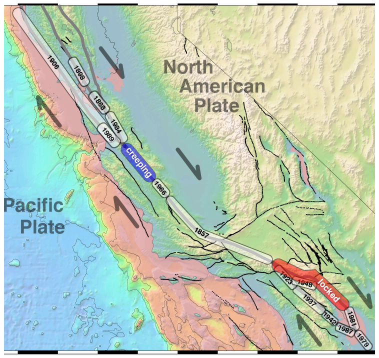
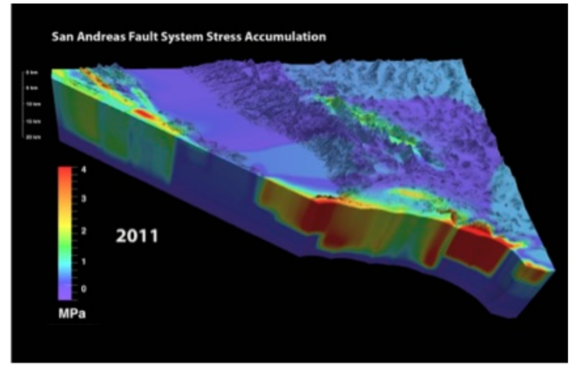


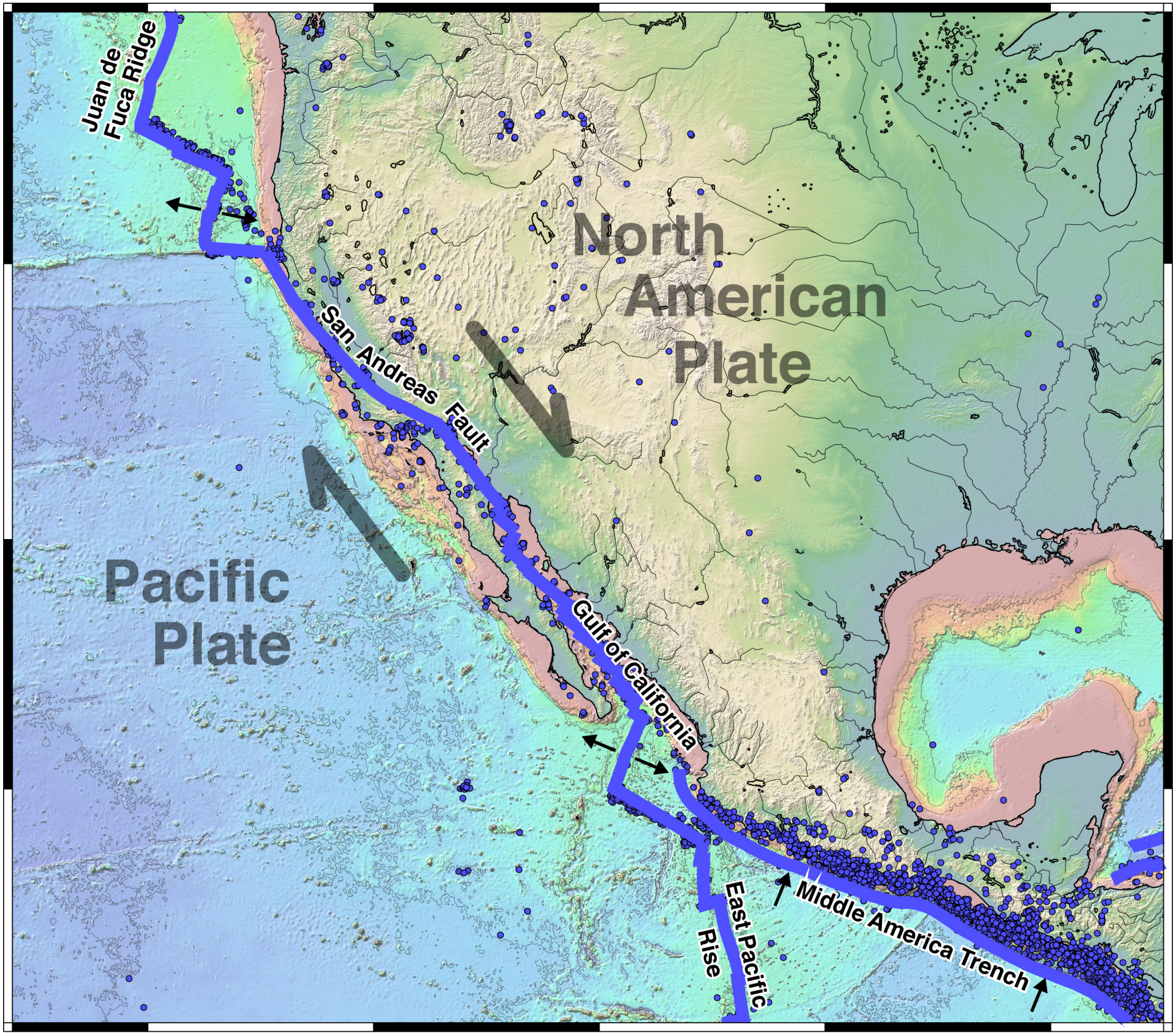
Earthquake Cycle: Heat Flow Paradox 4-D Model

David Sandwell
Bridget Smith-Konter
Xiaopeng Tong



- Review Heat Flow Paradox – Kang, DiPerna
- Semi-analytic 4-D viscoelastic earthquake cycle model developed using computer algebra.

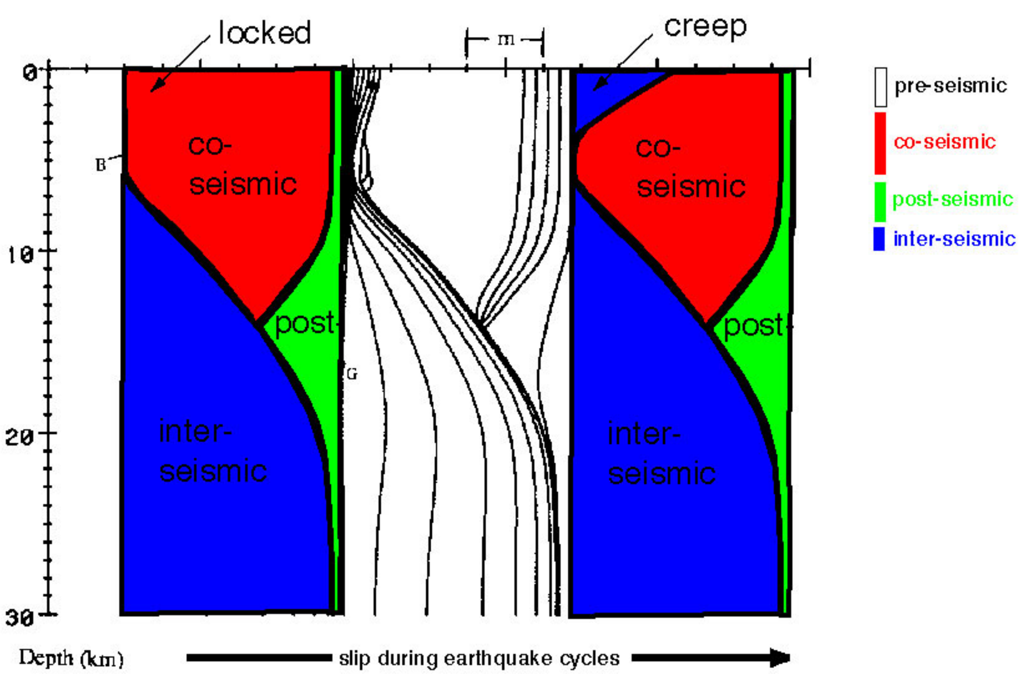
(IUGG June 30, 2015)

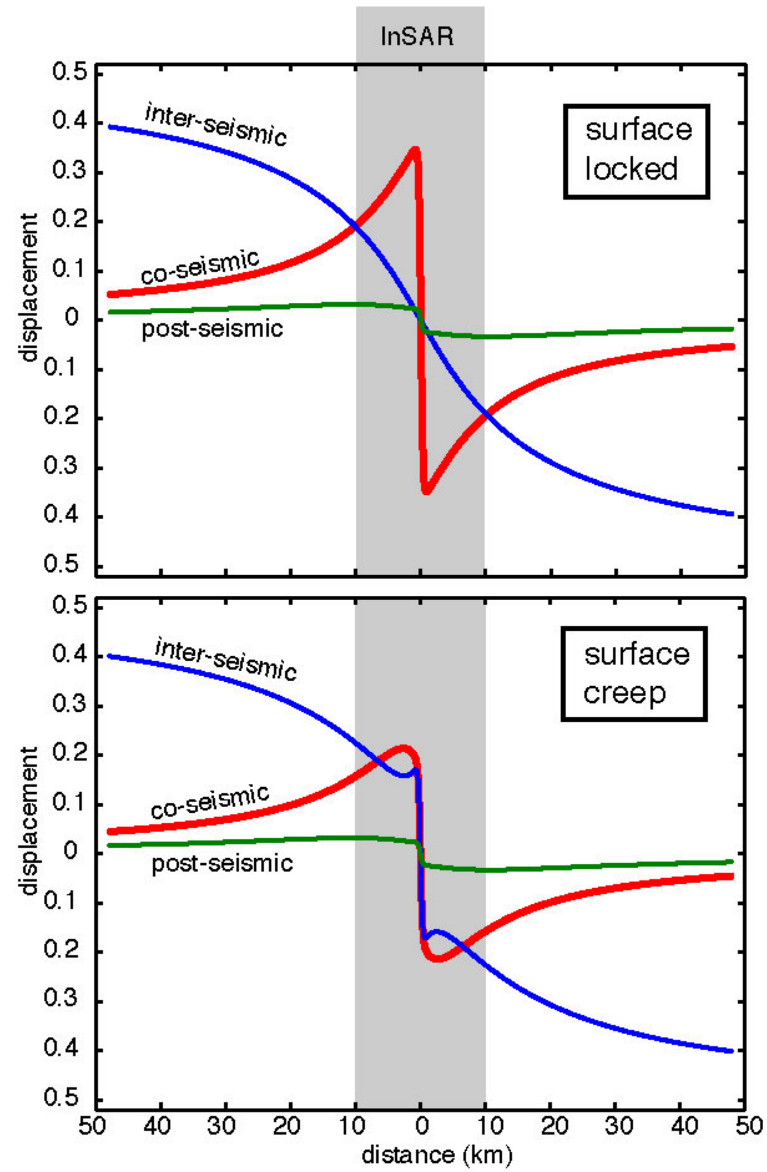


Juan de Fuca Ridge
North American Plate
San Andreas Fault
Pacific Plate
Gulf of California
East Pacific Rise
Middle America Trench

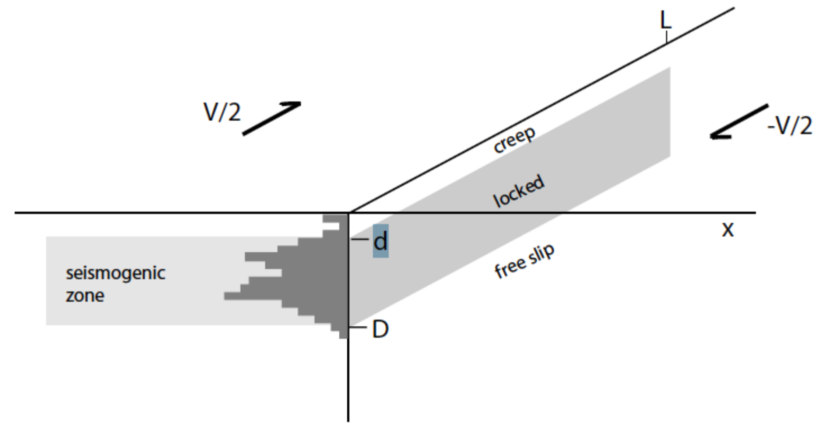
Earthquake Cycle
(modified from Tse and Rice, JGR, 1986)

100 to 10,000 yr





interseismic model



velocity $$v(x) = \frac{V}{\pi} \tan^{-1} \frac{x}{D}$$

strain rate $$\dot{\varepsilon}(x) = \frac{V}{\pi D} \frac{1}{1 + \left(\frac{x}{D} \right)^2}$$ = velocity / depth

moment rate $$\frac{\dot{M}}{L} = \mu V D$$ = velocity X depth

Modeling the Heat Flow Anomaly on the San Andreas Fault

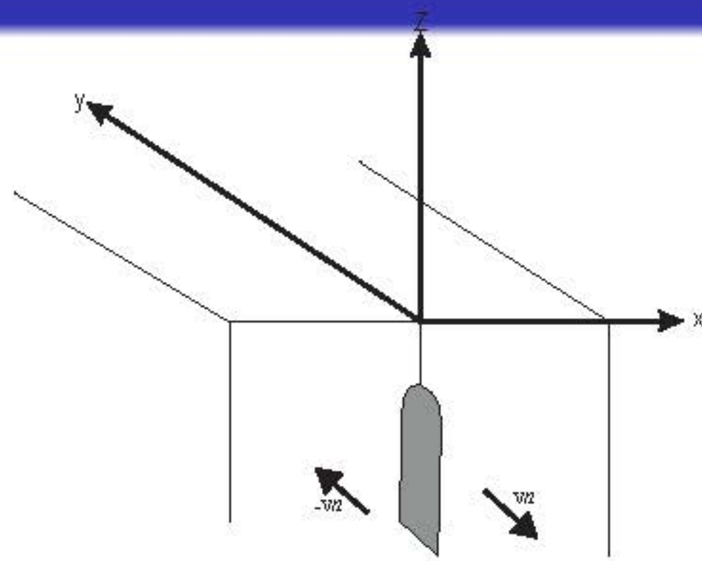
Kang Wang Lauren DiPerna

October 28, 2011

What we expect to find

- We measure temperature because it gives us information about faulting mechanism
- We believe that earthquakes generate heat through friction
- Therefore we should see heat at faults that exceed the Earth's ambient surface flow

Line sources



Assuming that the length of fault is infinity, and that there is no heat conduction along the strike direction. Then the 3-D problem reduces to a 2-D one

$$k(\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2}) + s(x, z) = 0 \quad (z < 0)$$

$$T(x, 0) = 0$$

$$\lim_{|x| \to \infty} T(x, z) = 0$$

$$\lim_{|z| \to \infty} T(x, z) = 0$$

Using Green's function to find the solution for any arbitrary source

Arbitrary source

$$\begin{cases} k\nabla^2 T = -s(M) \\ T|_{\partial V} = \phi(M) \end{cases}$$

Point source

$$\begin{cases} k\nabla^2 G = -\delta(M_0) \\ G|_{\partial V} = 0 \end{cases}$$

$$T(M) = -\int\int_{\partial V} \phi(M_0)\frac{\partial G}{\partial n} dS + \int\int\int_V G \quad s(M_0) dM$$

For the problem here, $T|_{\partial V} = 0$. Also, we have found the Green's function of heat flow at the surface,

$$G_q(x) = -\frac{1}{\pi}\frac{a}{x^2 + a^2}$$

the heat production (*source term*) for a fault with shear stress $\tau(z)$ and relative slip velocity V

$$q_s(z) = \tau(z) V$$

Thus, the surface heat flow generated by a fault extending from d to D is

$$q(x) = -\frac{V}{\pi}\int_d^D \frac{z\tau(z)}{x^2 + z^2} dz$$

Shear stress on the fault

- Assuming that the shear stress of the fault follows Byerlee's law,

$$\tau(z) = \mu(\rho_c - \rho_w)gz$$

and that water percolates to 12 km (depth of seimsmogenic zone), then we can estimate the average shear on the fault

$$\bar{\tau} = \frac{1}{D}\int_0^D \mu(\rho_c - \rho_w)gzdz = \frac{1}{2}\mu(\rho_c - \rho_w)gD \approx 56MPa$$

with coefficient of friction $\mu = 0.6$

- The observed stress drop during an earthquake ranges from 0.1 to 10 MPa with a typical value of 5 MPa, which is about 10 times smaller than the average stress from Byerlee's law.

