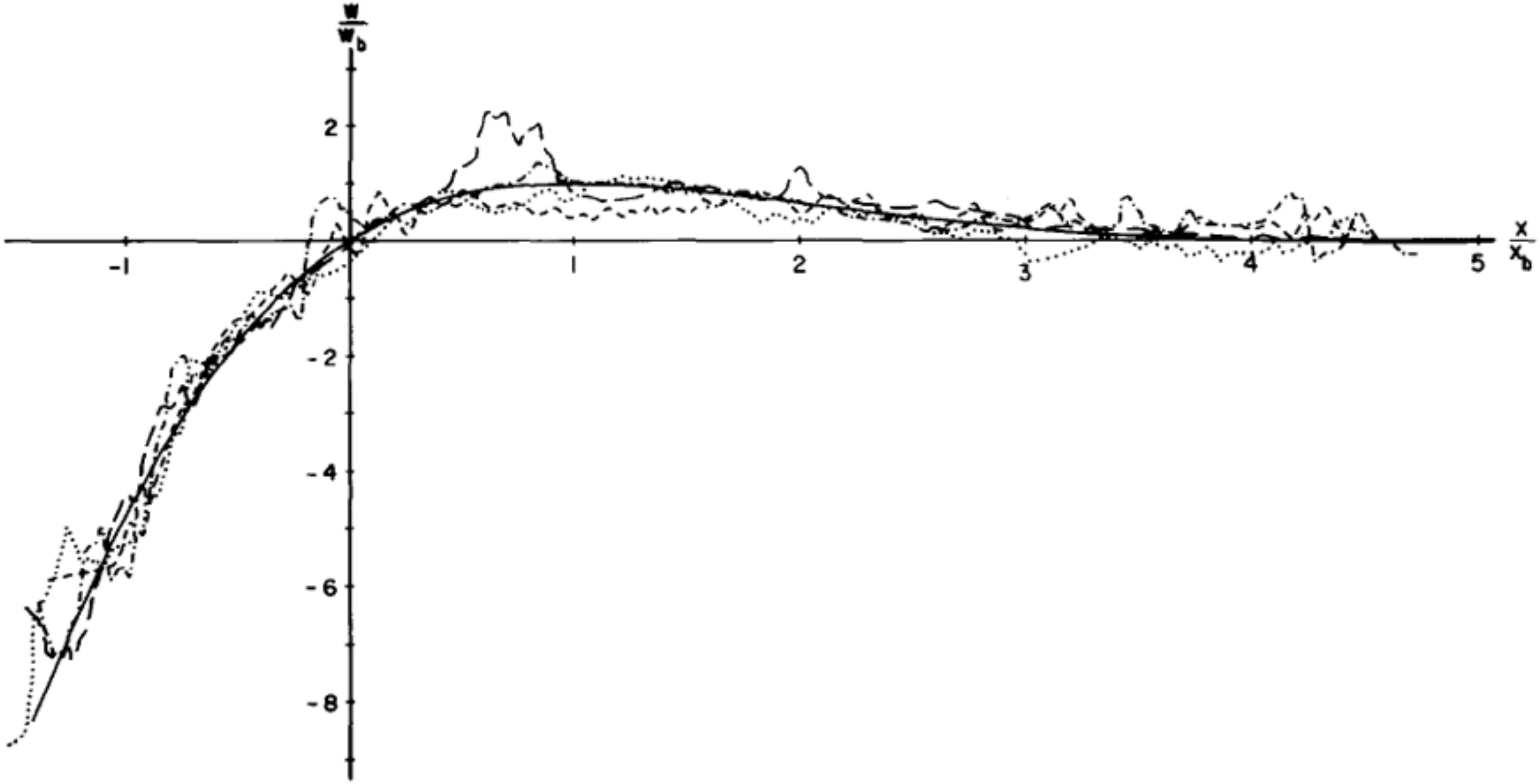


Fig. 6. The solid line is the universal deflection curve and the broken lines are the corrected and normalized bathymetric profiles. – – – is the Mariana profile (data from Scan 5 cruise and [19]), – · – · is the Bonin profile (Hunt 3 and Aries 7 cruises), · · · · · is the Kuril profile (Zetes 2 cruise and [18]), and - - - - - is the central Aleutian profile (Seamap 13 cruise and [17]).

Key Conclusions of Caldwell et al. [1976]

- Universal model fits well for several subduction zones
- Thin elastic plate under applied vertical force and bending moment can model trench flexure
- Not much difference between using 0 vs. 10 kilobars of horizontal force—plate behaves elastically anyway
- Caution: This study looked at mostly older subducting lithosphere—reason to believe it doesn't work well for younger subduction

6 Trench Flexure Models

- Forsyth, D. W. Comparison of mechanical models of the oceanic lithosphere. *J. Geosphys. Res.*, 85(B11), 6364-6368.

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  - Elastic
  - Elastic w/ Horizontal Compression
  - Elastic-Plastic
  - Elastic-Plastic w/Horizontal Compression
  - Elastic-Plastic w/Variable Yield Strength
  - Viscous

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How well do these models fit observations?

6 Trench Flexure Models

- Which observations?

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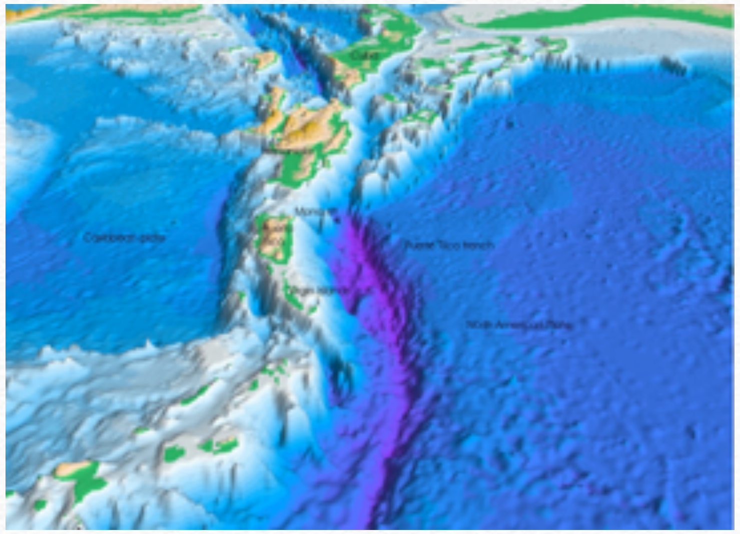
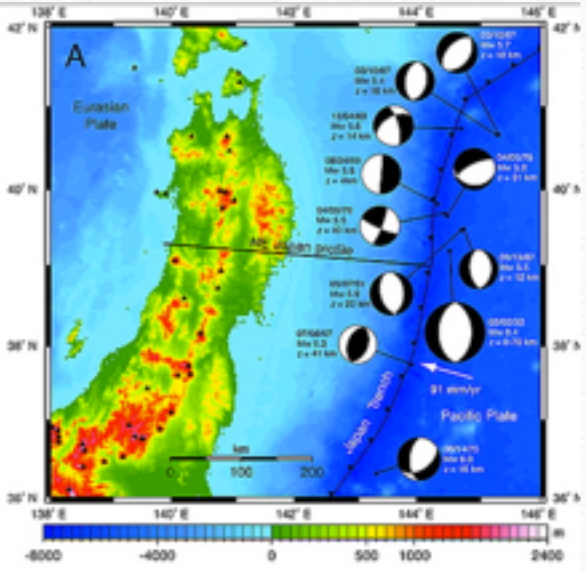
  - Topography

