Constraints on ***Yield Strength*** in the Oceanic Lithosphere derived from Observations of Flexure

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@Geodynamics - Homework 6 Group G

Bending of the plate

- The deflection of the plate w satisfies the PDE:

$$\frac{d^2M}{dx^2} - N\frac{d^2w}{dx^2} - \Delta\rho g w = 0$$

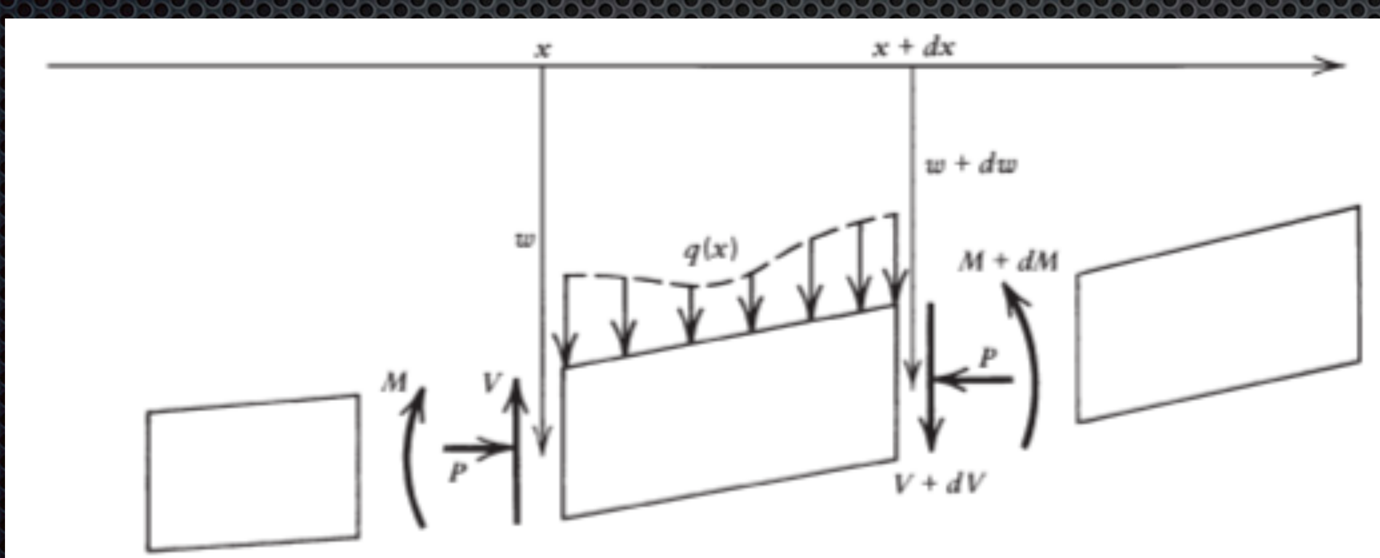
end load

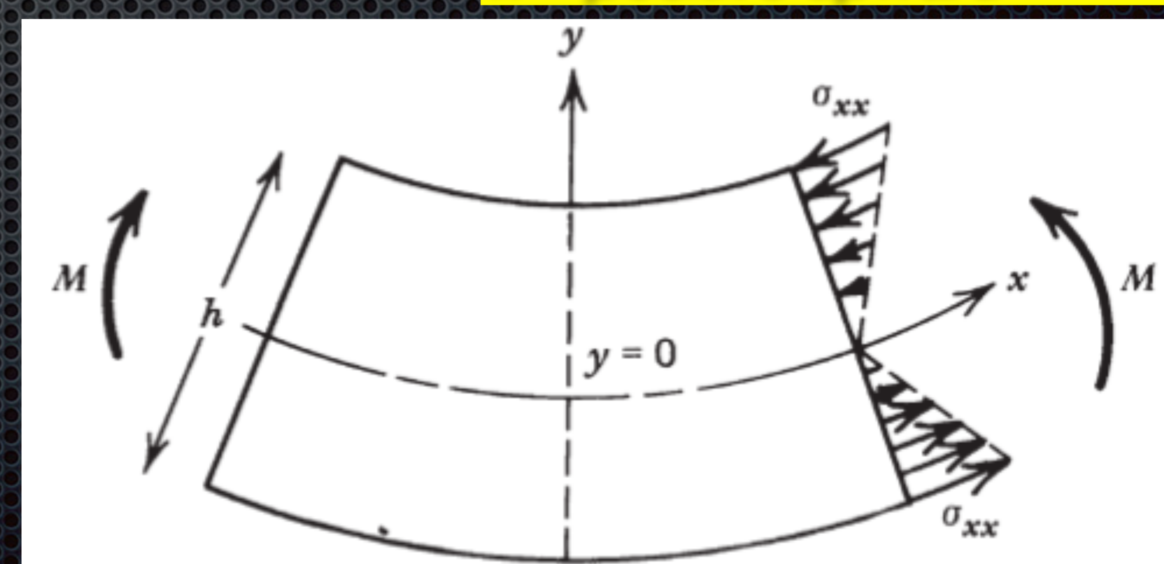
restoring force

- The fiber stress is caused by the bending moment

$$M = \int_0^H \sigma_f (z - z_n) dz$$

(+) stress - tensional
(-) stress - compressional
z - positive downward
w - positive upward





(Turcotte and Schubert, 2014)

