

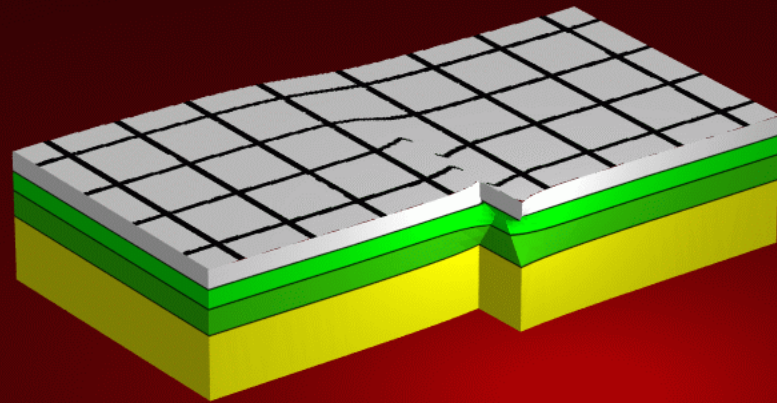
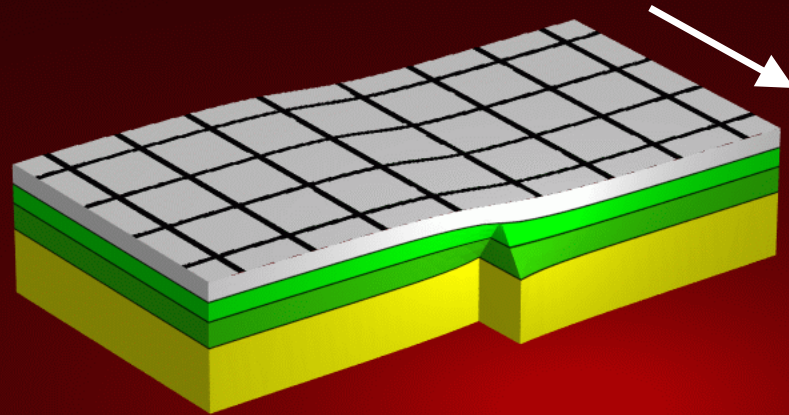
Earthquake Cycle