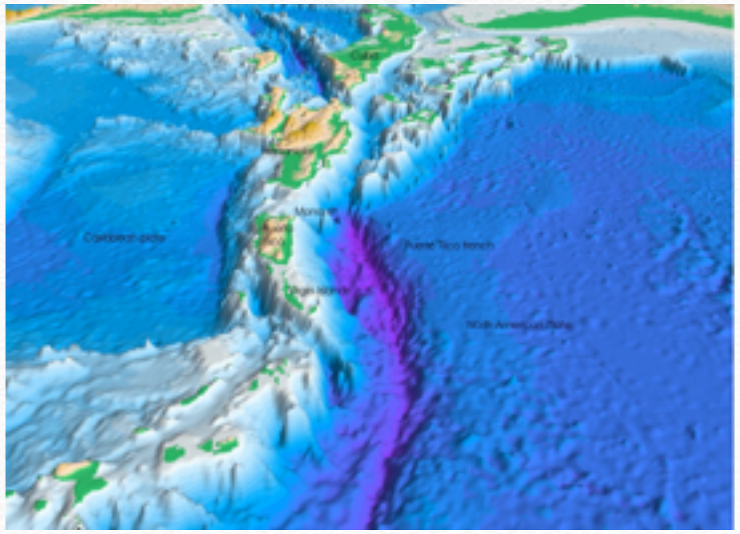


http://academic.emporia.edu/aberjame/student/brown2/Images/Atlantic_trench.jpg

6 Trench Flexure Models

- Which observations?
  - Topography
  - Seismicity





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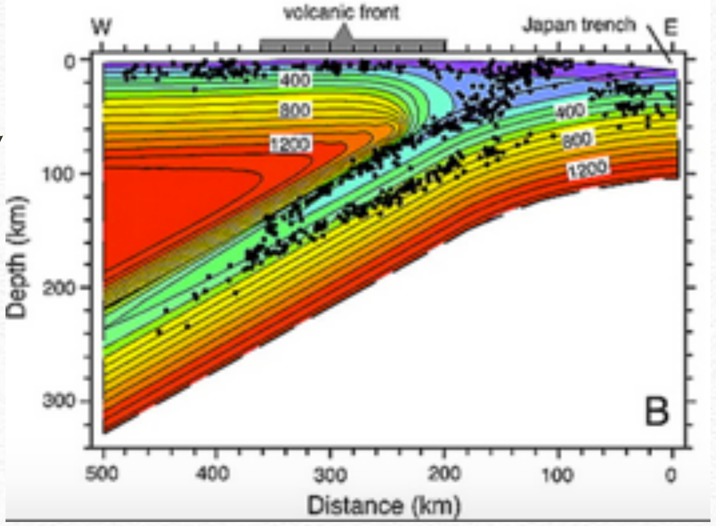
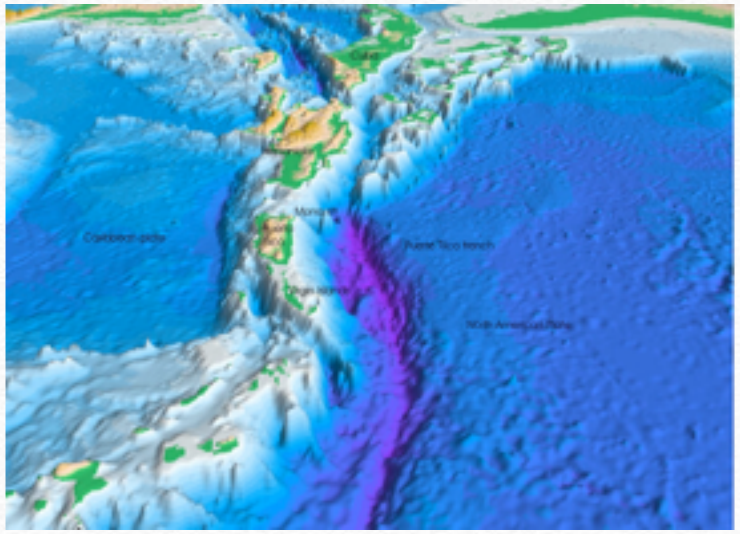
6 Trench Flexure Models

- Which observations?
  - Topography
  - Seismicity
  - Depth of Normal Faults





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- Which observations?
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  - Seismicity
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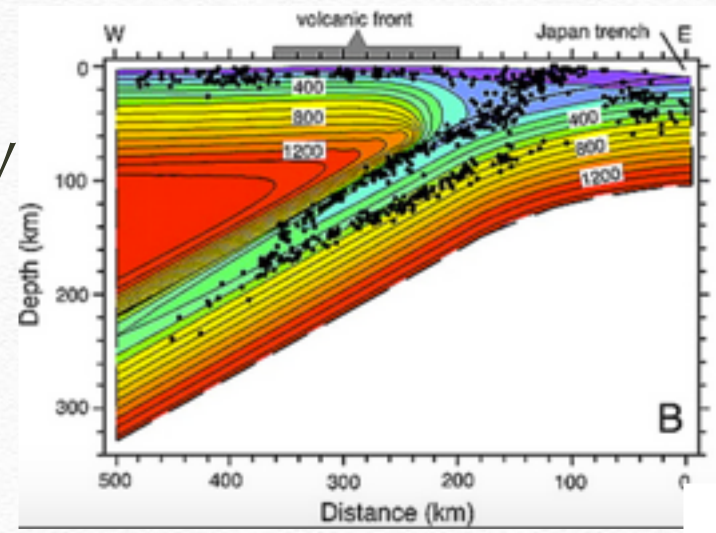
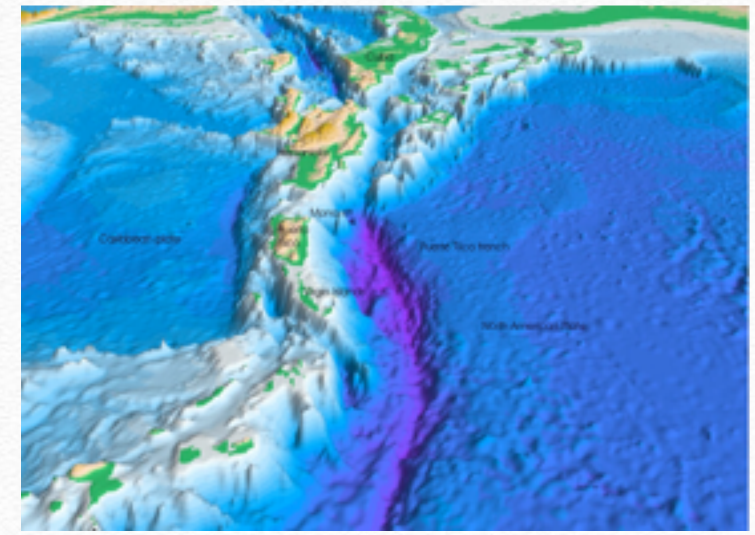
  - Seamount Loading





