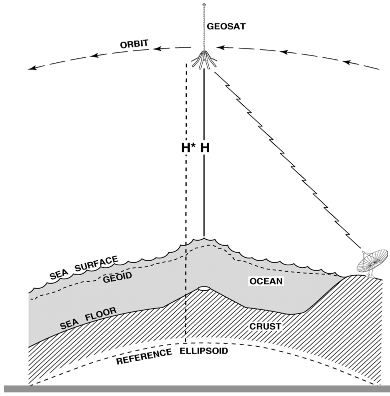
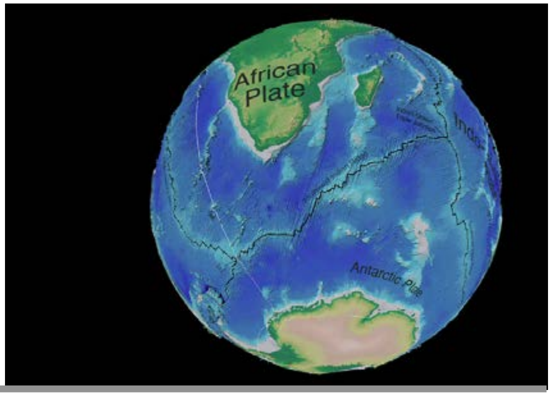


Marine Gravity from Satellite Altimetry

Geodynamics, November, 2014



- basic theory
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reference model

total potential = spherical + rotation/flattening +
spherical harmonic model +
flat earth residual

spherical + rotation/flattening (WGS84)

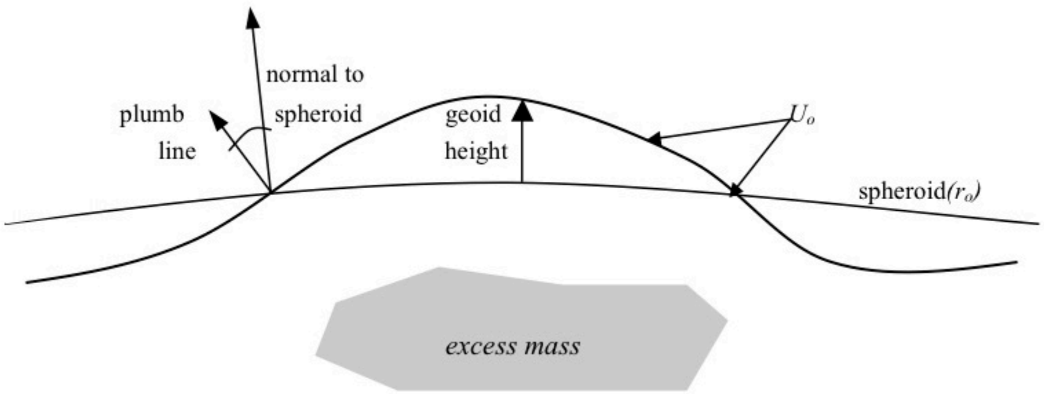
$$U_o(r) = -\frac{GM_e}{r} + \frac{GM_e J_2 a^2}{2r^3}\left(3\sin^2\theta\right) - \frac{1}{2}\omega^2 r^2 \cos^2\theta$$

spherical harmonic (EGM96)

$$U(r) = \frac{GM_e}{r}\sum_{l=2}^{360}\sum_{m=0}^{l}\left(\frac{a}{r}\right)^l C_{lm} Y_{lm}\left(\theta,\lambda\right)$$

flat earth residual - wavelengths 14 km to 600 km

geoid and slope of ocean surface



Disturbing potential Φ

U	$=$	U_o	$+$	Φ	(19)
total potential	$=$	reference potential		disturbing potential	

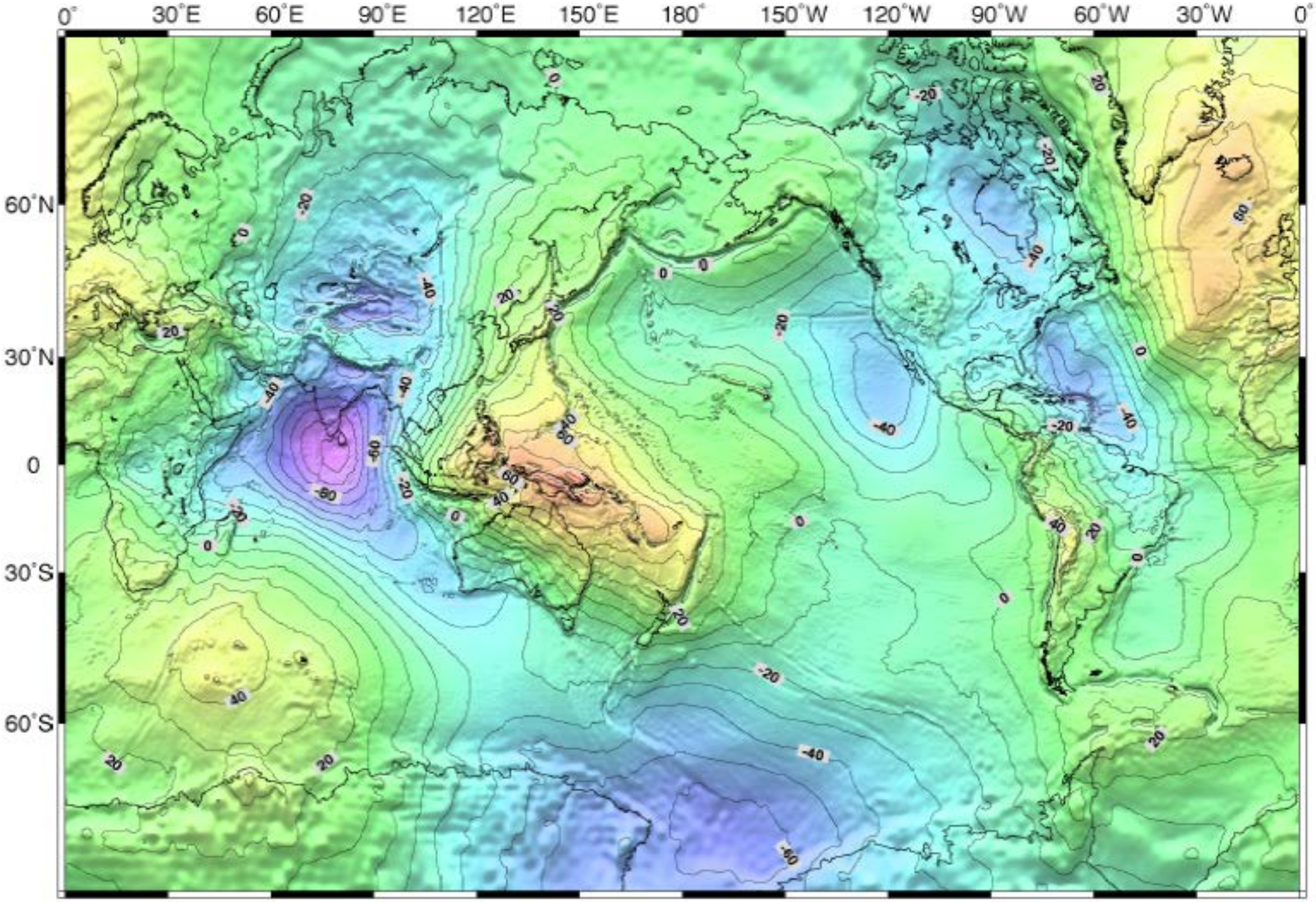
Geoid height N

$$N = \frac{\Phi}{g_o(\theta)} \tag{20}$$

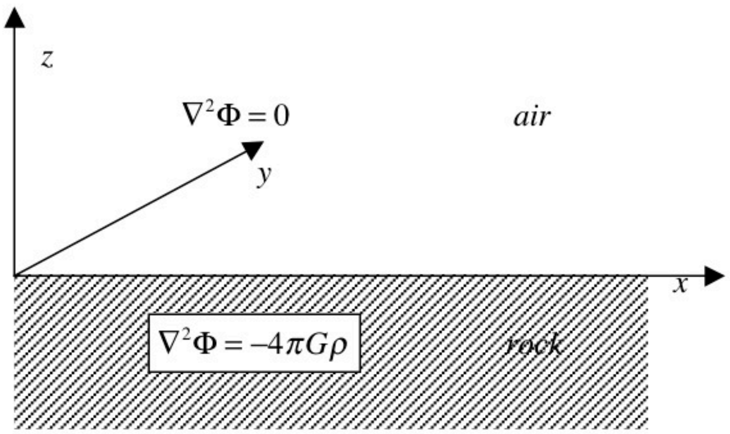
Free-air gravity anomaly

$$\Delta g = -\frac{\partial \Phi}{\partial r} - \frac{2g_o(\theta)}{r(\theta)} N \tag{21}$$

EGM96 Geoid - wavelengths 220 km to 40,000 km



flat-earth potential



$\Phi(x,y,z)$ -- disturbing potential (total - reference)
G -- gravitational constant
ρ -- density anomaly (total - reference)

Laplace's equation is a second order partial differential equation in three dimensions.

$$\frac{\partial^2\Phi}{\partial x^2}+\frac{\partial^2\Phi}{\partial y^2}+\frac{\partial^2\Phi}{\partial z^2}=0, \qquad z>0 \tag{2}$$

Six boundary conditions are needed to develop a unique solution. Far from the region, the disturbing potential must go to zero; this accounts for 5 of the boundary conditions

$$\lim_{|x|\to\infty}\Phi=0, \quad \lim_{|y|\to\infty}\Phi=0, \quad \lim_{z\to\infty}\Phi=0 \tag{3}$$

At the surface of the earth (or at some elevation), one must either prescribe the potential or the vertical derivative of the potential.

$$\Phi(x,y,0)=\Phi_o(x,y) \qquad \text{-- Dirichlet}$$

$$\frac{\partial\Phi}{\partial z}=-\Delta g(x,y) \qquad \text{-- Neumann} \tag{4}$$

flat-earth potential

$$-4\pi^2\left(k_x^2+k_y^2\right)\Phi(\mathbf{k},z)+\frac{\partial^2\Phi}{\partial z^2}=0 \qquad (6)$$

$$\lim_{z\to\infty}\Phi(\mathbf{k},z)=0, \qquad \Phi(\mathbf{k},0)=\Phi_o \qquad (7)$$

The general solution is

$$\Phi(\mathbf{k},z)=A(\mathbf{k})e^{2\pi|\mathbf{k}|z}+B(\mathbf{k})e^{-2\pi|\mathbf{k}|z} \qquad (8)$$

To satisfy the boundary condition as $z\to\infty$, the $A(\mathbf{k})$ term must be zero. To satisfy the boundary condition on the z=0 plane, $B(\mathbf{k})$ must be $\mathbf{\Phi}(\mathbf{k},0)$. The final result is

$$\Phi(\mathbf{k},z) \quad = \quad \Phi_o(\mathbf{k},0) \qquad e^{-2\pi|\mathbf{k}|z} \qquad (9)$$

potential at altitude	=	potential at $z=0$	×	upward continuation

λ	z	gain
8	4	.043
8	400	10^{-137}
100	400	10^{-11}

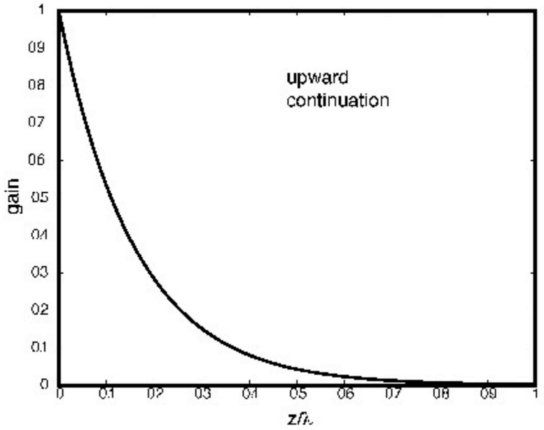
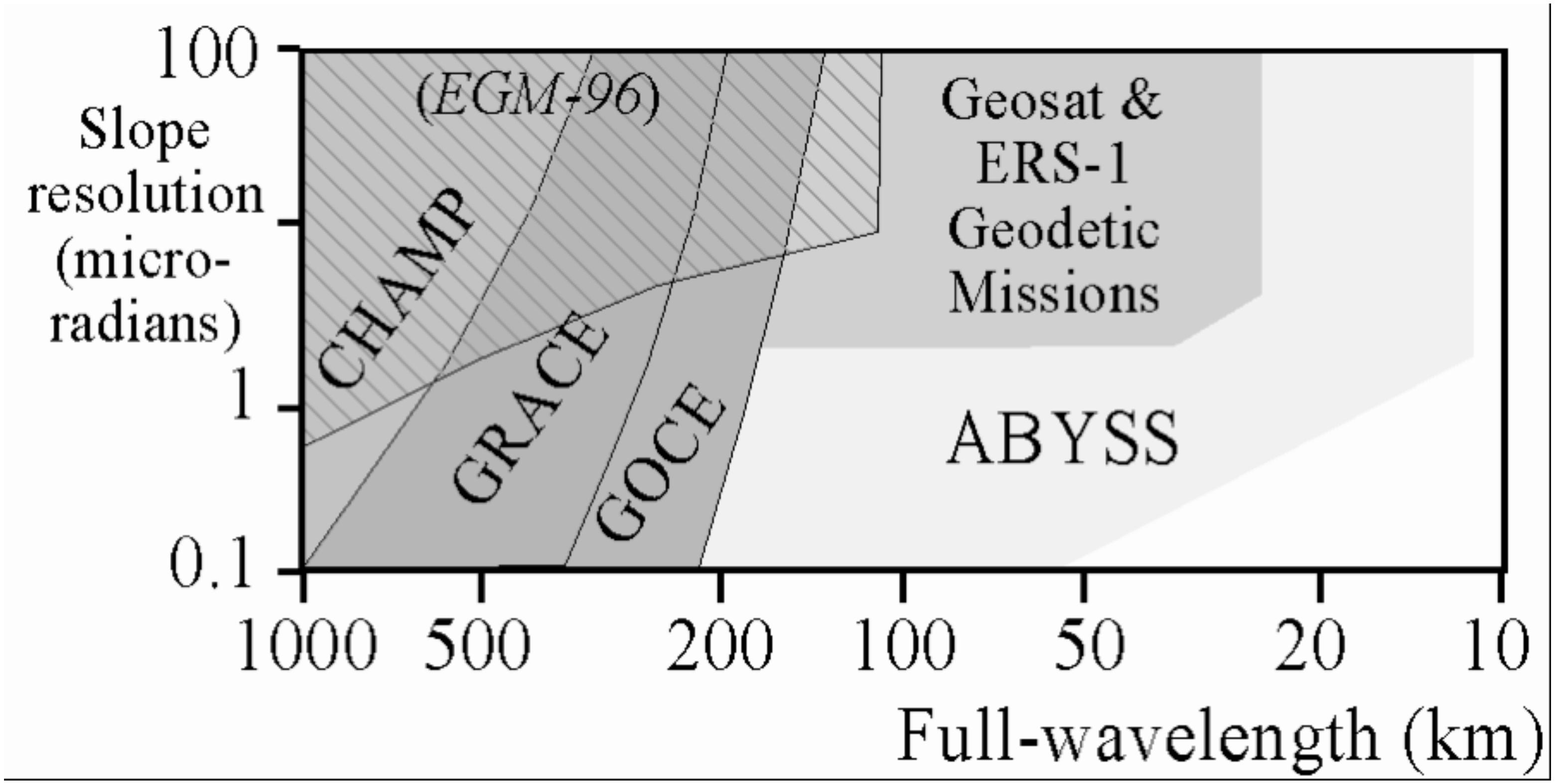
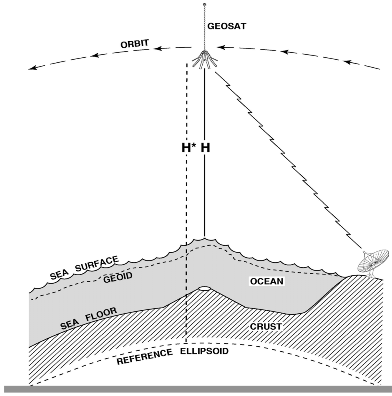


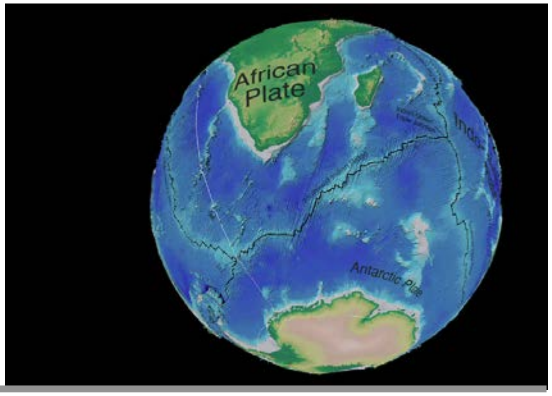
Figure 1. Gain of upward continuation kernel as a function of the altitude of the observation z, divided by the wavelength of the anomaly λ.

ABYSS will extend short wavelength coverage by an order of magnitude beyond that of CHAMP, GRACE, and GOCE





Marine Gravity from Satellite Altimetry



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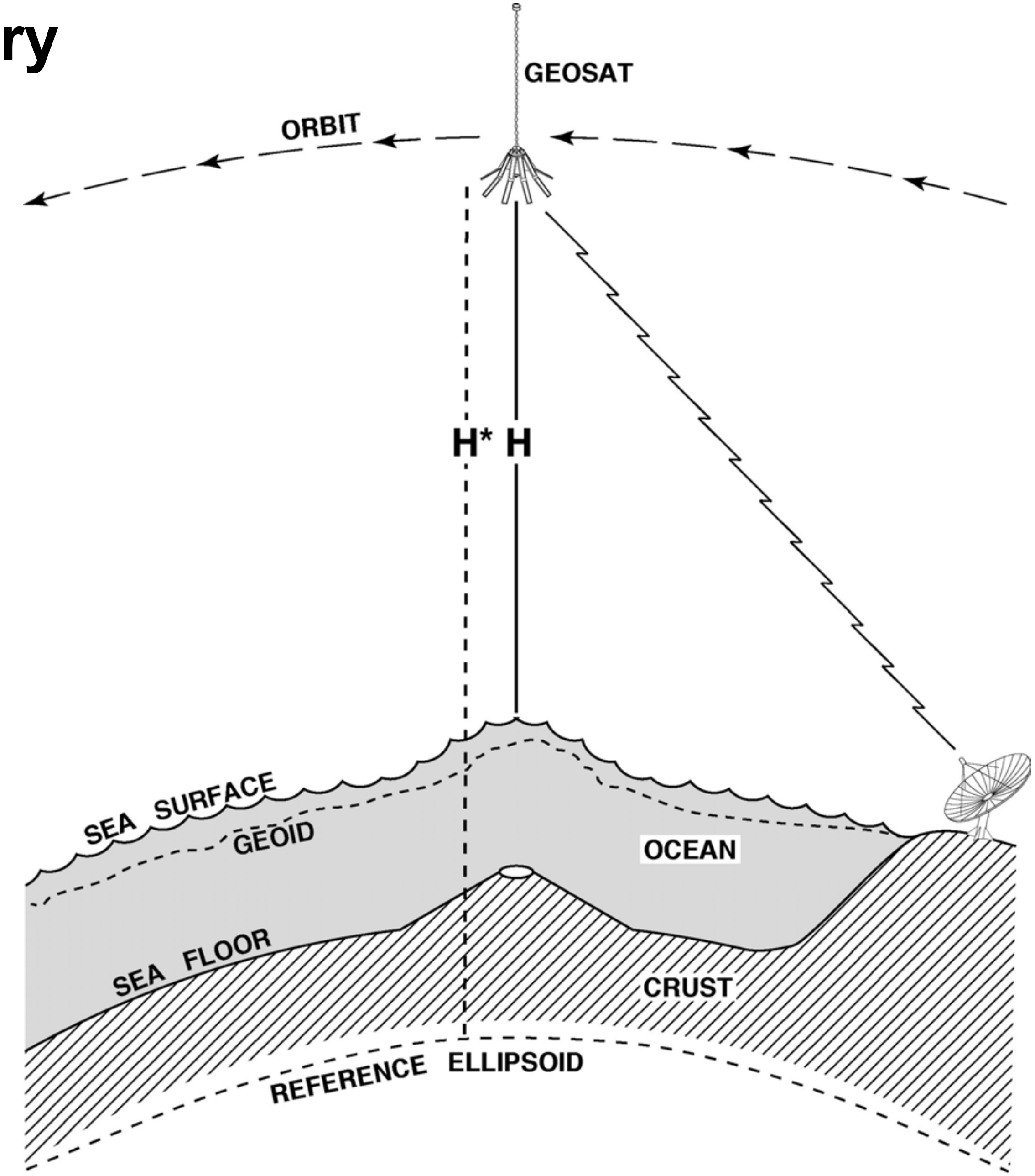
radar altimetry



slope requirement

Φ	disturbing potential
$N = \frac{1}{g_o}\Phi$	geoid height
$\Delta g = -\frac{\partial \Phi}{\partial z}$	gravity anomaly
$\eta = -\frac{\partial N}{\partial x}$	slope of ocean surface

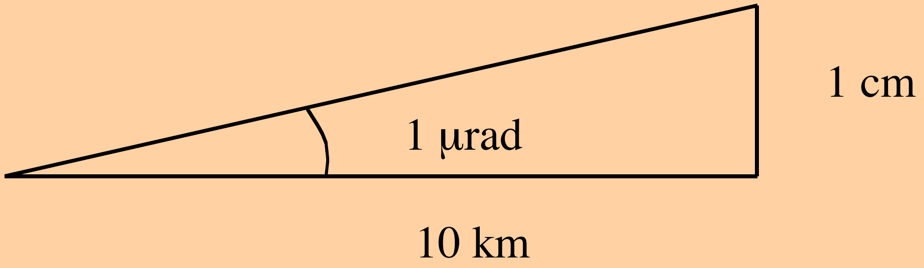
Laplace's equation, assume 2-D anomaly

$$\nabla^2 \Phi = 0 \qquad \Rightarrow \qquad g_o \frac{\partial \eta}{\partial x} + \frac{\partial g}{\partial z} = 0$$

take fourier transform w.r.t. x

$$\Delta g(k) = i g_o \frac{k}{|k|} \eta(k)$$

1 μrad of slope error ⇔ 1 mGal gravity error



orbit inclination controls error anisotropy

- **Error propagation**

  θ - local inclination of track

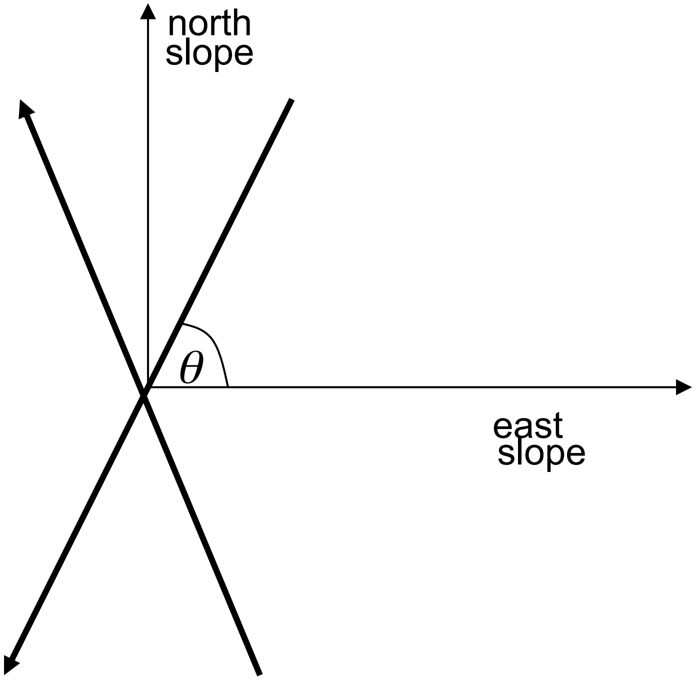
  σ - error in along-track slope

  σ_x - error in east slope

  σ_y - error in north slope

$$\sigma_x = \frac{\sigma}{\sqrt{2}\cos\theta}$$

$$\sigma_y = \frac{\sigma}{\sqrt{2}\sin\theta}$$



- **Orthogonal tracks are optimal**

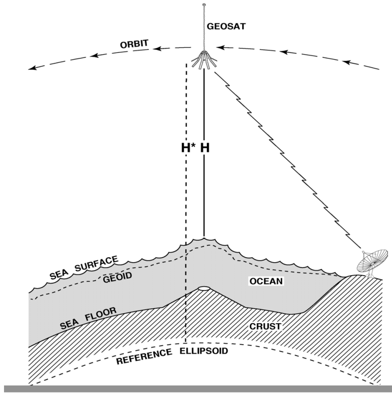
Achieving 1 mGal Gravity Accuracy

- **Improved range precision** -- A factor of 2 or more improvement in altimeter range precision, with respect to Geosat and ERS-1, is needed to reduce the noise due to ocean waves.

