Practice final quiz 2015

Geodynamics – 2015

1 (a) Complete the following table.

parameter	**symbol**	**units**
temperature	T	˚C or ˚K
thermal conductivity		
heat capacity		
density		
coefficient of thermal expansion (volumetric)		
acceleration of gravity		
gravitational constant		
Young's modulus		
Poisson's ratio		
shear modulus		
bulk modulus		
viscosity, dynamic		

(b) Devise a thought experiment to measure each quantity. This has to be physically realistic but not necessarily practical.

2 (a) What two measurements must be made to determine the conductive heat flow at the bottom of the ocean?
(b) Why is it OK to measure heat flow in the upper few meters of sediment on the seafloor while one needs a borehole hundreds of meters deep obtain a reliable measure of heat flow on the continents.

3 (a) Abyssal hills on the seafloor have a characteristic wavelength of 10 km and produce a gravity anomaly amplitude of 5 mGal at the bottom of the ocean. What is the amplitude of the gravity anomaly at the sea surface where the mean ocean depth is 4 km?

(b) What is the amplitude at the altitude of a satellite of 400 km?

4 Calculate the slip rate along the San Andreas Fault in San Francisco (latitude 38°, longitude –127°). The North America – Pacific rotation pole is 48.7 latitude, -78.2 longitude, 1.36×10^{-8} rad/yr. (90% credit if you setup the problem and provide an upper bound on the slip rate.)

5 (a) Provide an approximate formula for the magnitude of the shear stress that is needed to induce slip on a dry fault at 10 km depth in continental crust (density 2800 kg m^{-3})? Which parameter is least well known and what is a possible range for this parameter.

(b) Suppose the crust is saturated with water to 10 km depth. How does this change the stress magnitude?

6 Solve for the temperature T as a function of time t and depth z in a cooling half space.

The differential equation for heat diffusion is

$$\frac{\partial^2 T}{\partial z^2} = \frac{1}{\kappa}\frac{\partial T}{\partial t}$$

and the boundary/initial conditions are

$$T(0,\ t) = T_o; \quad T(\infty,\ t) = T_m; \quad T(z,\ 0) = T_m.$$

Use the following similarity variable η to reduce the partial differential equation to an ordinary differential equation

$$\eta = \frac{z}{2\sqrt{\kappa t}}$$

where κ is the thermal diffusivity, T_o is the surface temperature, and T_m is the initial temperature of the half space.

7 The equations below relate principal stress to principal strain. Derive formulas for Young's modulus E and and Poisson's ratio ν in terms of the Lame parameters λ and μ by setting up a uniaxial stress case.

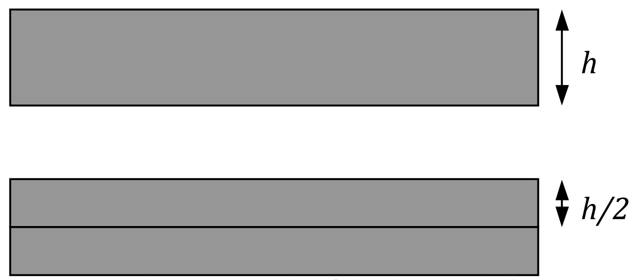
$$\begin{pmatrix}\sigma_1\\ \sigma_2\\ \sigma_3\end{pmatrix} = \begin{pmatrix}\lambda+2\mu & \lambda & \lambda\\ \lambda & \lambda+2\mu & \lambda\\ \lambda & \lambda & \lambda+2\mu\end{pmatrix}\begin{pmatrix}\varepsilon_1\\ \varepsilon_2\\ \varepsilon_3\end{pmatrix}$$

Geodynamics - 2014

1) Assume the lithosphere of Venus has evolved to a steady-state temperature profile. Given a current heat flow of 4×10^{-2} W m^{-2}, a surface temperature of 450˚C, a mantle temperature of 1500˚C and a thermal conductivity of 3.3 W m^{-1} C^{-1} calculate the thickness of the lithosphere.