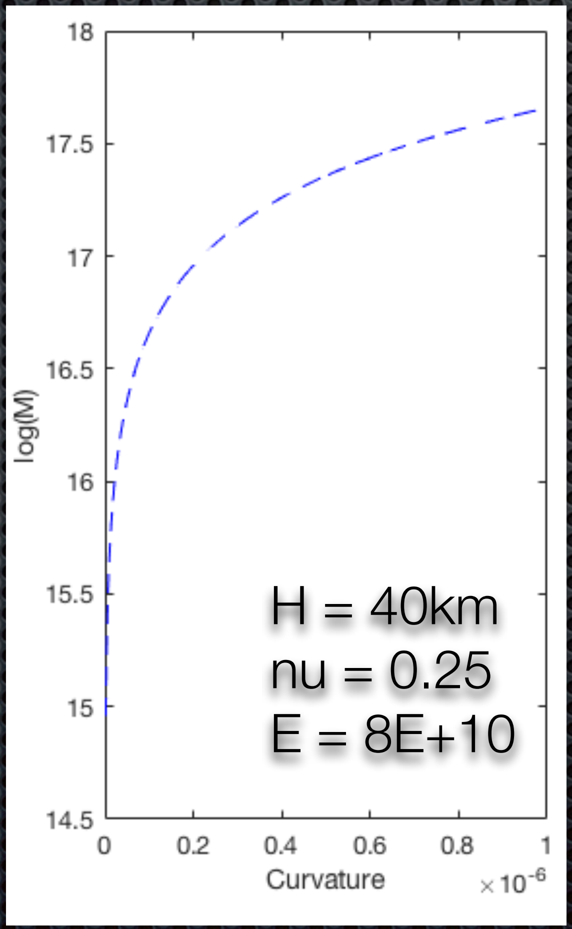
Imagine a totally elastic world…

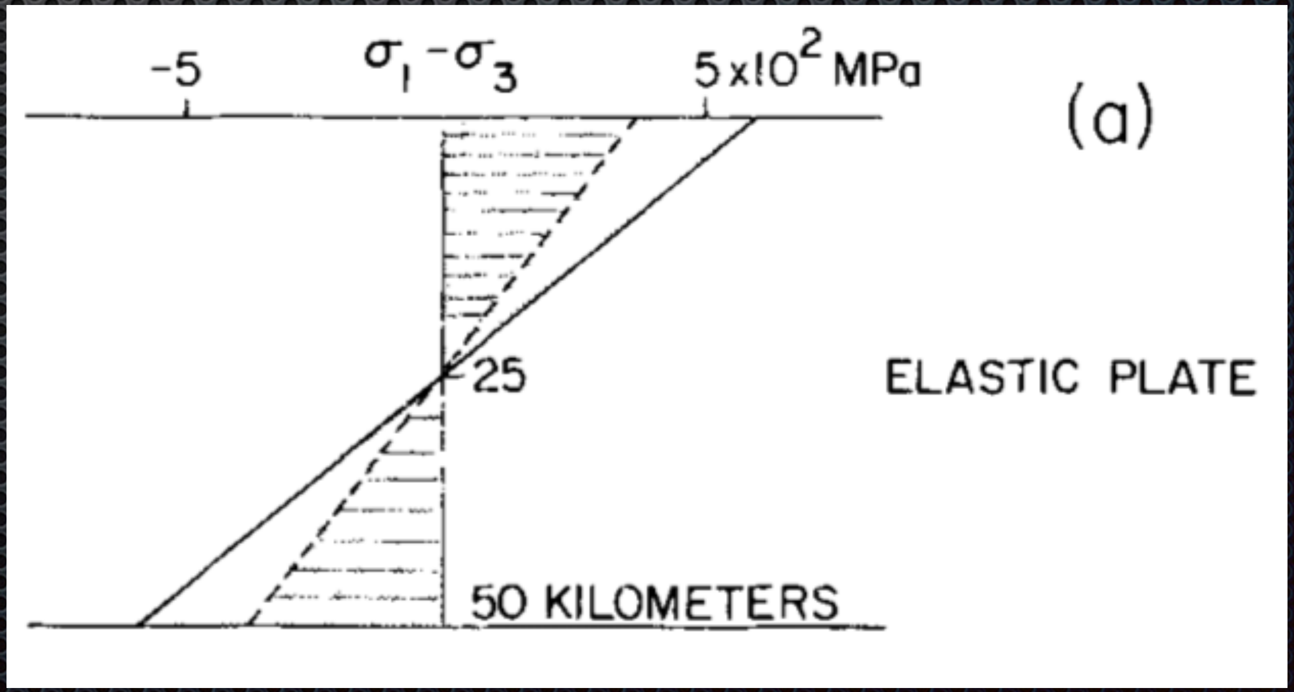
- Bending moment is expressed by

$$M(x) = -DK(x)$$

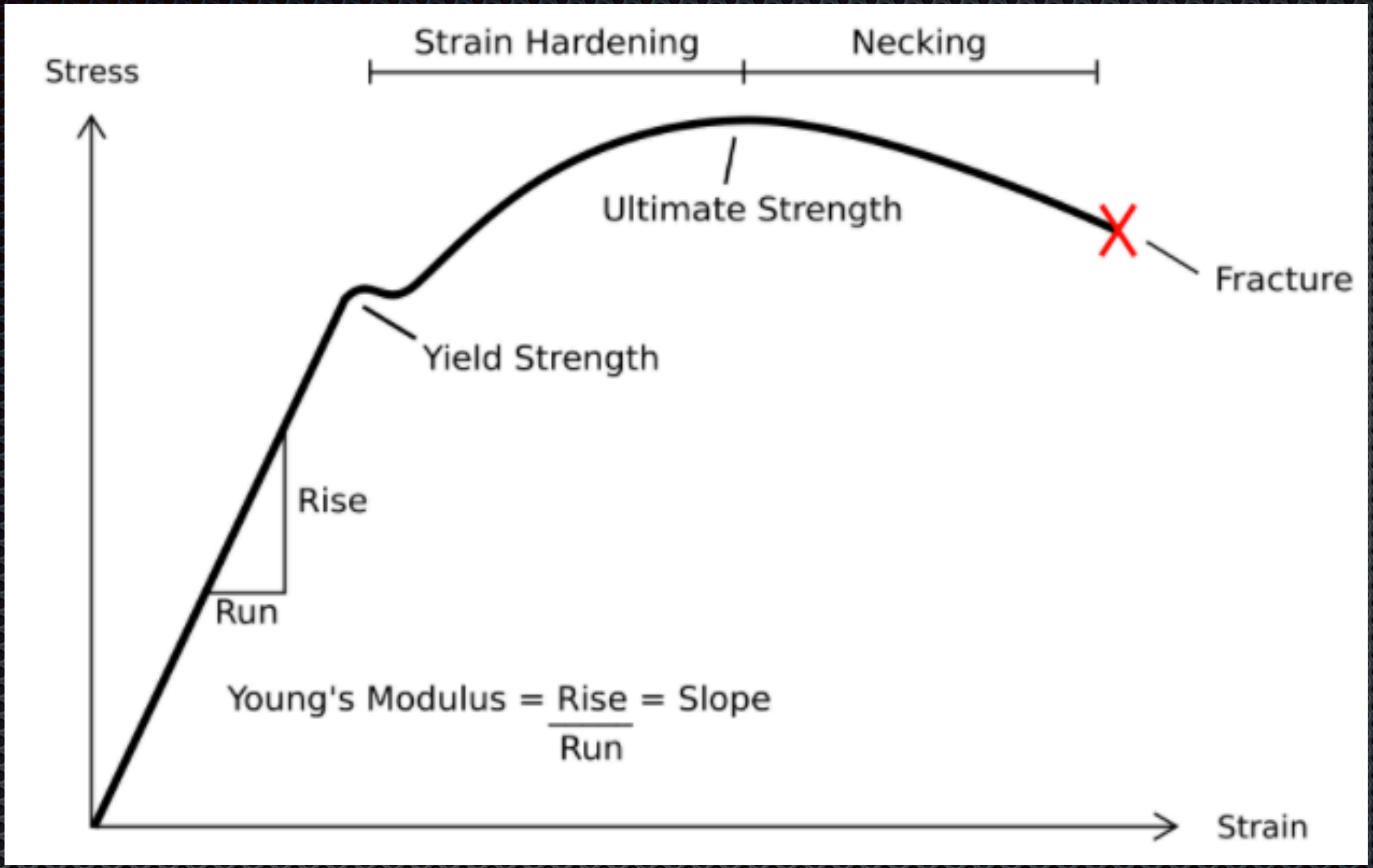
- **D** flexure rigidity, the mechanical stiffness of the plate, is controlled by effective elastic thickness **(Te)** of the plate.
- **K(x)** is the curvature of the plate

$$D = \frac{ET_e^3}{12(1-v^2)} \qquad K(x) = \frac{d^2w}{dx^2}$$





(McNutt and Menard, 1982)