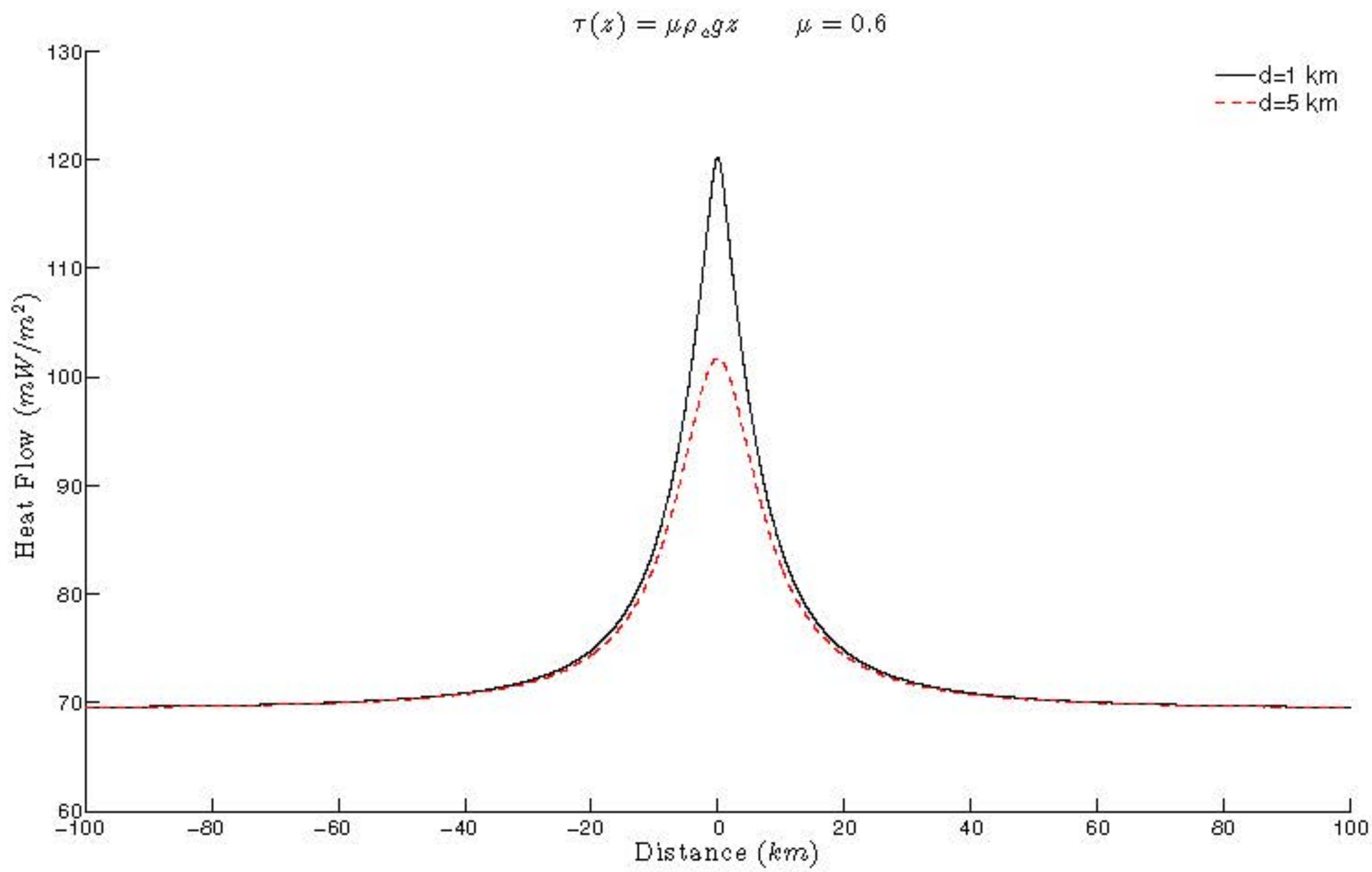
Heat flow base on Byerlee's law

$$q(x) = -\frac{V}{\pi}\int_d^D \frac{z\tau(z)}{x^2+z^2}dz$$

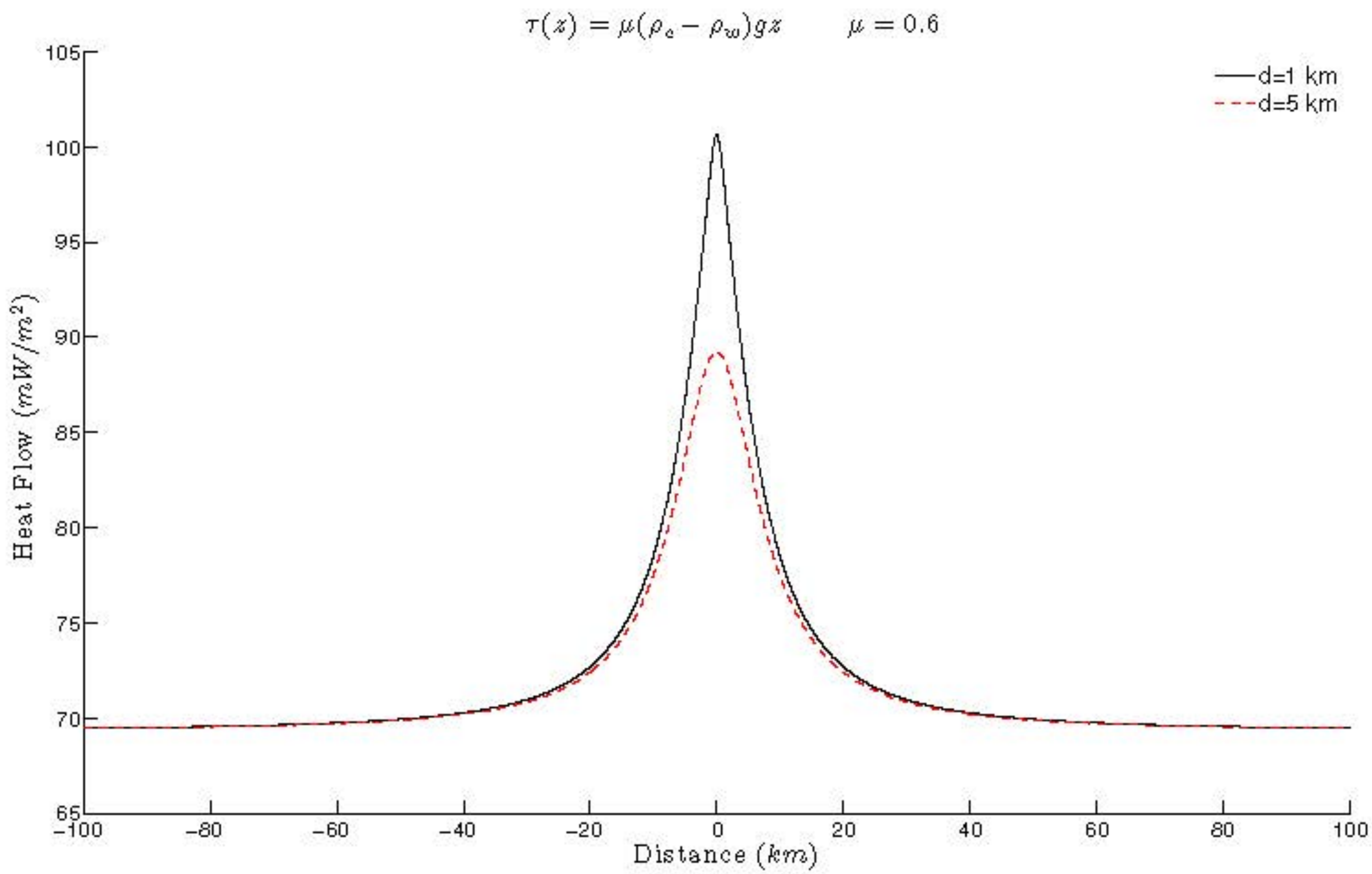
Assuming that the hydrothermal circulation removes the heat generation from surface to some depth d, then the surface heat flow generated by the fault slip is

$$\begin{aligned} q(x) &= -\frac{\mu(\rho_c-\rho_w)gV}{\pi}\int_d^D \frac{z^2}{x^2+z^2}dz \\ &= -\frac{\mu(r_c-r_w)gV}{\pi}[(D-d)+(x\arctan\frac{d}{x}-x\arctan\frac{D}{x})] \end{aligned}$$

Model predications



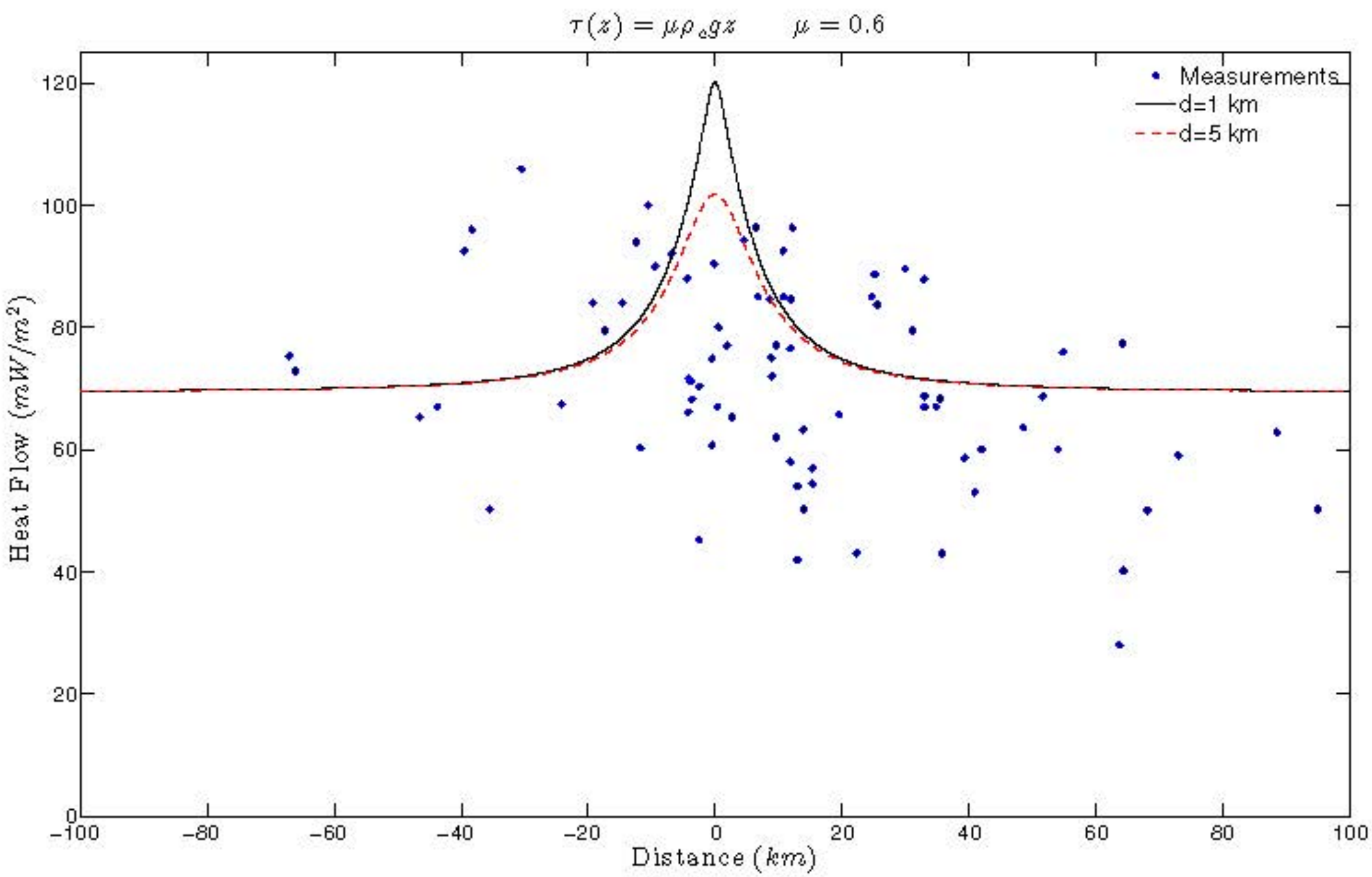
Model predications(cont.)