2) Continental yield strength envelope model. The continental yield strength has been described as a jelly sandwich consisting of a weak layer (jelly) between two strong layers (bread). The flexural rigidity of a single strong later is.

$$D_o = \frac{Eh^3}{12(1-\nu^2)}$$



(a) What is the flexural rigidity of two strong layers, each of thickness *h/2*, that are not bonded along their common interface?
(b) What is the effective elastic thickness for this layered case?

3 (a) Use the formula for isostatic geoid height $N = \frac{2\pi G}{g}\int_{-\infty}^{0} \Delta\rho(z) z\,dz$ to calculate the geoid height for the following 2-layer density model

$$\Delta\rho = \sigma\left[\delta(z) - \delta(z+a)\right]$$

where σ is the surface density. Check the dimensions of your solution.

(b) What is the value of geoid height for a surface density of 2800 kg m^{-3} X 1000 m and a compensation depth *a* of 30,000 m? (g=9.8 ms^{-2} and G=6.67x10^{-11} m^3 kg^{-1} s^{-2}).

4 Given the rotation pole between the African and South American plates (pole; latitude=62.5°, longitude=320.6°, rate= 5.58 x 10^{-9} radian/yr), calculate the spreading rate at a point on the northern Mid-Atlantic Ridge (lat= 30°, lon= 319°).

5 Abyssal hills, which form at seafloor spreading ridges, have a characteristic spacing of 4 km and have peak-to-trough amplitude of 400 m and a density relative to water of 1600 kg m^{-3}. Assume they are infinitely long in the ridge-parallel direction. What is the amplitude of their gravity anomaly at the surface of the ocean 4 km above the seafloor? What is the amplitude at the altitude of a satellite (400 km)? (assume a flat earth, G=6.67x10^{-11} m^3 kg^{-1} s^{-2})

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7 The differential equation for the deflection of a thin elastic plate $w(x)$ in response to a line load $V_o\delta(x)$ is:

$$D\frac{d^4 w}{dx^4} + \Delta\rho g w(x) = V_0 \delta(x)$$

Take the fourier transform of this differential equation and solve for $W(k)$.

Geodynamics - 2013

1 Derive the following relationship between the rate of increase in seafloor depth with age $\frac{\partial d}{\partial t}$ and the difference between the surface and basal heat flow $\left(q_s - q_L\right)$.

$$\frac{\partial d}{\partial t} = \frac{\alpha}{C_p\left(\rho_m - \rho_w\right)}\left(q_s - q_L\right)$$

You will need Fourier's law, energy conservation, and isostasy as follows:

$$q = k\frac{\partial T}{\partial z}$$

$$\frac{\partial T}{\partial t} = \frac{k}{\rho_m C_p}\frac{\partial^2 T}{\partial z^2}$$

$$d(t) = \frac{-\alpha\rho_m}{\left(\rho_m - \rho_w\right)}\int_o^L T\,dz$$

where

L - asymptotic lithospheric thickness and also the depth of compensation (m)
d - seafloor depth (m)
q - heat flow (W m^{-2})
α - coefficient of thermal expansion (˚C^{-1})
C_p - heat capacity (J kg^{-1})
ρ_m, ρ_w- mantle and seawater density (kg m^{-3})
k - thermal conductivity (W m^{-1} ˚C^{-1})

Geodynamics - 2012

1 Assume the density of the earth is uniform and the earth is a prefect sphere.

(a) Develop a formula for the gravitational acceleration as a function of radius and check the dimensions.

(b) Develop a formula for the pressure at the center of this earth and check dimensions.

OR – a bit more challenging

2 Assume the density of the earth is uniform increases with radius as $\rho = Ar$ and the earth is a prefect sphere.

(a) Develop a formula for the gravitational acceleration as a function of radius and check the dimensions.

(b)Develop a formula for the pressure at the center of this earth and check dimensions.