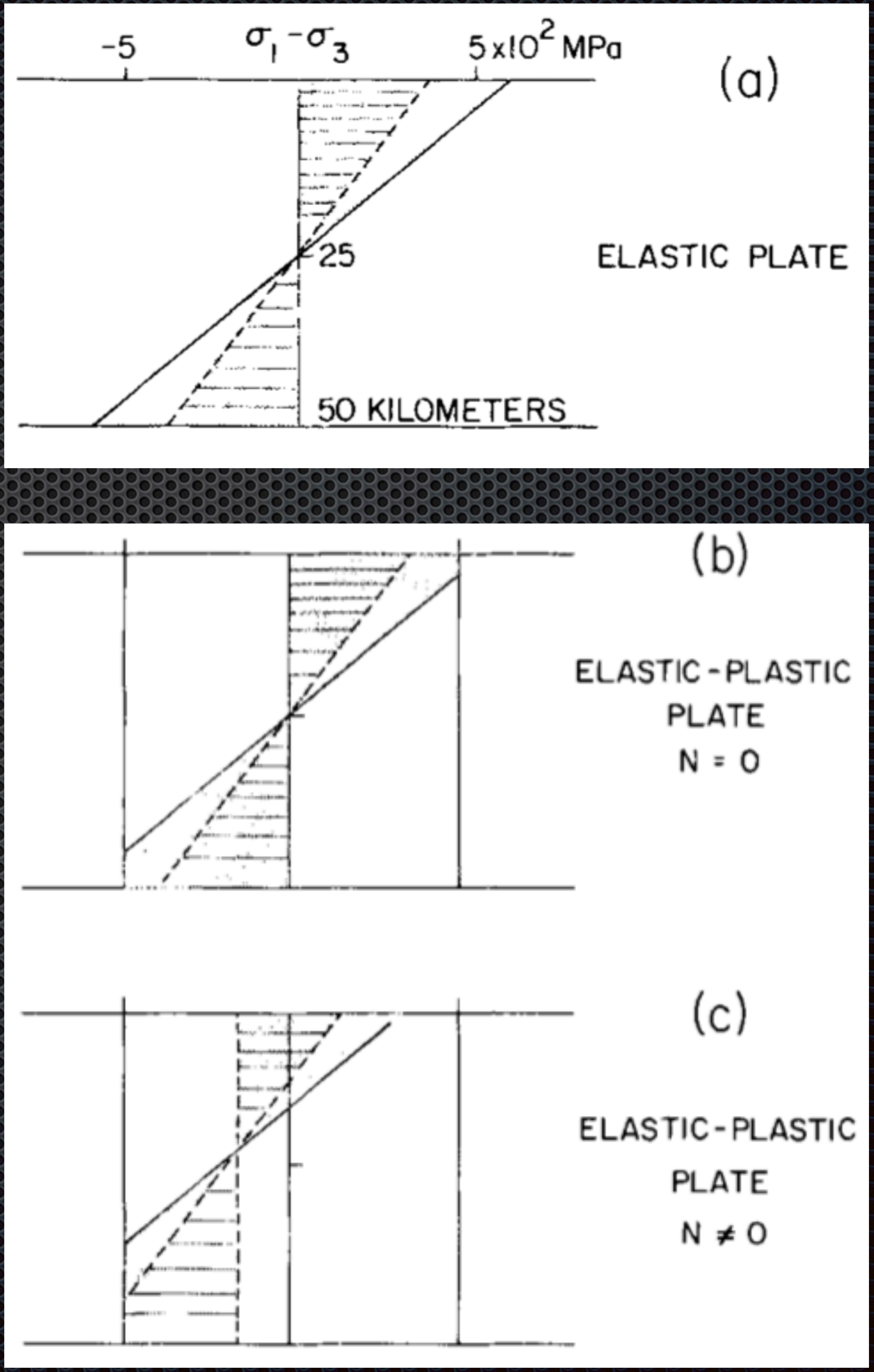


What is yield strength?

-The maximum stress that can be applied to a material without causing ***plastic deformation***

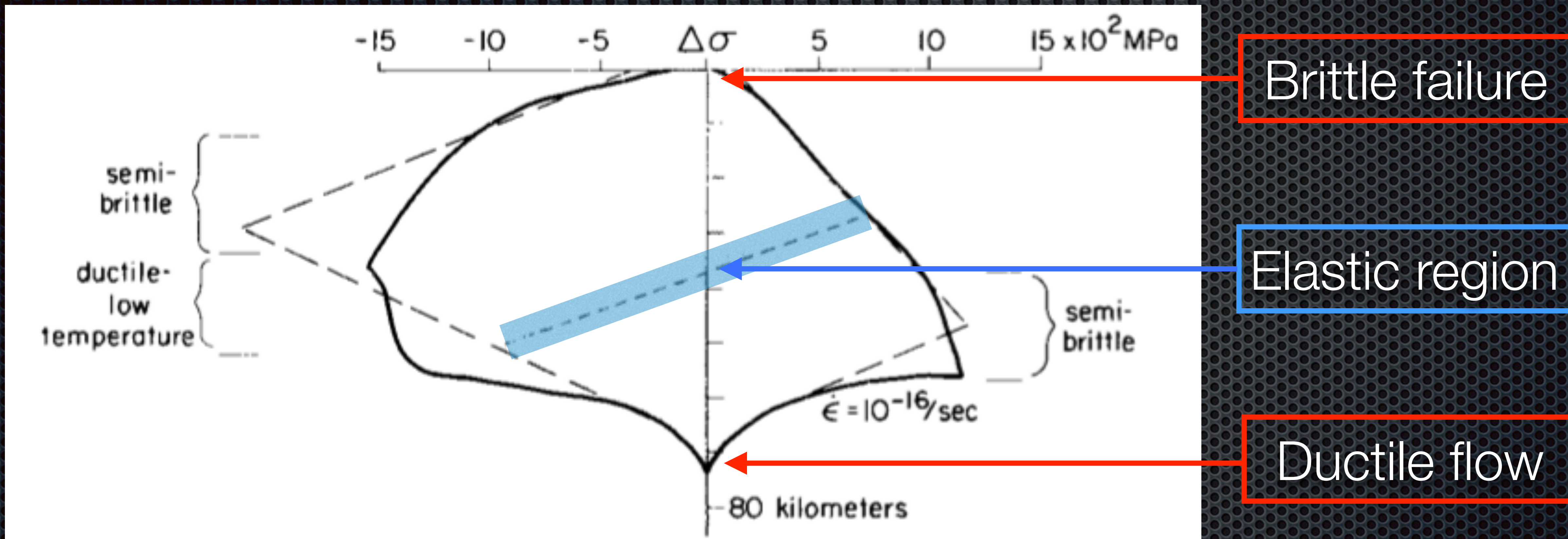
More realistic case

- Real earth materials do have a **finite strength**
- Plate behaves elastically up to the yield stress, then additional strain causes **no increase** in stress
- Finite strength **reduces** effective elastic thickness of the plate
- Saturated moment-curvature curve



(McNutt and Menard, 1982)

Plate bending vs. yielding strength envelope



(McNutt and Menard, 1982)

(+) stress - tensional
(-) stress - compressional

- Yield strength in the oceanic lithosphere is **depth dependent**
- Constrained by results of rock experiments under various temperature, pressure, and strain rate conditions
- For the modeling of oceanic plate flexure, only three regimes are considered, **brittle**, **semi-brittle**, and **ductile**.