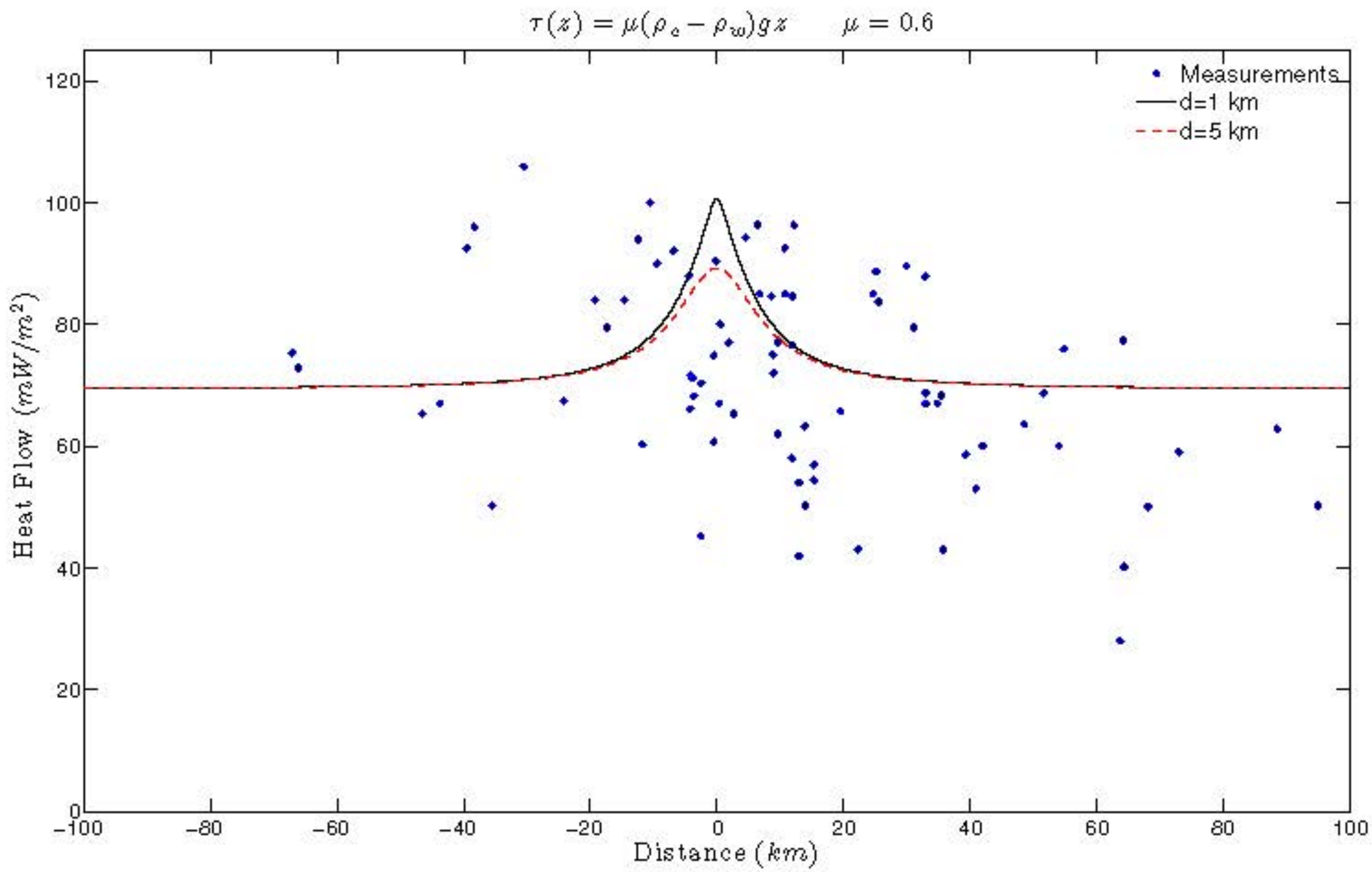
Measurements

- Data is from *Lachenbruch et al, 1980*

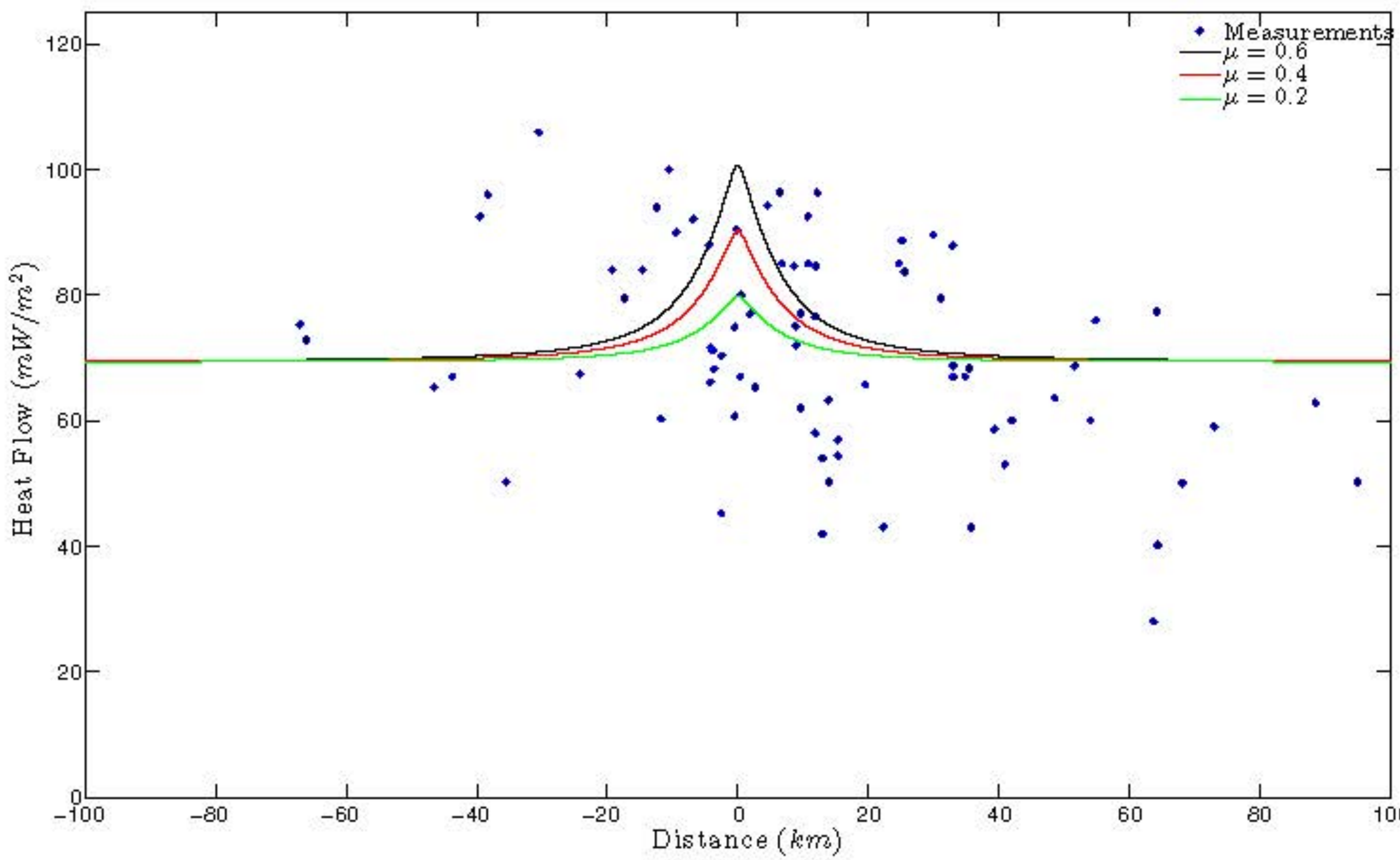
Comparison of model and measurements



Comparison of model and measurements(cont.)



Comparison of model and measurements(cont.)

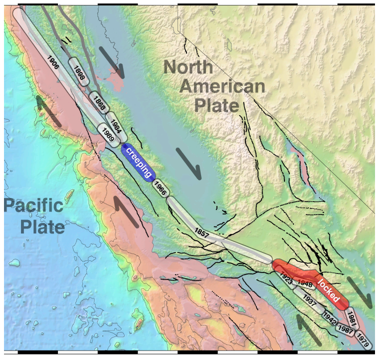


Conclusions

The comparison reveals an inconsistency between the modeled predictions and the measurements.

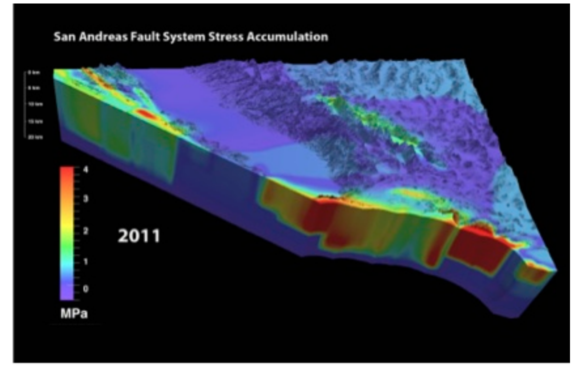
To make a model matching the data, we need

- a lower coefficient of friction; a friction coefficient of 0.6 is too high.
- to consider a hydrological system that can remove heat
- to change the model to include nonlinear terms



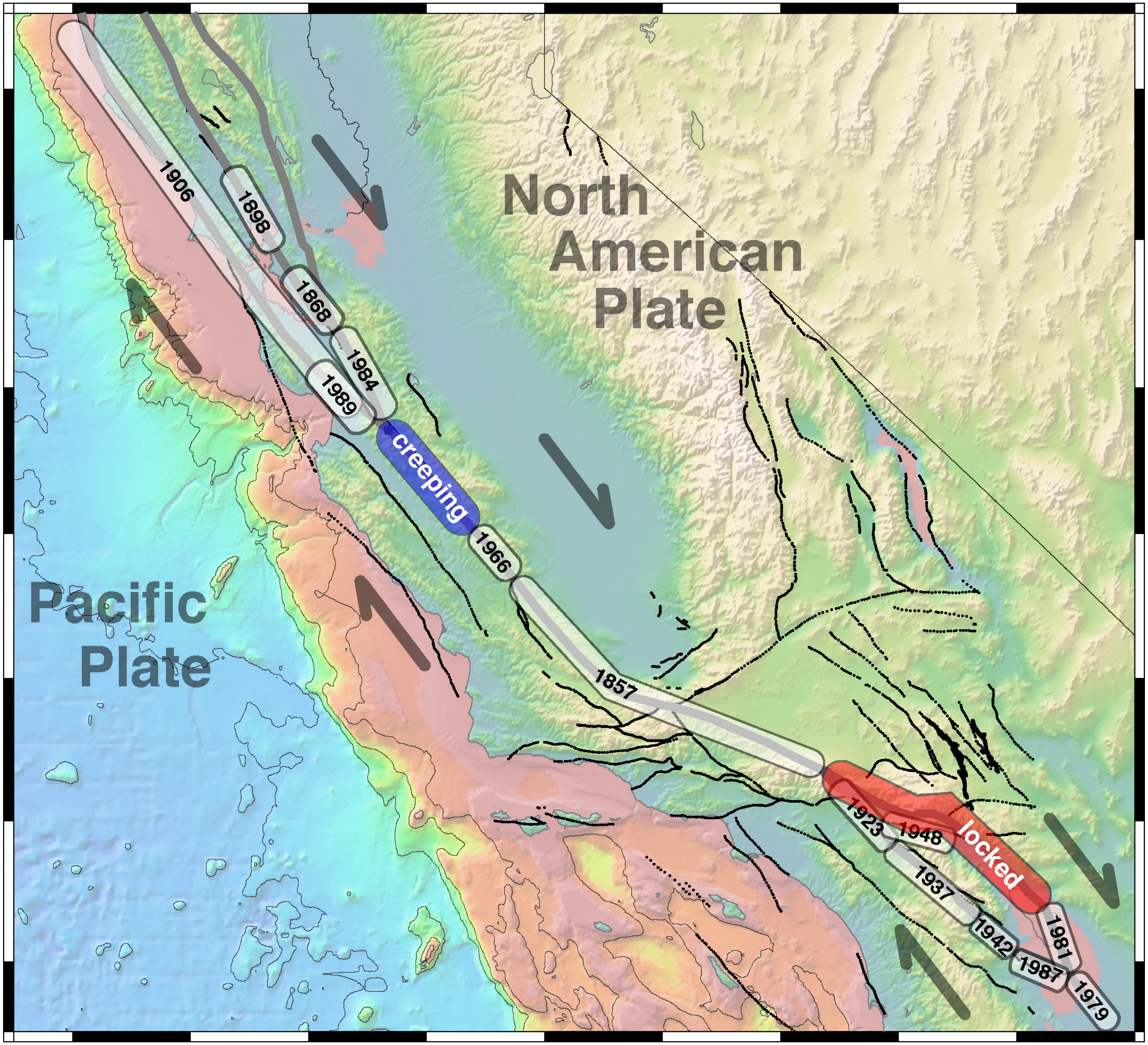
4-D Earthquake Cycle Model for Bounding Seismic Moment Accumulation Rate

David Sandwell
Bridget Smith-Konter
Xiaopeng Tong



- What is the present-day seismic potential (moment) and stress along the main faults in the San Andreas system?
- Geodesy provides a direct measure of 2-D seismic moment accumulation rate.
- Semi-analytic 4-D viscoelastic earthquake cycle model developed using computer algebra.
- Estimate moment rate with geodesy, geology, paleo-seismology.
- What is missing?

(IUGG June 30,2015)

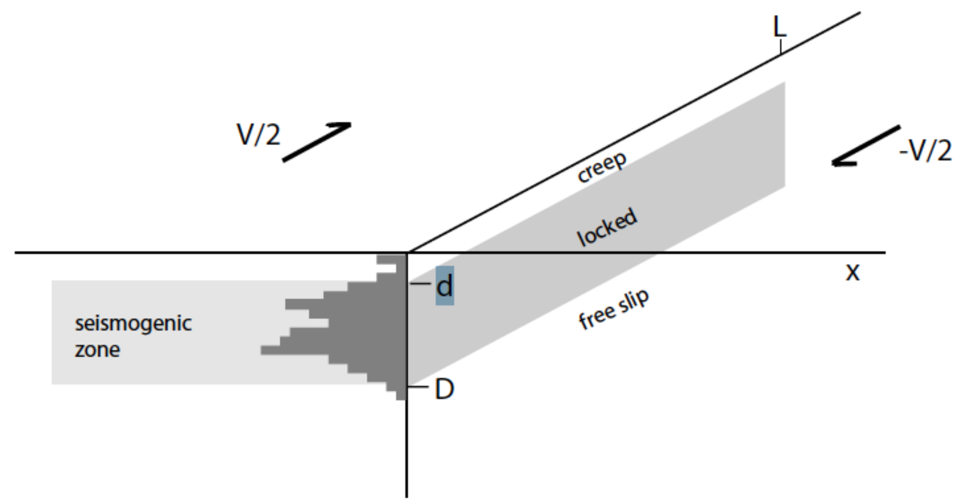


North American Plate
Pacific Plate
creeping
locked
1906
1898
1868
1984
1989
1966
1857
1923
1948
1937
1942
1987
1981
1979

seismic moment (2D)

interseismic moment accumulation rate

$$\frac{\dot{M}}{L} = \mu VD$$

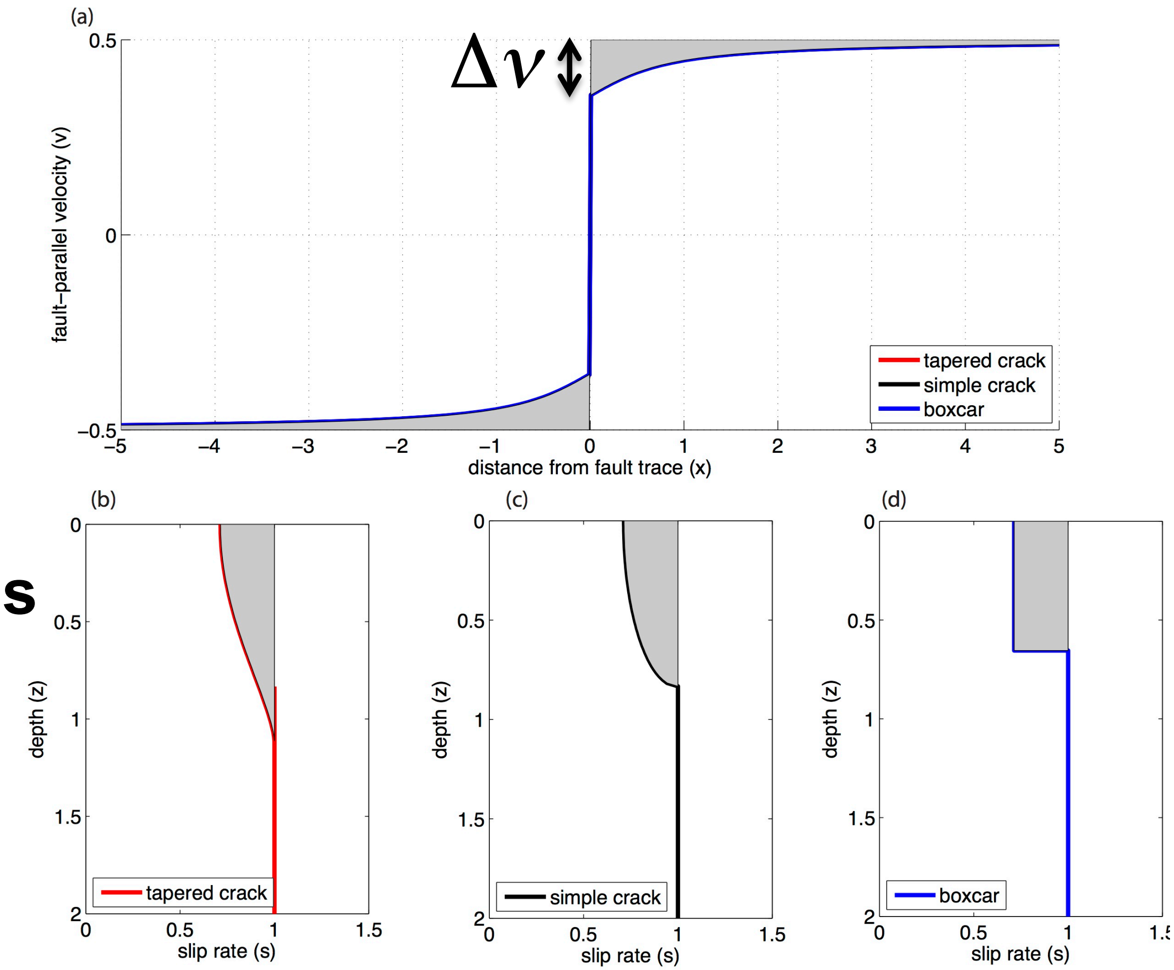


The size of the next earthquake depends on: the **slip rate** beneath the fault; times the **depth** of fault locking; times the **length** of the rupture; times the number of **years** since the last earthquake.

$$M = \mu VDL\ \Delta t$$

three different slip models have equal moment accumulation rate and similar velocity

inverting for slip versus depth is useless!