- **Fine cross-track spacing and long mission duration** -- A ground track spacing of 6 km or less is required.

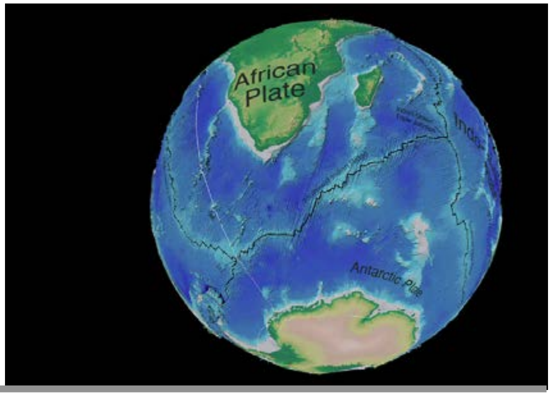
- **Moderate inclination** -- Current non-repeat-orbit altimeter data have high inclination and thus poor accuracy of the E-W slope at the equator.

- **Near-shore tracking** -- For applications near coastlines, the ability to track the ocean surface close to shore is desirable.



Marine Gravity from Satellite Altimetry

Geodynamics, November, 2014



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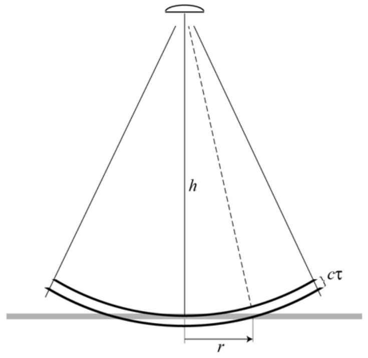
Waveform Model

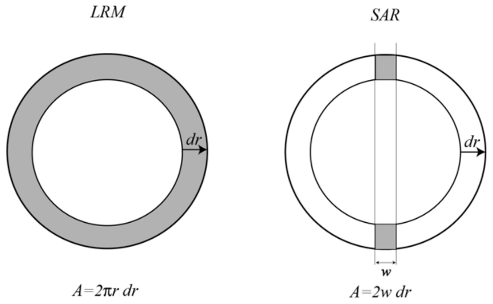
$$M(\tau) = P(\tau) * A(\tau) * G(\tau)$$

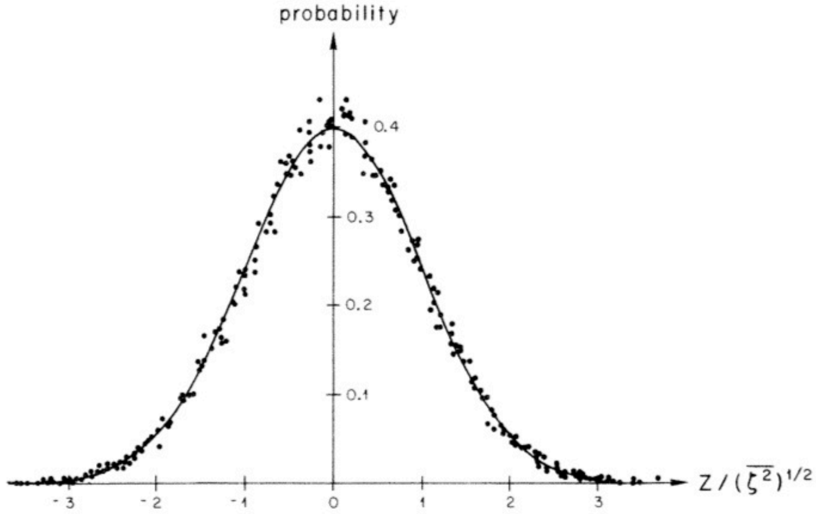
outgoing pulse (assume Gaussian)

flat surface response

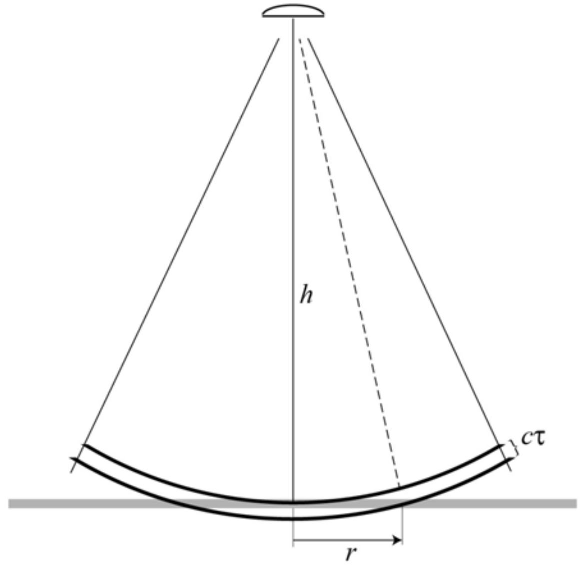
Gaussian waves

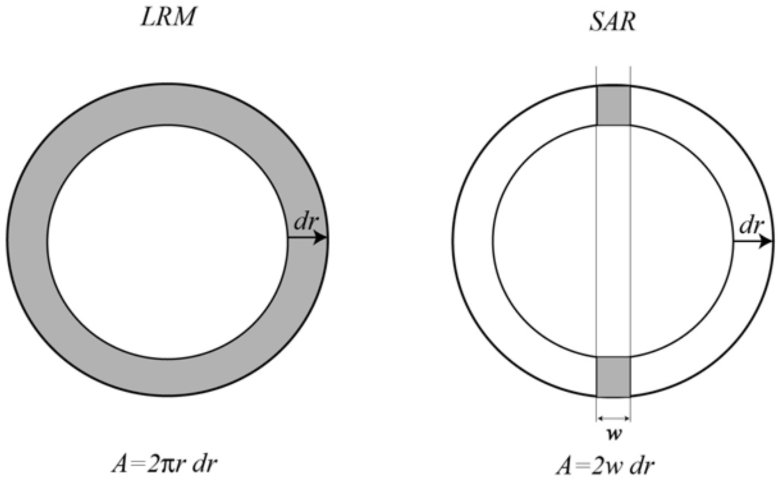


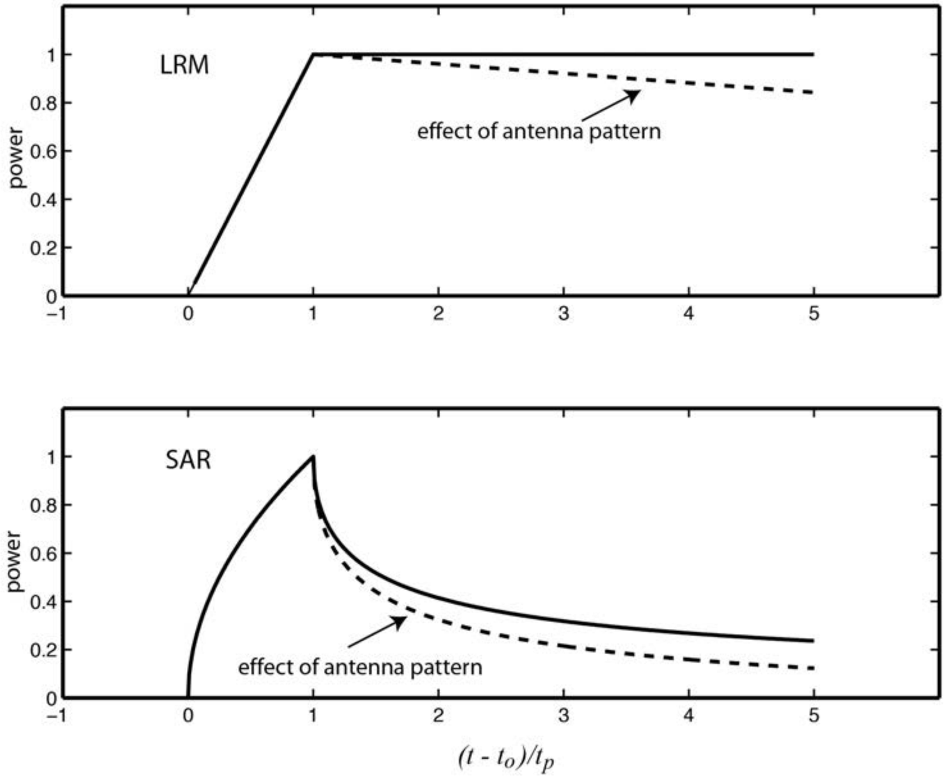




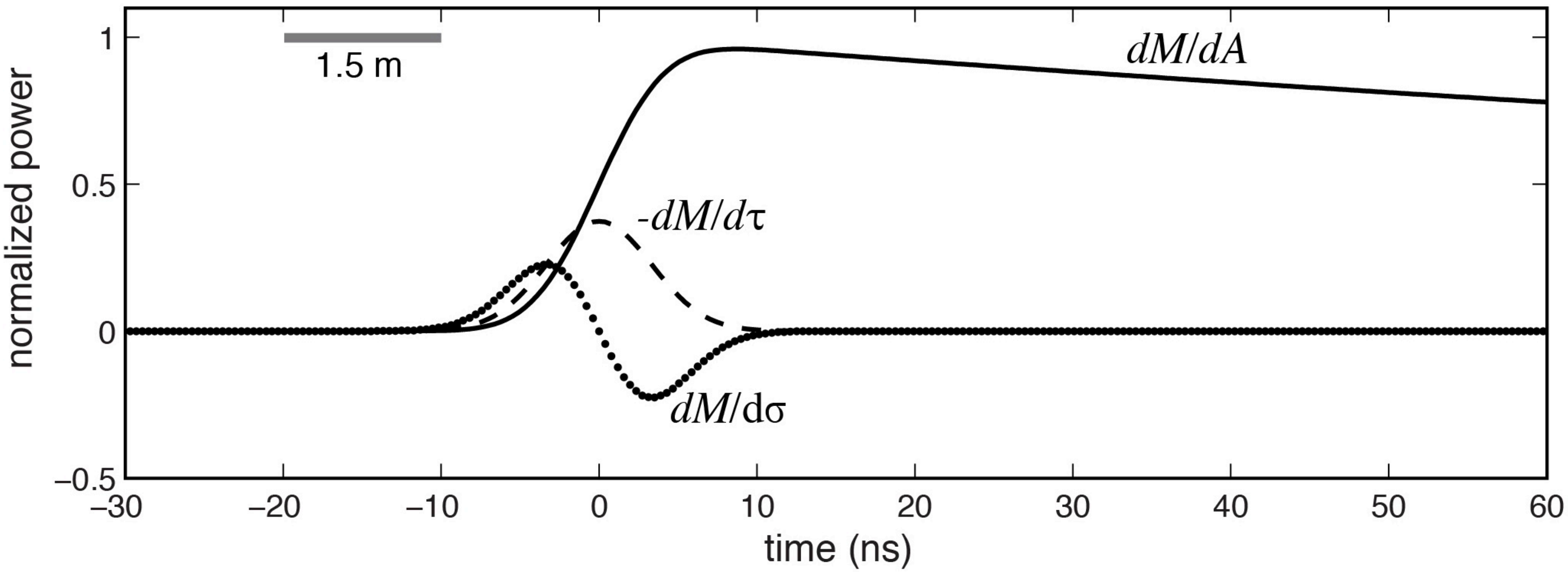
flat surface response – LRM vs. SAR







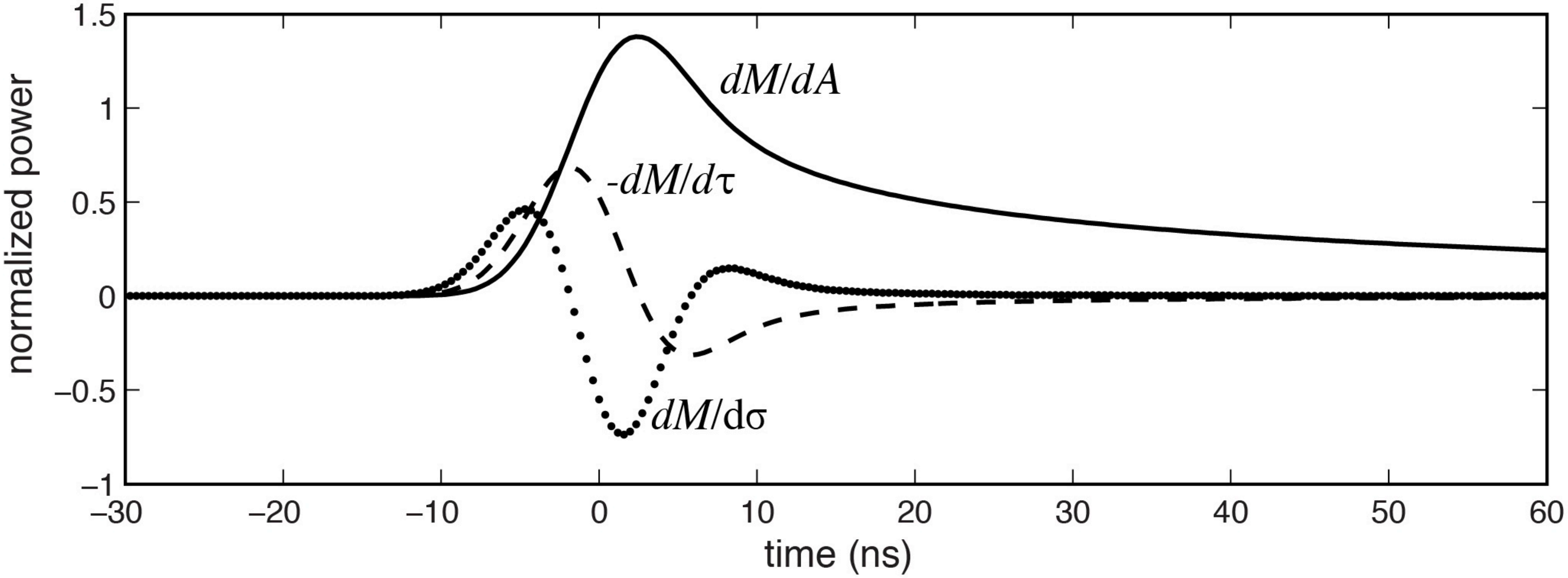
LRM waveform



Estimate 3 parameters: arrival time (τ), rise time (σ), and power (A).

$$M(A,\tau,\sigma) = \frac{A}{2}\left\{1 + erf(\eta)\right\}; \qquad \eta = \frac{\tau}{\sqrt{2}\sigma}$$

SAR waveform

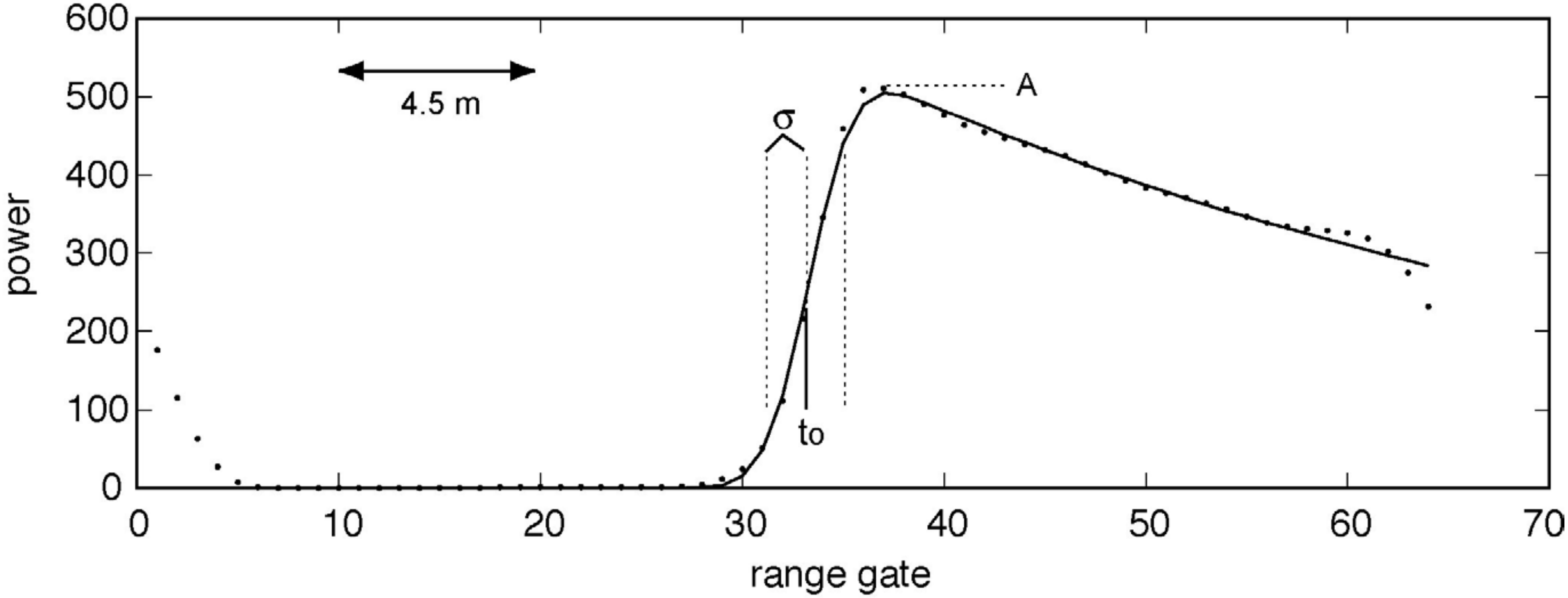


Estimate 3 parameters: arrival time (τ), rise time (σ), and power (A).

$$M(A,\tau,\sigma) = \frac{A}{2}\exp\left(-\frac{1}{4}z^2\right)D_{-1/2}(z); \qquad z = \frac{\tau}{\sigma}$$

D – parabolic cylinder function

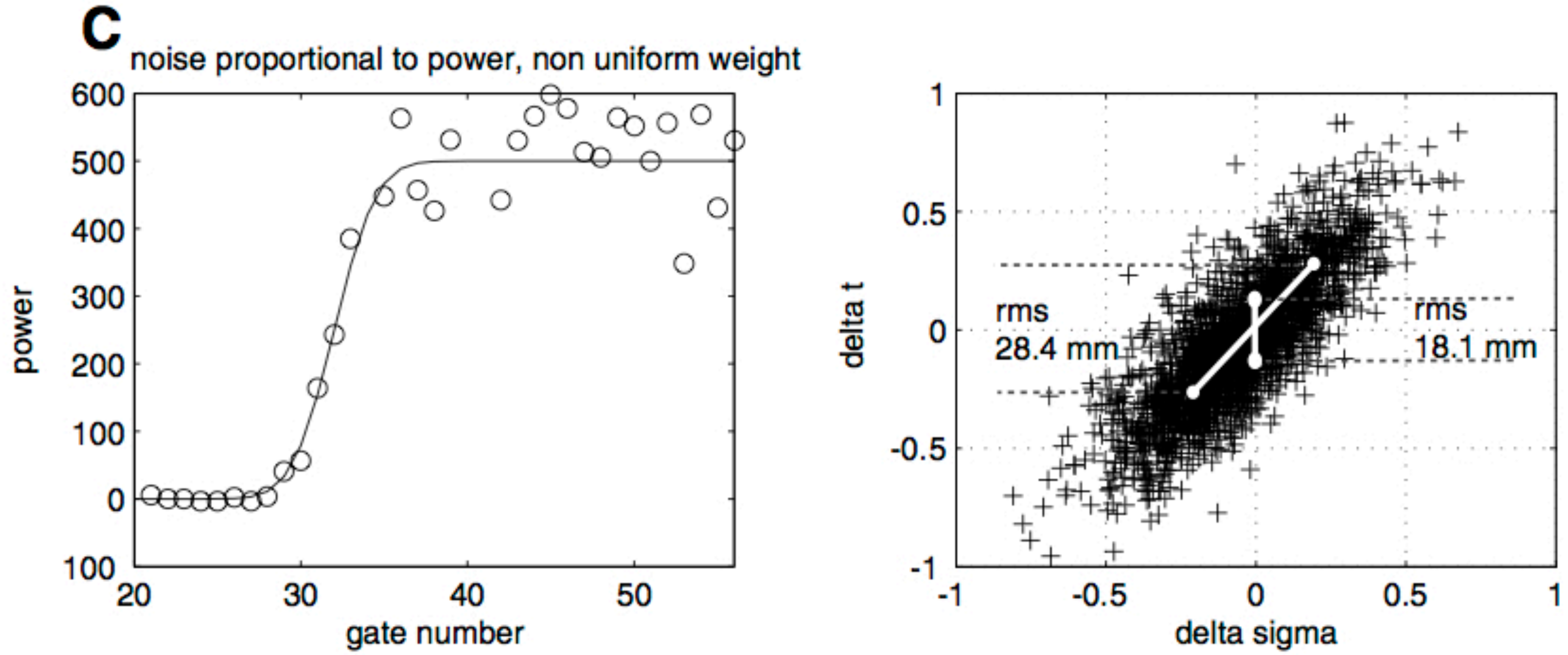
waveform retracking



Estimate 3 parameters: arrival time (t_o), rise time (σ), and power (*A*).

$$M(A, t_o, \sigma) = \frac{A}{2}\left\{1 + erf(\eta)\right\}; \qquad \eta = \frac{t_o}{\sqrt{2}\sigma}$$

least squares correlation of arrival time and rise time



If the wave height were known then one could estimate the arrival time 1.56 times more precisely.