Brittle & semi-brittle regime

- Uppermost, cool regions of the lithosphere
- Strength increases with **overburden pressure**
- Insensitive to temperature, strain rate, and rock type
- Assuming that rocks fails by movement along localized **fractures**

- Byerlee's law

$$\tau = 0.85\bar{\sigma}_n, \qquad 3 < \bar{\sigma}_n < 200MPa$$

$$\tau = 60 + 0.6\bar{\sigma}_n, \qquad 200MPa < \bar{\sigma}_n$$

- Relationship between *principle stresses* and stresses on the fault

$$\begin{cases} \bar{\sigma}_n = \frac{1}{2}(\sigma_1 + \sigma_3) + \frac{1}{2}(\sigma_1 - \sigma_3)\cos(2\theta) \\ \tau = \frac{1}{2}(\sigma_1 - \sigma_3)\sin(2\theta) \end{cases}$$

- Yield strength in terms of principle stresses

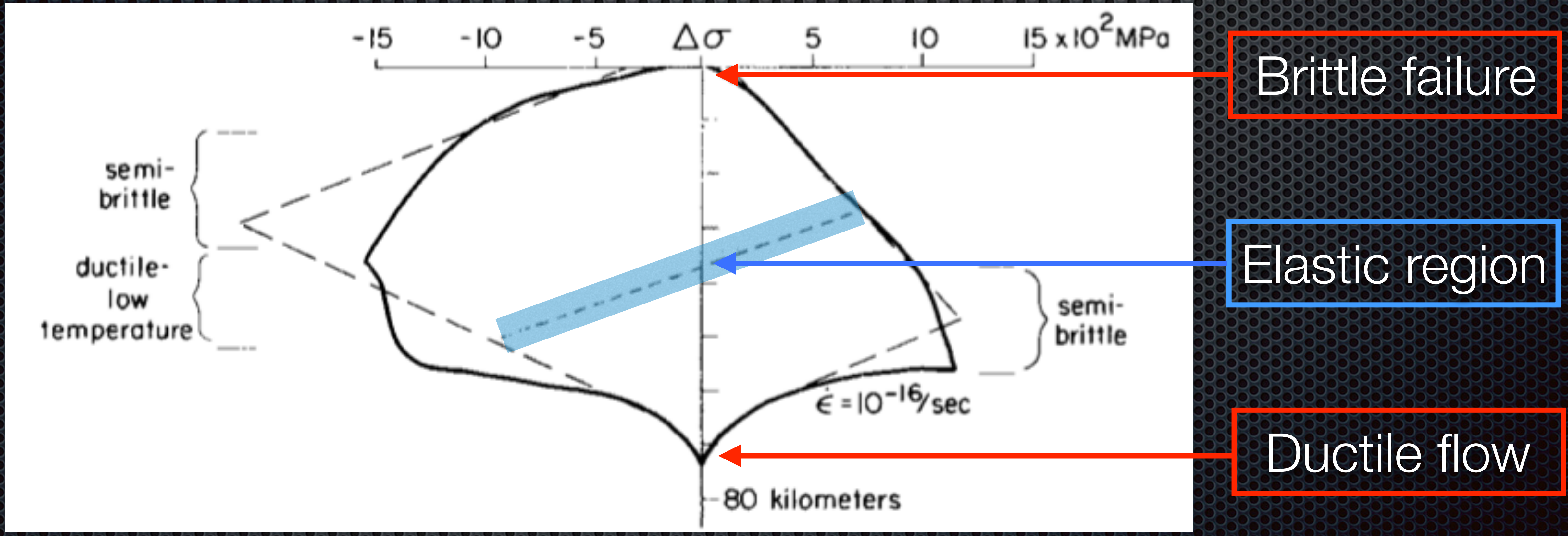
$$\bar{\sigma}_1 \cong 5\bar{\sigma}_3, \quad \bar{\sigma}_3 < 110MPa$$

$$\bar{\sigma}_1 \cong 3.1\bar{\sigma}_3 + 210, \quad \bar{\sigma}_3 > 110MPa$$

(Brace and Kohlsteld, 1980)

Ductile regime

- Dominated by elevated **temperature**
- Yield strength is insensitive to pressure
- Dominant mechanism for failure is **ductile flow**



(+) stress - tensional
(-) stress - compressional

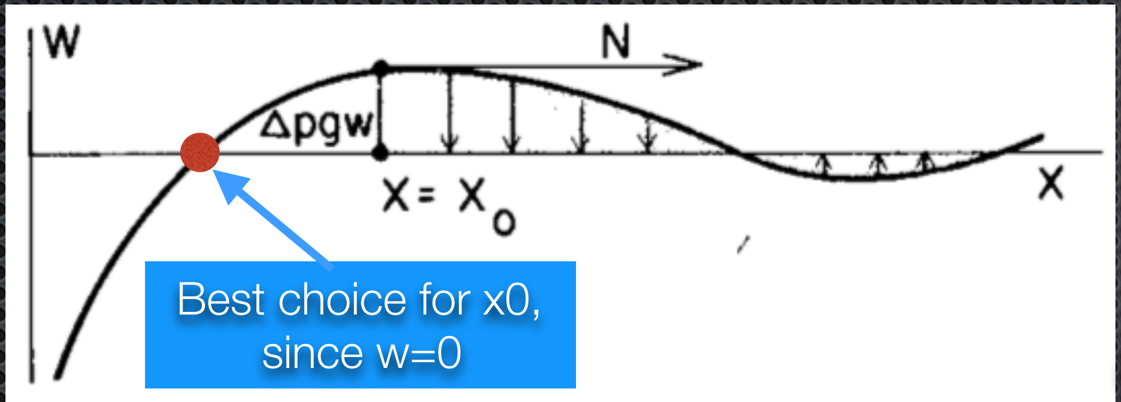
(McNutt and Menard, 1982)

Moment-Curvature Formulation

A **rheologically independent** measurement of moment:

$$M(x_0) = \int_{x_0}^{\infty} \Delta\rho g w(x)(x - x_0)dx + Nw(x_0)$$

Physically, this formula sums up the torques about x0



(McNutt and Menard, 1982)

Letting the first zero-crossing point as x0,

the formula is simplified to a form that is only based on observed w(x):