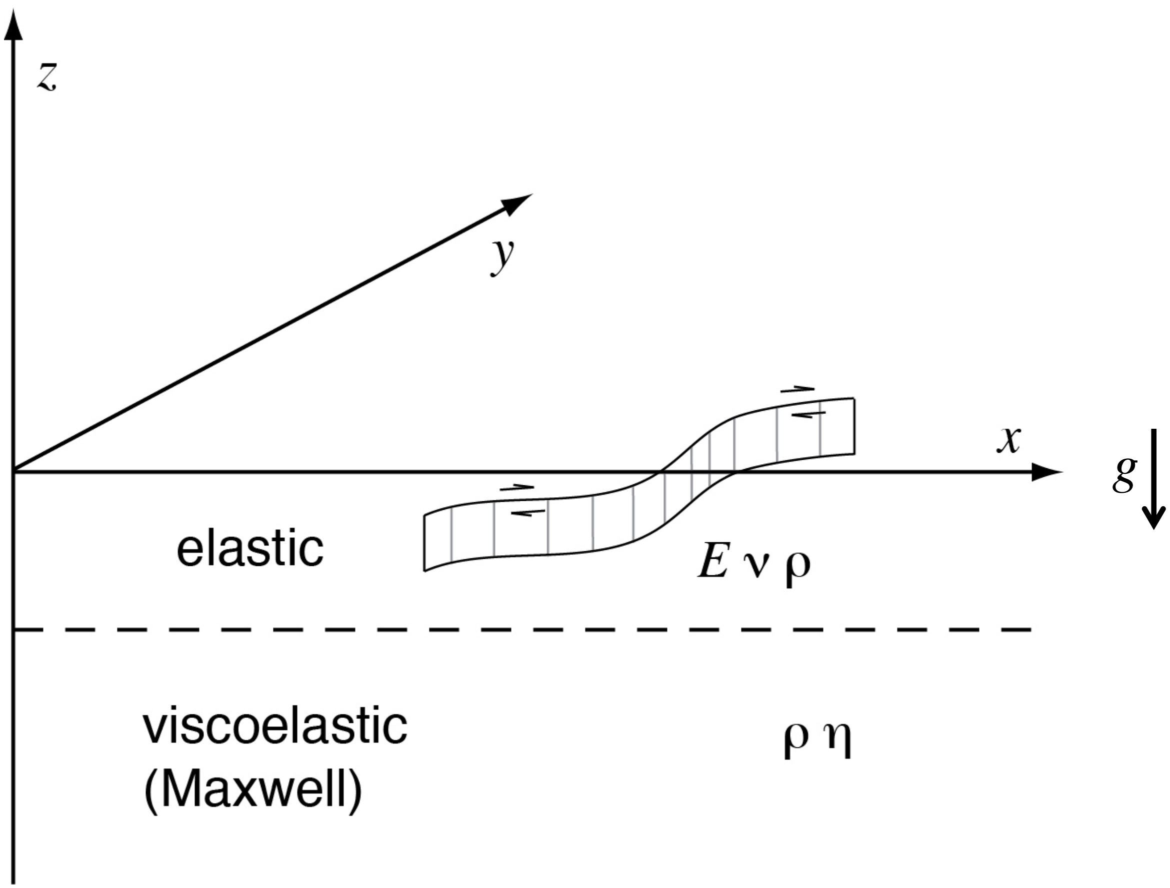
surface velocity provides a direct measure of moment accumulation rate.

$$\frac{\dot{M}}{L} = \frac{\mu\pi}{W}\int_{-W}^{W}\Delta v(x)\,x\,dx$$

4-D earthquake cycle models

	elastic block model	viscoelastic plate model	numerical model
pro	simple Green's function fast	time-dependent fast stress and strain are continuous	non-linear rheology existing codes
con	no time-dependent rheology stress singularities at fault intersections inaccurate vertical displacements	complicated algebra linear rheology only	complicated numerical codes difficult setup/meshing too slow for exhaustive parameter search

force couples in elastic plate over viscoelastic half space



outline of solution for vertical strike-slip fault

[*Smith-Konter and Sandwell*, 2004]

0) do all calculations in horizontal Fourier transform space (k_x, k_y, z)

1) solve for response of elastic full space to vector body force and integrate over fault depth

2) use method of images to simulate a layer over a half space [*Rybicki*, 1971].

3) match zero traction surface boundary condition using Galerkin vector approach [*Steketee*, 1958]

4) modify the *Steketee* [1958] approach to solve for layer over half space - **NEW**

5) use the elastic-viscoelastic correspondence principle to map half-space viscoelastic parameters to elastic parameters

6) create grids of double-couple forces to simulate faults [*Burridge and Knopoff*, 1964]

NEW

Boussinesq problem for point load on layer over half space with gravity restoring force

$\tau_{zz1} = -\tau_{33} + \rho g W_1 \big|_{z=0}$ $\quad \tau_{xz1} = \tau_{yz1} = 0 \big|_{z=0}$ surface boundary conditions (2 – radial symmetry)

$\tau_{xz1} = \tau_{xz2} \big|_{z=-h}$ $\quad U_1 = U_2 \big|_{z=-h}$

$\tau_{yz1} = \tau_{yz2} \big|_{z=-h}$ $\quad V_1 = V_2 \big|_{z=-h}$

$\tau_{zz1} = \tau_{zz2} \big|_{z=-h}$ $\quad W_1 = W_2 \big|_{z=-h}$

continuous displacement and
stress at base of layer (4 – radial symmetry)

Need to invert this 6X6 algebraic system analytically. – GOOD LUCK!!!!

$$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \chi\beta(2\mu_1\beta - \rho g) & \chi\beta(2\mu_1\beta + \rho g) & 2\chi\left(\mu_1\beta(3 - 1/\eta_1) - \rho g(1 - 1/\alpha_1)\right) & -2\chi\left(\mu_1\beta(3 - 1/\eta_1) + \rho g(1 - 1/\alpha_1)\right) & 0 & 0 \\ \beta & -\beta & (2 - 1/\alpha_1) & (2 - 1/\alpha_1) & 0 & 0 \\ \mu_1\alpha_1\beta e^{-\beta h} & -\mu_1\alpha_1\beta e^{\beta h} & \mu_1\alpha_1(2 - \beta h - 1/\alpha_1)e^{-\beta h} & \mu_1\alpha_1(2 + \beta h - 1/\alpha_1)e^{\beta h} & -\mu_2\alpha_2\beta e^{-\beta h} & -\mu_2\alpha_2(2 - \beta h - 1/\alpha_2)e^{-\beta h} \\ \mu_1\alpha_1\beta e^{-\beta h} & \mu_1\alpha_1\beta e^{\beta h} & \mu_1\alpha_1(3 - \beta h - 1/\eta_1)e^{-\beta h} & -\mu_1\alpha_1(3 + \beta h - 1/\eta_1)e^{\beta h} & -\mu_2\alpha_2\beta e^{-\beta h} & -\mu_2\alpha_2(3 - \beta h - 1/\eta_2)e^{-\beta h} \\ \alpha_1\beta e^{-\beta h} & \alpha_1\beta e^{\beta h} & \alpha_1(1 - \beta h)e^{-\beta h} & -\alpha_1(1 + \beta h)e^{\beta h} & -\alpha_2\beta e^{-\beta h} & -\alpha_2(1 - \beta h)e^{-\beta h} \\ \alpha_1\beta e^{-\beta h} & -\alpha_1\beta e^{\beta h} & \alpha_1(2 - \beta h - 2/\alpha_1)e^{-\beta h} & \alpha_1(2 + \beta h - 2/\alpha_1)e^{\beta h} & -\alpha_2\beta e^{-\beta h} & -\alpha_2(2 - \beta h - 2/\alpha_2)e^{-\beta h} \end{bmatrix} \begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \\ A_2 \\ C_2 \end{bmatrix}$$

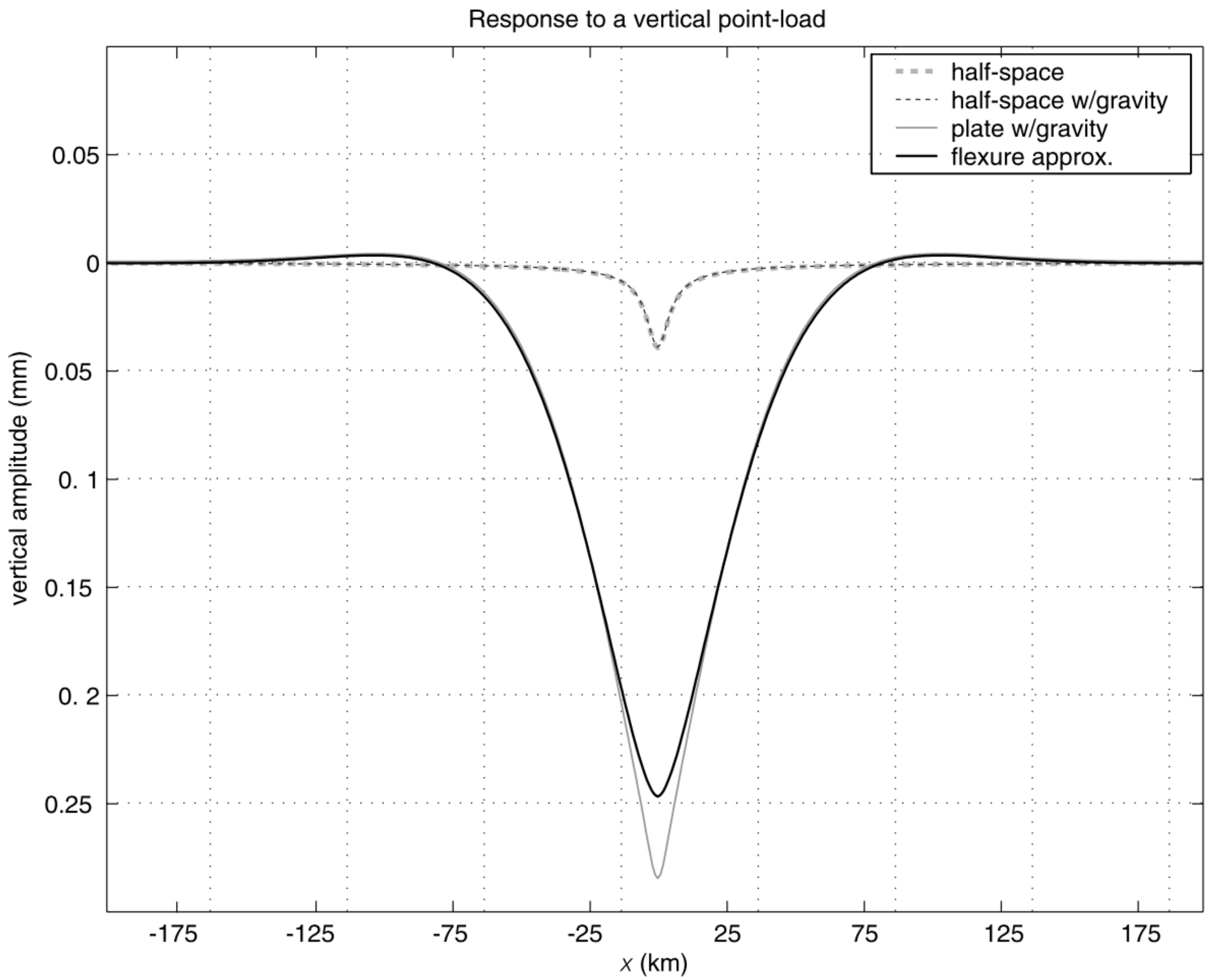
$$
\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} =
\begin{bmatrix}
\chi\beta(2\mu_1\beta-\rho g) & \chi\beta(2\mu_1\beta+\rho g) & 2\chi\left(\mu_1\beta(3-1/\eta_1)-\rho g(1-1/\alpha_1)\right) & -2\chi\left(\mu_1\beta(3-1/\eta_1)+\rho g(1-1/\alpha_1)\right) & 0 & 0 \\
\beta & -\beta & (2-1/\alpha_1) & (2-1/\alpha_1) & 0 & 0 \\
\mu_1\alpha_1\beta e^{-\beta h} & -\mu_1\alpha_1\beta e^{\beta h} & \mu_1\alpha_1(2-\beta h-1/\alpha_1)e^{-\beta h} & \mu_1\alpha_1(2+\beta h-1/\alpha_1)e^{\beta h} & -\mu_2\alpha_2\beta e^{-\beta h} & -\mu_2\alpha_2(2-\beta h-1/\alpha_2)e^{-\beta h} \\
\mu_1\alpha_1\beta e^{-\beta h} & \mu_1\alpha_1\beta e^{\beta h} & \mu_1\alpha_1(3-\beta h-1/\eta_1)e^{-\beta h} & -\mu_1\alpha_1(3+\beta h-1/\eta_1)e^{\beta h} & -\mu_2\alpha_2\beta e^{-\beta h} & -\mu_2\alpha_2(3-\beta h-1/\eta_2)e^{-\beta h} \\
\alpha_1\beta e^{-\beta h} & \alpha_1\beta e^{\beta h} & \alpha_1(1-\beta h)e^{-\beta h} & -\alpha_1(1+\beta h)e^{\beta h} & -\alpha_2\beta e^{-\beta h} & -\alpha_2(1-\beta h)e^{-\beta h} \\
\alpha_1\beta e^{-\beta h} & -\alpha_1\beta e^{\beta h} & \alpha_1(2-\beta h-2/\alpha_1)e^{-\beta h} & \alpha_1(2+\beta h-2/\alpha_1)e^{\beta h} & -\alpha_2\beta e^{-\beta h} & -\alpha_2(2-\beta h-2/\alpha_2)e^{-\beta h}
\end{bmatrix}
\begin{bmatrix} A_1 \\ B_1 \\ C_1 \\ D_1 \\ A_2 \\ C_2 \end{bmatrix}
$$

Today we have symbolic algebra – no sweat, no errors!!

```
% set up left hand side

Y=[1; 0; 0; 0; 0; 0;];

% set up right hand side

M=[ X*m*(2*u1*m-p*g)  X*m*(2*u1*m+p*g)  2*X*( u1*m*(3-ie1) -p*g*(1-ia1) )   -2*X*( u1*m*(3-ie1) + p*g*(1-ia1) )   0    0;
   m       -m       (2-ia1)            (2-ia1)          0        0;
   ma1*m*en  -ma1*m*ep  ma1*(2-mh-ia1)*en     ma1*(2+mh-ia1)*ep  -ma2*m*en  -ma2*(2-mh-ia2)*en;
   ma1*m*en   ma1*m*ep  ma1*(3-mh-ie1)*en    -ma1*(3+mh-ie1)*ep  -ma2*m*en  -ma2*(3-mh-ie2)*en;
   a1*m*en   a1*m*ep   a1*(1-mh)*en        -a1*(1+mh)*ep      -a2*m*en   -a2*(1-mh)*en;
   a1*m*en   -a1*m*ep   a1*(2-mh-(2*ia1))*en   a1*(2+mh-2*ia1)*ep -a2*m*en   -a2*(2-mh-(2*ia2))*en];

% invert matrix

Z=M\Y;
```