[Sandwell and Smith, GJI, 2005]

recipe for 40% improvement

1) retrack waveforms with standard 3-parameter model

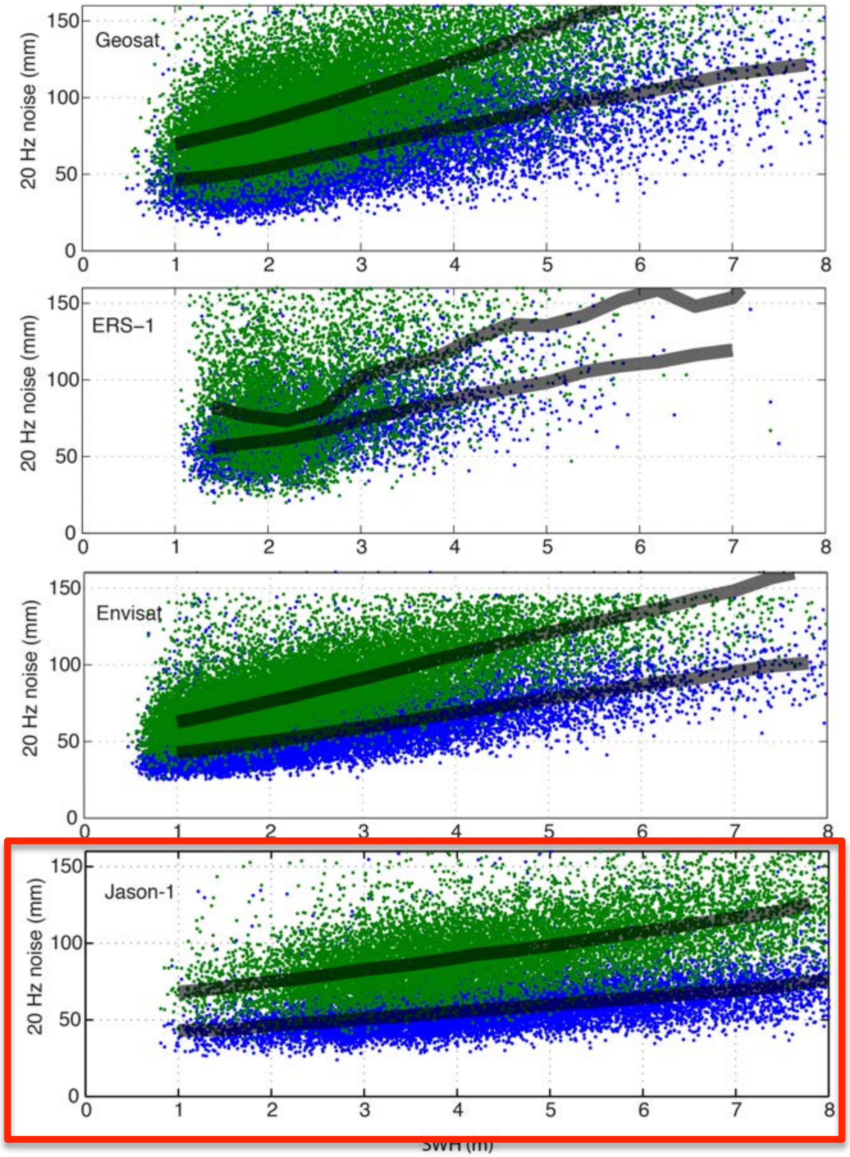
2) smooth wave height and amplitude over 90-km wavelength

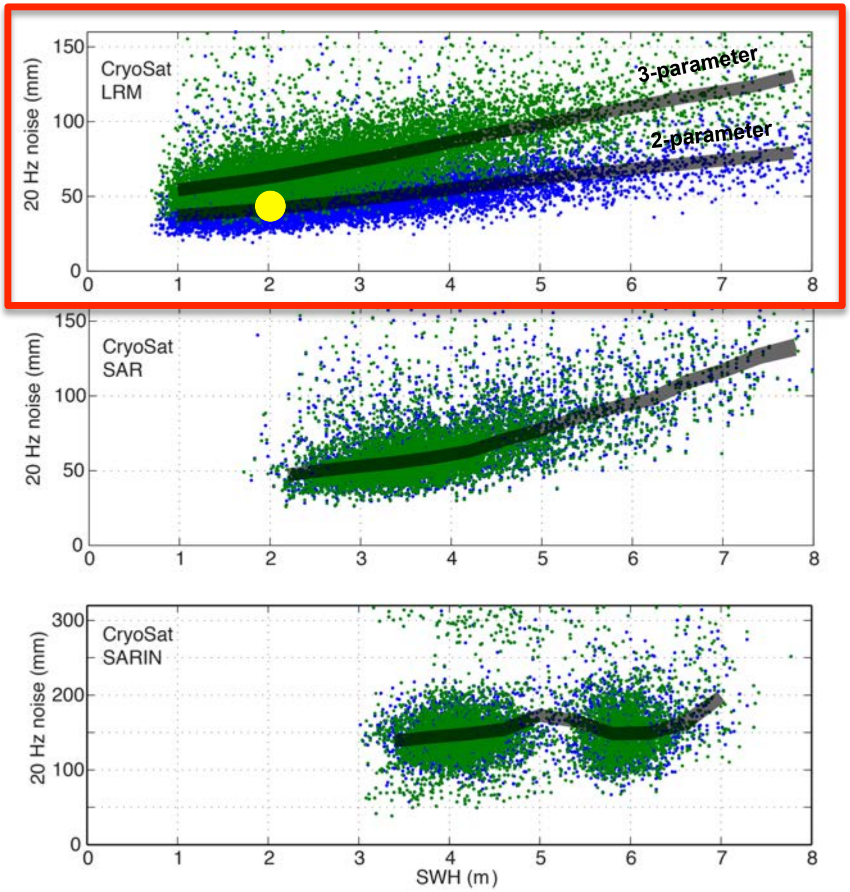
3) retrack waveforms with 1-parameter model

Note: this assumes wave height/amplitude vary smoothly along track.

[Garcia et al., 2014]

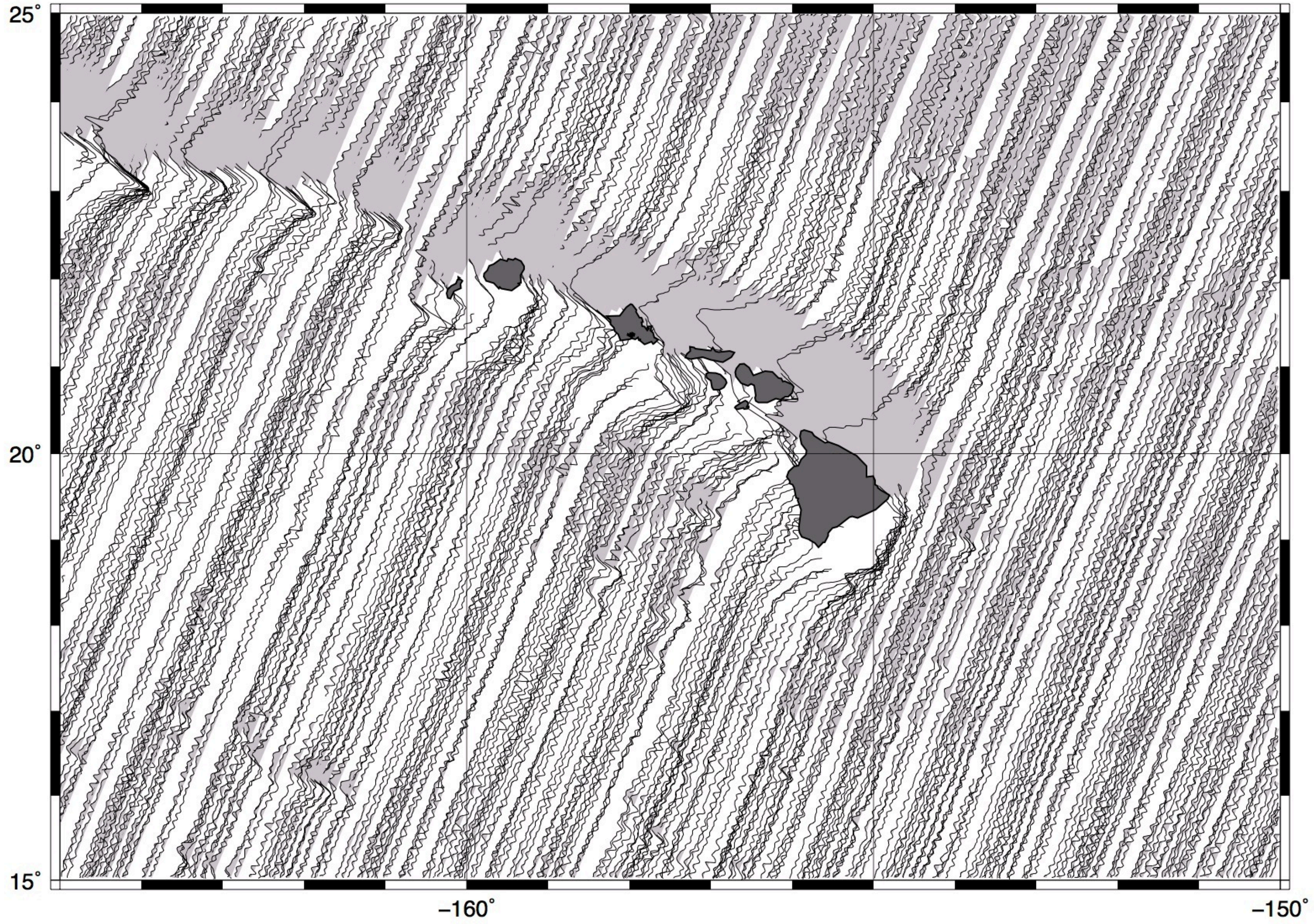
20 Hz range precision of all altimeters



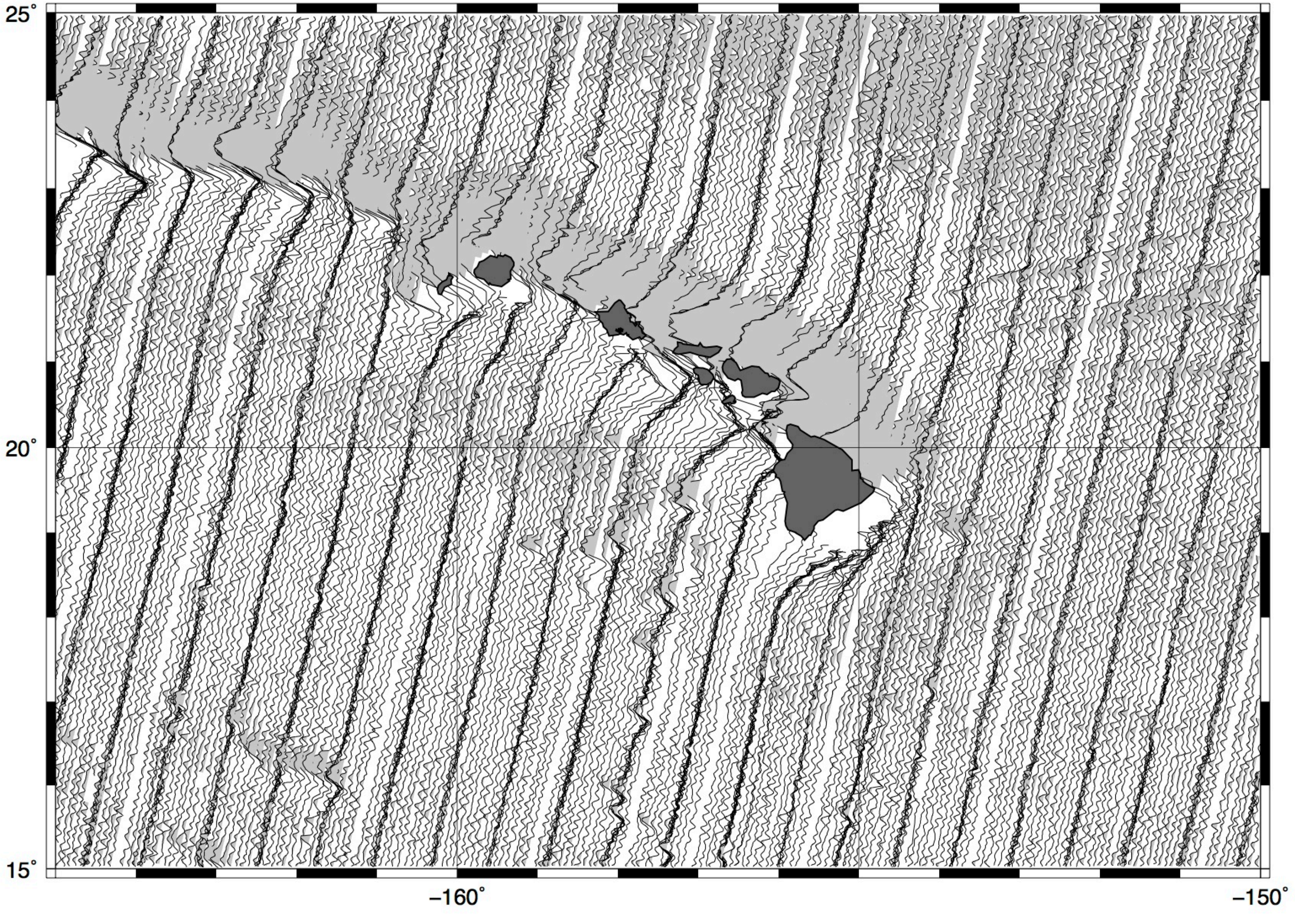


Altimeter	2-PAR @ 2 m
Geosat	57.0
ERS-1	61.8
Envisat	51.8
Jason-1	46.4 ←
CryoSat LRM	42.7 ←
CryoSat SAR	49.7
CryoSat SARIN	138.7

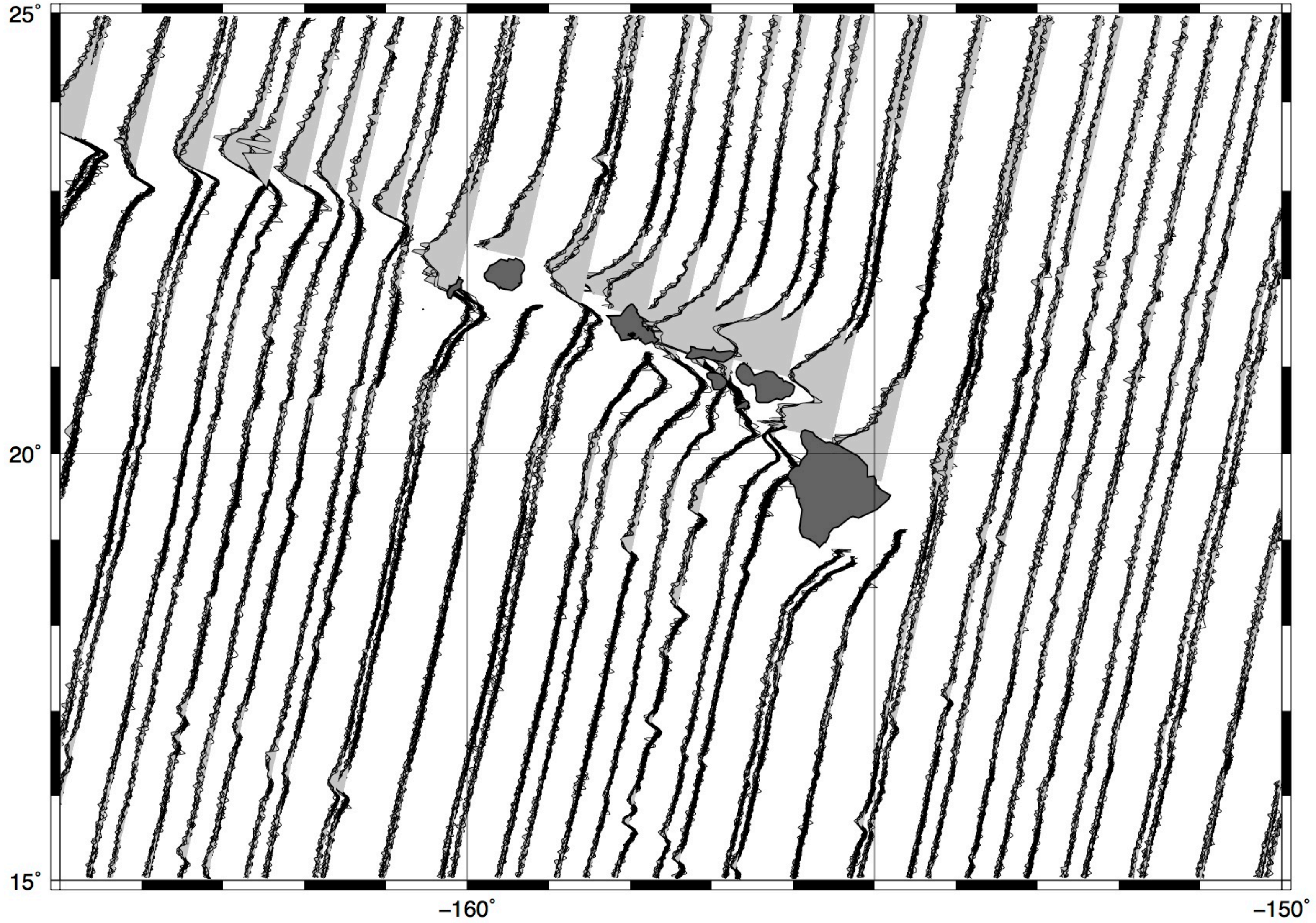
Geosat – descending (18 mo. old)



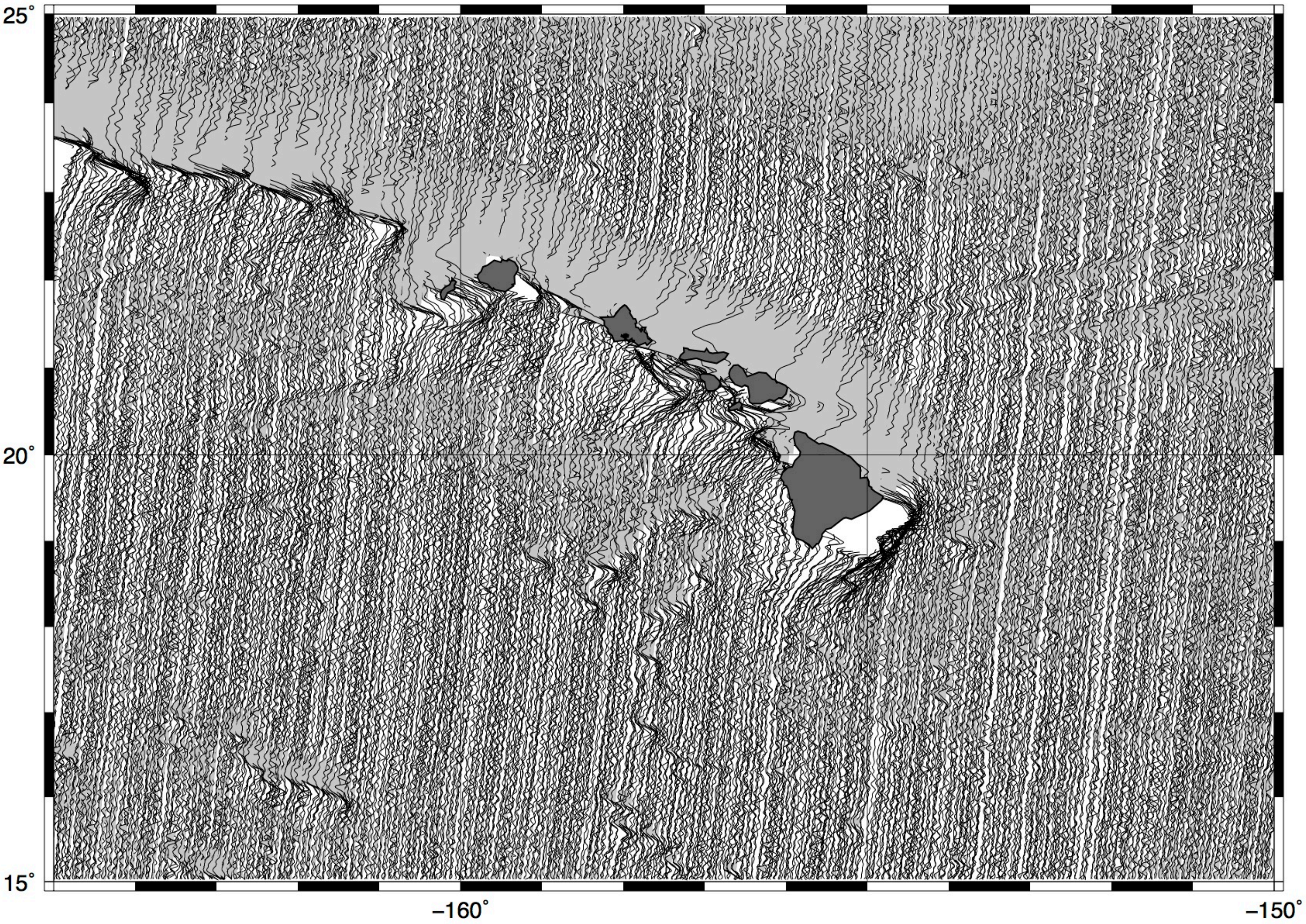
ERS-1 – descending (12 mo. old)