$$M(x_0) = \int_{x_0}^{\infty} \Delta\rho g w(x)(x - x_0)dx$$

$$M = \int_{0}^{H} \sigma_f (z - z_n)dz$$

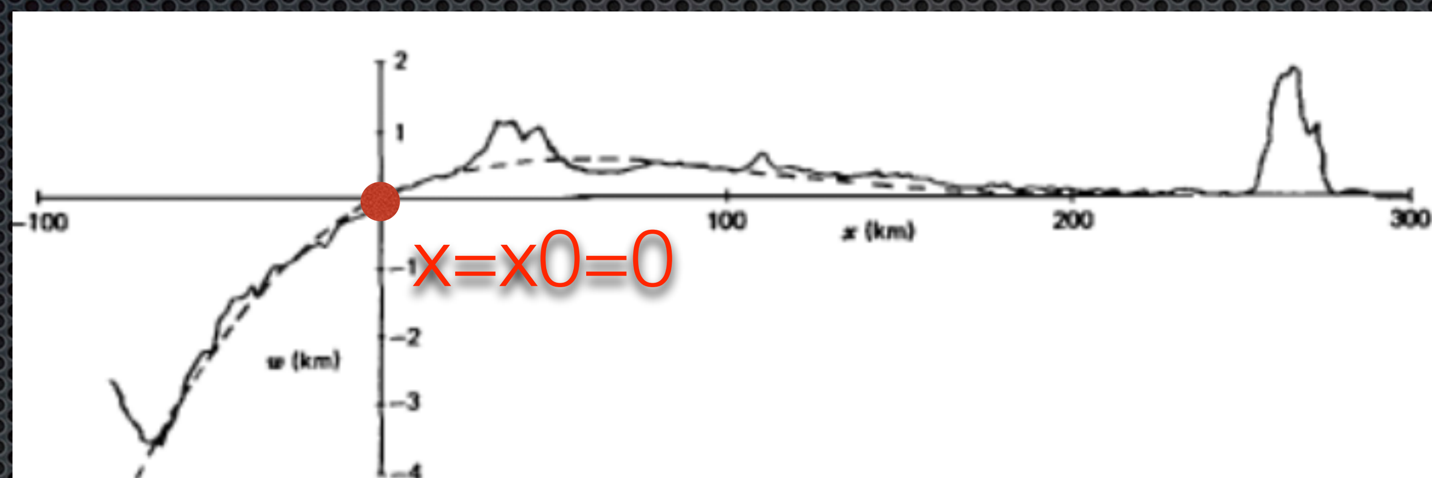
Rheologically dependent measurement

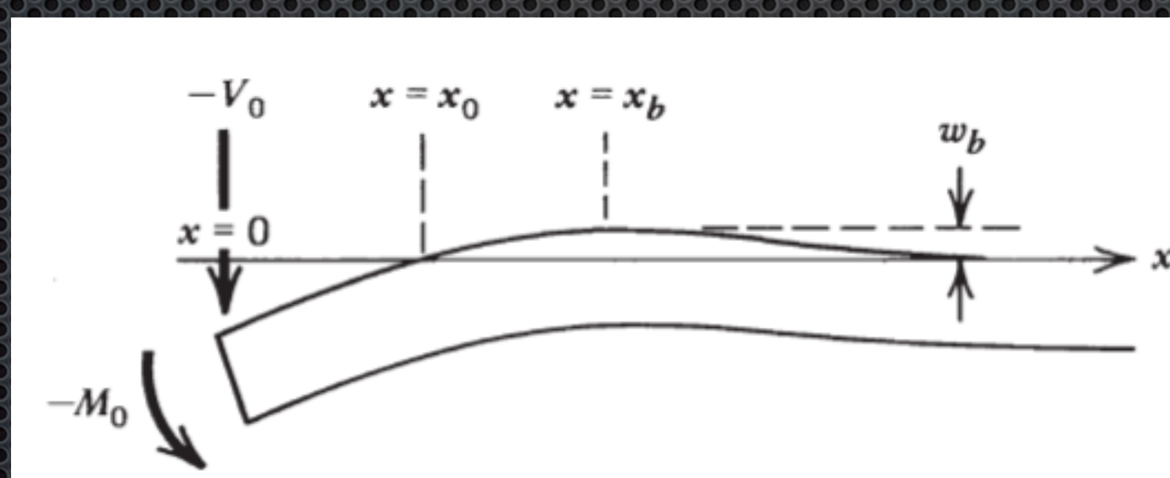
Data parameterization

Theory predicts that the deflection will be the form of a *damped, sinusoidal function*:

$$w(x) = A\, exp(\frac{-x}{\alpha}) \sin(\frac{x}{\alpha})$$

Fitting the data to determine 2 unknown parameters.





$$w_b = A exp(\frac{-\pi}{4})/\sqrt{2}$$

$$x_b = \frac{\pi \alpha}{4}$$

Such that, one could translate the observations into **bending moment**, **curvature**, and **strain rate** at x=0 in terms of xb and wb.

(Caldwell et al., 1976; McNutt and Menard, 1982; Turcotte and Schubert, 2014)

Application to trench data

Table 2. Trench profiles.

No.	Location	Profile	Age (Myr)	Velocity (mm yr^{-1})	Strain rate $\times 10^{-16}$ s^{-1}	W_b m	X_b km	Curvature $\times 10^{-7}$ m^{-1}	Moment $\times 10^{16}$ N	Source
1	Marianas	Scan 5	>165	73	1.4	500	55	6.3	8.6	Caldwell *et al.* (1976)
2	Marianas		>165	73	1.2	550	63	5.4	12	Carey & Dubois (1981)
3	Bonin	Bent 2–2	130	87	2.1	640	55	8.0	11	Jones *et al.* (1978)
4	Bonin	Japanyon 4	130	87	1.5	820	78	5.1	29	Jones *et al.* (1978)
5	Bonin	Hunt 1–4	130	87	1.6	570	59	6.3	11	Jones *et al.* (1978)
6	Bonin	Bent 1–3	130	87	1.6	420	49	6.5	5.8	Jones *et al.* (1978)
7	Bonin	Antipode 3	130	87	1.9	350	40	8.2	3.3	Jones *et al.* (1978)
8	Bonin	Hunt 3 Aries 7	130	87	1.4	400	53	5.5	6.4	Caldwell *et al.* (1976)
9	Japan	Bent 1–1	130	87	0.6	620	115	1.8	47	Jones *et al.* (1978)
10	Kuril	Zetes 2	100	83	1.4	280	42	6.1	2.8	Caldwell *et al.* (1976)
11	Kermadec	Geo 318	100	83	0.5	240	71	1.8	6.9	Carey & Dubois (1981)
12	Aleutian	Seamap 13–4	55	72	1.0	350	53	4.8	5.6	Caldwell *et al.* (1976)
13	Middle America	Iguana 4–2	20	84	0.8	129	36	3.9	0.92	Jones *et al.* (1978)
14	Middle America	Iguana 2	20	84	0.8	106	34	3.5	0.70	Jones *et al.* (1978)