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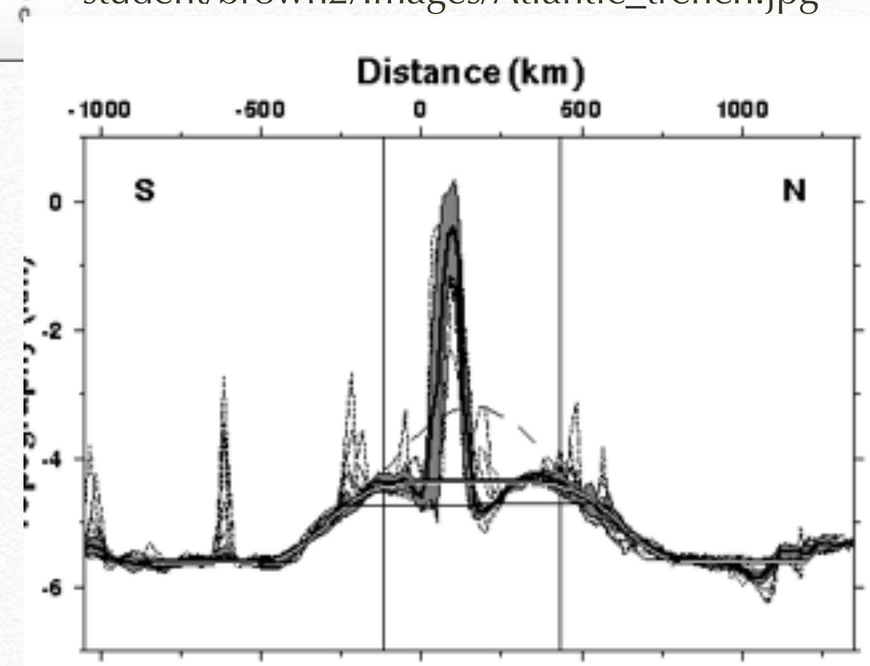

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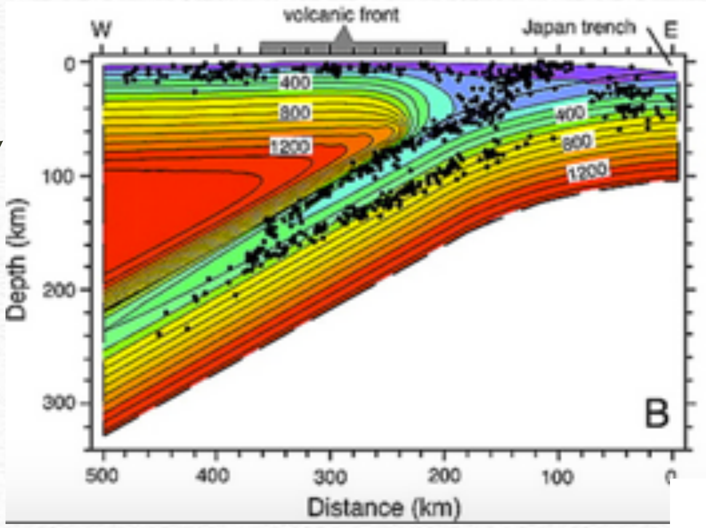
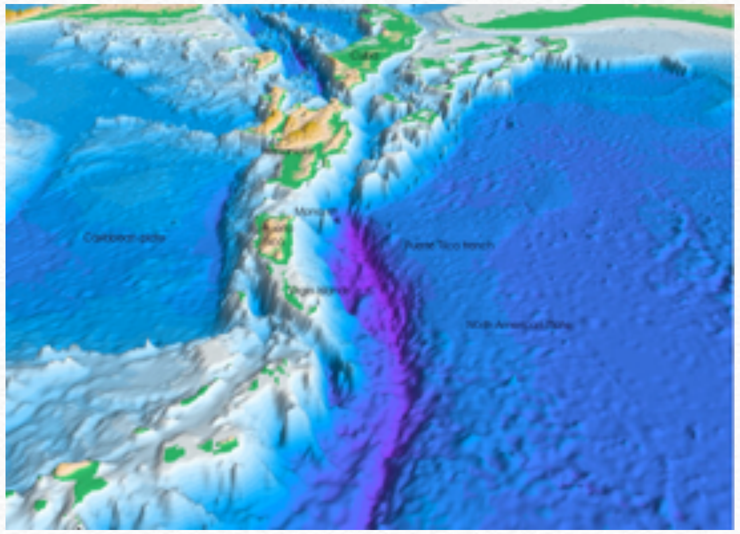
6 Trench Flexure Models

- Which observations?
  - Topography
  - Seismicity
  - Depth of Normal Faults
  - Seamount Loading
  - And more…





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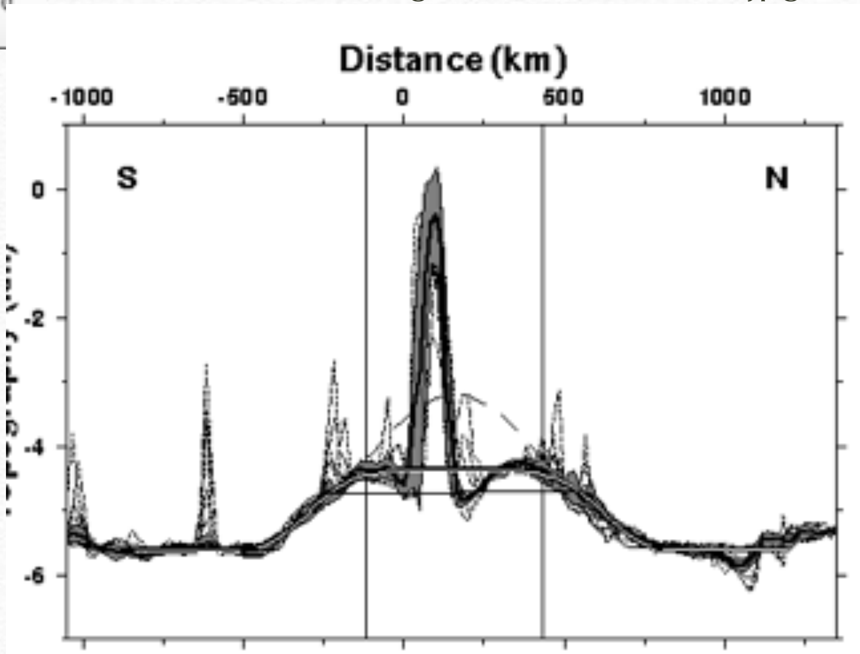