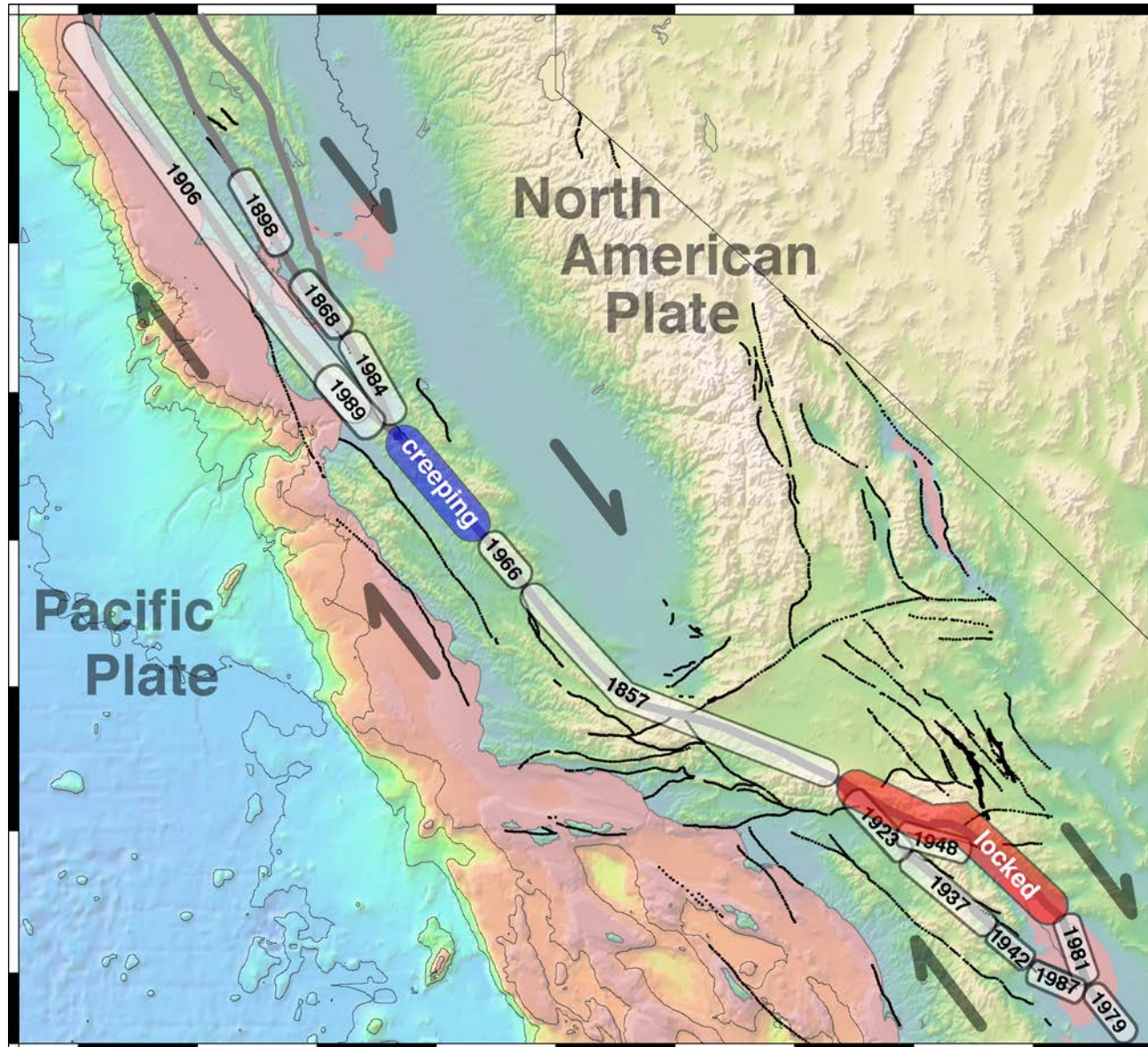
Stress today =

**Interseismic
Accumulation**

+

**Earthquake stress
release**

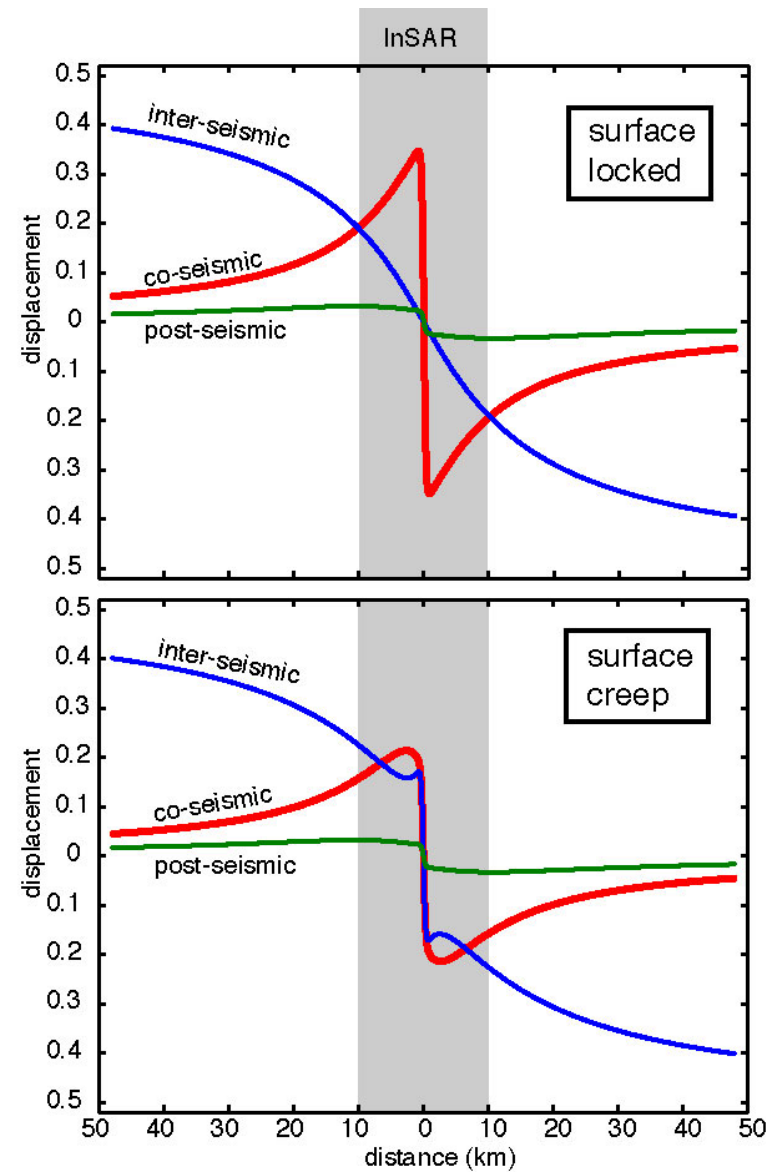