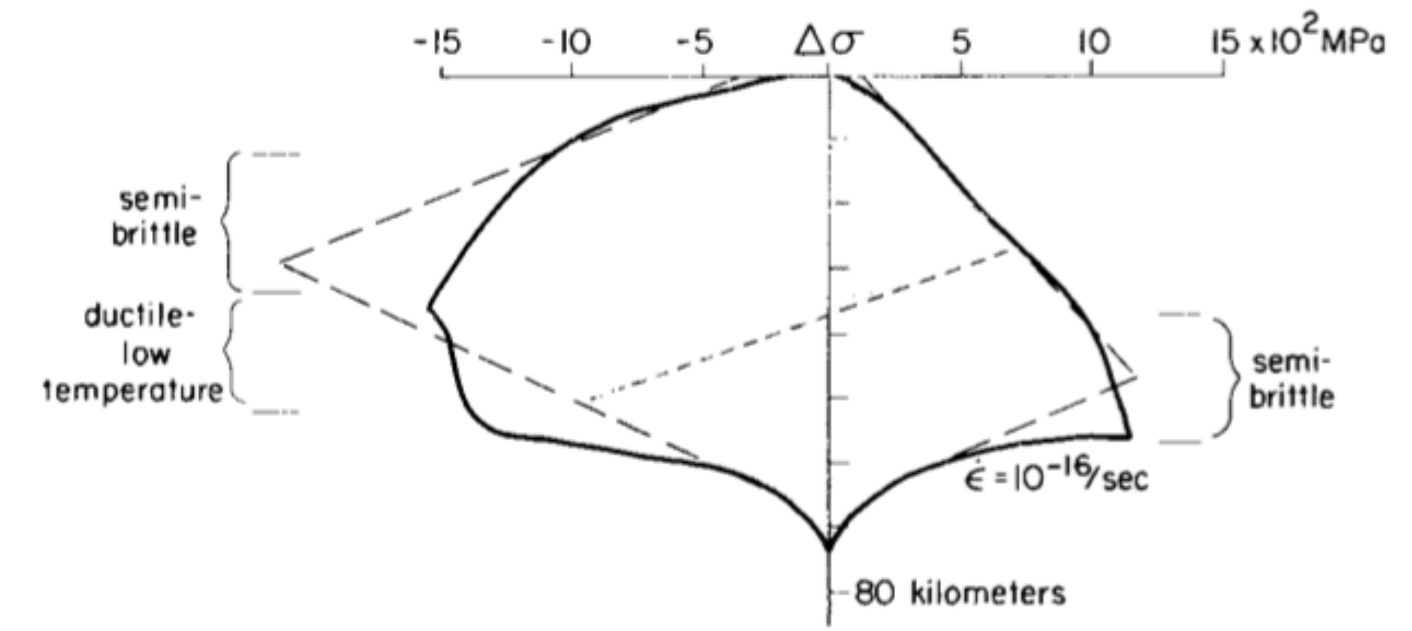
Measured from bathymetry profile and/or free-air gravity anomaly

McNutt and Menard, 1982

Weaker lithosphere

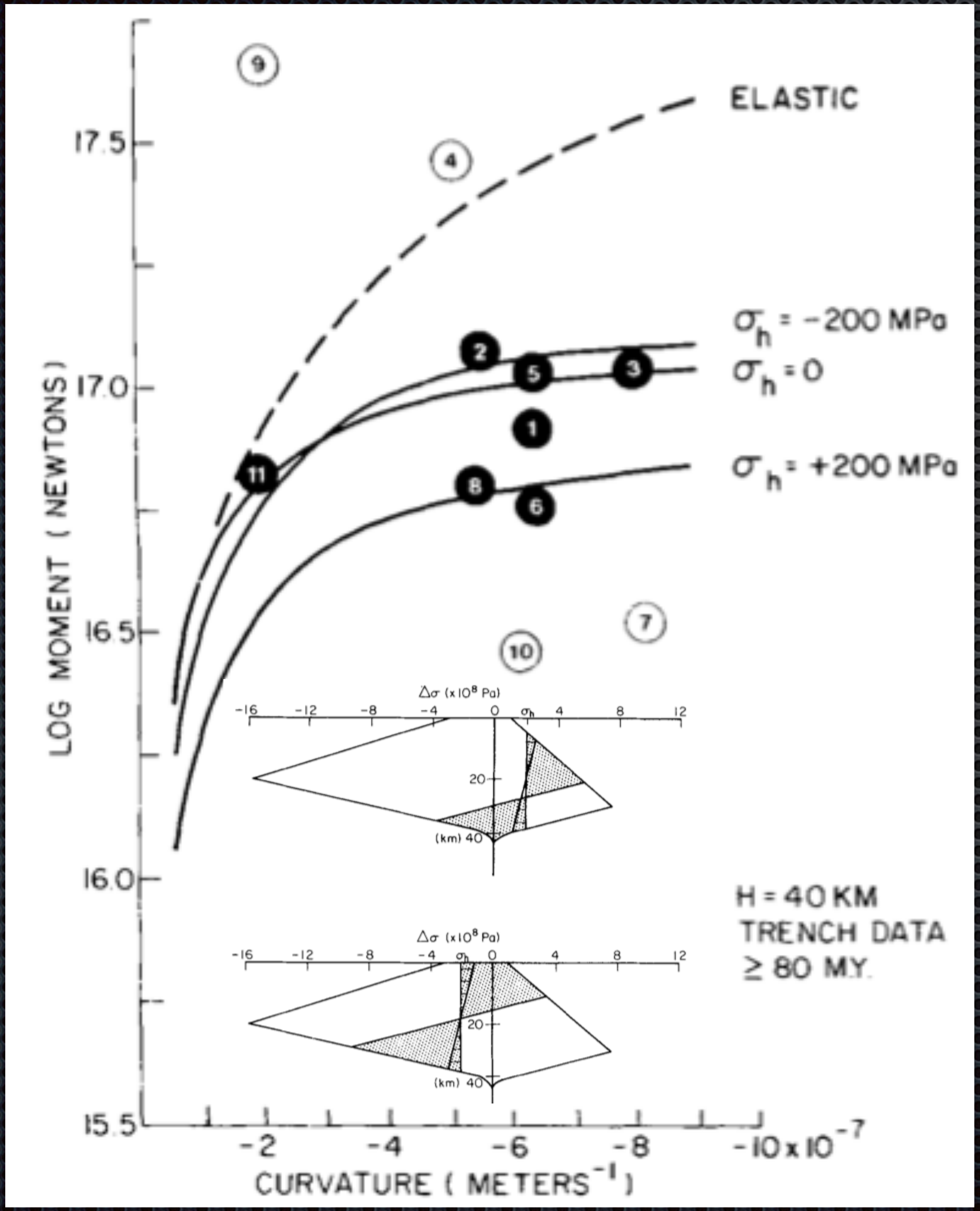
- The yield envelope with 70 km depth of base produces a moment that is too large to explain the data points
- It implies the lithosphere is **weaker** than the laboratory yield strength

Two possible solutions:
(1) Decrease the depth of the base of the yield envelope
(2) Increase the slope of the yield envelope by adding pore-fluid pressure