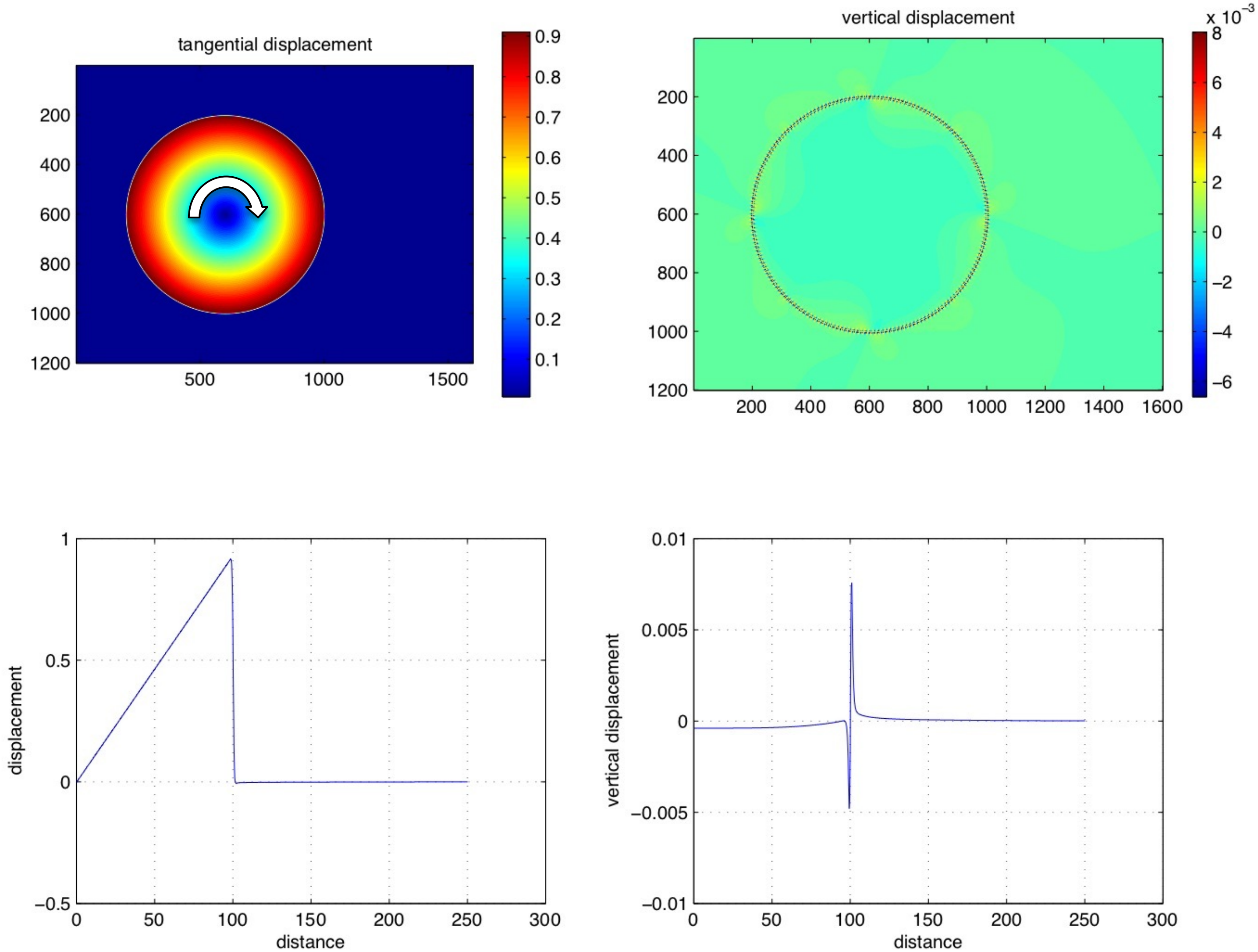
[*Smith-Konter and Sandwell*, 2004]

tests with half-space [*Love,* 1944] and thin-plate flexure



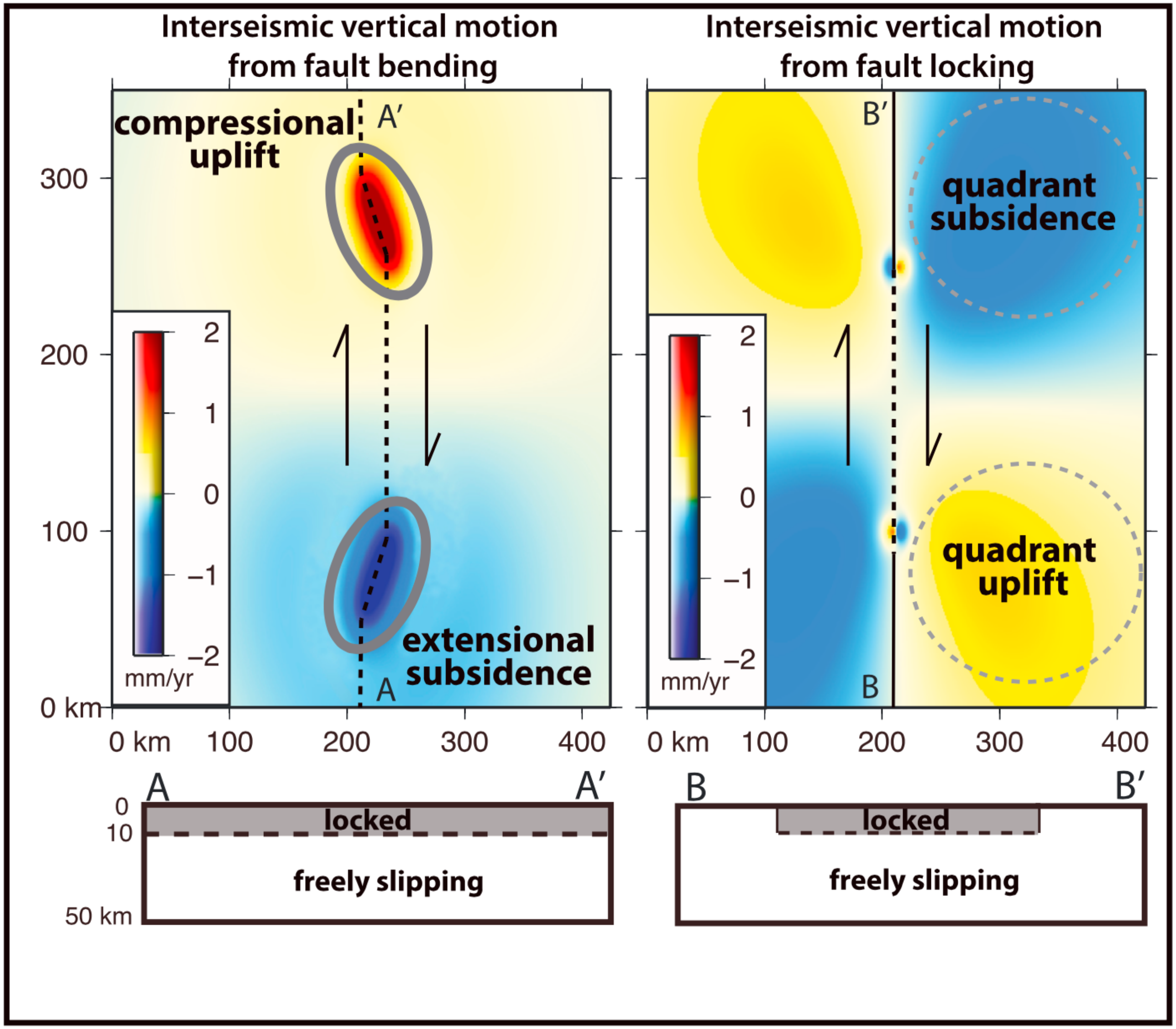
gravity dominates for plate model but is unimportant for half space model.

spinning plate benchmark



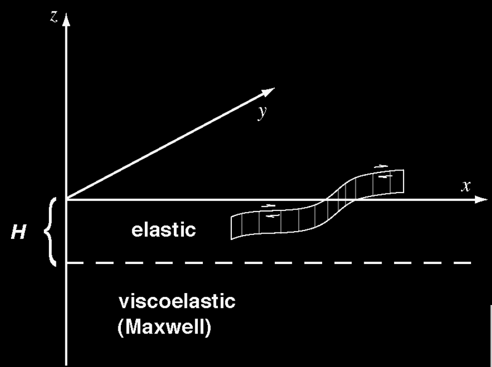
Don’t need fake blocks with backslip but can have true plate-like behavior that couples far-field velocity to near-fault stress.

need thick plate and gravity to simulate vertical deformation

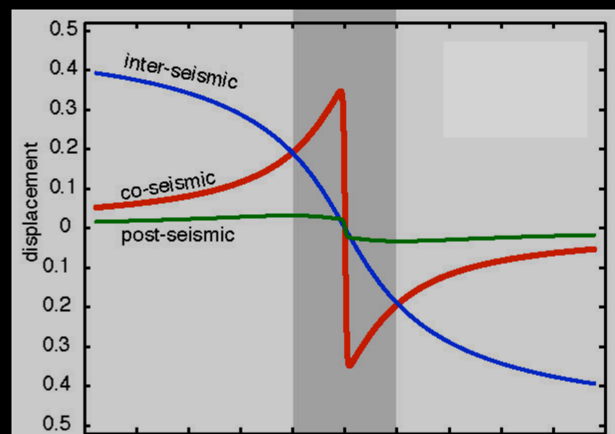


Kinematic Earthquake Cycle Model

[Smith and Sandwell, JGR, 2006]



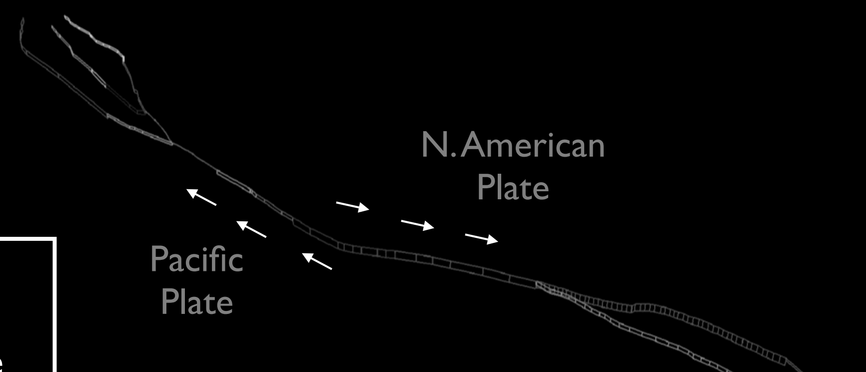
Simple model



[Tse and Rice, JGR, 1986]

displacement(t) =

interseismic + Σ earthquakes
(deep slip) (shallow slip)



San Andreas Fault elements

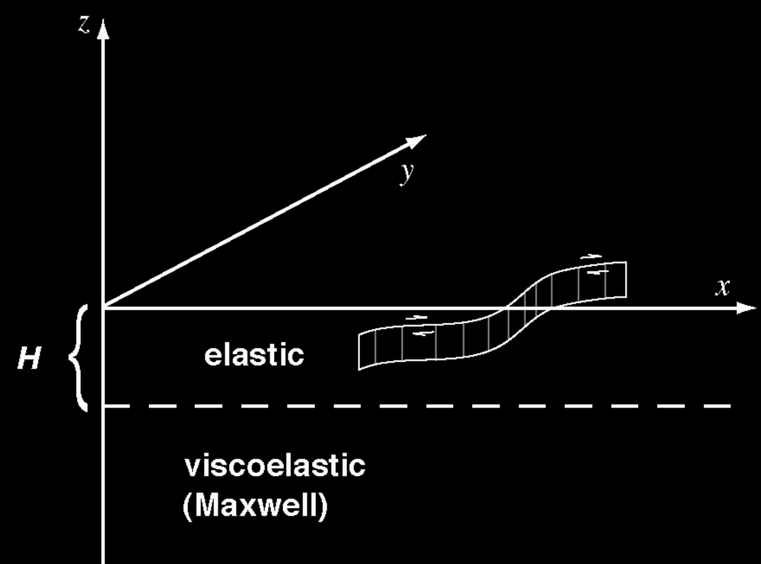
Model efficiency

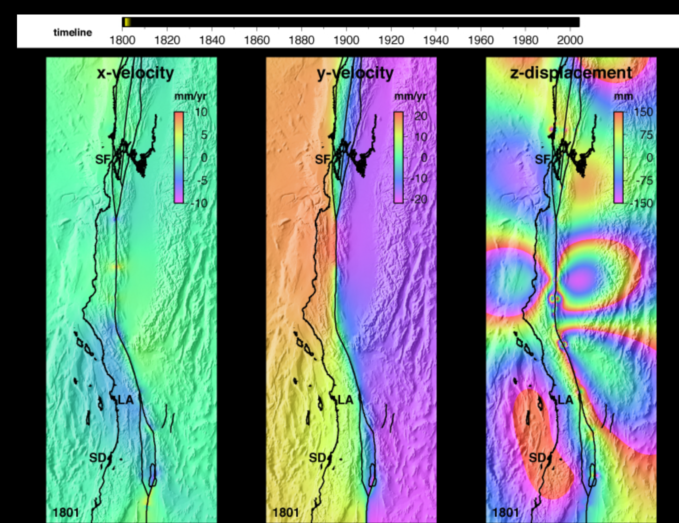
- 2048 x 2048 grid cells, 400 fault elements
- common locking depth, single event: ~ 1s of CPU time
- 27 locking depths, 70 events over 1000 years: < 30 min

Building a 4-D Model of the Earthquake Cycle

1. Physical model: 4-D Maxwell viscoelastic
2. Initial slip rate estimates (geology)
3. Crustal velocities (GPS/InSAR)
4. Historical earthquakes (earthquake record)
5. Pre-historical earthquakes and recurrence intervals (paleoseismology)

(items 2–3: elastic; items 2–5: viscoelastic)