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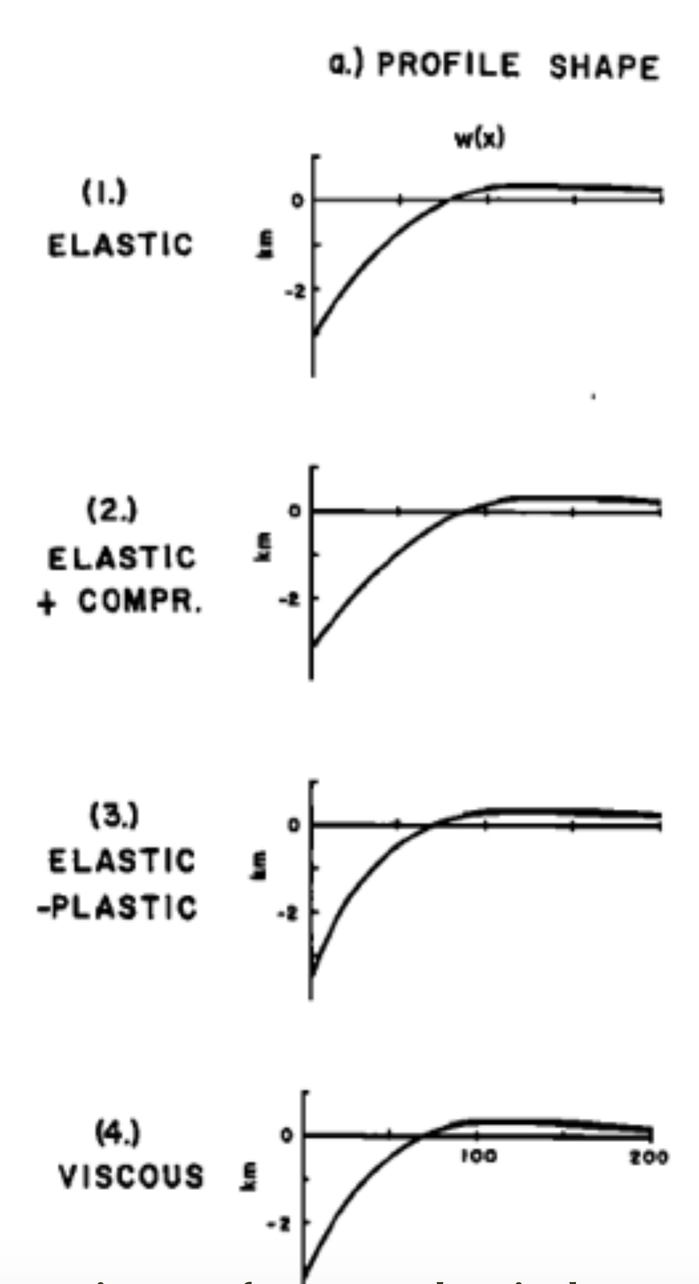
6 Trench Flexure Models

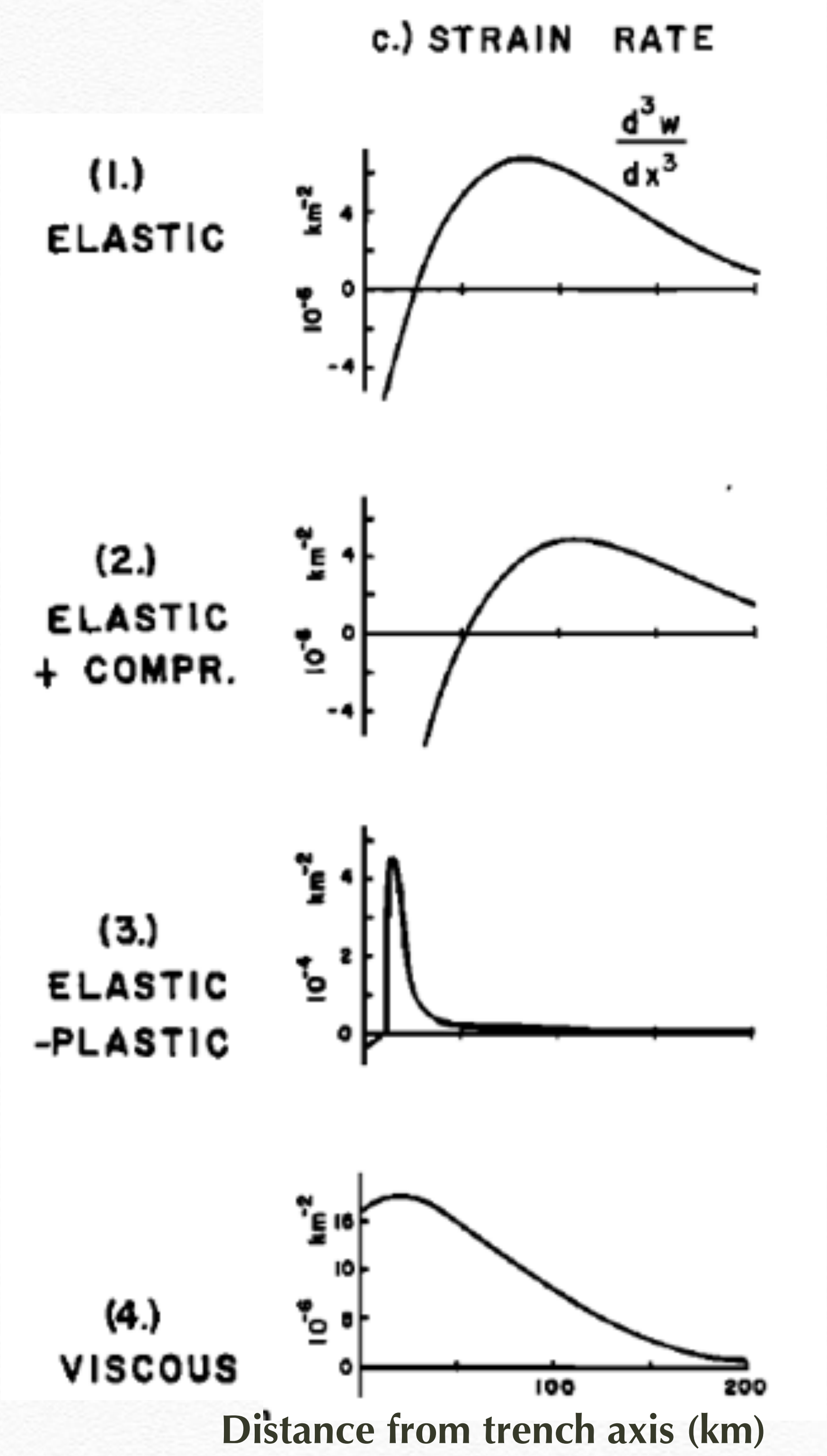
- TOPOGRAPHY—Similar for all 6



Distance from trench axis (km)

6 Trench Flexure Models

- SEISMICITY—3rd derivative of deflection tells us about extension (normal faulting)