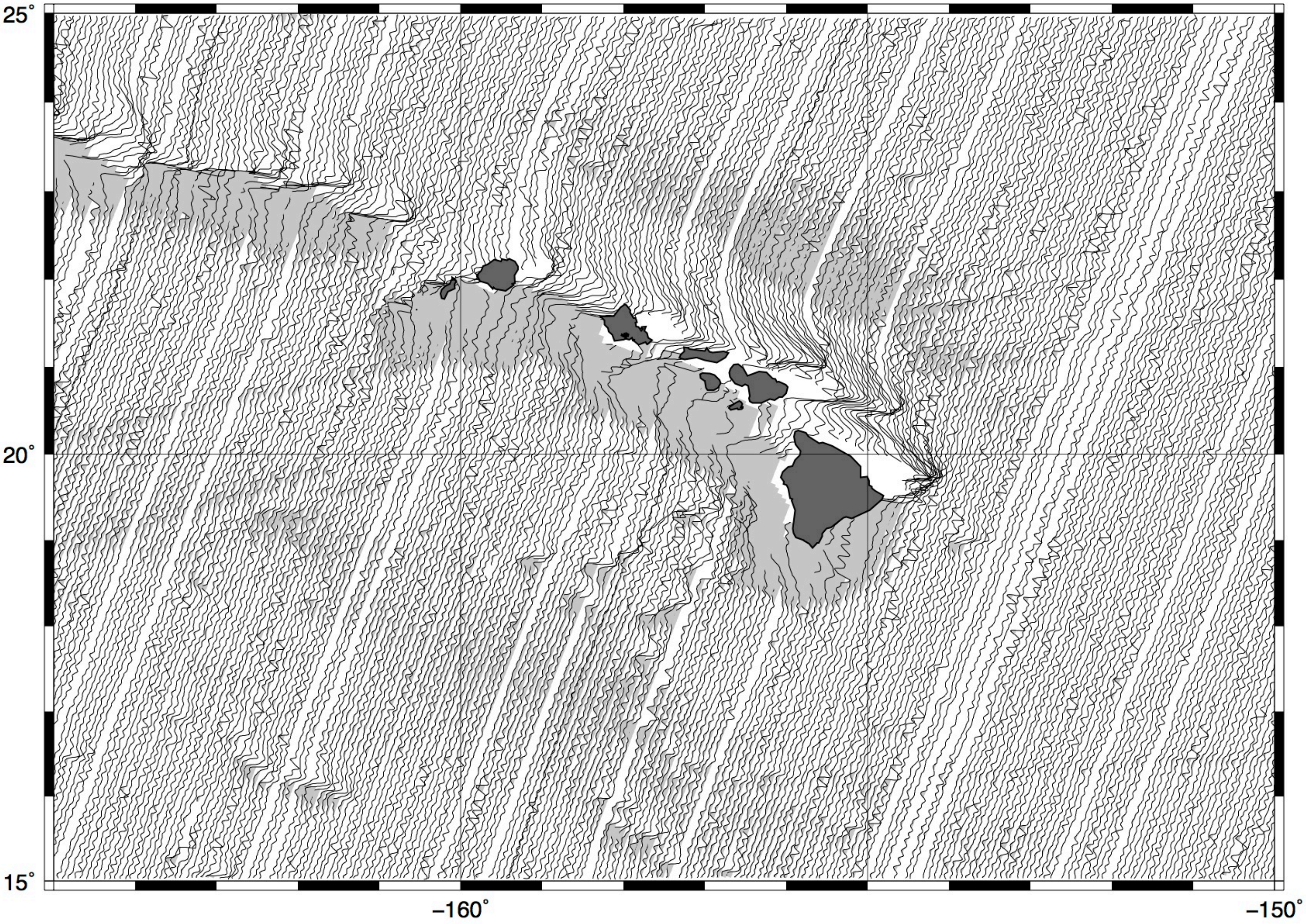
Envisat – descending (new)

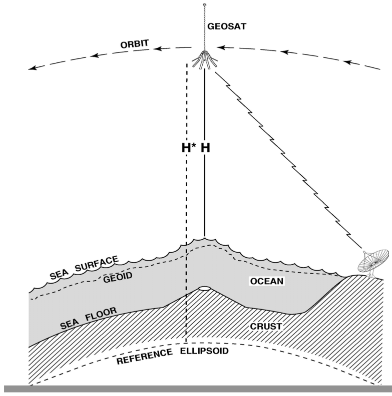


CryoSat-2 – descending (45 mo. new)



Jason-1 – ascending (14 mo. new)





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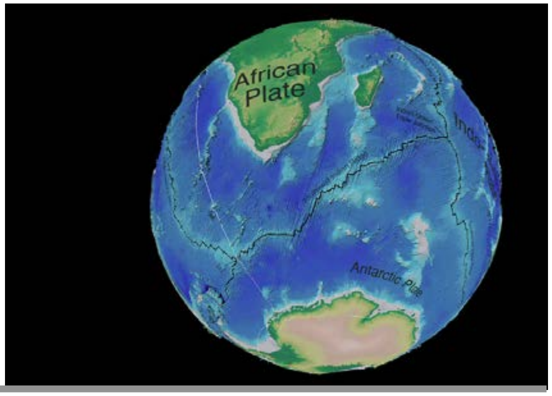


- basic theory
- retracking altimeter waveforms and CryoSat
- **gravity from satellite altimetry**
- predicting bathymetry from gravity

construction of gravity anomaly from satellite altimeter data

1. Acquire a spherical harmonic model of the gravitational potential of the Earth and generate models of the relevant quantities (e.g., geoid height, gravity anomaly, deflection of the vertical, . . .) out to say harmonic 80. You may want to taper the harmonics between say 60 and 120 to avoid Gibb's phenomenon; this depends on the application.

2. **Remove** that model from the satellite altimeter profiles.

3. Project the residual data onto a Mercator grid so the cells are approximately square and use the central latitude of the grid to establish the dimensions of the grid for Fourier analysis.

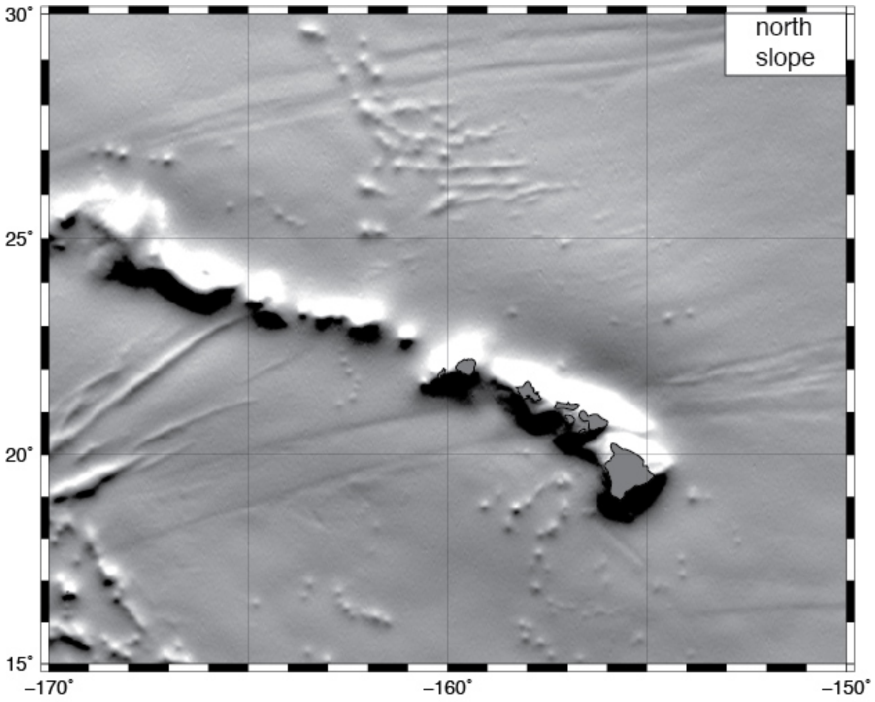
4. Perform the desired calculation (e.g., upward continuation, gravity/topography transfer function, . . .).

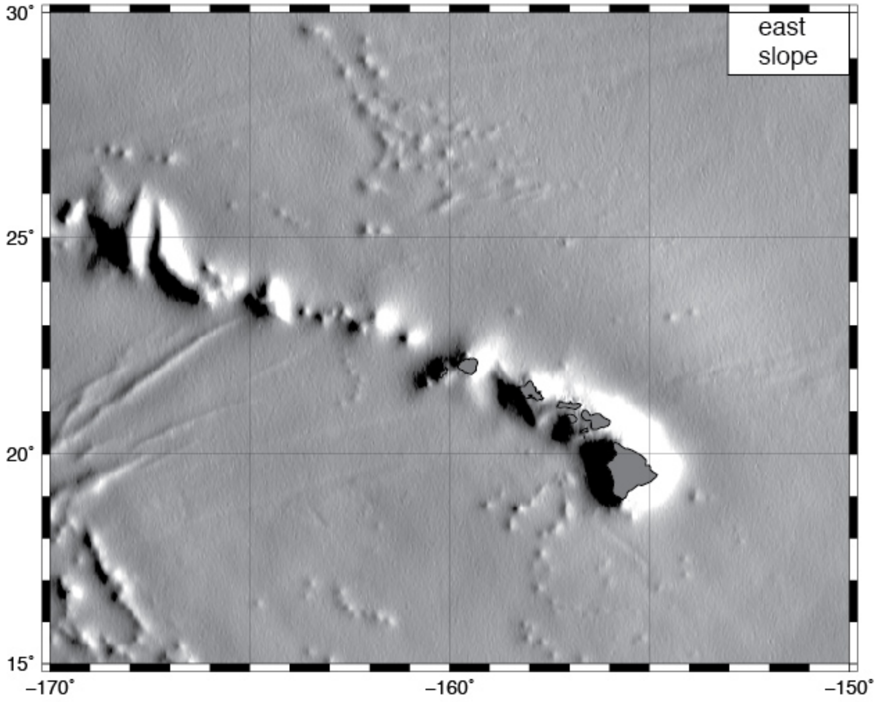
5. **Restore** the appropriate spherical harmonic quantity using the exact model that was removed originally.

north and east slope

– combine along-track slopes from all available satellite altimeters to form north and east slope grids.

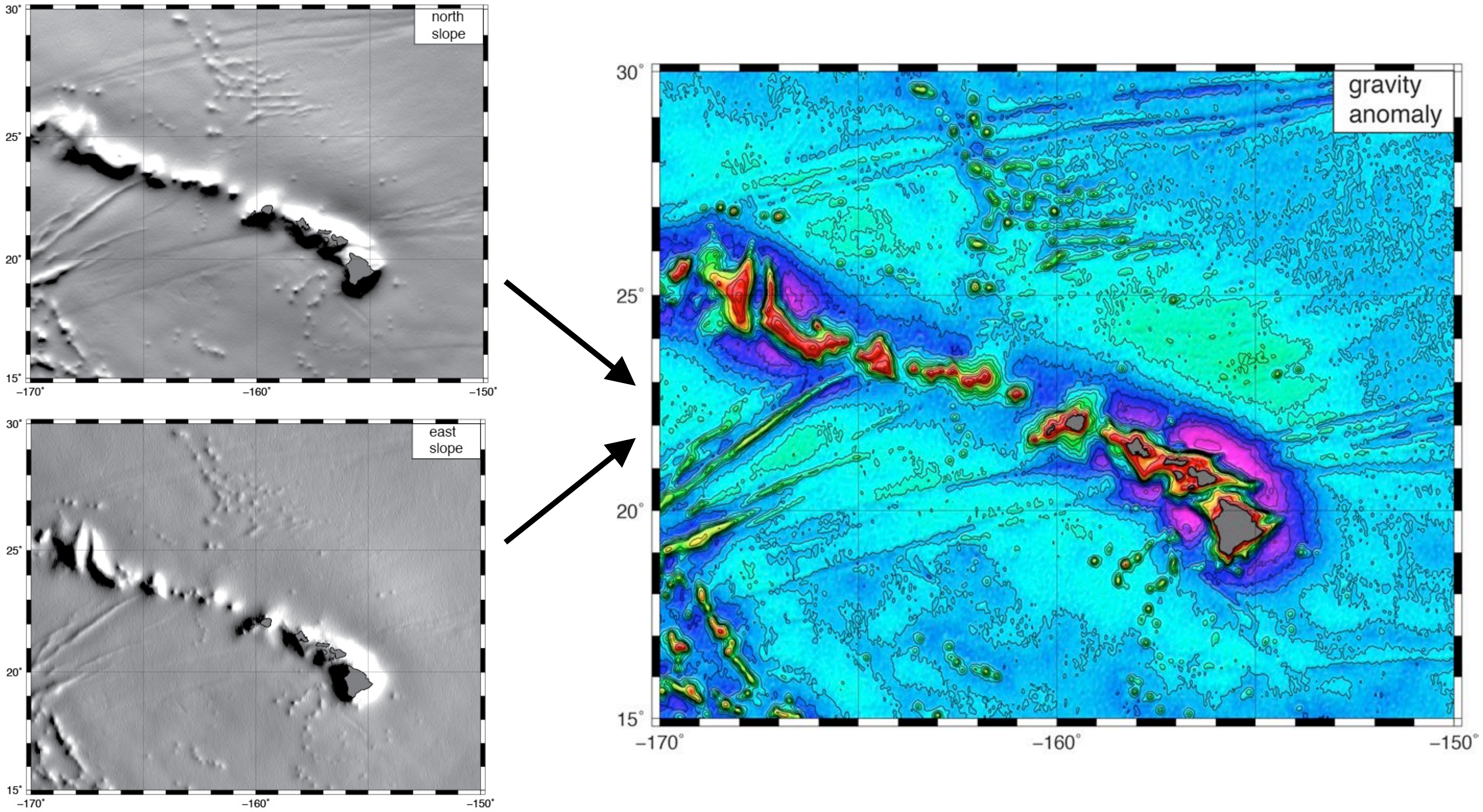
current altimeters provide ~2 x higher noise in the east slope than in the north slope because of their high inclination orbits.



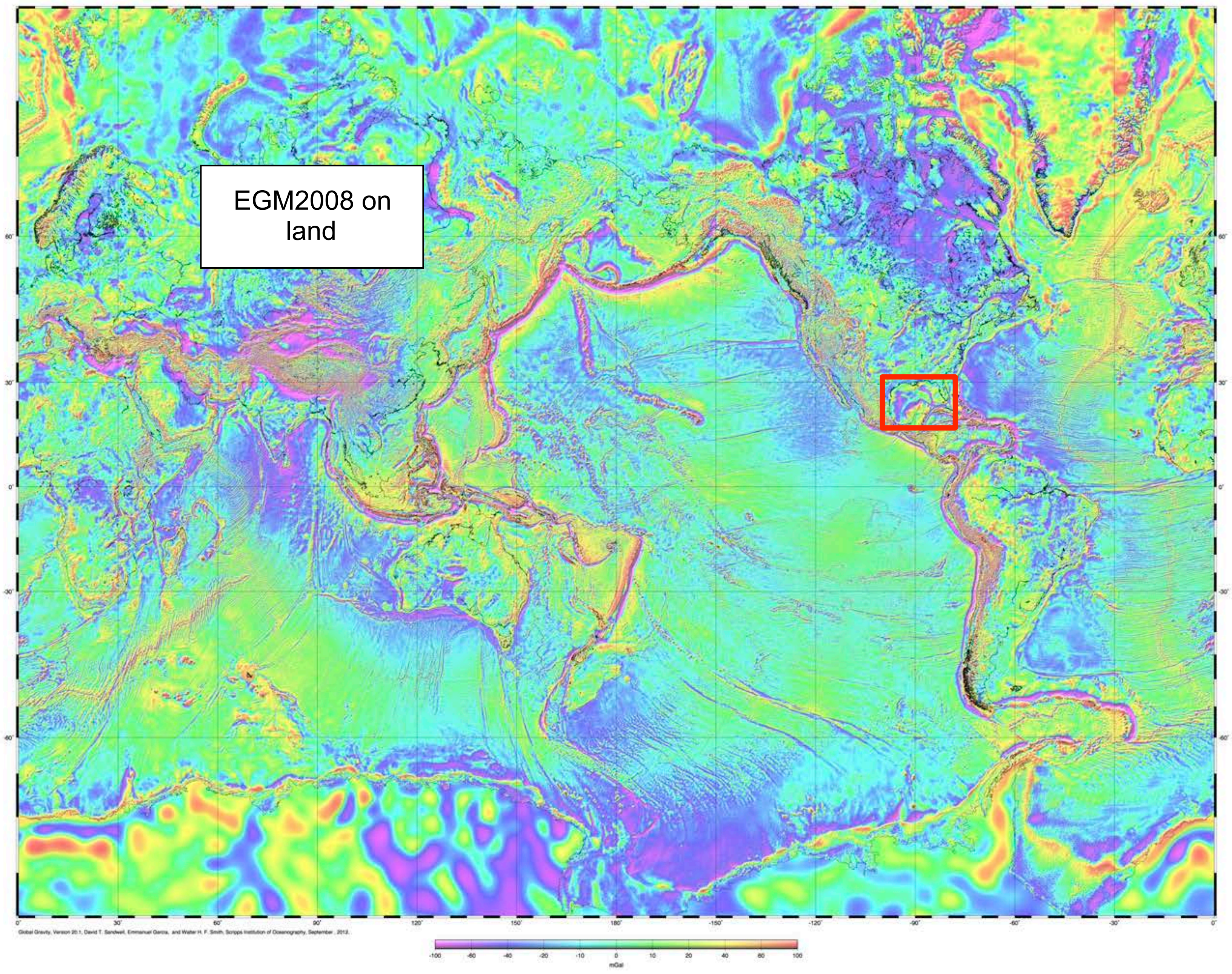


gravity anomaly

- use Laplace equation to convert slopes to gravity anomaly.
- restore long-λ gravity model.



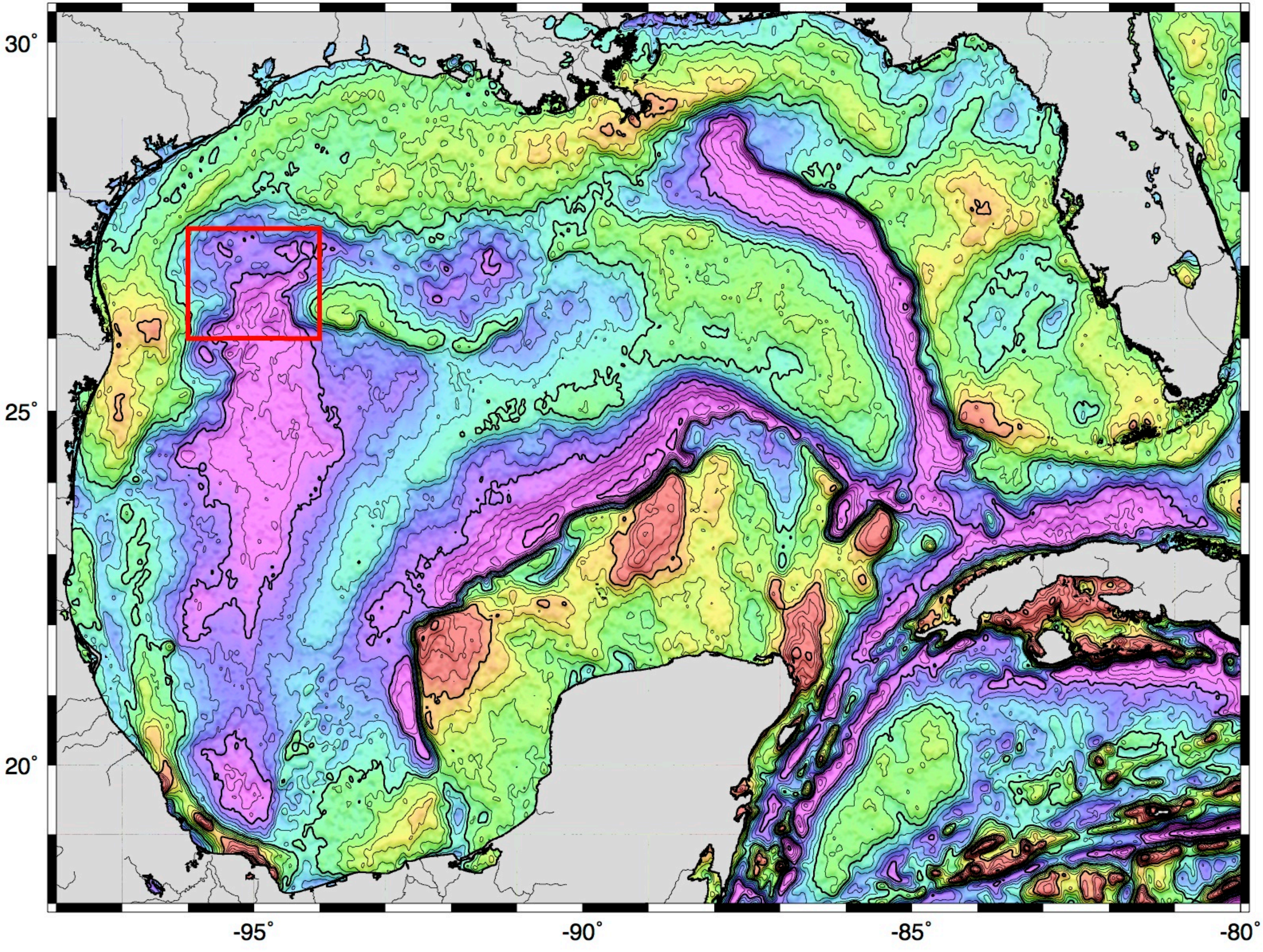
Global Gravity Anomaly V22.1



EGM2008 on land

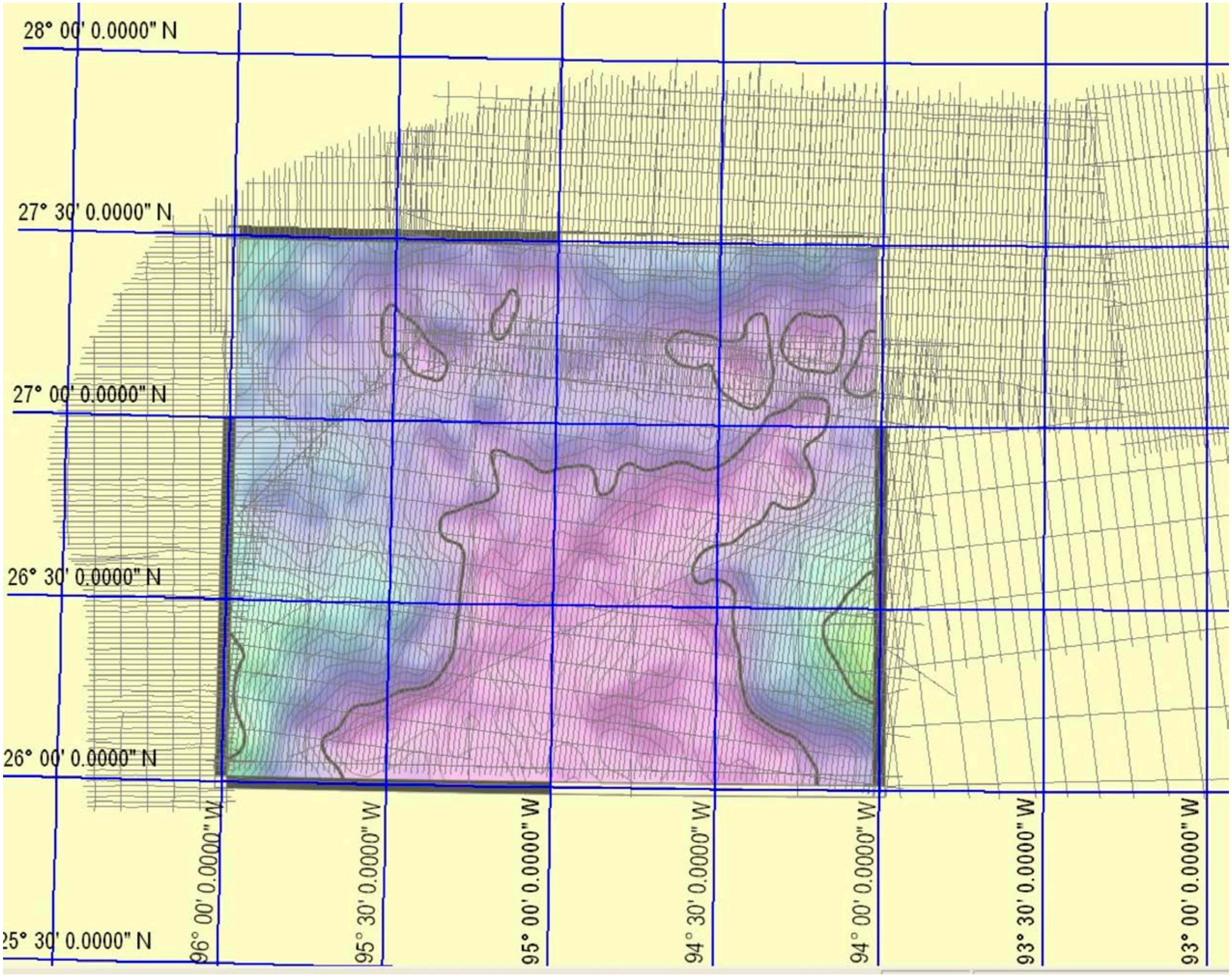
Global Gravity, Version 20.1, David T. Sandwell, Emmanuel Garcia, and Walter H. F. Smith, Scripps Institution of Oceanography, September , 2012.