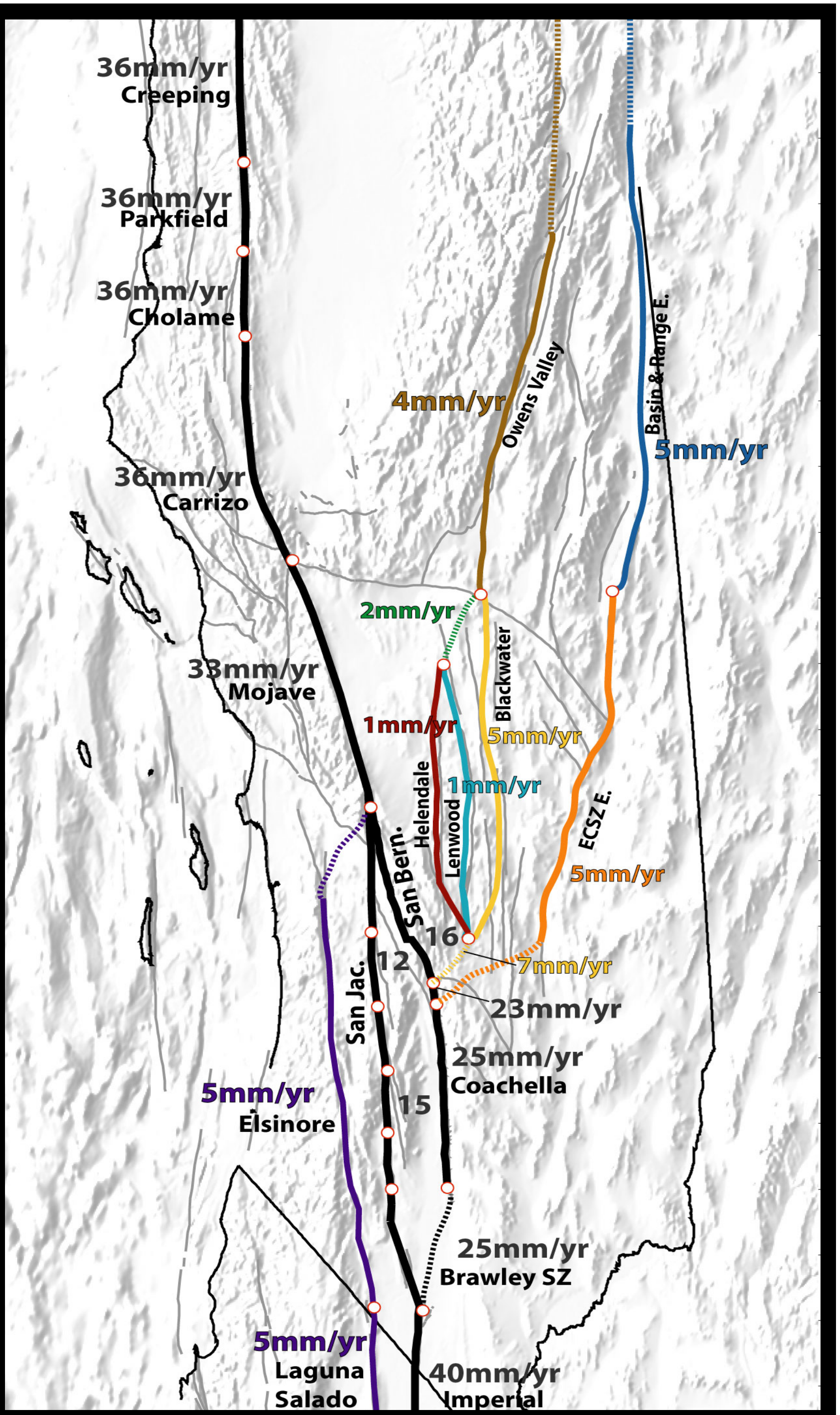




[Smith-Konter & Sandwell, JGR 200

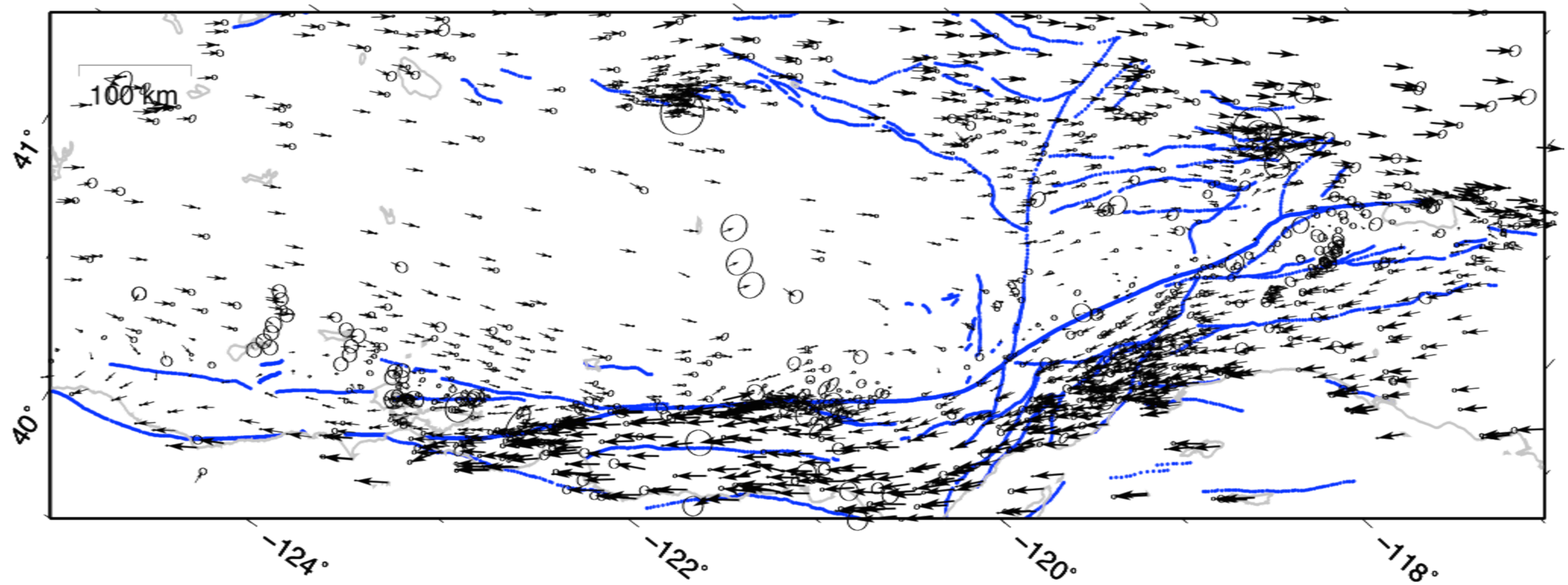
Geological Slip Rates

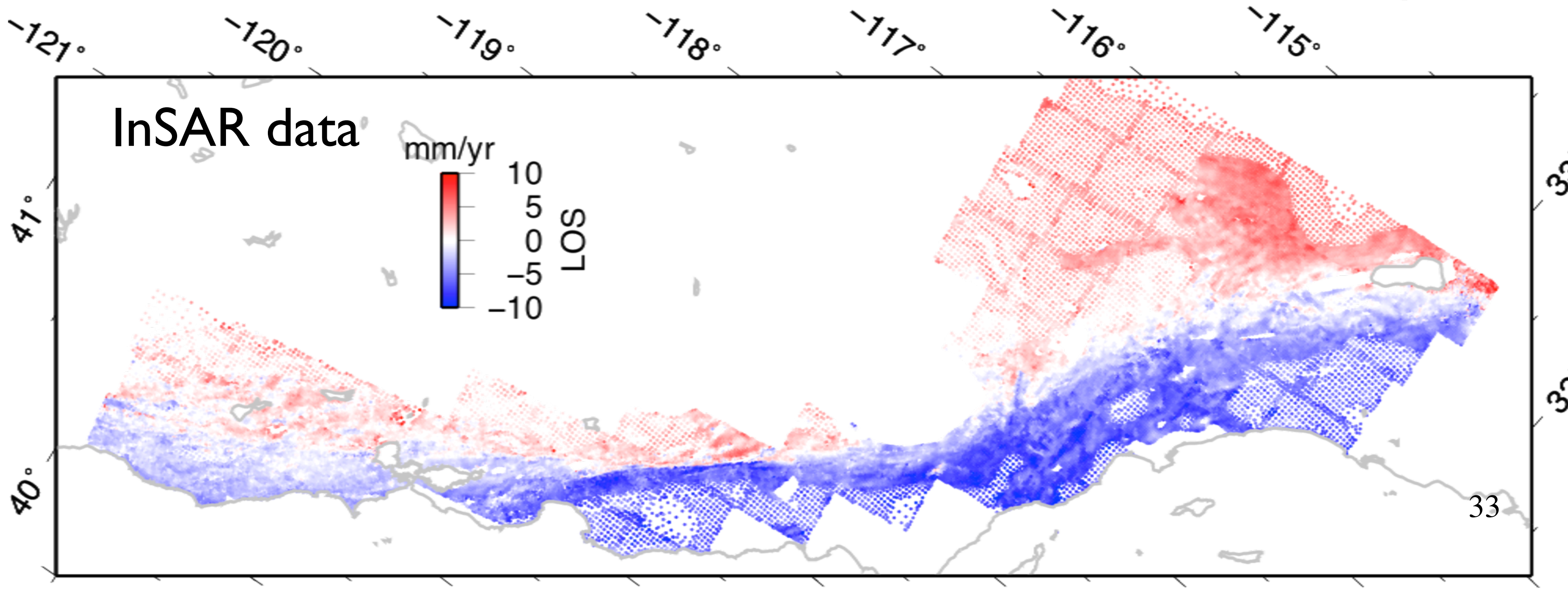
- Provides block motion
- Far-field velocity must match North America-Pacific plate motion (45 mm/yr)



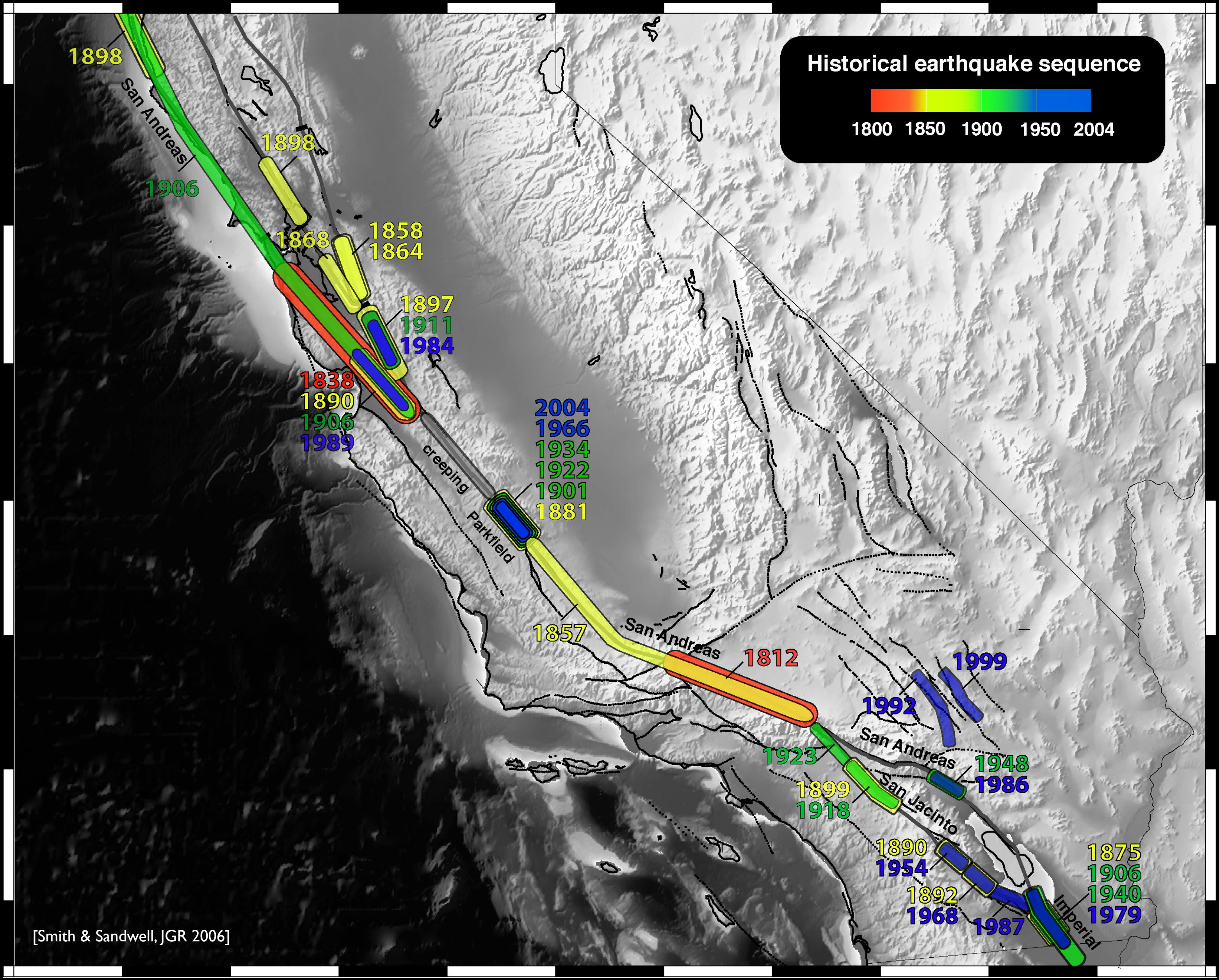
GPS data

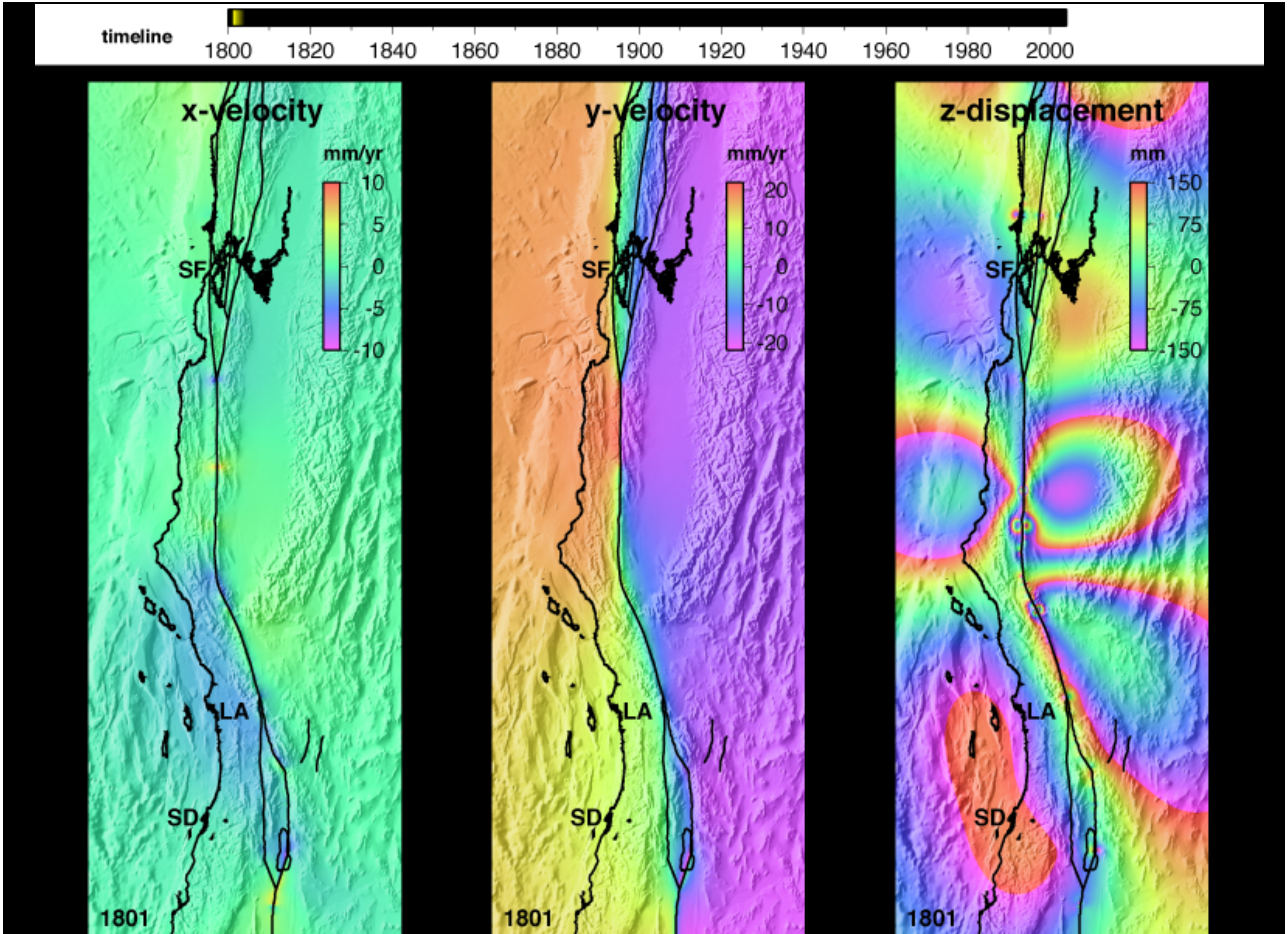
[*Tong et al., JGR* 2013]