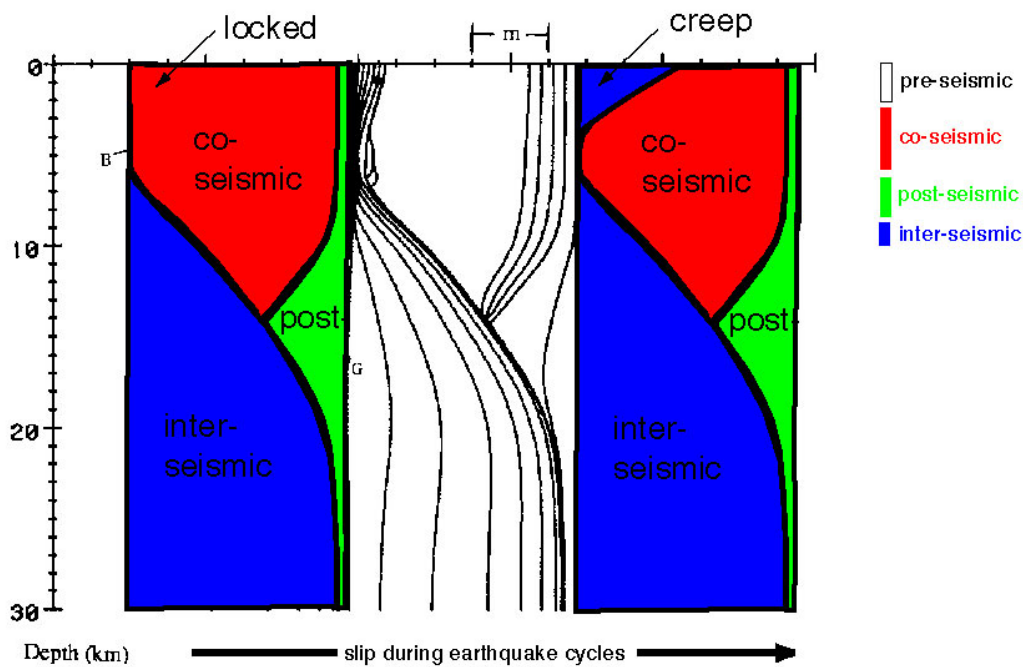




Earthquake Cycle

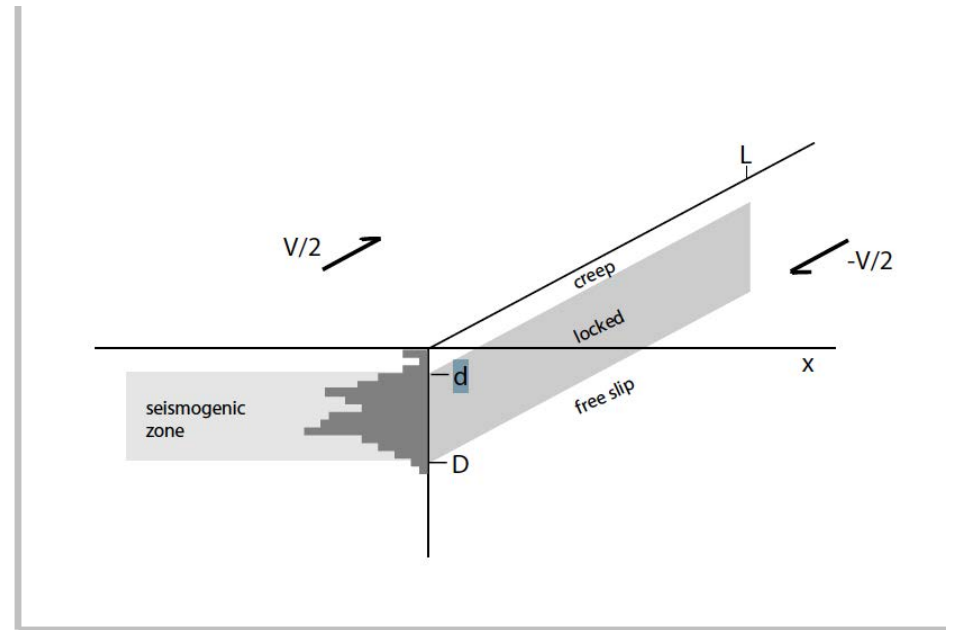
(modified from Tse and Rice, JGR, 1986)