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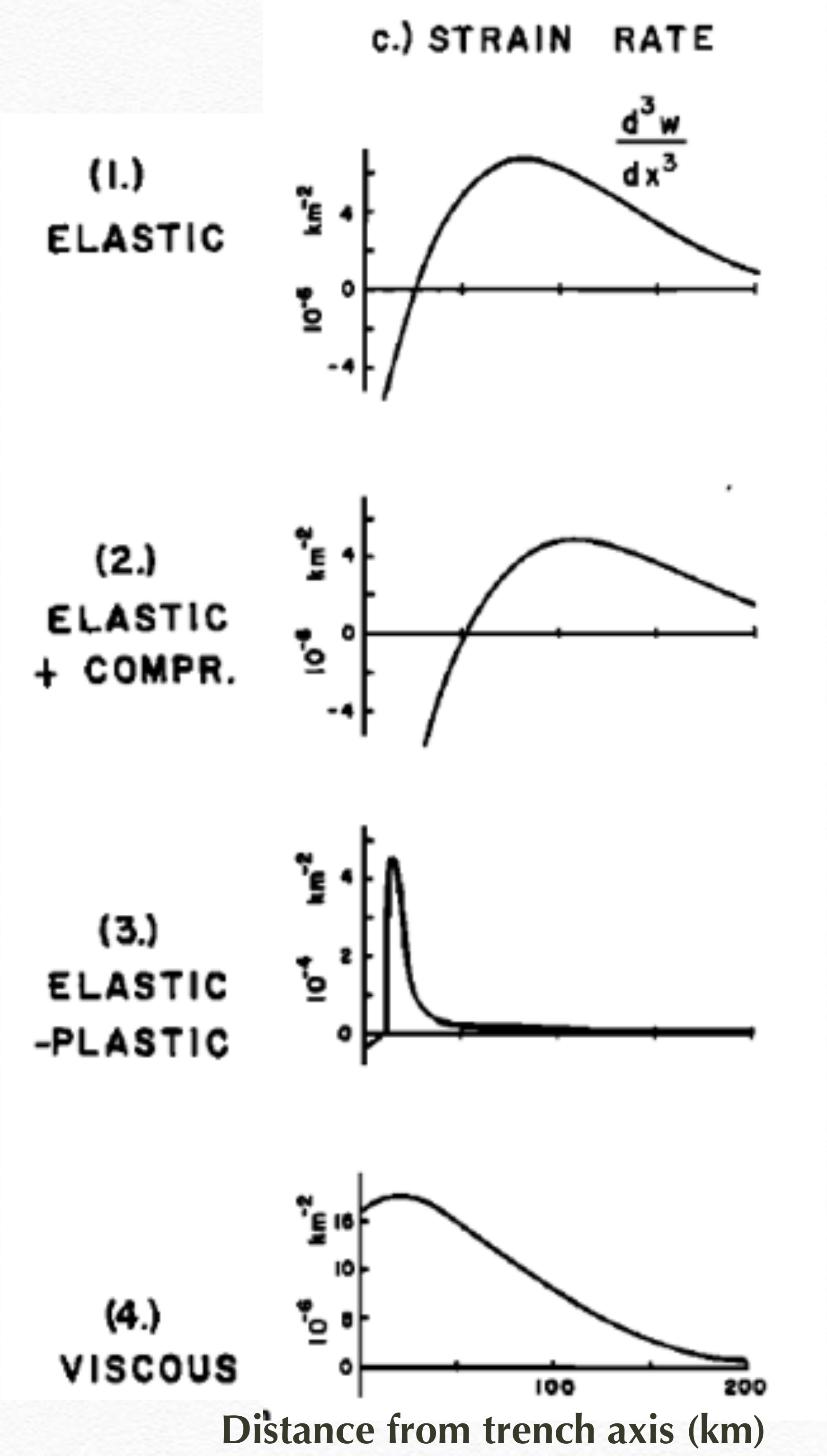
- SEISMICITY—3rd derivative of deflection tells us about extension (normal faulting)
- Should peak near trench axis



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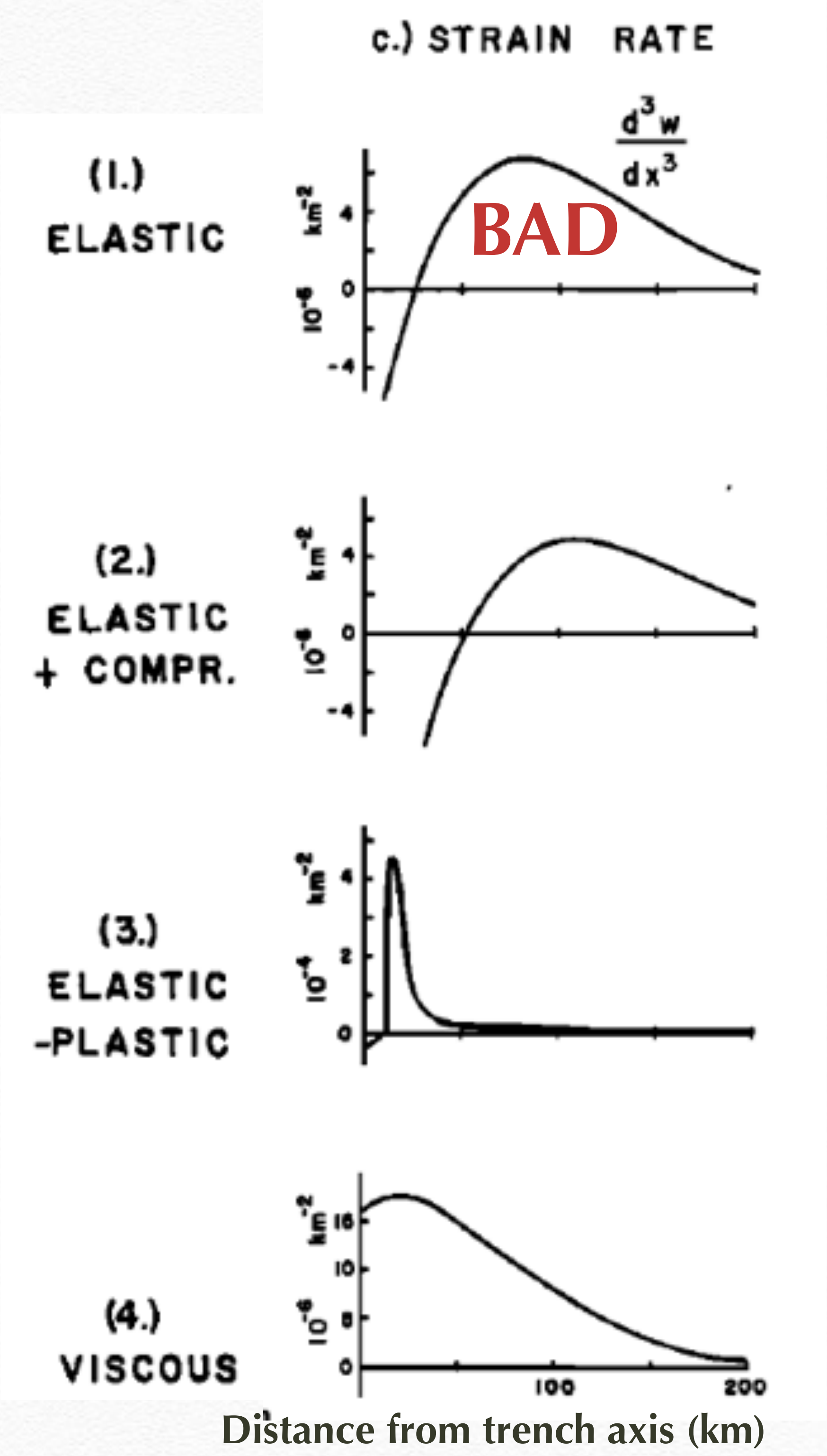