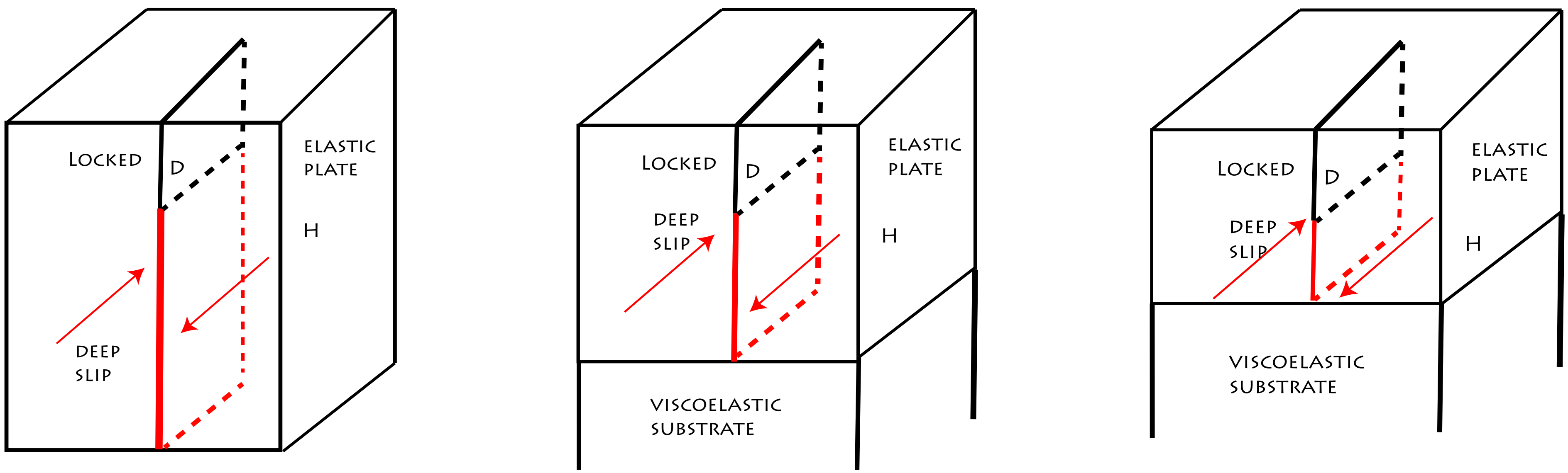
Historical Earthquakes





[Smith & Sandwell, JGR 2006]

3 candidate models



Half-space model
H=9999 km

Thick plate model
H=60 km

Thin plate model
H=30 km

None — Weak — Strong

Earthquake cycle effects

Inverse problem

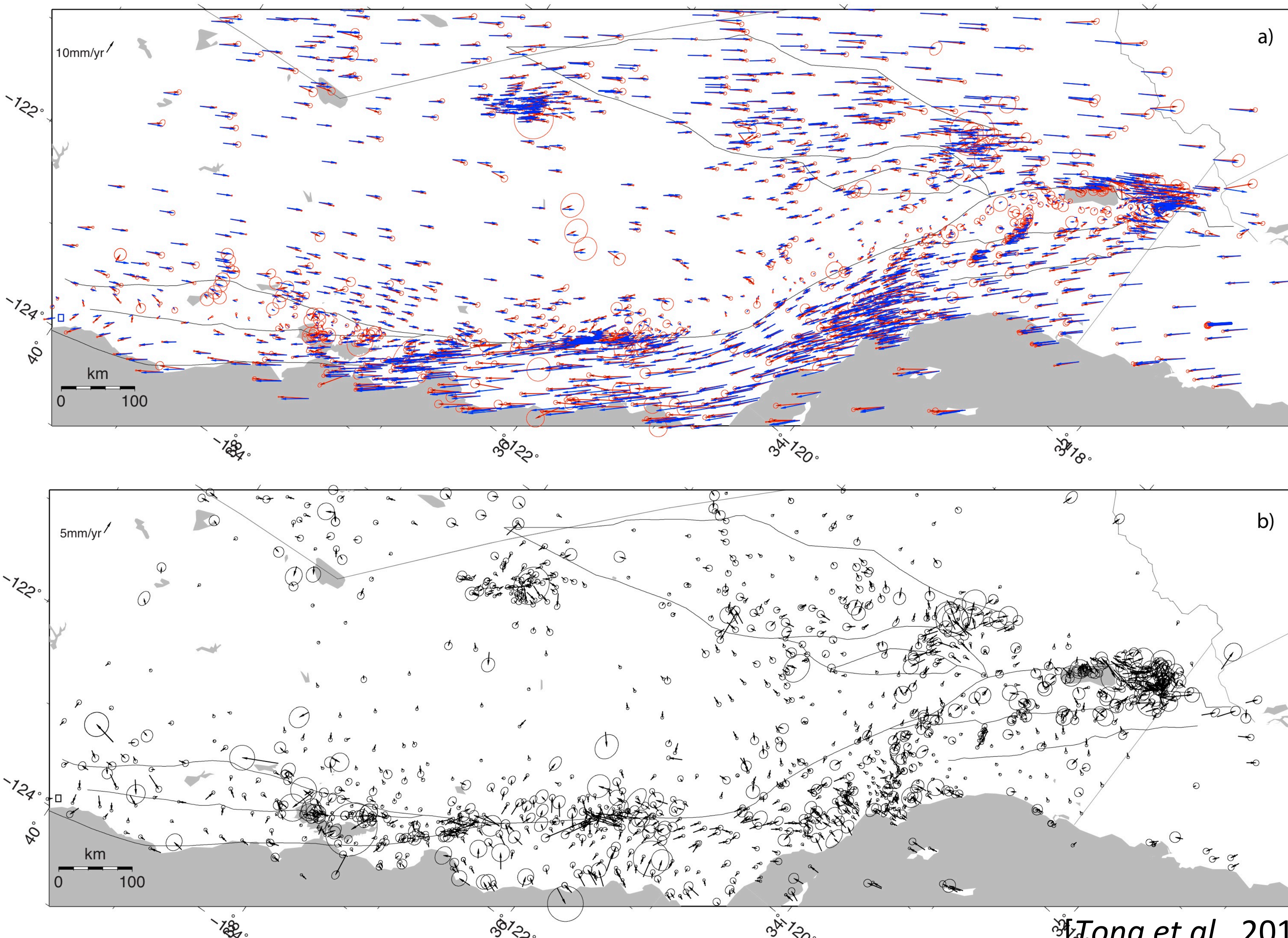
- Green function
  - Deep slip in the **earthquake cycle model** $\overline{\overline{G_g}}$ $\overline{\overline{G_i}}$
  - Fault creep from layered elastic model $\overline{\overline{E_g}}$ $\overline{\overline{E_i}}$
- Geological constraint $\overline{\overline{C}}$
- Smoothing factor $\overline{\overline{S}}$
- Invert for:
  - deep slip rate $\overline{s}$
  - fault creep rate $\overline{p}$
- Data:
  - GPS $\overline{v_g}$
  - InSAR $\overline{l}$
  - Geological slip rate $\overline{s_c}$

$$\begin{bmatrix} \overline{\overline{G}}_g & \overline{\overline{E}}_g \\ \overline{\overline{G}}_i & \overline{\overline{E}}_i \\ \overline{\overline{C}} & 0 \\ 0 & \overline{\overline{S}} \end{bmatrix} \begin{bmatrix} \overline{s} \\ \overline{p} \end{bmatrix} = \begin{bmatrix} \overline{v_g} \\ \overline{l} \\ \overline{s_c} \\ 0 \end{bmatrix}$$

[*Tong et al.*, 2014]

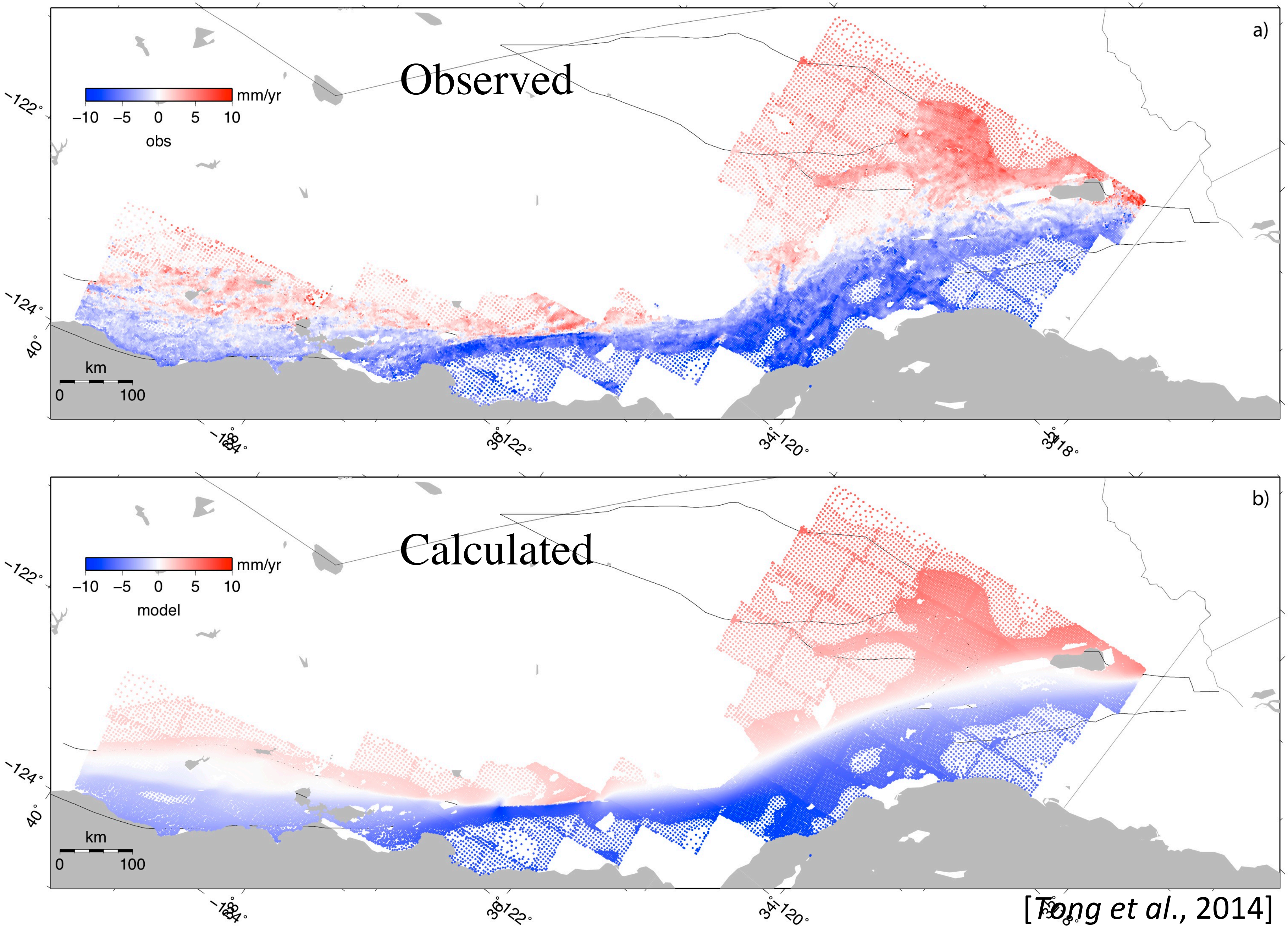
GPS velocity for the thick plate

Observed calculated residual

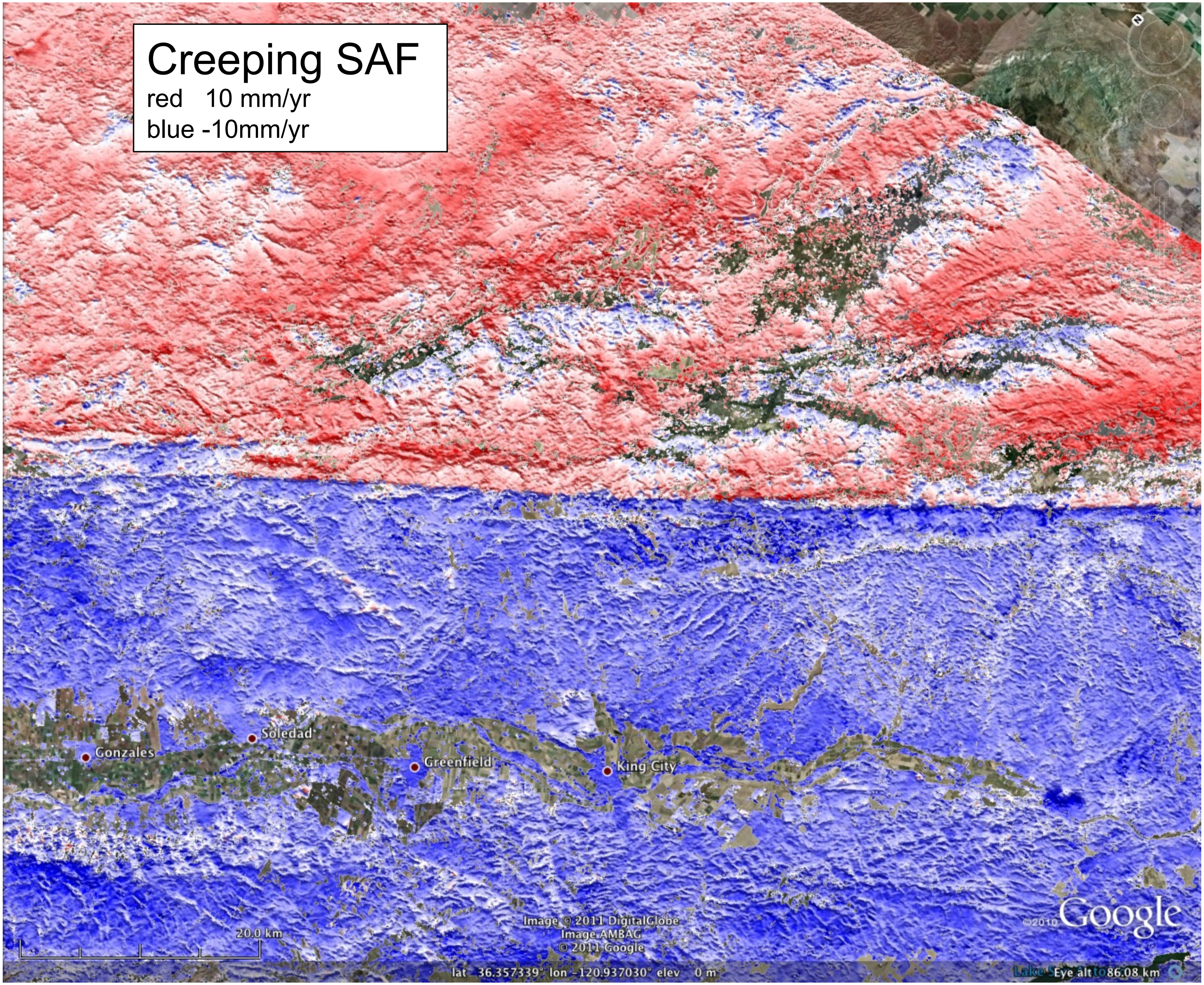


[*Tong et al.*, 2014]

InSAR velocity for the thick plate



[*Tong et al.*, 2014]


Creeping SAF
red 10 mm/yr
blue -10mm/yr
Soledad
Gonzales
Greenfield
King City
20.0 km
Image © 2011 DigitalGlobe
Image AMBAG
© 2011 Google
©2010
Google
lat 36.357339° lon -120.937030° elev 0 m
Eye alt 86.08 km