Gravity Anomaly V22.1 (10 mGal contours)

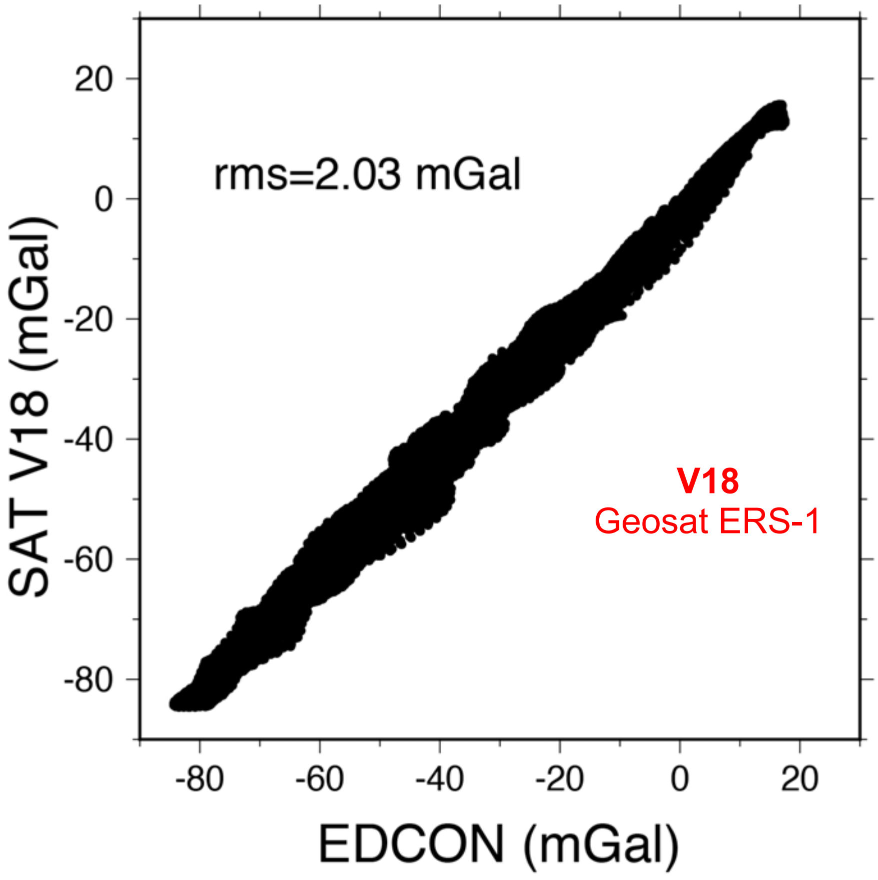


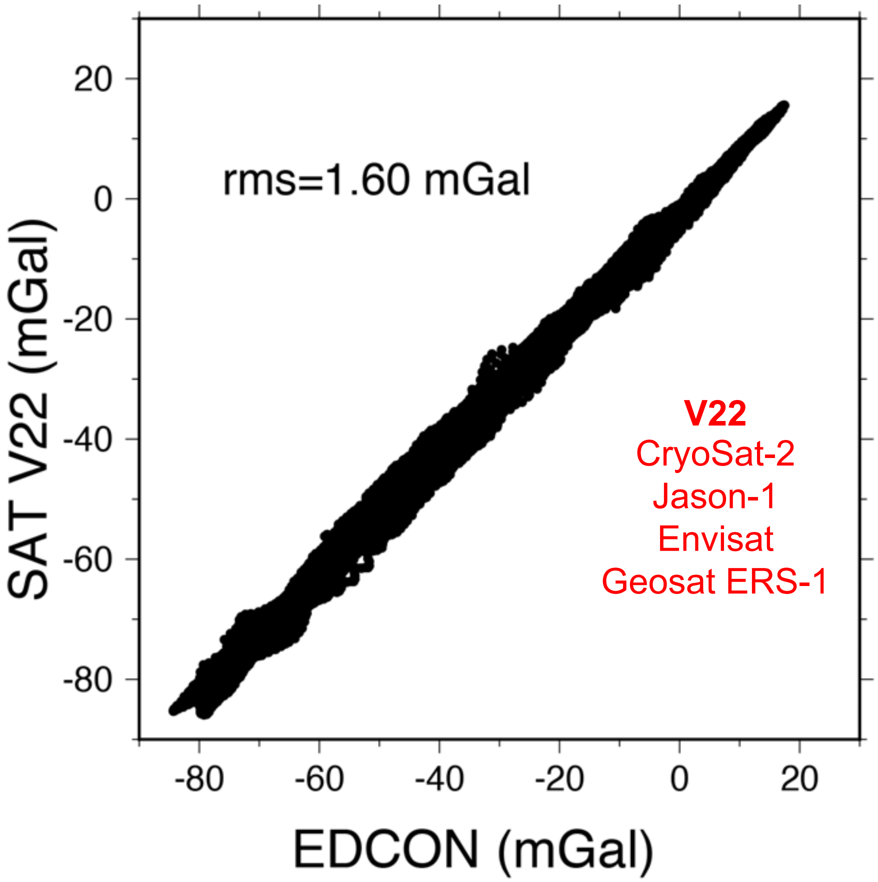
GPS Navigated Gravity survey of Alaminos Canyon

(Alan Herring, personal communication, Dec. 2011.)



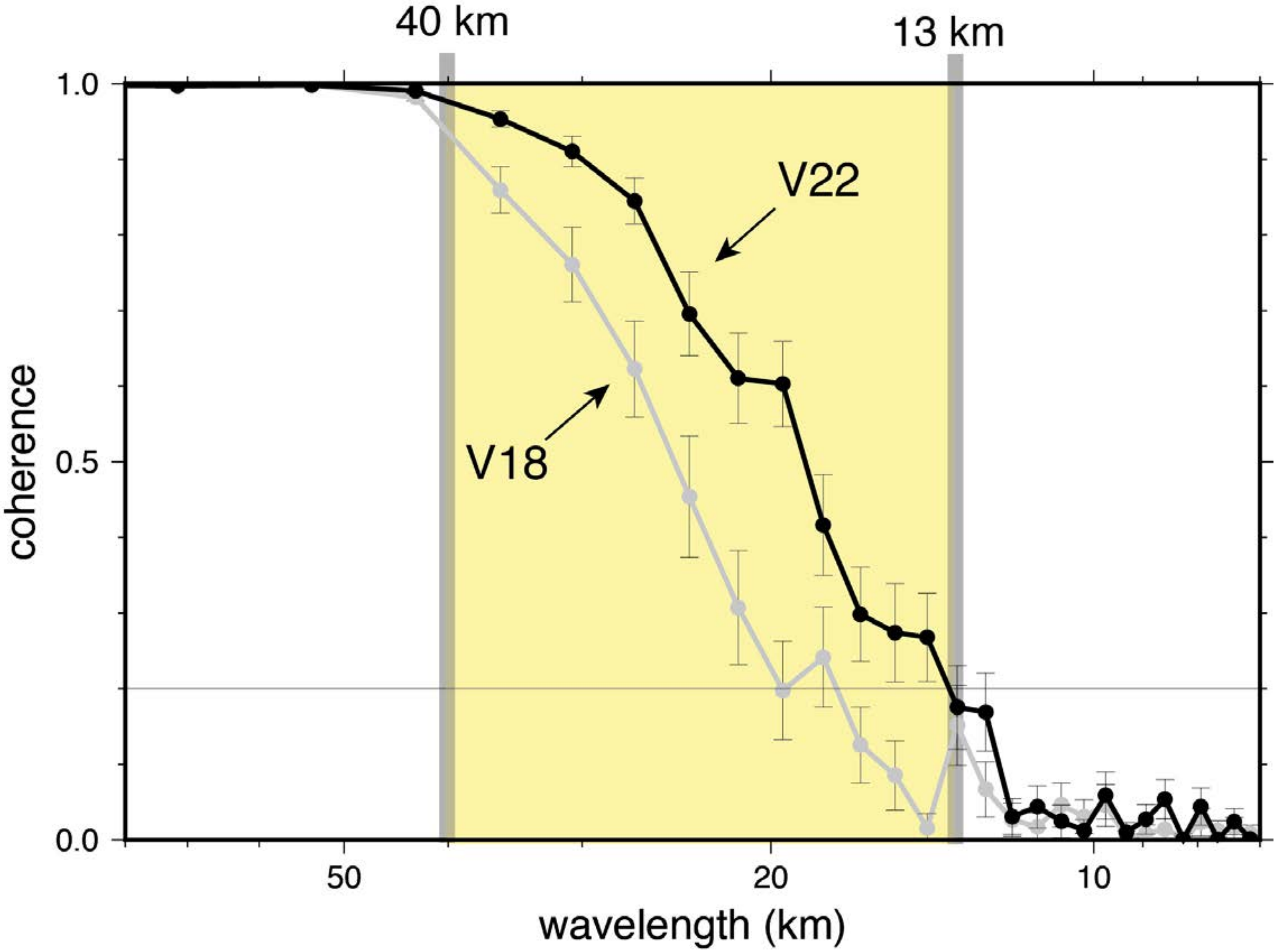
spatial comparisons in the Gulf of Mexico





Noise contribution from the EDCON data is 0.51 mGal.

spectral comparisons in the Gulf of Mexico



NGA Accuracy Assessment

Blind comparison of altimetry gravity (V22) with 30 million of
NGA's "BEST" shipboard gravity measurements.

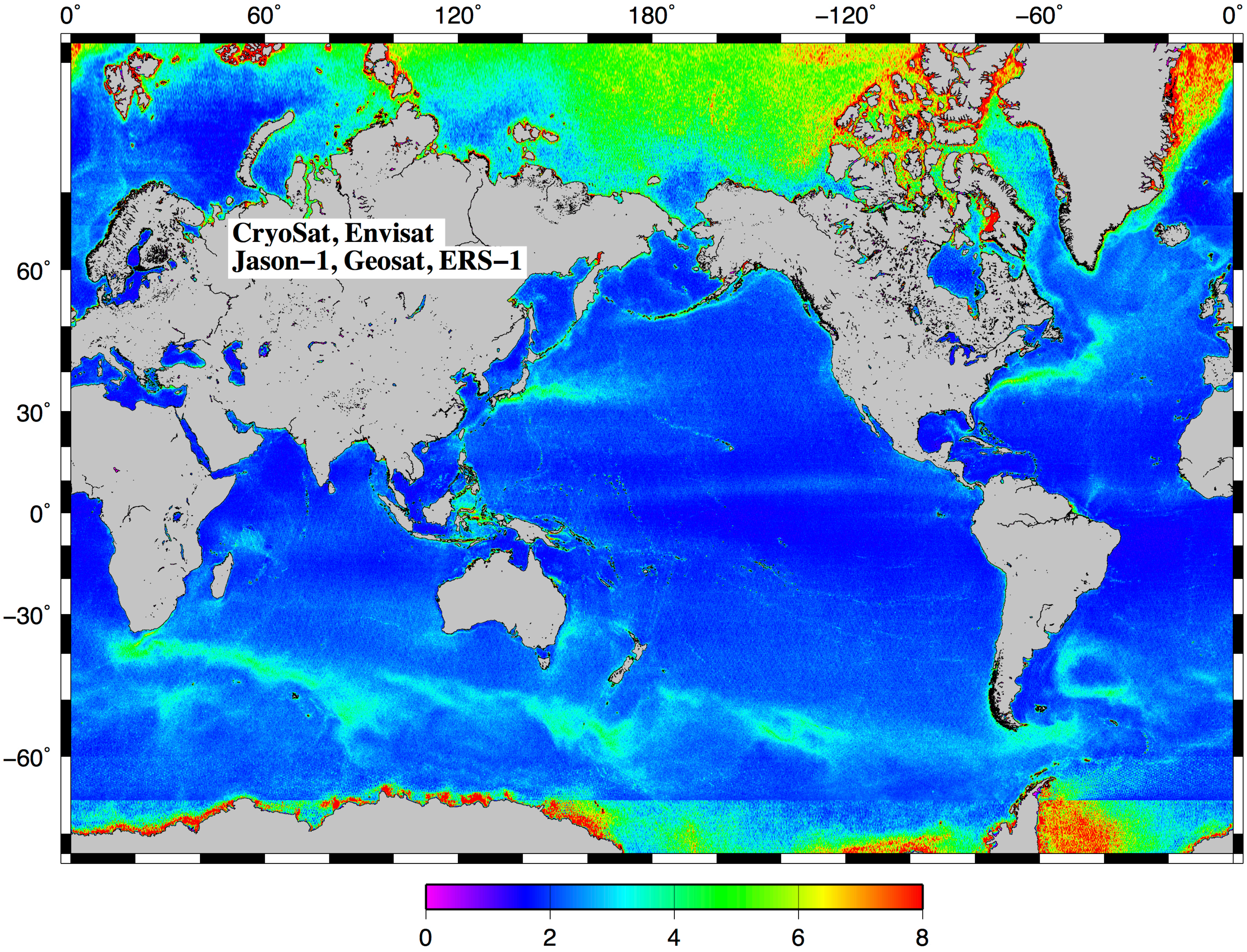
Analysis performed by John Factor at NGA, February, 2014.

gravity model	all depths	10 - 1000 m	1 – 3 km	3 to 6 km
SS22	**2.61**	**3.58**	**2.97**	**2.31**
EGM08	3.07	4.04	3.85	2.65
SS-Uncertainty	2.08	4.18	2.22	1.67

Shallow gravity errors 3.58 mGal >> deep ocean gravity errors 2.31 mGal.

NGA shipboard data has 1-2 mgal error so mean error of SSV22 is ~2 mGal.

median along track slope difference in microradian (0.98 mGal)



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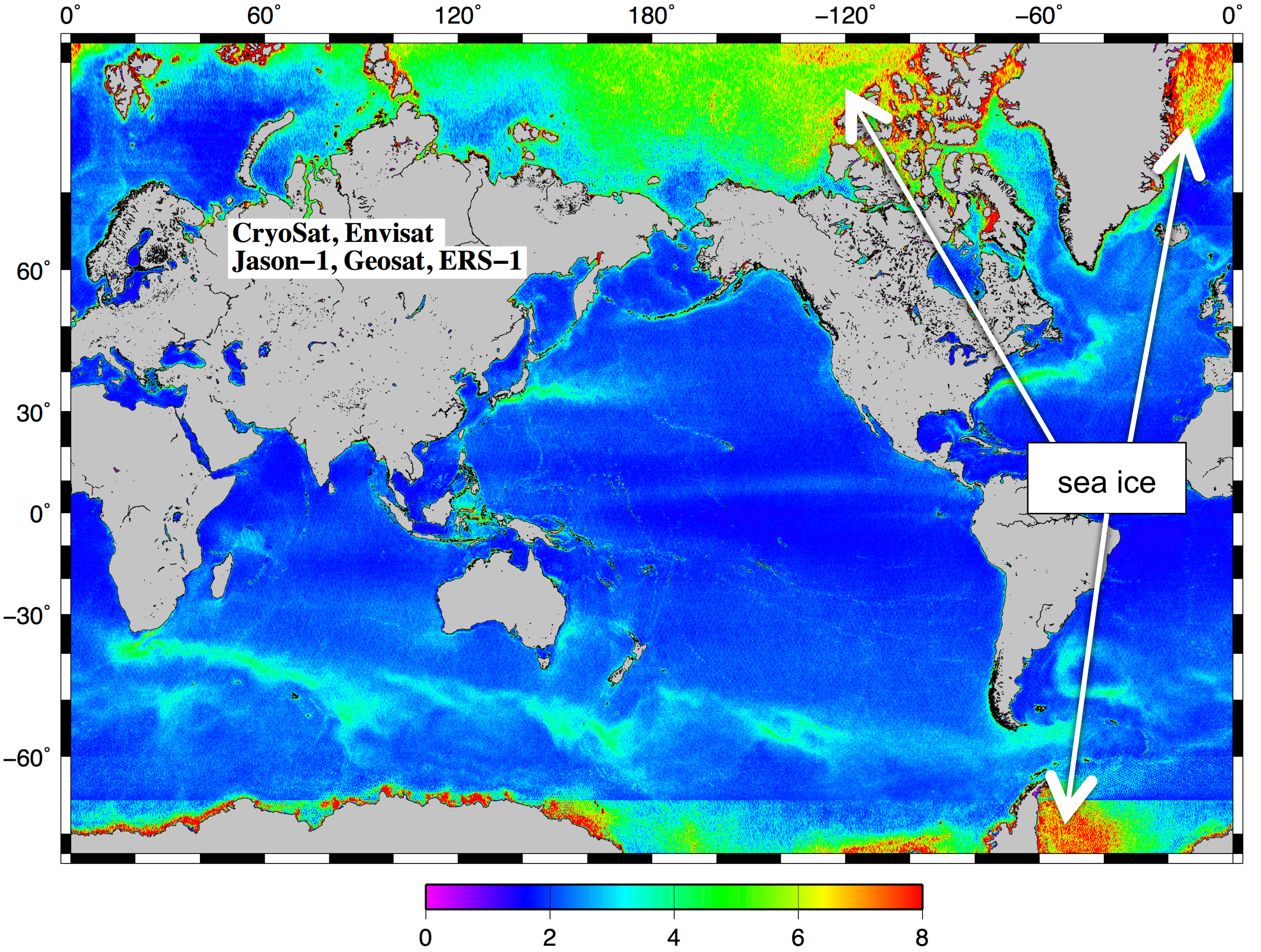
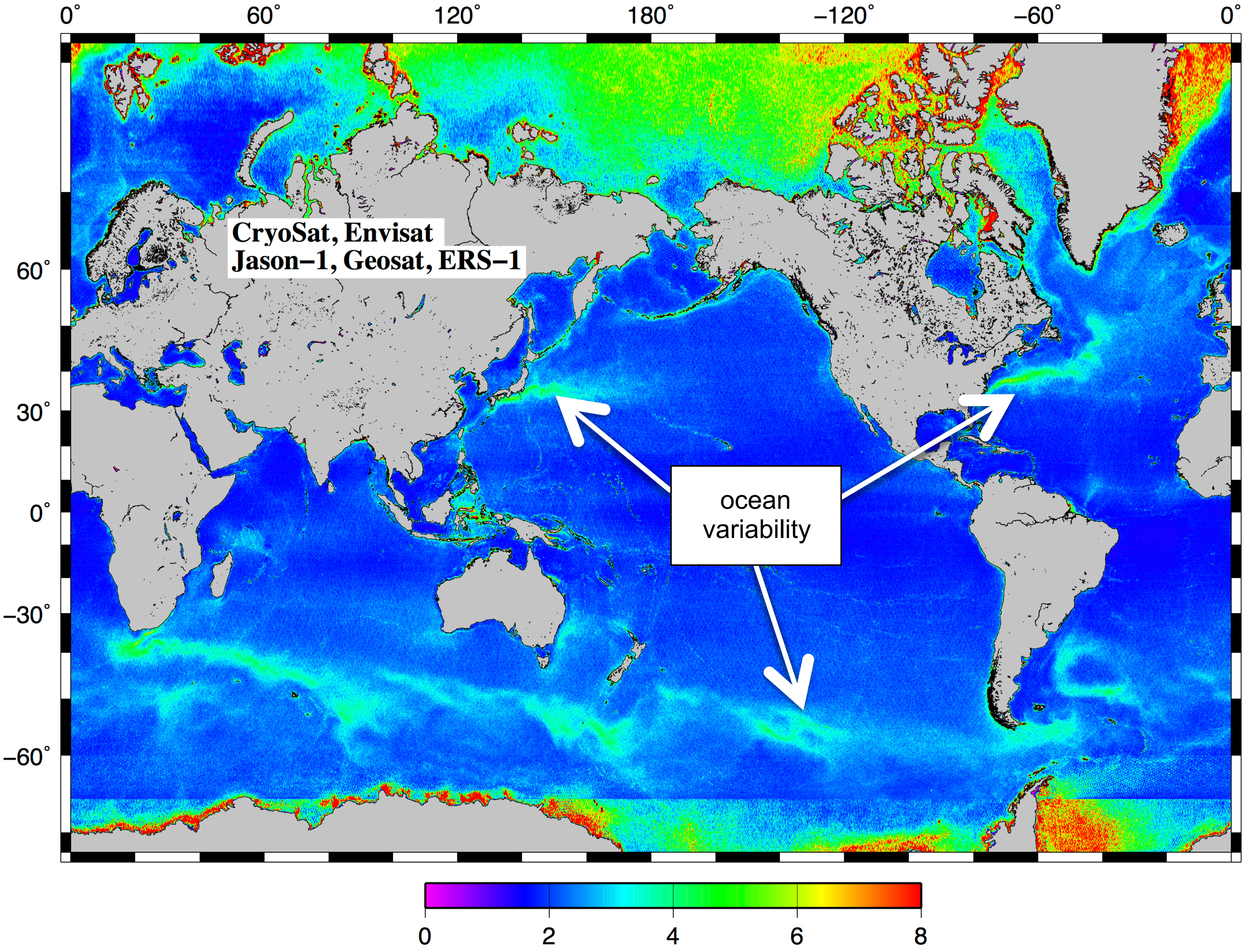
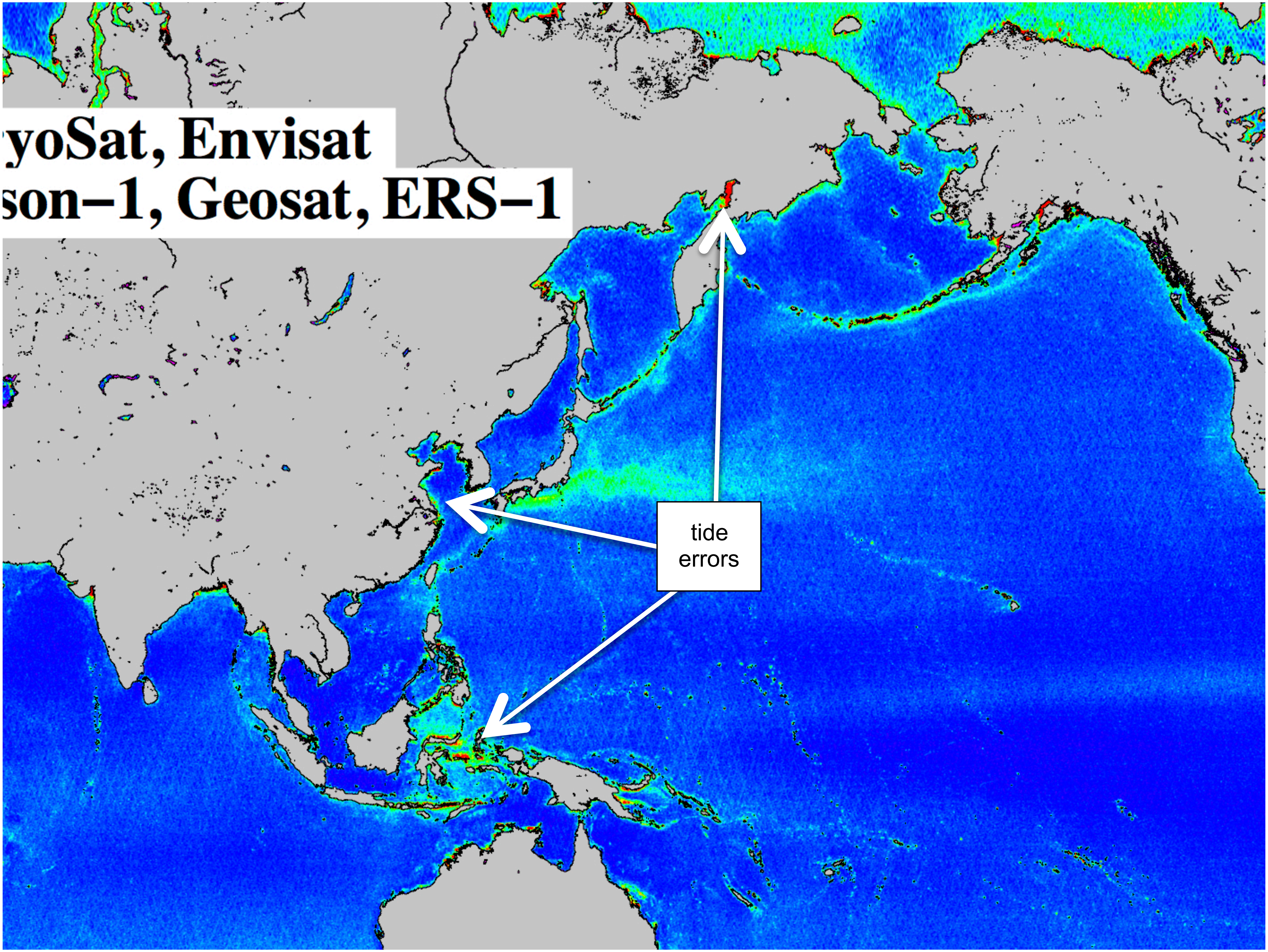