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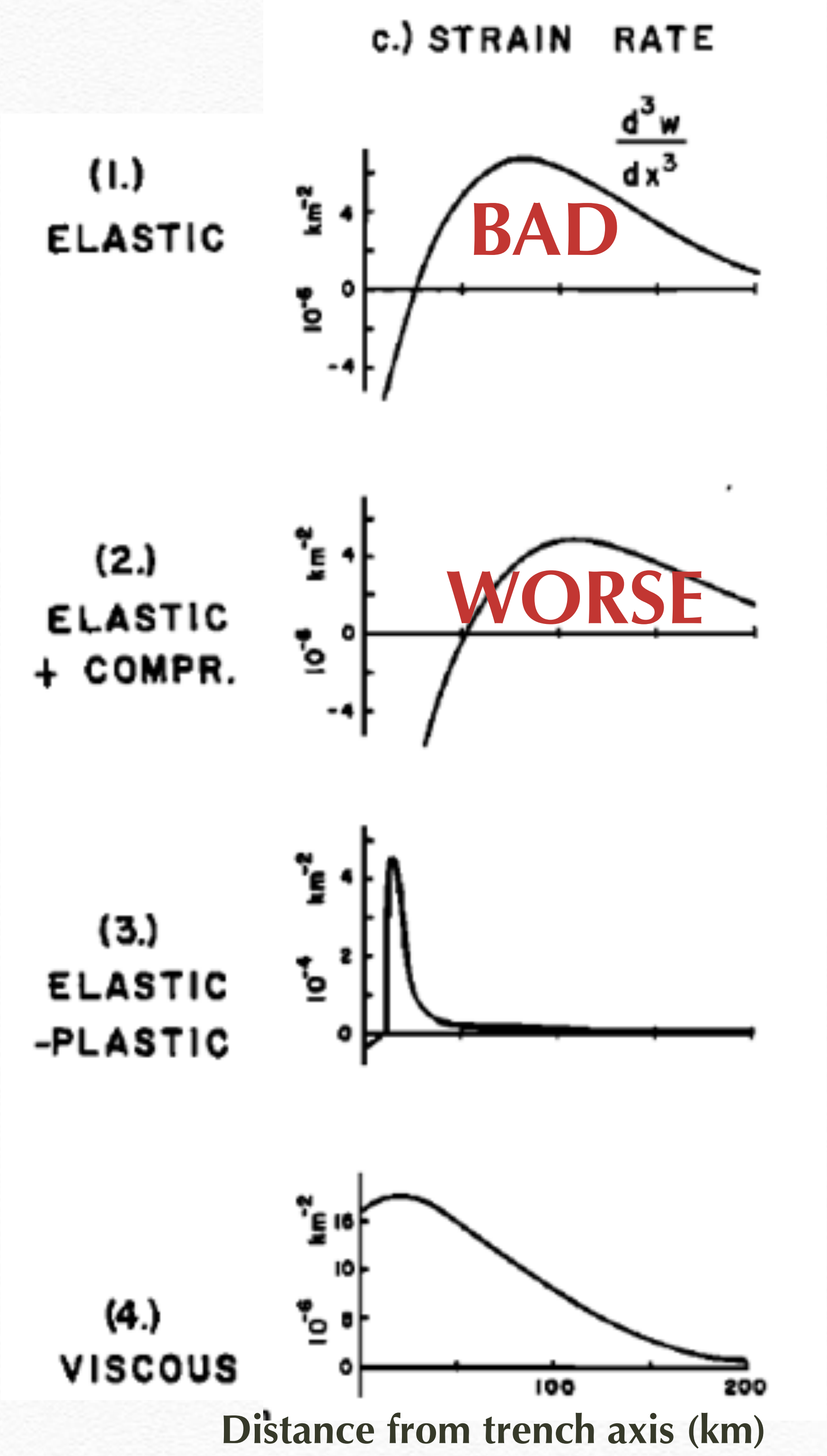

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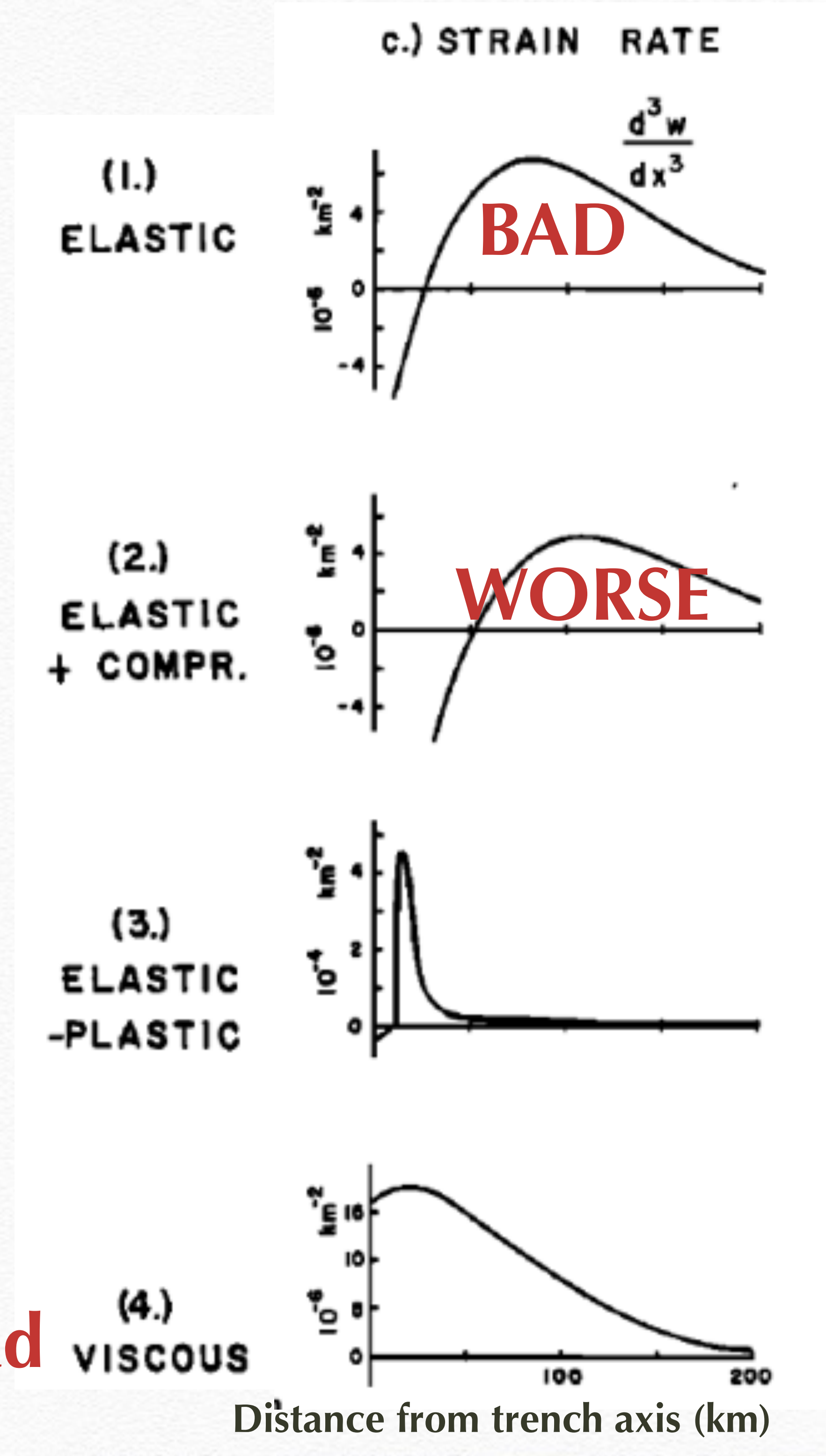

