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QUIZ 02
10/21/11

Open book, need calculator, no notes.

Complete as much as you can in class and hand in at 9:55AM.

1) Exercise from class. Show $\frac{1}{V} \left(\frac{\partial V}{\partial T} \right) = -\frac{1}{\rho} \left(\frac{\partial \rho}{\partial T} \right)$ where V is volume, T is temperature, and ρ is density.

$$\frac{\partial \rho}{\partial T} = \frac{\partial \rho}{\partial V} \frac{\partial V}{\partial T} \quad \rho = m V^{-1} \Rightarrow \frac{\partial \rho}{\partial V} = -m V^{-2} \Rightarrow \frac{\partial \rho}{\partial T} = -m V^{-2} \frac{\partial V}{\partial T} = -\frac{\rho}{V} \frac{\partial V}{\partial T}$$
$$\Rightarrow -\frac{1}{\rho} \frac{\partial \rho}{\partial T} = \frac{1}{V} \frac{\partial V}{\partial T}$$

2) Given the solid line in Figure 4-11, calculate the mantle heat flow and the length scale for the decrease in heat production with depth.

intercept of line is mantle heat flow = 21 mW m^{-2}

$$\text{slope of line is } h_m = \frac{60 - 21}{6} = 6.5 \frac{10^{-2} \text{ W m}^{-2}}{10^6 \text{ V m}^{-3}} = 6.5 \times 10^3 \text{ m}$$

Do the following problems from Chapter 4 in Turcotte and Schubert. Show all work.
4-13, 4-24, 4-31

4-13 simple heat balance $qA = H\rho V$

$$q = \frac{H\rho V}{A} = \frac{H\rho \frac{4}{3}\pi a^3}{4\pi a^2} = \frac{H\rho a}{3}$$

A - area of sphere

V - volume of sphere

a - radius of sphere

4-24 use conservation of energy

total surface heat loss = change in total heat content

$$q_0 A = \rho C V \frac{\partial T}{\partial t} \quad \rho V = m_e \text{ mass of earth}$$

$$\frac{\partial T}{\partial t} = \frac{q_0 A}{C m_e} = \frac{8.2 \times 10^{-2} \text{ W m}^{-2} 4\pi (6.3 \times 10^6 \text{ m})^2}{10^3 \text{ J K}^{-1} \text{ } ^\circ\text{K}^{-1} 5.97 \times 10^{24} \text{ kg}} = 7.4 \times 10^{-15} \text{ } ^\circ\text{K s}^{-1}$$

4-31

$$\Theta = \text{erfc}(\eta) \quad \eta = \frac{z}{2\sqrt{\kappa t}} \quad \text{erfc}(\eta) = 0.01 \quad \eta_T = 1.8$$

$$z = 3.6\sqrt{\kappa t}$$