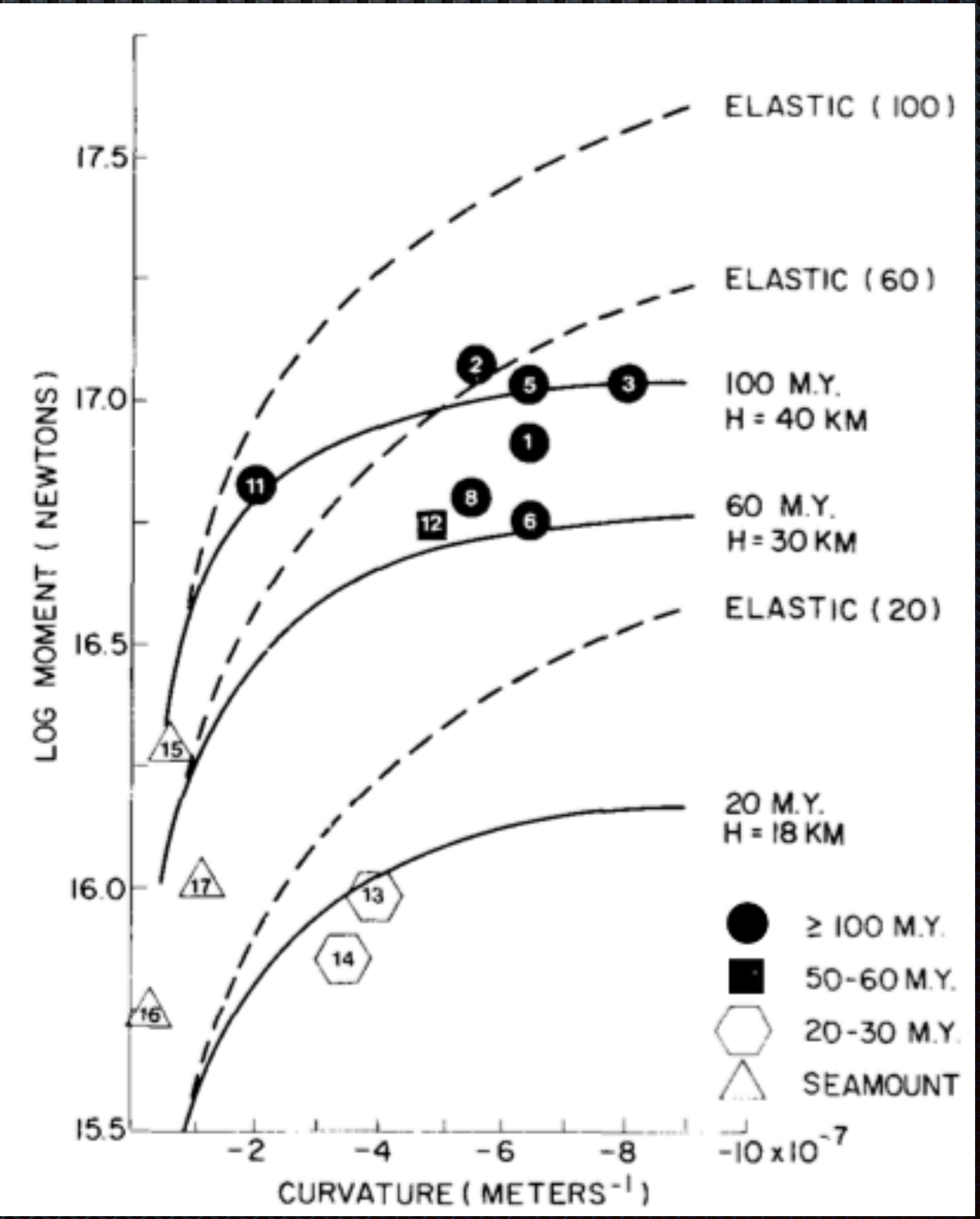


Possible solution 1: Decreasing the depth of base

H = 70 km -> H = 40 km



Age-dependent depth

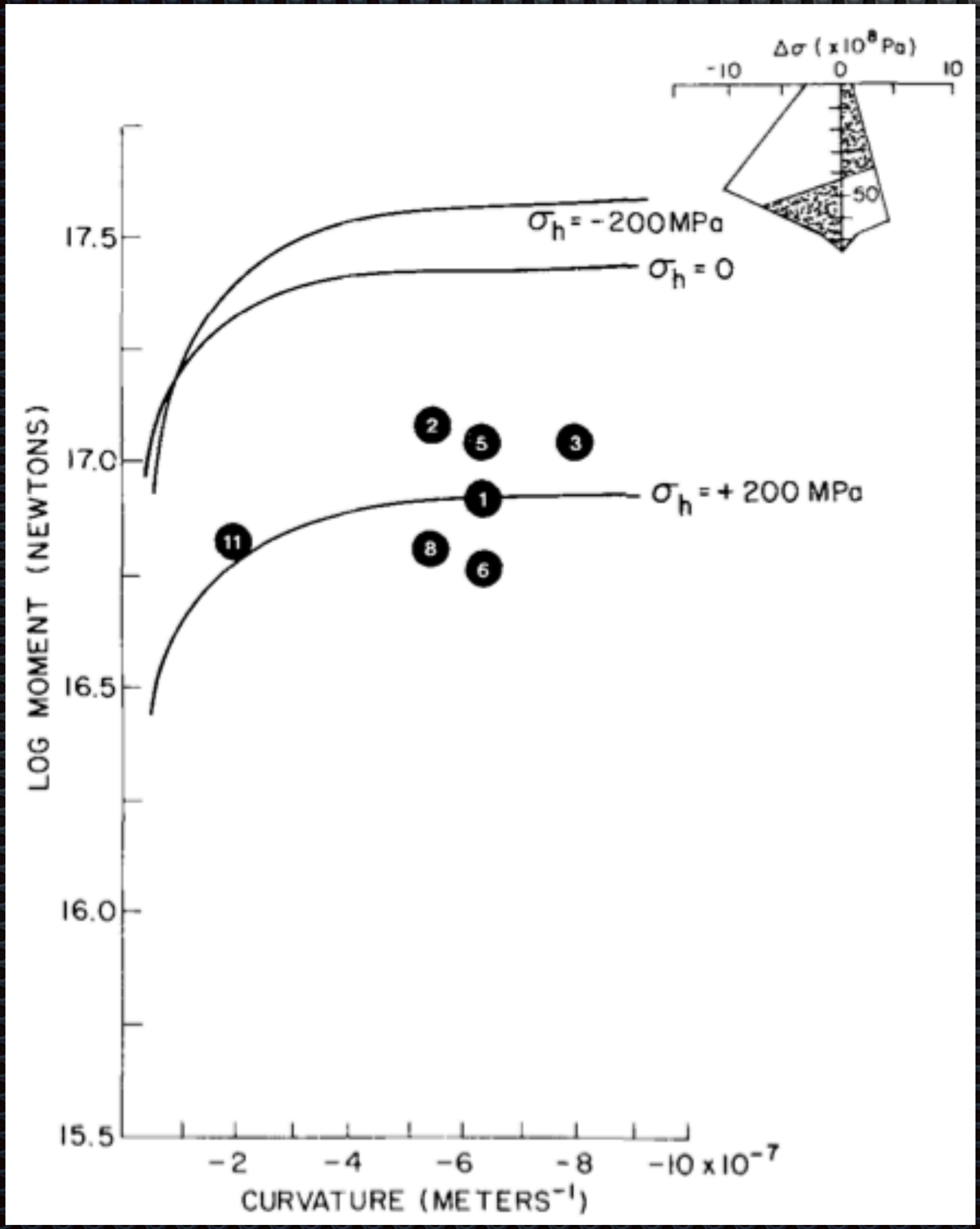


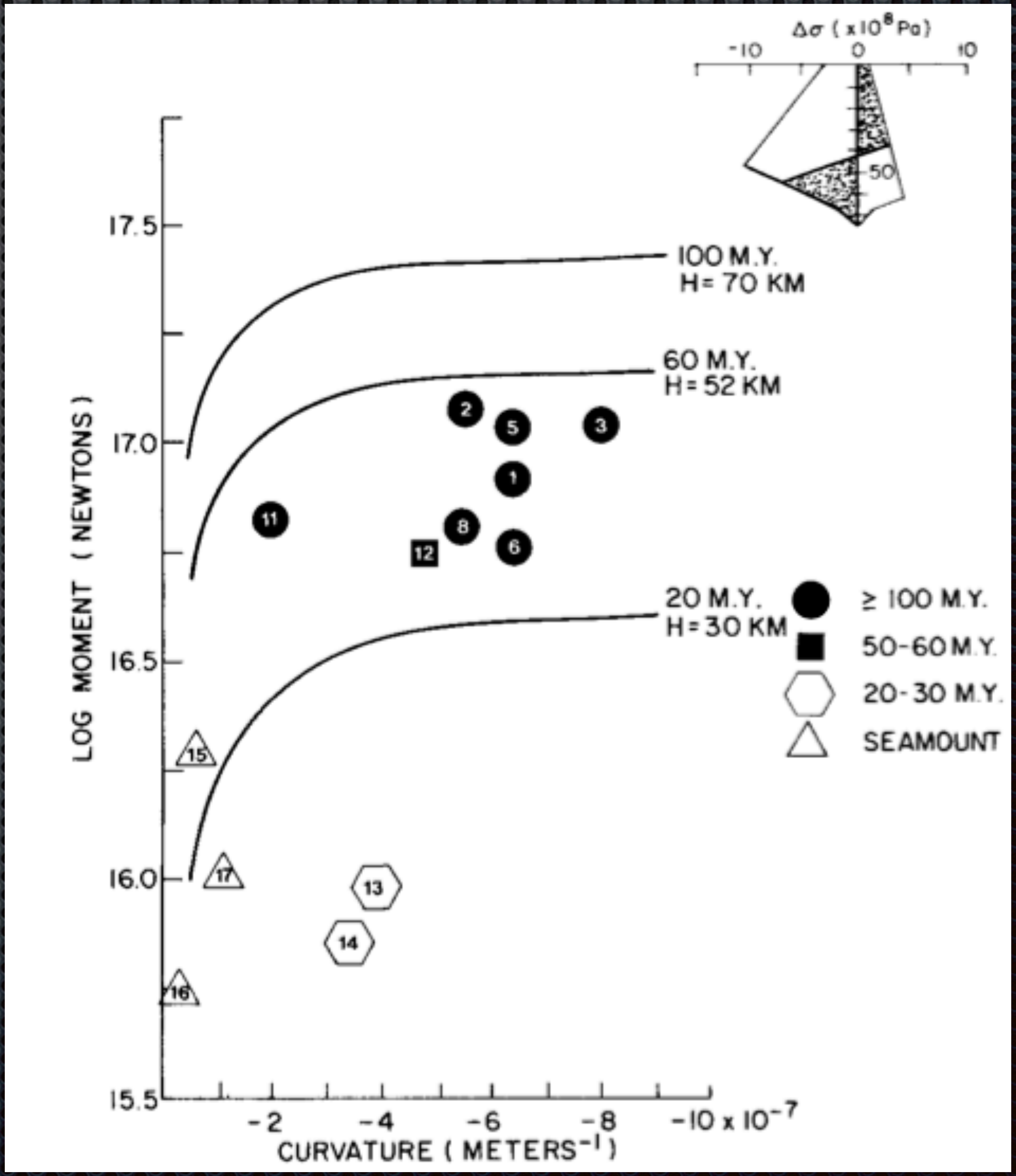
McNutt and Menard, 1982