median along track slope difference in microradian (0.98 mGal)



CryoSat, Envisat
Jason−1, Geosat, ERS−1

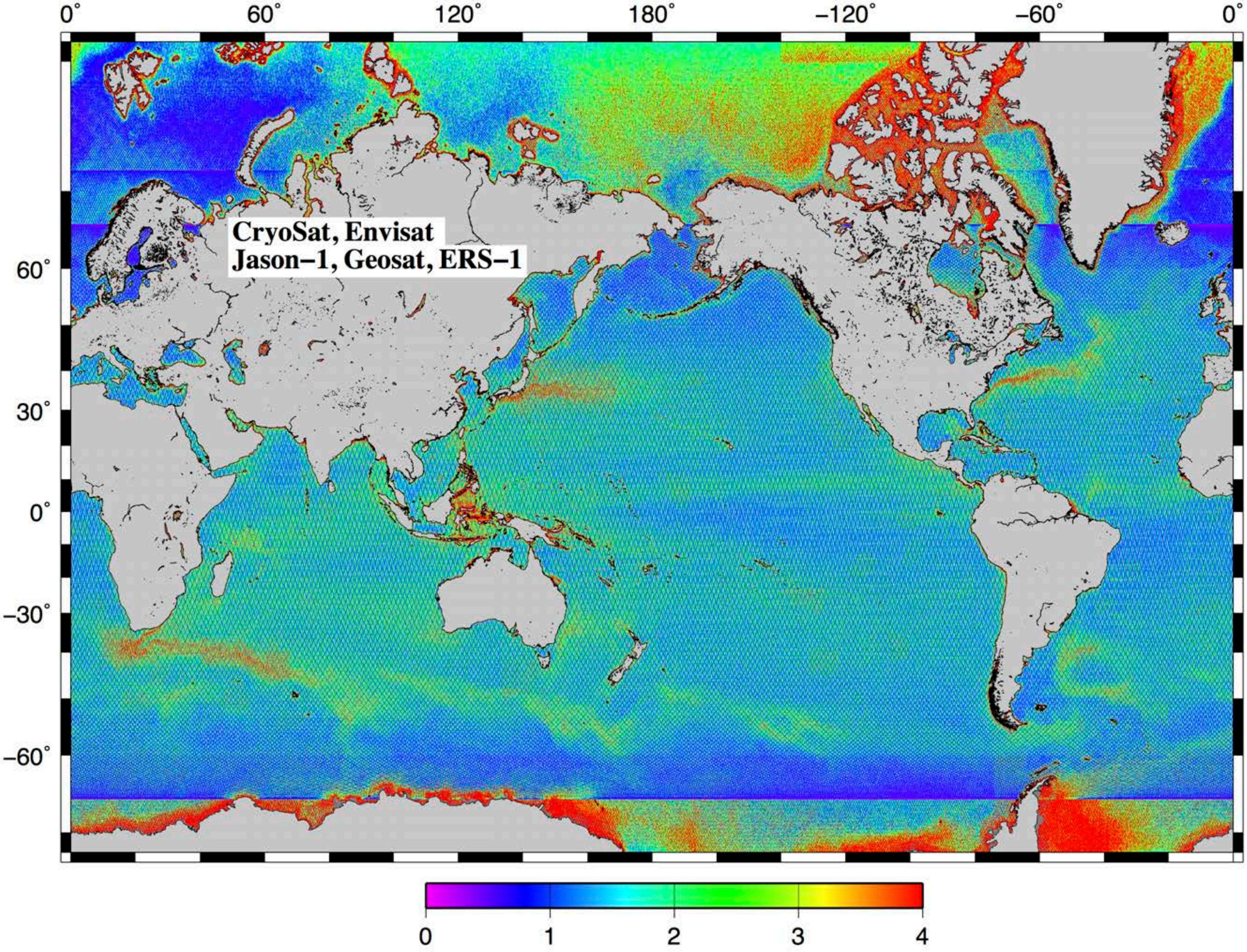
ocean variability



yoSat, Envisat
son–1, Geosat, ERS–1

tide errors

gravity anomaly error (mGal) calibrated using NGA analysis



applications of radar altimetry

(non-repeat orbit)

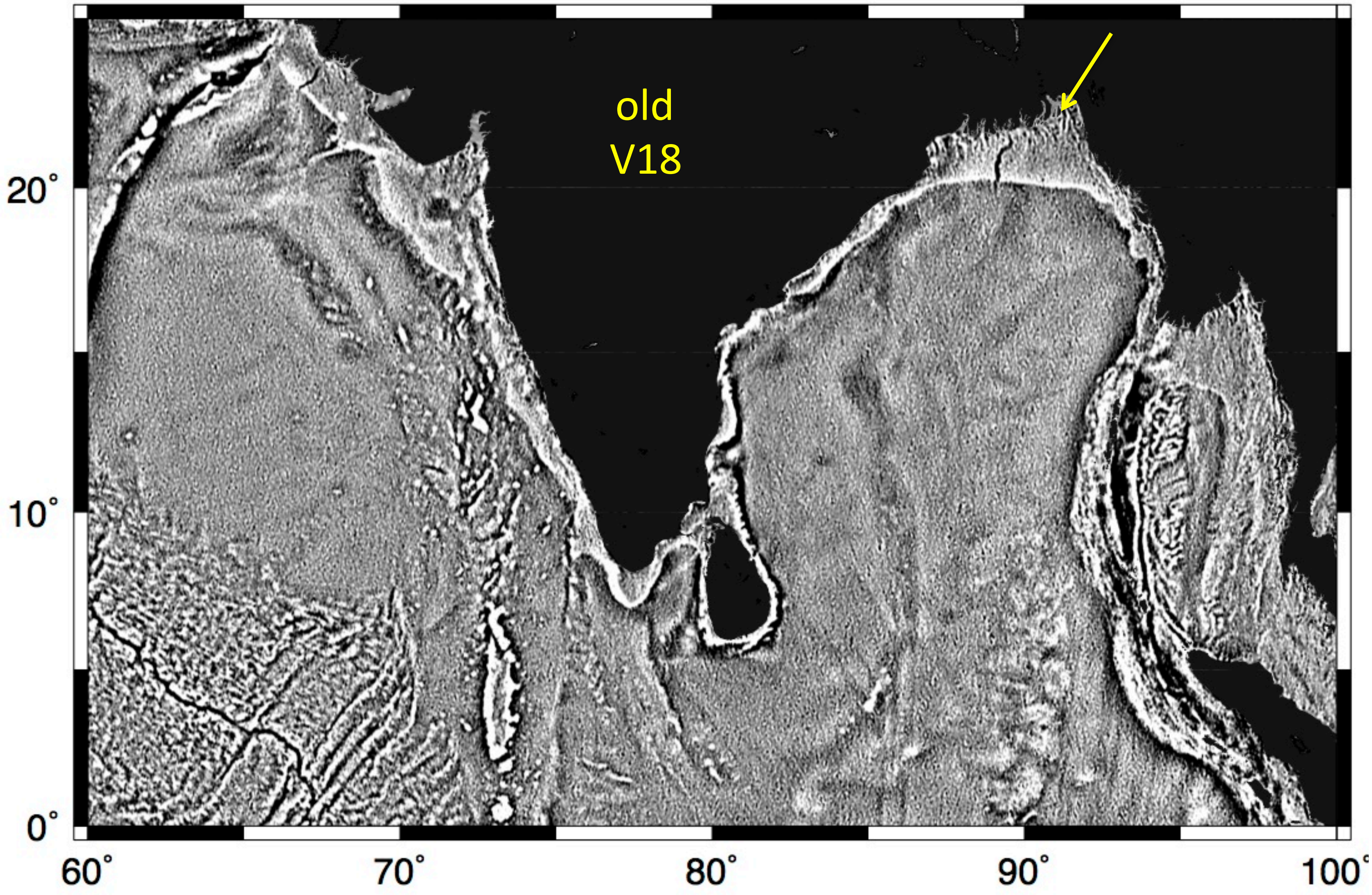
Gravity:

- plate tectonics
- planning ship surveys
- inertial guidance (mostly military)
- petroleum exploration

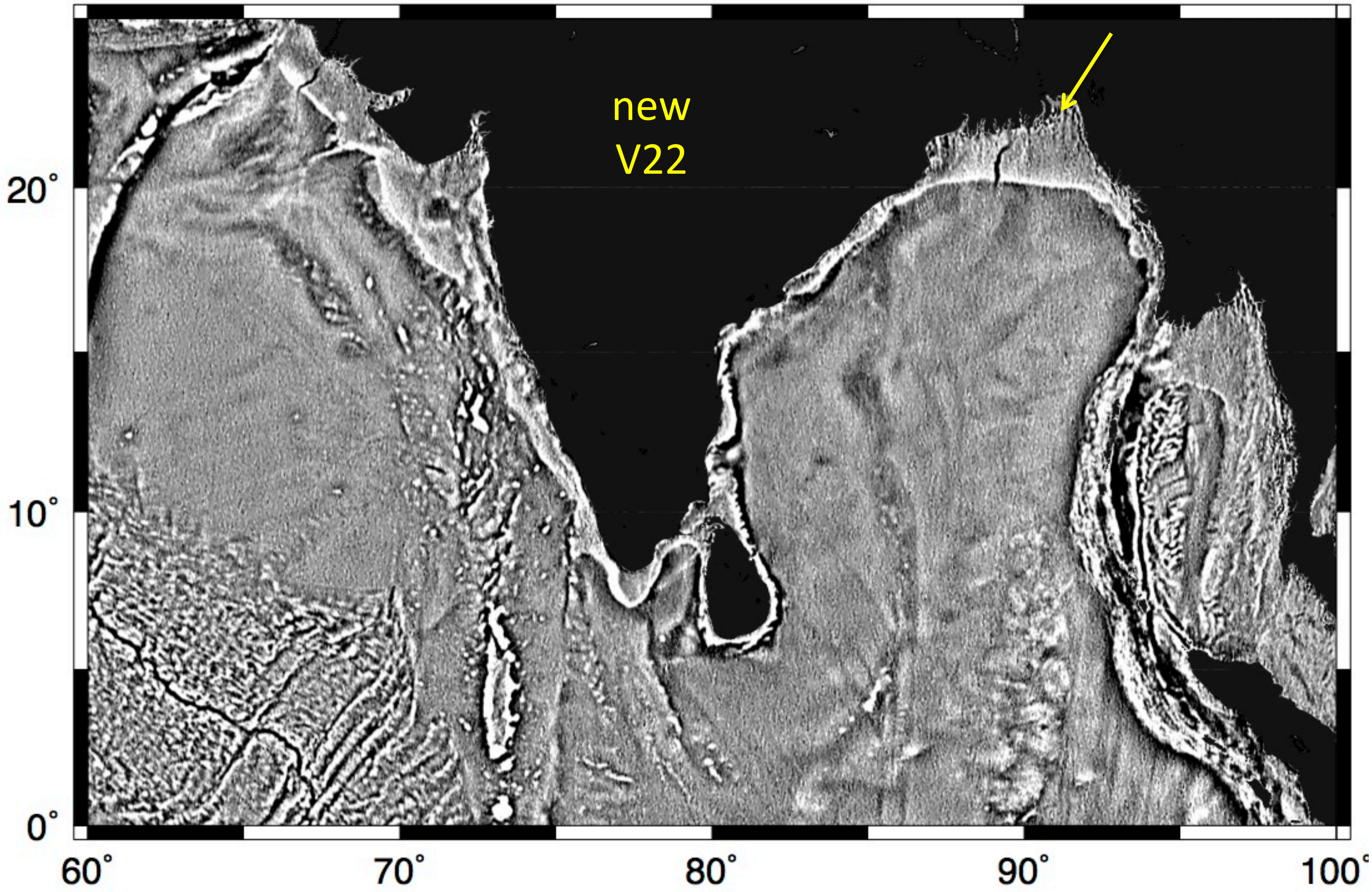
Topography:

- seafloor roughness
- linear volcanic chains
- tsunami models
- tide models, tidal friction, thermohaline circulation
- planning undersea cables
- law of the sea
- education and outreach