100 to 10,000 yr



interseismic model

velocity $v(x) = \frac{V}{\pi} \tan^{-1} \frac{x}{D}$



strain rate $\dot{\epsilon}(x) = \frac{V}{\pi D} \frac{1}{1 + \left(\frac{x}{D}\right)^2} = \frac{\text{velocity}}{\text{depth}}$

moment rate $\frac{\dot{M}}{L} = \mu V D = \text{velocity} \times \text{depth}$

see notes on
Elastic solutions for strike-slip faulting

Case 3. – The third case considered also has shallow slip between depth $-d$ and the surface. However, in this case we consider a so-called *crack model* where the slip versus depth function results in zero stress on the fault. This derivation will lead to the crack solution given in equation 8-110 of Turcotte and Schubert [2002]. The Case-2 solution has uniform slip with depth. This leads to a stress singularity at the base of the fault. In contrast, the model in Turcotte and Schubert has a stress-free crack imbedded in a pre-stressed elastic half space. Using the Green's function developed above it can be shown that the two solutions are in fundamental agreement. The only difference is related to the slip versus depth function.

From the dislocation theory developed in equation (19), the y -displacement as a function of distance from the fault is given by

$$v(x) = \frac{1}{\pi} \int_{-d}^0 \frac{s(z)x}{x^2 + z^2} dz \quad (26)$$

where z is depth, x is distance from the fault, $s(z)$ is the slip versus depth, and $v(x)$ is the displacement. Now consider the two slip versus depth functions between the surface and $-d$.

$$s_1 = S$$

$$s_2 = S(1 - z^2 / d^2)^{1/2} \quad (27)$$

The integral of the slip function for the crack model f_2 is given by

$$\nu(x) = \frac{S}{\pi} x \int_{-d}^0 \frac{(1 - z^2/d^2)^{1/2}}{x^2 + z^2} dz = \frac{S}{\pi} x \int_0^d \frac{(1 - z^2/d^2)^{1/2}}{x^2 + z^2} dz. \quad (29)$$

Now we let $x' = x/d$ and $z' = z/d$ so the integral becomes

$$\nu(x') = \frac{S}{\pi} x' \int_0^1 \frac{(1 - z'^2)^{1/2}}{x'^2 + z'^2} dz'. \quad (30)$$

This integral can be performed in Matlab using the following code with the symbolic toolbox.

```
%
clear
syms x positive
syms z
arg=sqrt(1-z*z)/(x*x+z*z);
int(arg,z,0,1)
%
% ans=-1/2*pi*(x-(x^2+1)^(1/2))/x
%
```

Note that the integrand contains x'^2 so the results for positive and negative x' are identical. Therefore in the integrated result, the x' should be replaced by $|x'|$. The result is

$$\nu(x') = \frac{S}{\pi} x' \frac{\pi}{2|x'|} \left[(1 + x'^2)^{1/2} - |x'| \right]. \quad (31)$$

Finally substitute for x' and we arrive at

$$\nu(x) = \frac{x}{|x|} \frac{S}{2} \left[\left(1 + \frac{x^2}{d^2} \right)^{1/2} - \frac{|x|}{d} \right]. \quad (32)$$

This matches equation (8-110) given in Turcotte and Schubert.

$$Sd \int_0^1 (1-z^2)^{1/2} dz = Sd\pi / 4 . \quad (33)$$

The following plot compares the two displacement functions when the depth of faulting for the arctangent model is reduced $\pi / 4$ by so the moments are matched; at this scale the plots are nearly identical. This illustrates the fact that measurements of displacement versus distance across a fault are not very sensitive to the shape of the slip versus depth function although they do provide an important constraint on the overall seismic moment. In the next section we highlight this issue that geodetic measurements of surface displacement are relatively insensitive to the shape of the slip versus depth function but provide a good estimate of the overall seismic moment.