Possible solution 2: Invoking pore fluid pressure

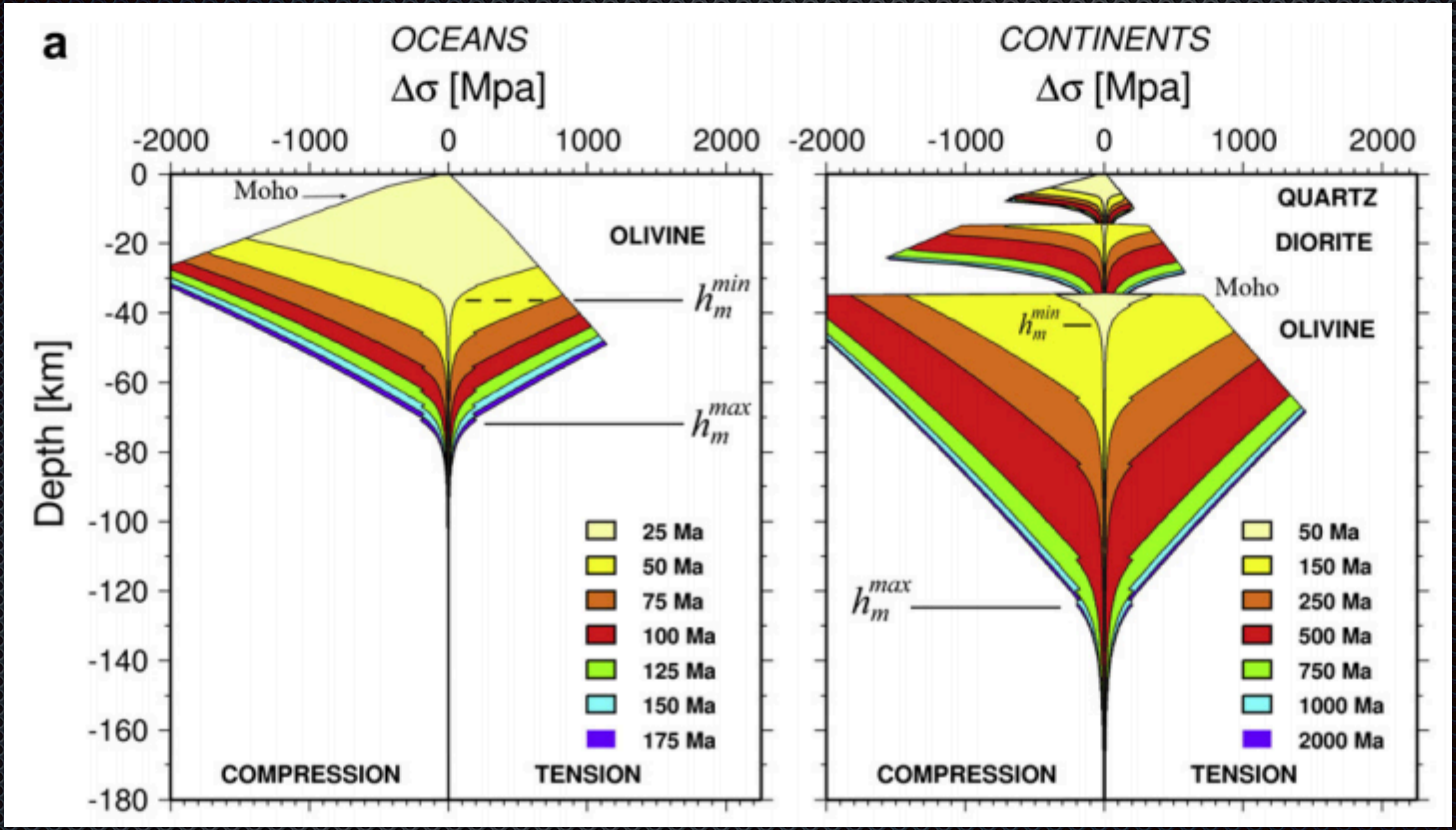
H = 70 km (Laboratory depth)

Larger slope -> weaker lithosphere





Age dependence of mechanical thickness



(Burov, 2011)

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