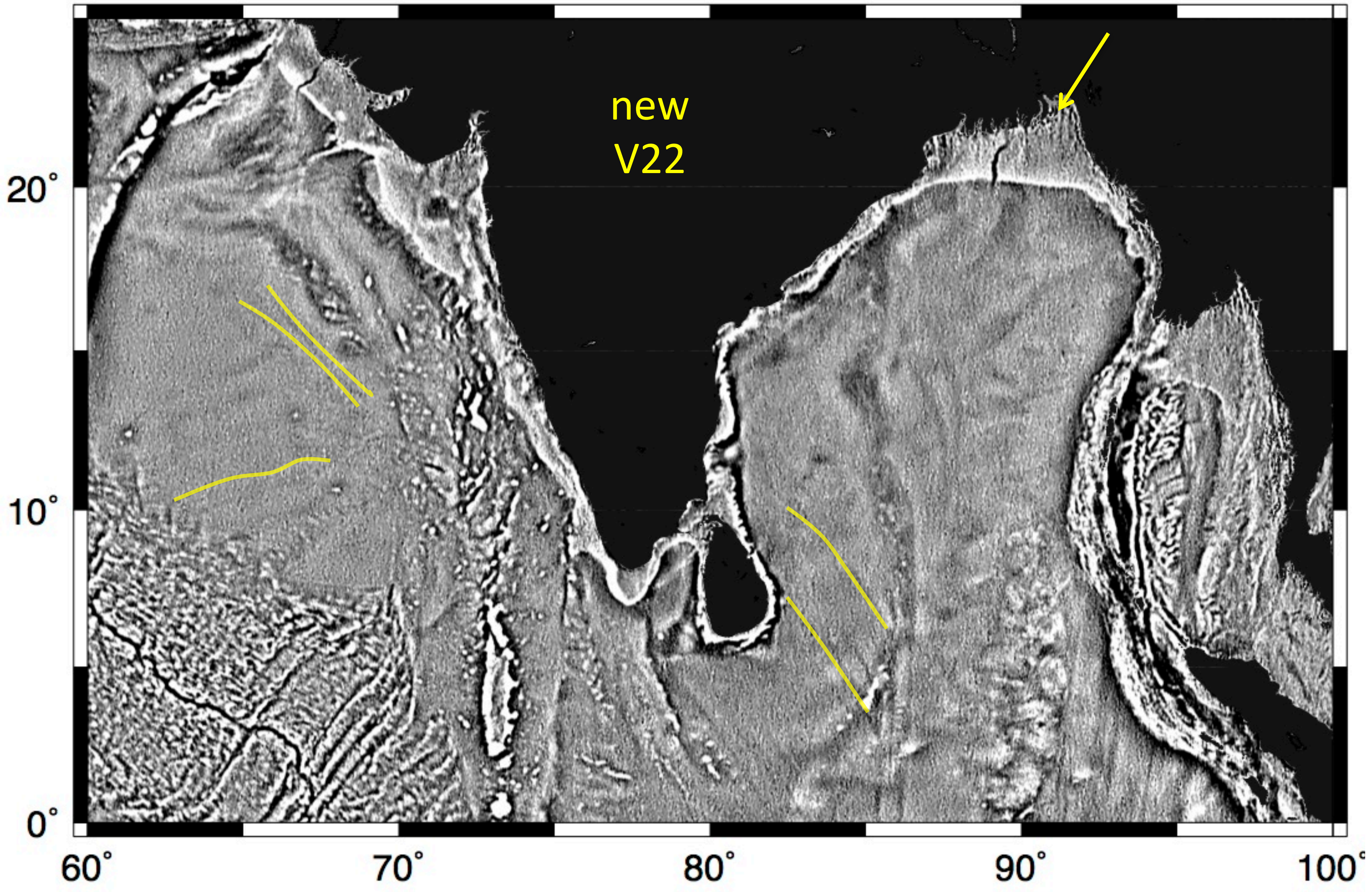
tectonics buried by sediments

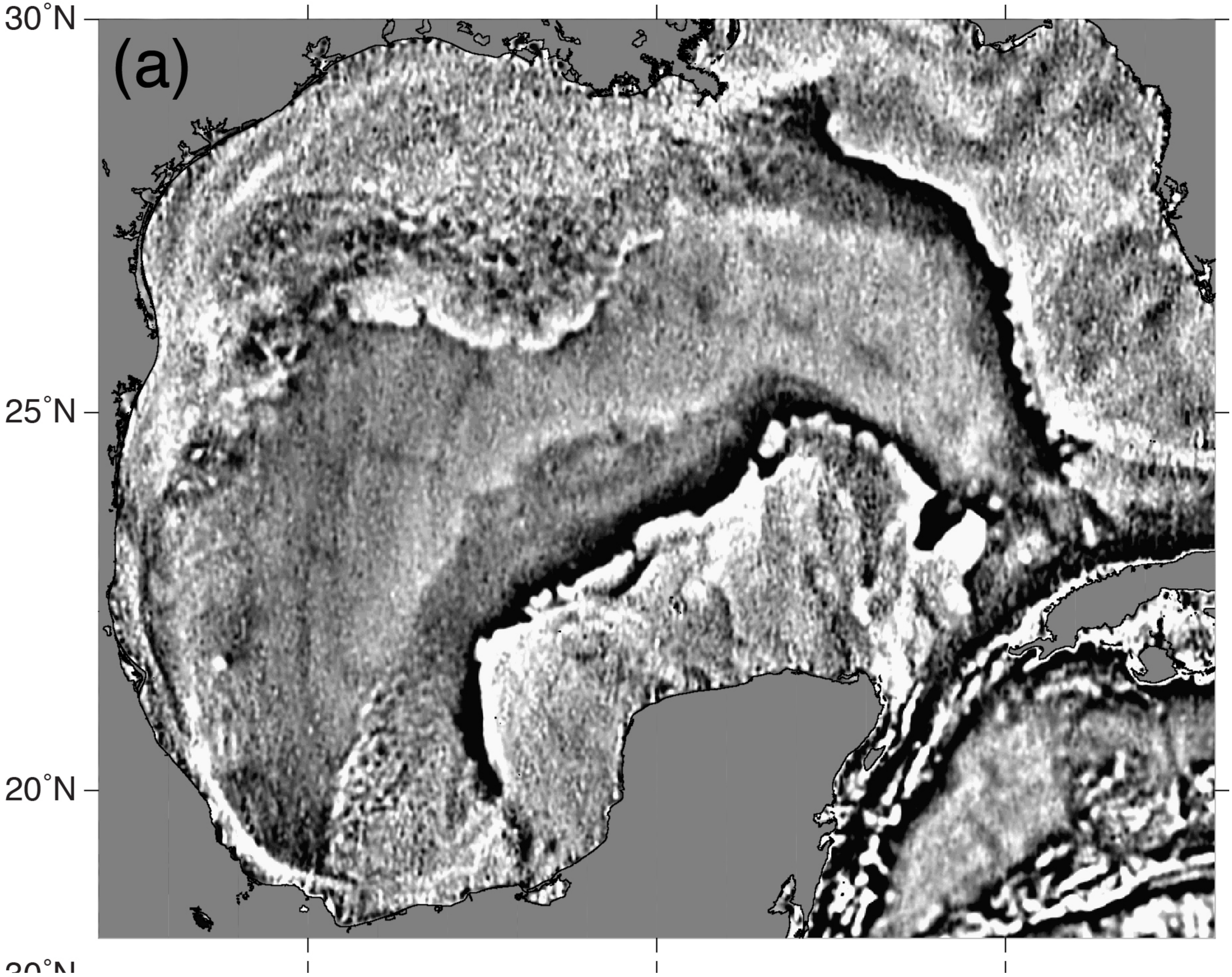


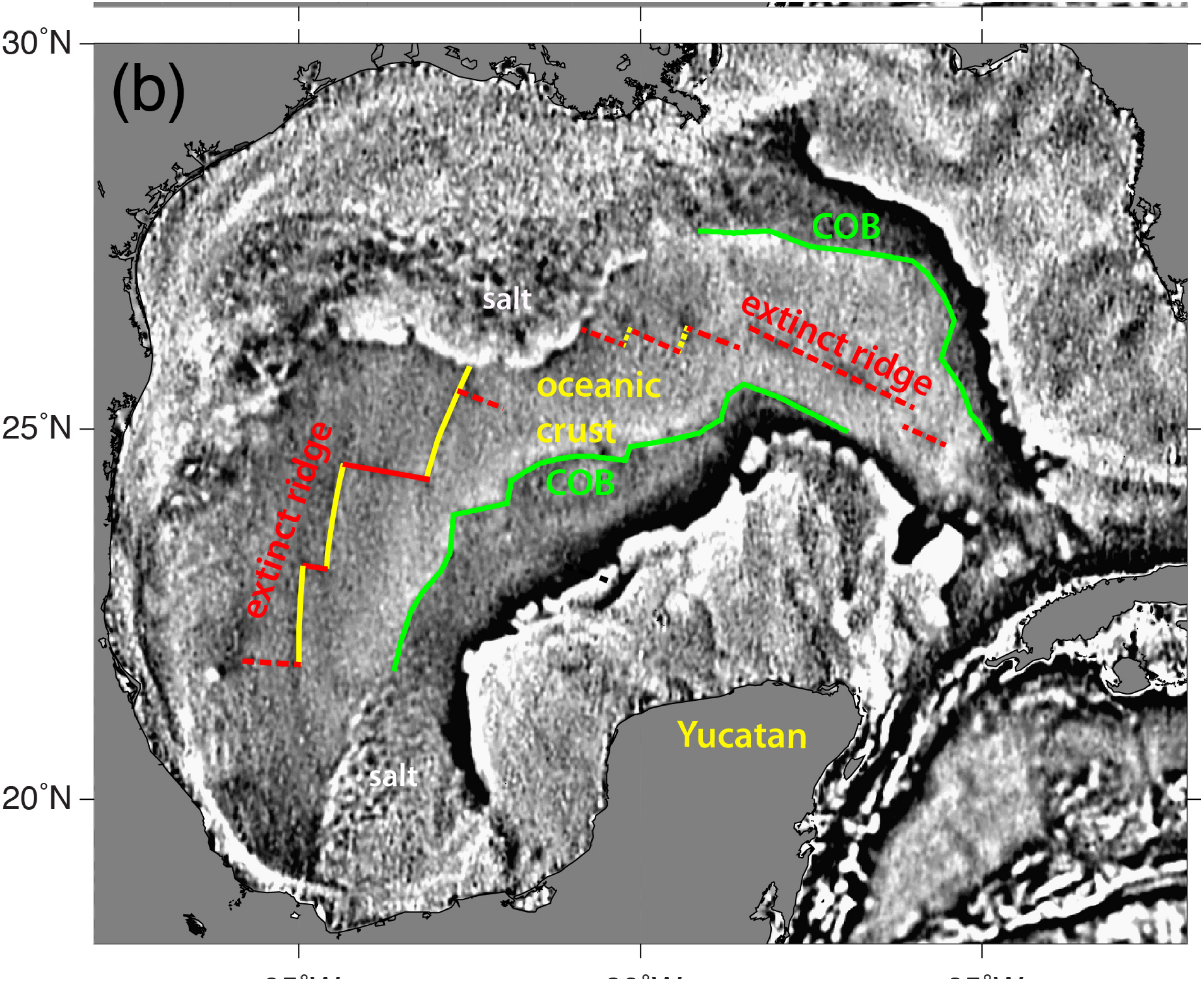
tectonics buried by sediments

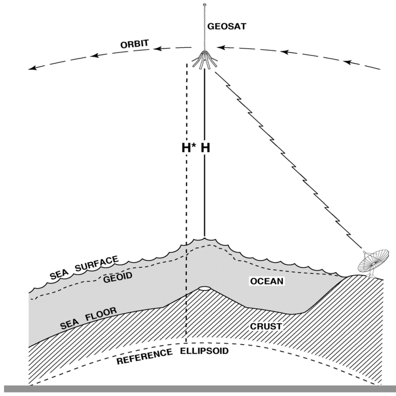


tectonics buried by sediments



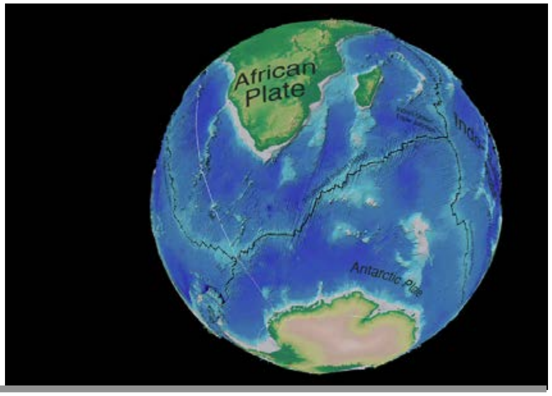

(a)
30°N
25°N
20°N
30°N


(b)
30°N
25°N
20°N
COB
salt
extinct ridge
oceanic
crust
extinct ridge
COB
Yucatan
salt



Marine Gravity from Satellite Altimetry

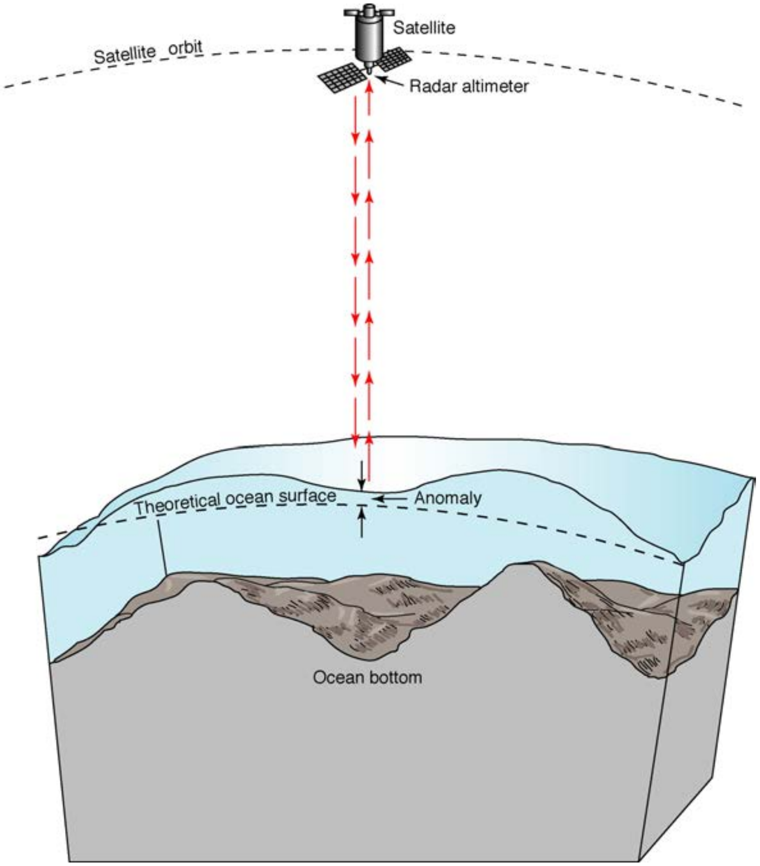
Geodynamics, November, 2014



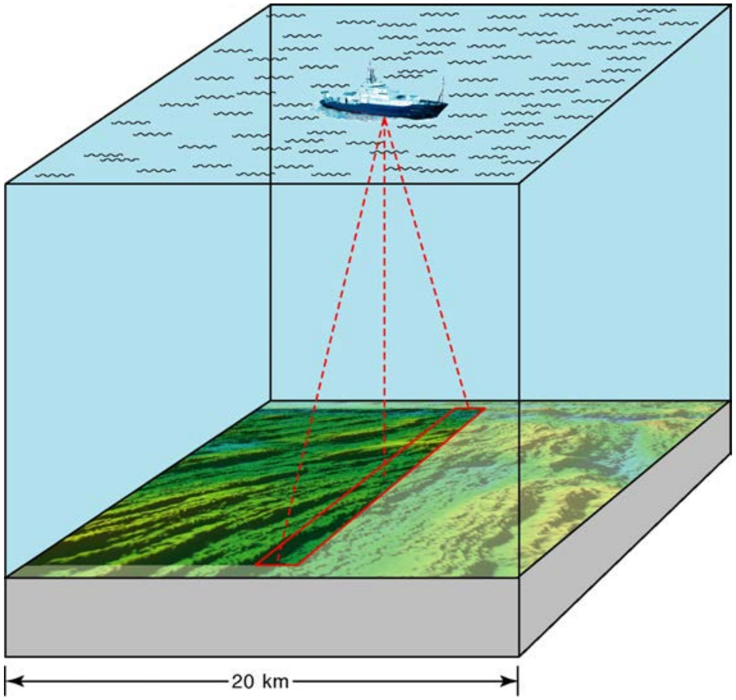
- basic theory
- retracking altimeter waveforms and CryoSat
- gravity from satellite altimetry
- **predicting bathymetry from gravity**

modern mapping tools

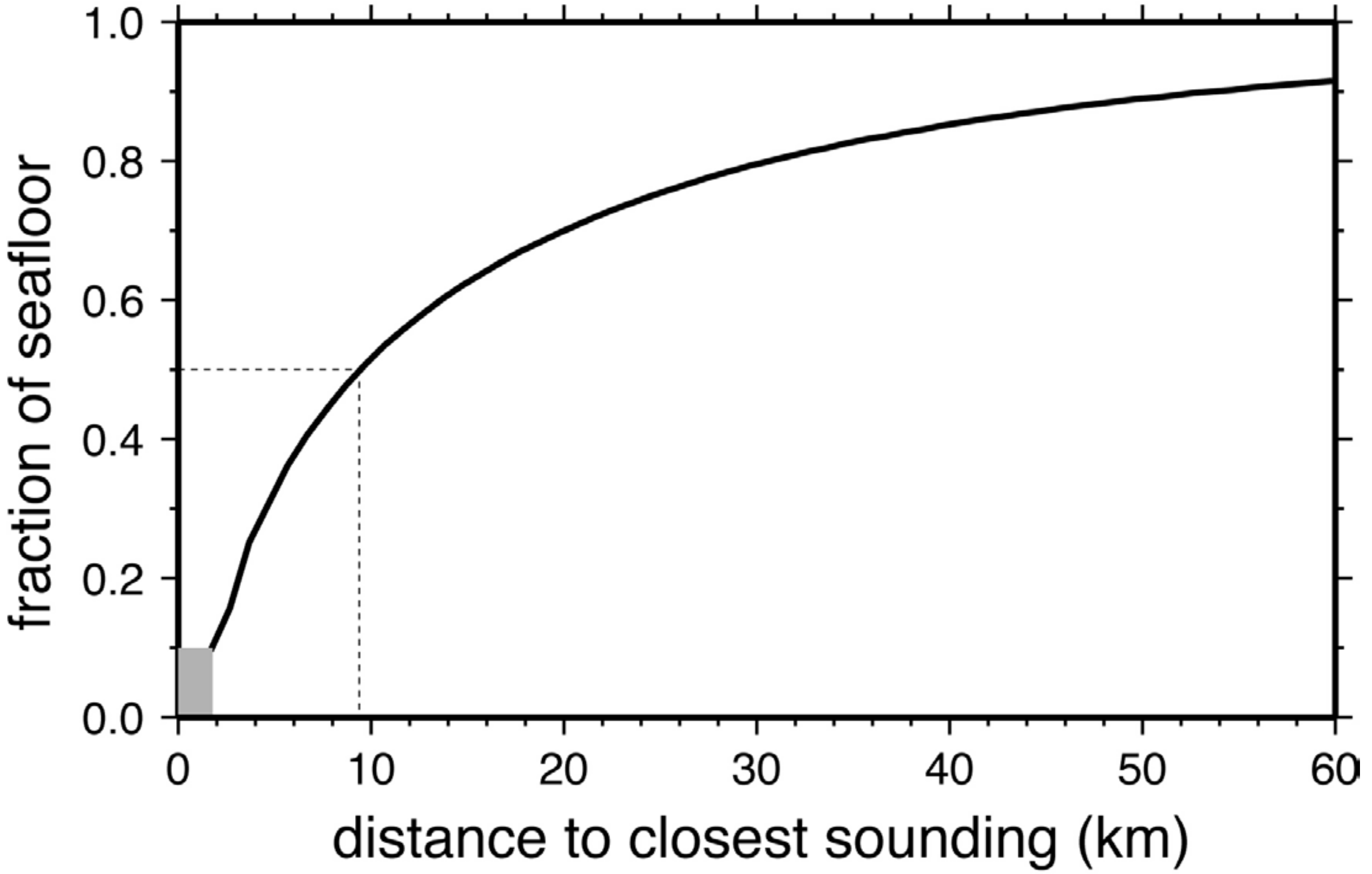
satellite altimeter



multibeam
echo sounder

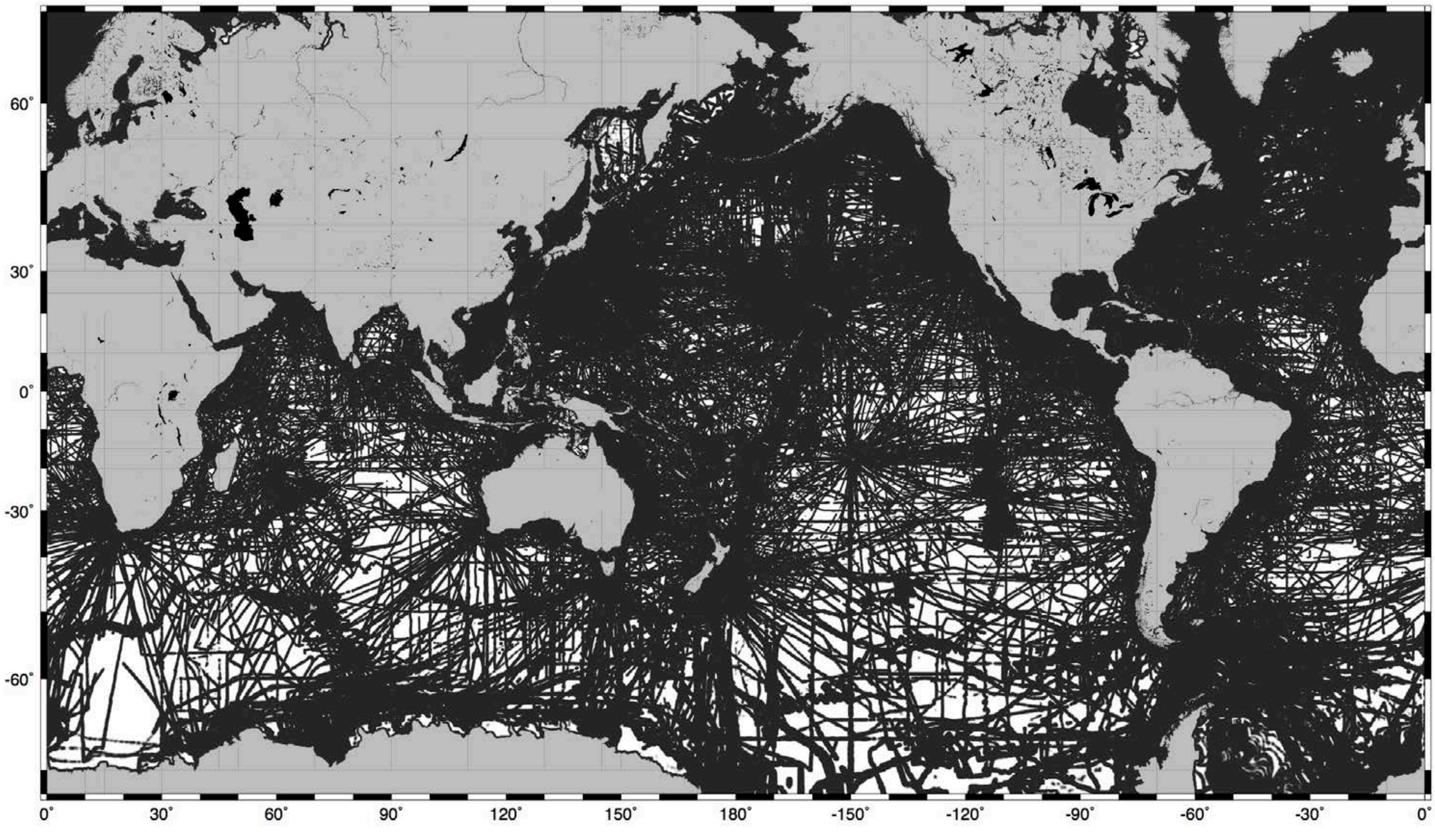


1/2 of global seafloor is more than 10 km from a depth sounding

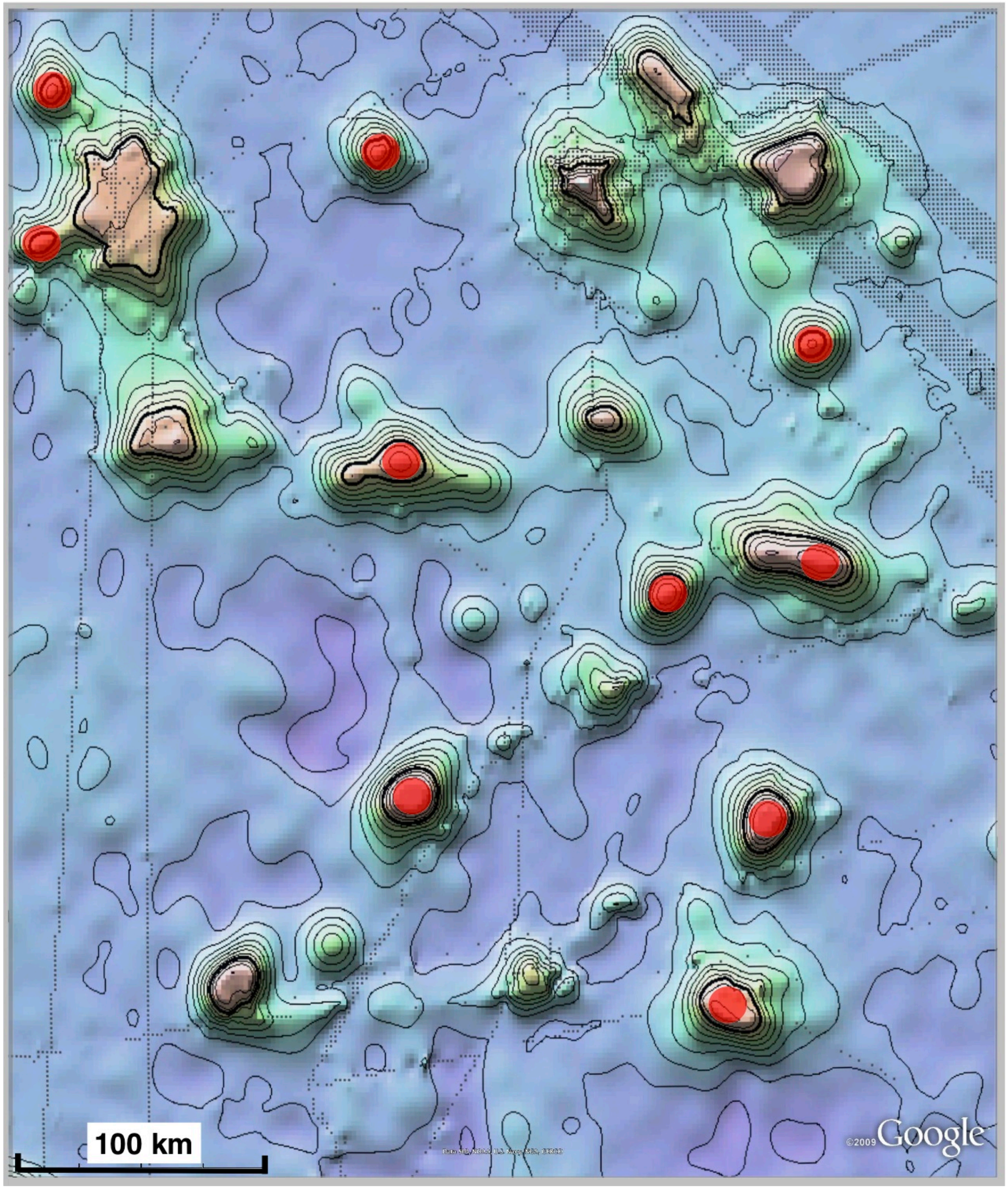


[Smith and Marks, 2009]