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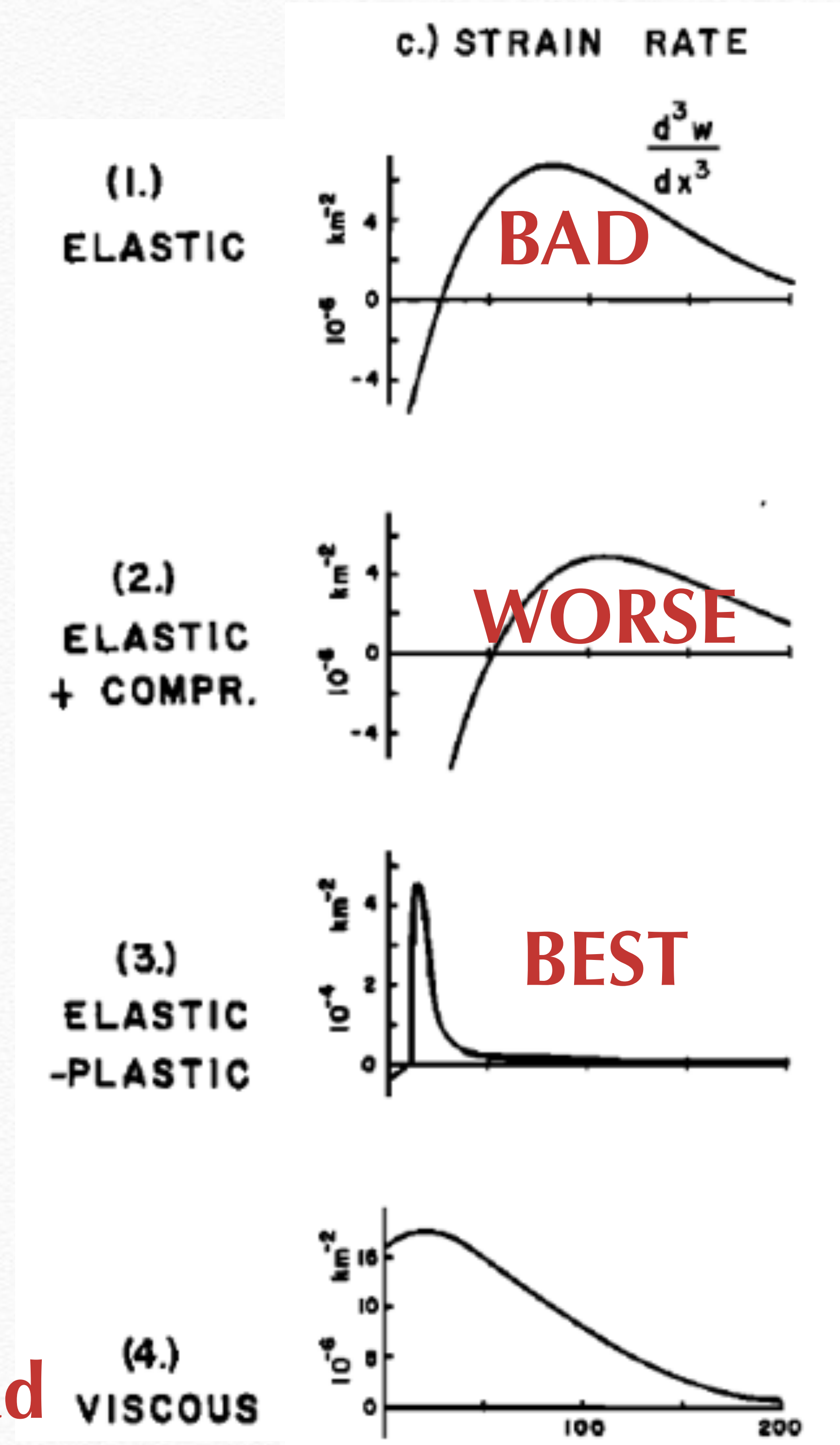
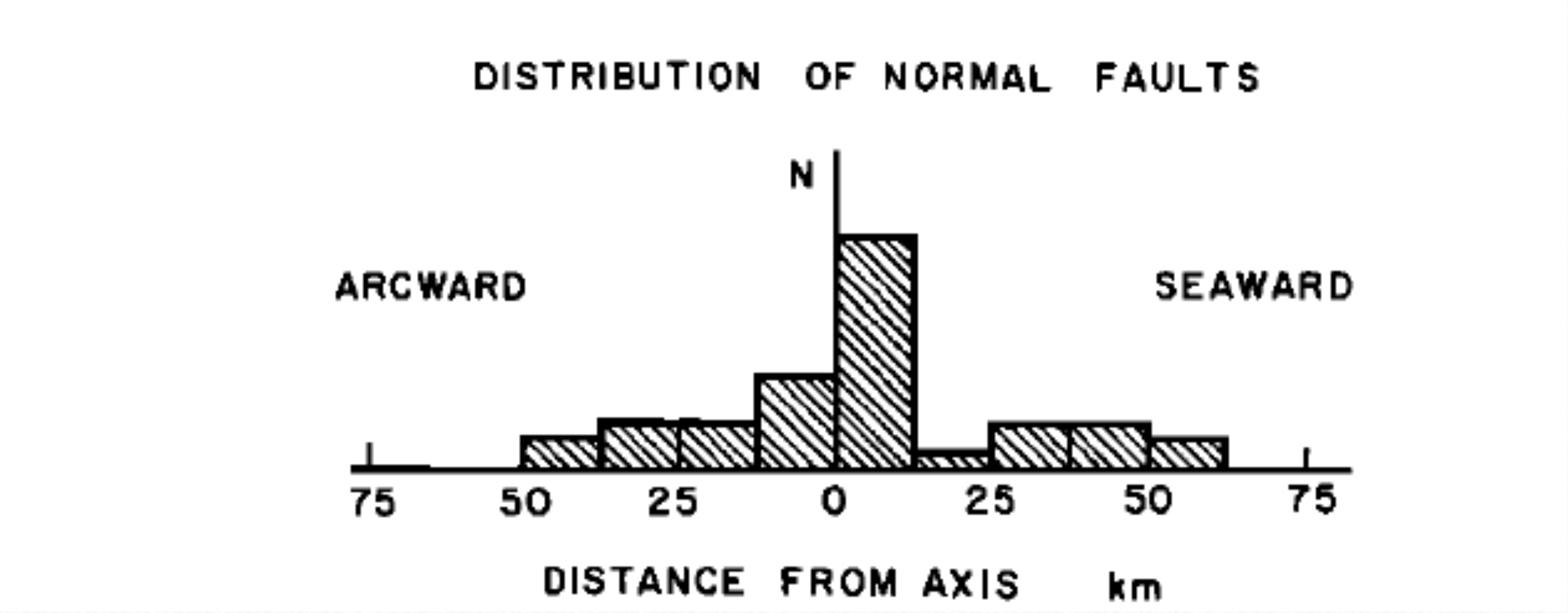


Pretty good, but peak is too broad

Distance from trench axis (km)

6 Trench Flexure Models

- NORMAL FAULTING AT DEPTH—There are normal faults as deep as 25 km (maybe even deeper) [*Chapple & Forsyth*, 1979]