areas of seafloor more than 10 km from a sounding



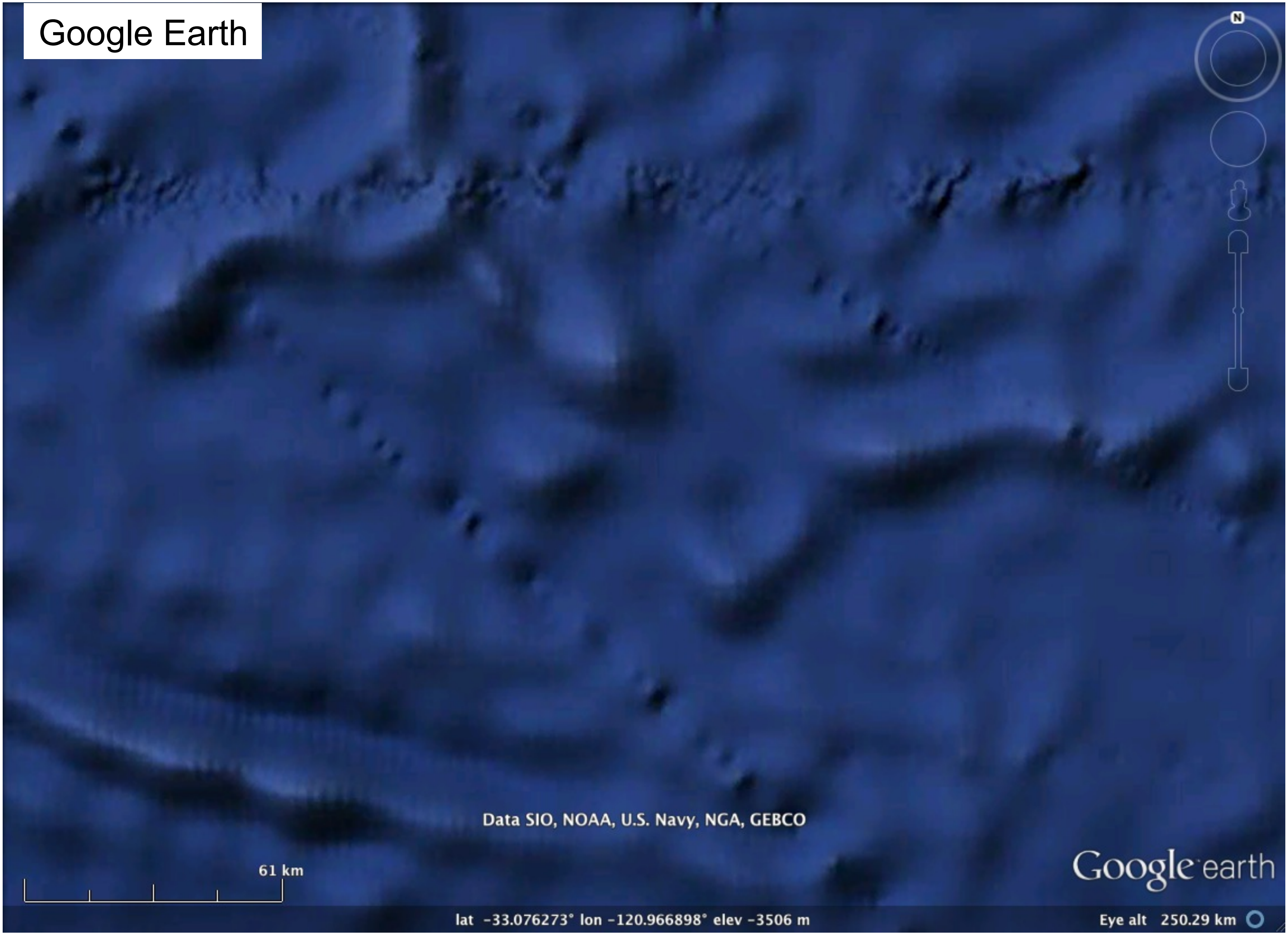
uncharted seamounts > 3 km tall



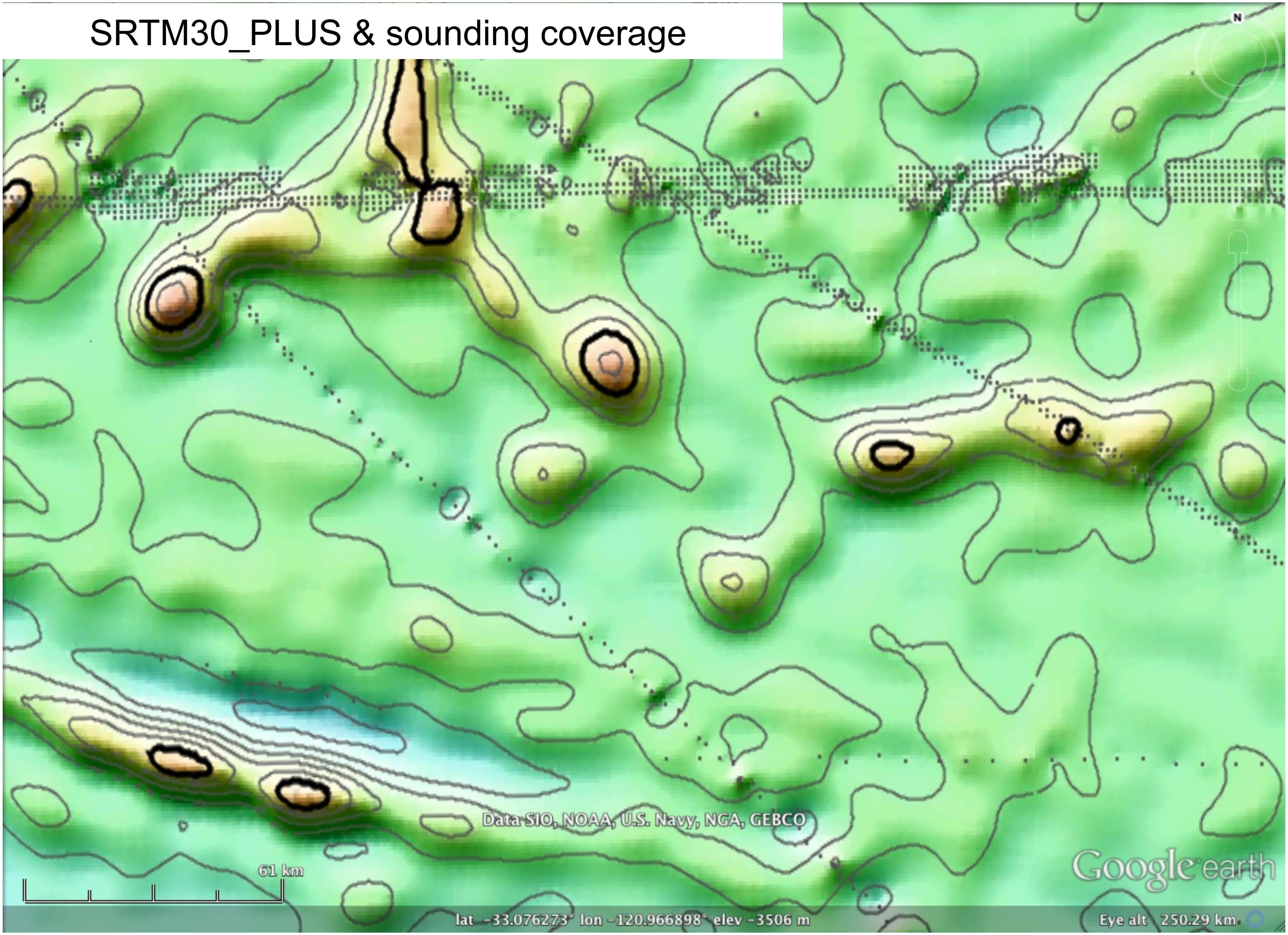
Grounding of USS San Francisco on Uncharted Guyot

- Los Angeles class Submarine ran aground in route from Guam to Brisbane, Australia - 8 January, 2005
- One sailor killed, 120 injured
- Crash depth ~160 m, speed 33 kn, Sonar measured a depth of 2000 m, 4 minutes before crash
- 30-hour trip back to Guam, crew managed to keep the sub from sinking
- Area of discolored water noted on navigational chart 4.8 km, south of crash site
- Navy began basing attack submarines in Guam in 2002

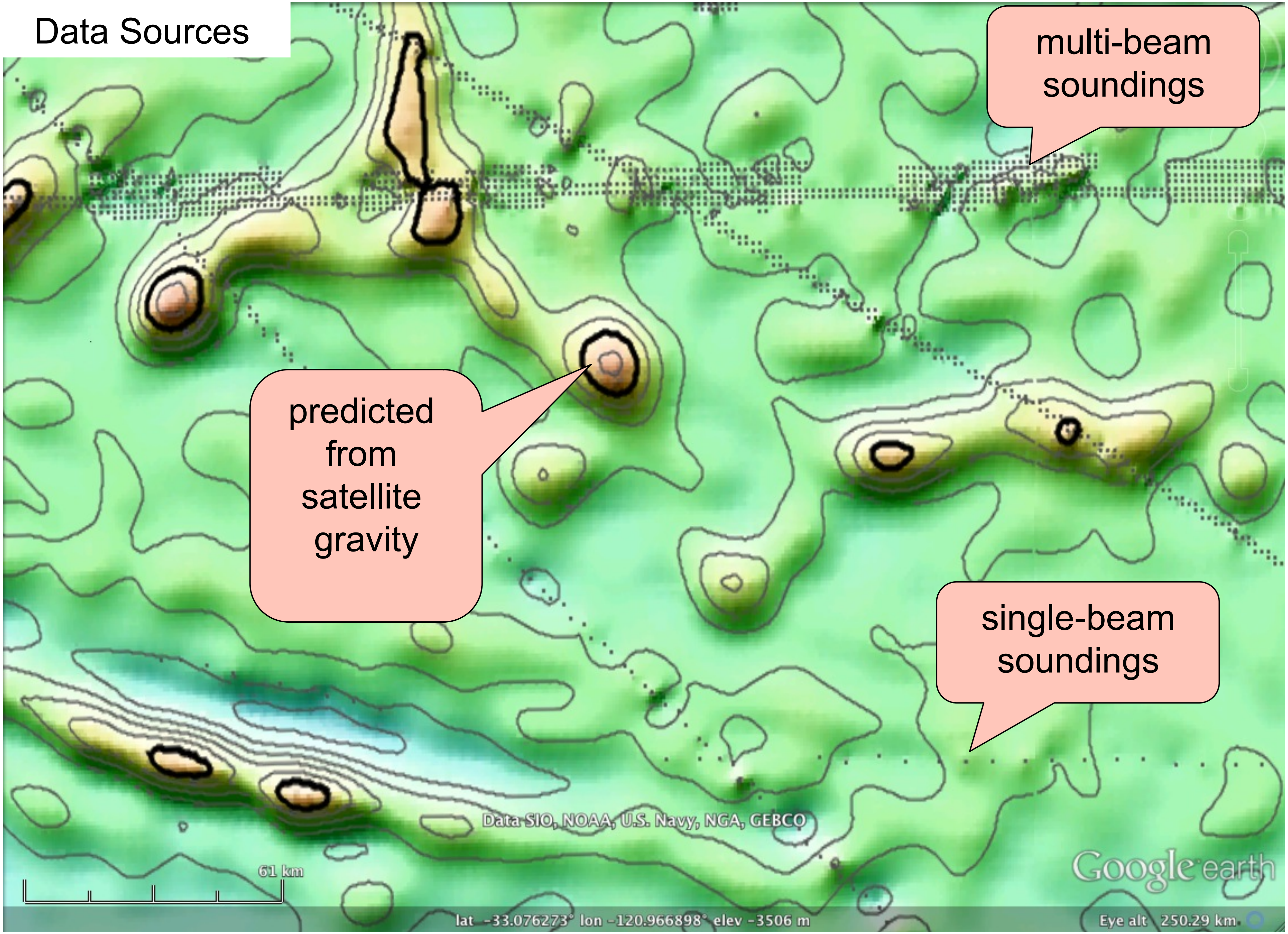
Google Earth



SRTM30_PLUS & sounding coverage



Data Sources



multi-beam soundings

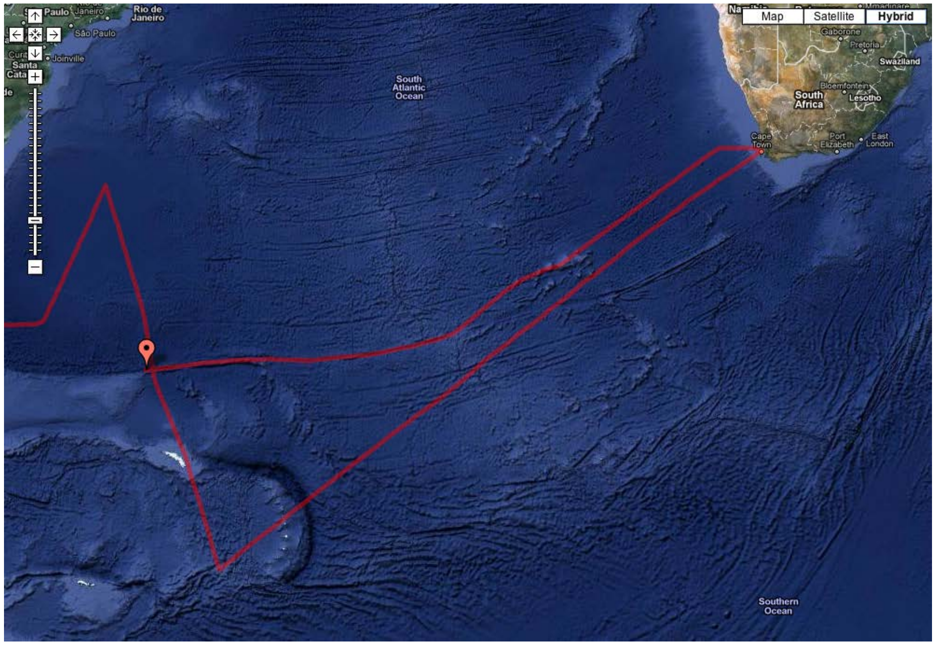
predicted from satellite gravity

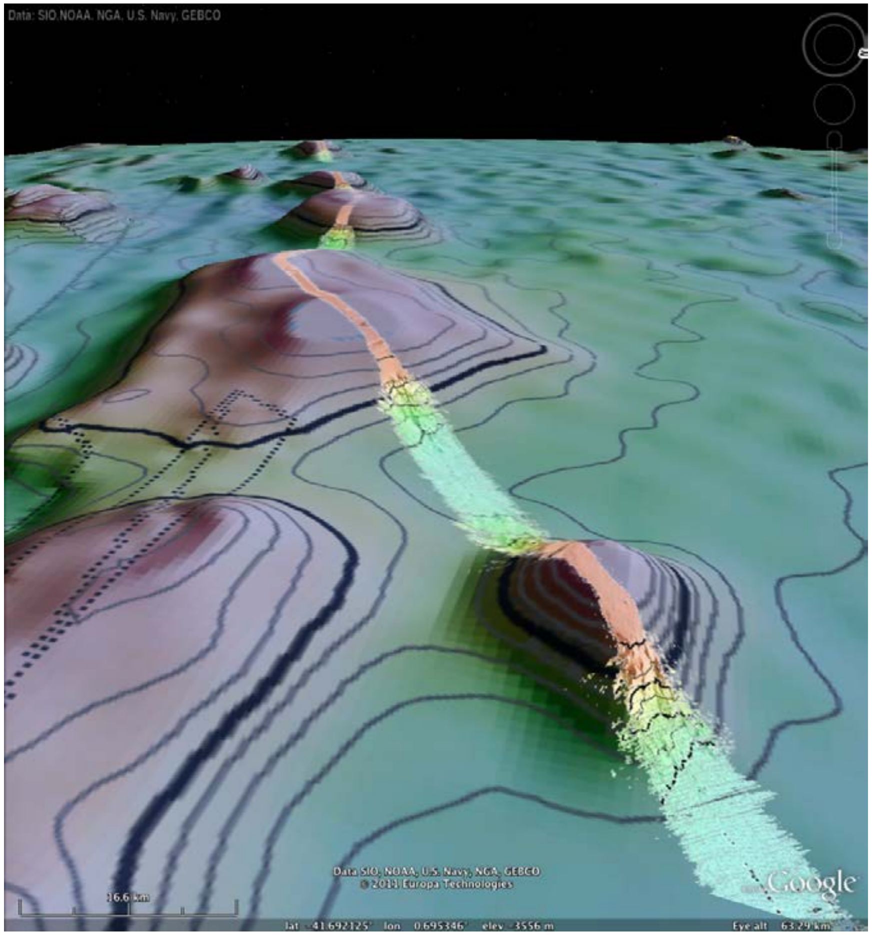
single-beam soundings

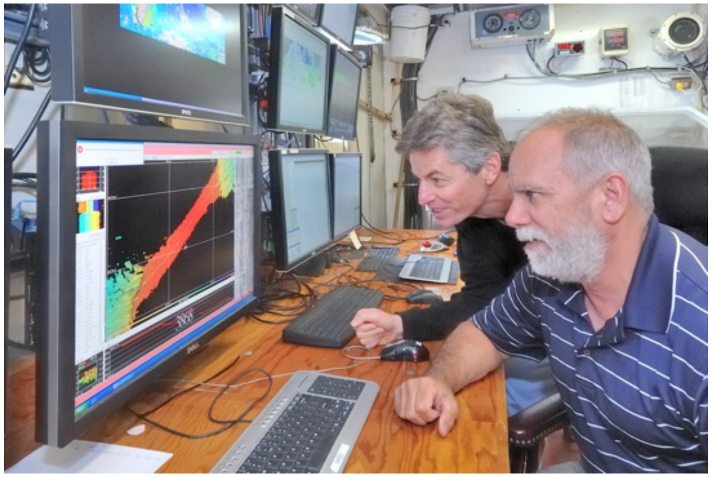
How can we do better?

- Improve public archives of bathymetry.
- Map the oceans with multibeam echosounders
  - ships of opportunity.
- Acquire new satellite altimeter data.
- Declassify US Navy bathymetry data.

Capetown to Punta Arenas - Melville – March, 2011

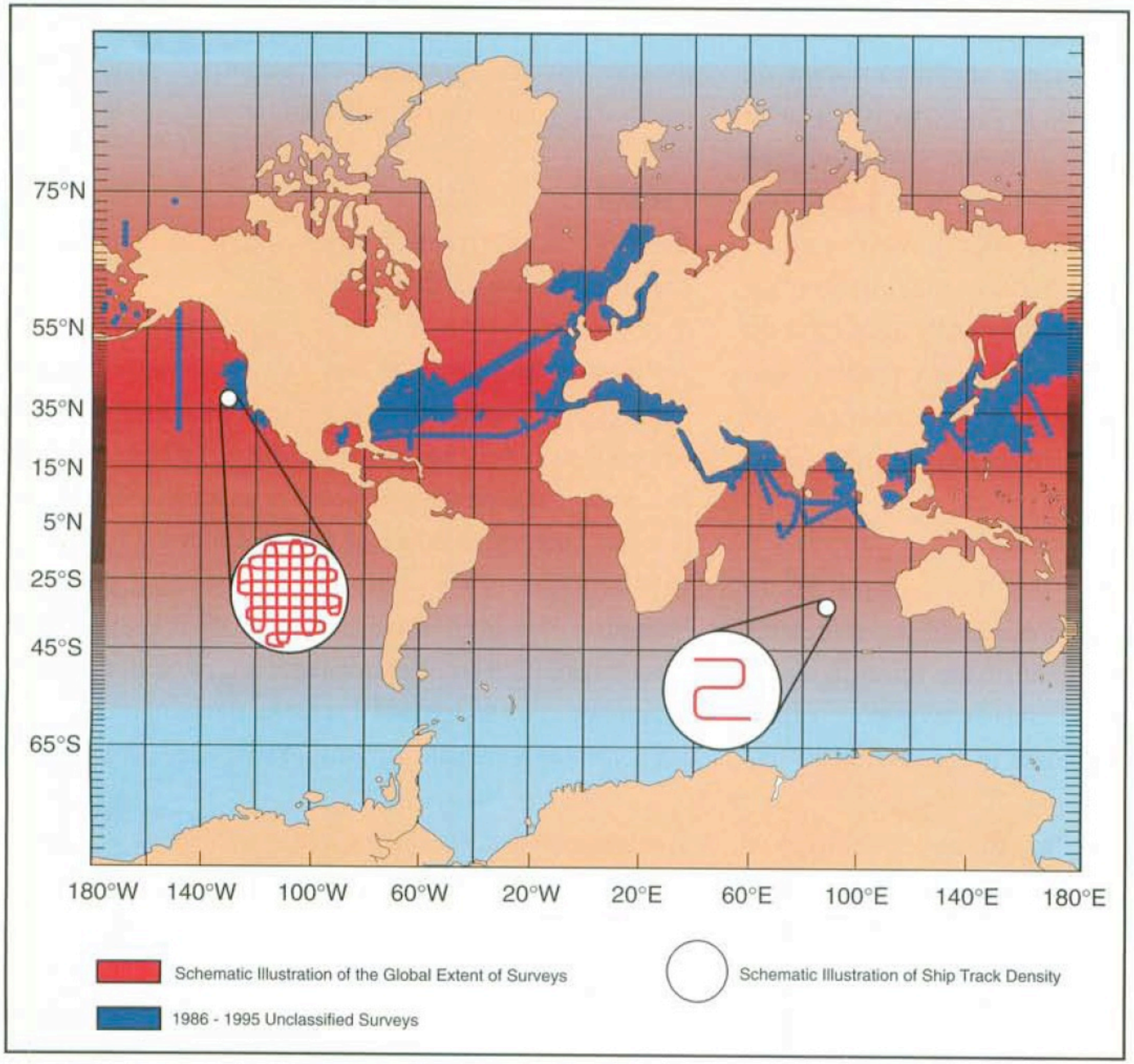






MEDEA: Scientific Utility of Naval Environmental Data,
(Mitre, Co., June, 1995)

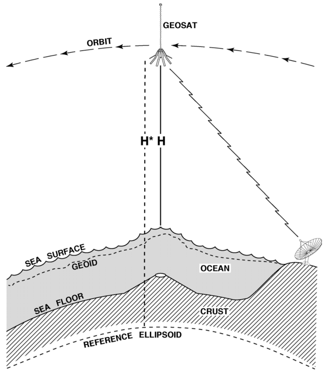
FIGURE 2. WORLDWIDE SURVEY OPERATIONS



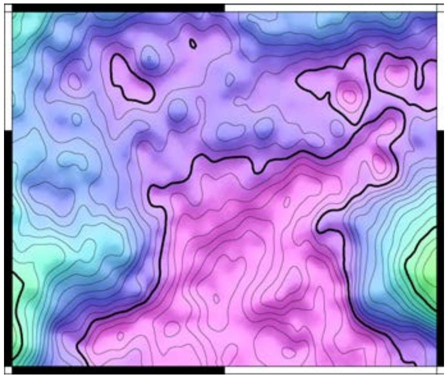
The red shaded area illustrates the global scope of the Navy's oceanographic survey and measurement program. As the tapered shading suggests, there has been a greater concentration of resources in the Northern Hemisphere than the Southern Hemisphere.

Also shown schematically are inset illustrations of how different the densities of ship tracks might be in different ocean areas. Tracks of naval oceanographic surveys (blue) from just the unclassified cruises covering the years from 1986 to 1995 show the worldwide nature of the sources of data.

Examination of all survey ship tracks from the entire cold war history of naval survey operations would show both a global breadth of coverage and a scientifically well founded spatial sampling of ocean processes.



Conclusions



- “Newer” altimeters have 1.4 times better range precision and 2 times better coverage.
- 2-pass waveform retracking provides 1.5 times range precision for all altimeters.
- Marine gravity accuracy is currently 1.6 – 3.5 mGal with most improvement in the 13 – 40 km wavelength band.
- Gravity accuracy is now limited by sea ice, ocean variability, and coastal tides.
- 16% of seafloor has been mapped by ships.
- 50% of the seafloor lies more than 10 km from a depth sounding.