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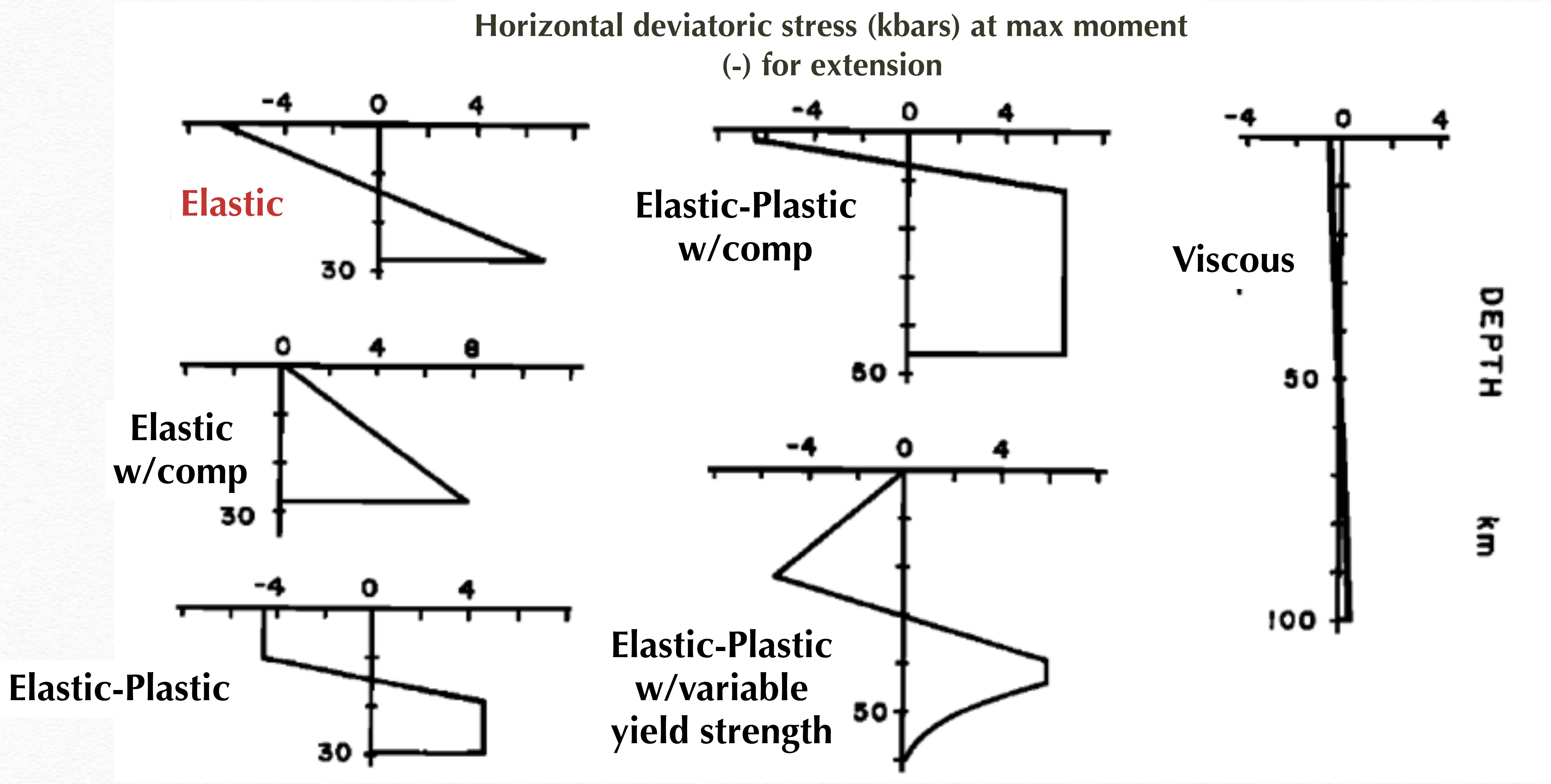

6 Trench Flexure Models

- SEAMOUNT LOADING—Models that describe flexure at trench should also describe behavior of other topographies like seamounts

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- Elastic and Elastic-Plastic models do this well

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    - Horizontal compression—does not accurately describe most observations

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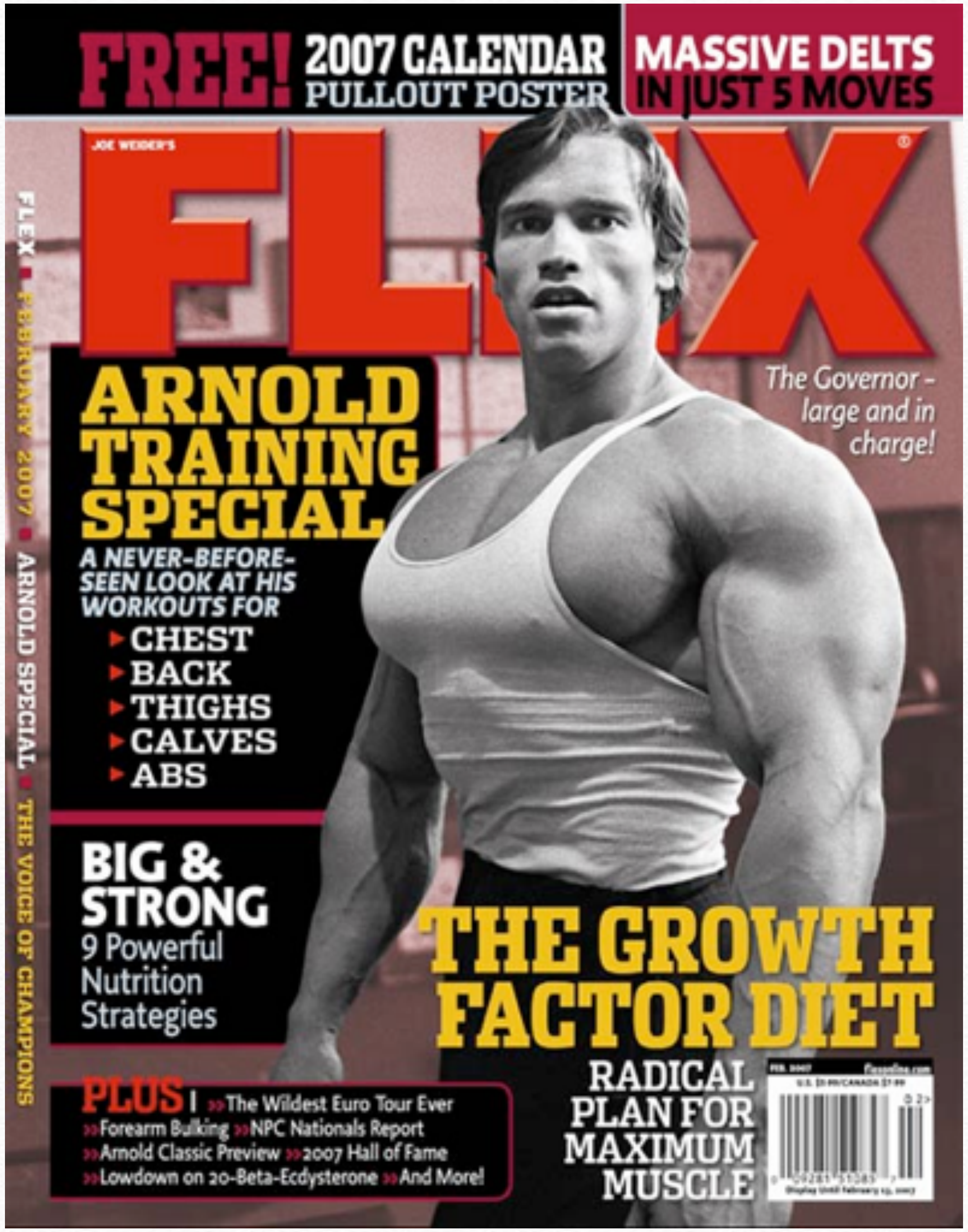
6 Trench Flexure Models

- Takeaways from *Forsyth*, 1980:
  - Horizontal compression—does not accurately describe most observations
  - Best model is Elastic-Plastic model with variable yield strength
  - Elastic and Elastic-Plastic models seem to be decent simplifications

Thanks for Flexing!



http://www.bodybuilding.com/fun/images/2007/flex_feb07cover.jpg

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