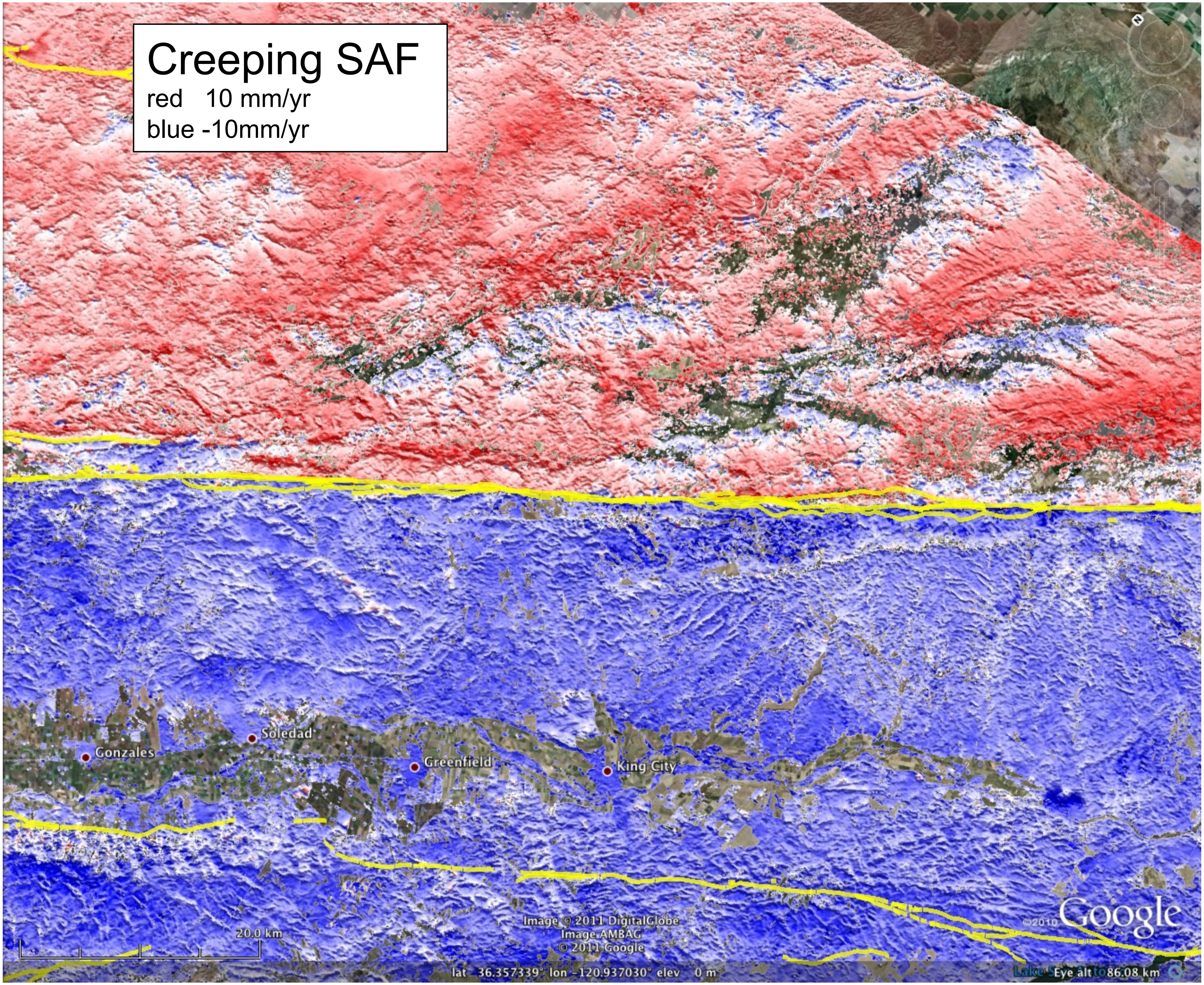

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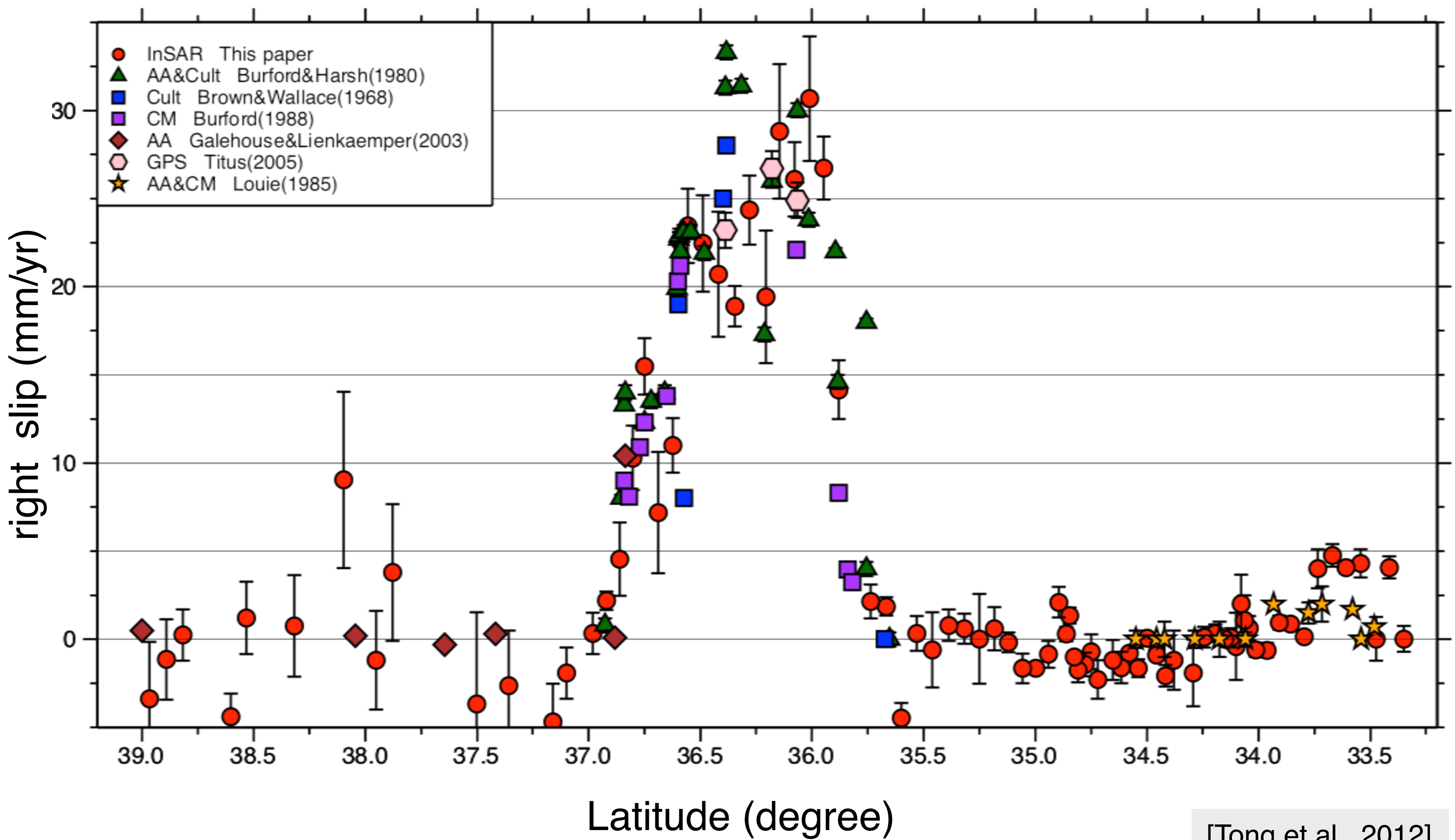
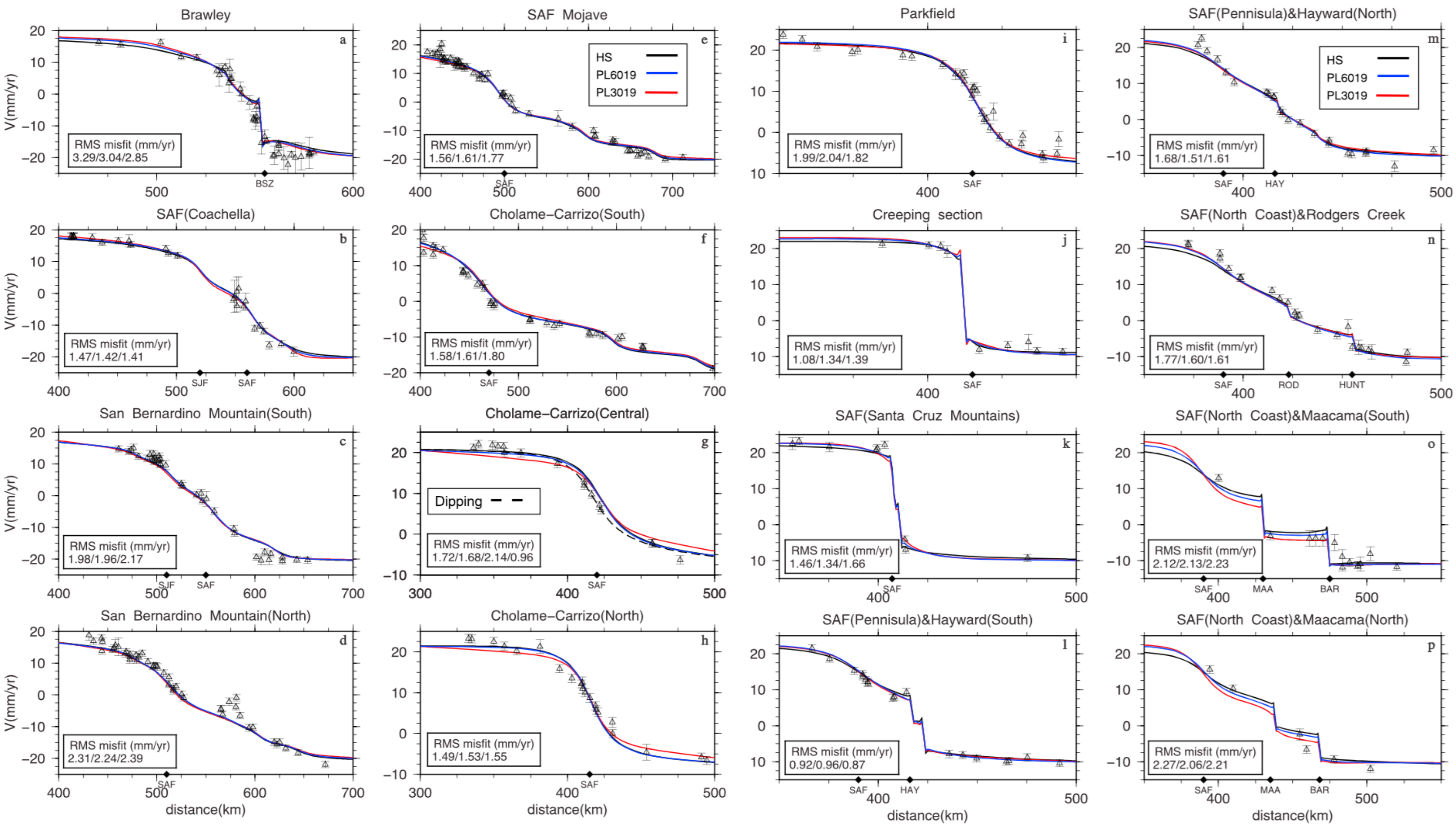
Image © 2011 DigitalGlobe
Image AMBAG
© 2011 Google
lat 36.357339° lon -120.937030° elev 0 m
Eye alt 86.08 km
©2010
Google

Fault Creep along entire SAF from ALOS vs. creep meters

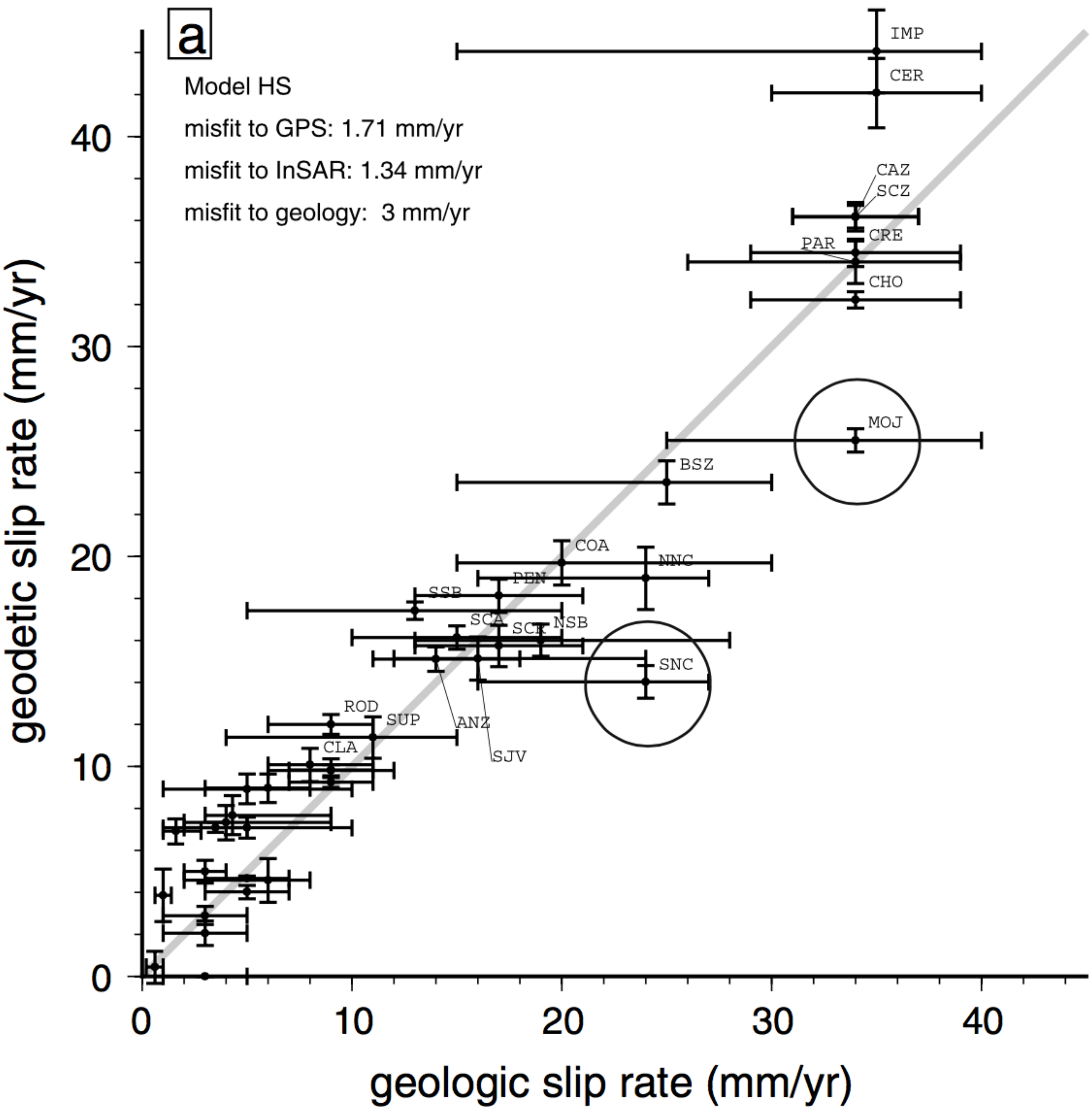


[Tong et al., 2012]

present-day velocities and fit to GPS



[*Tong et al.*, 2014]



half space model

[*Tong et al.*, 2014]

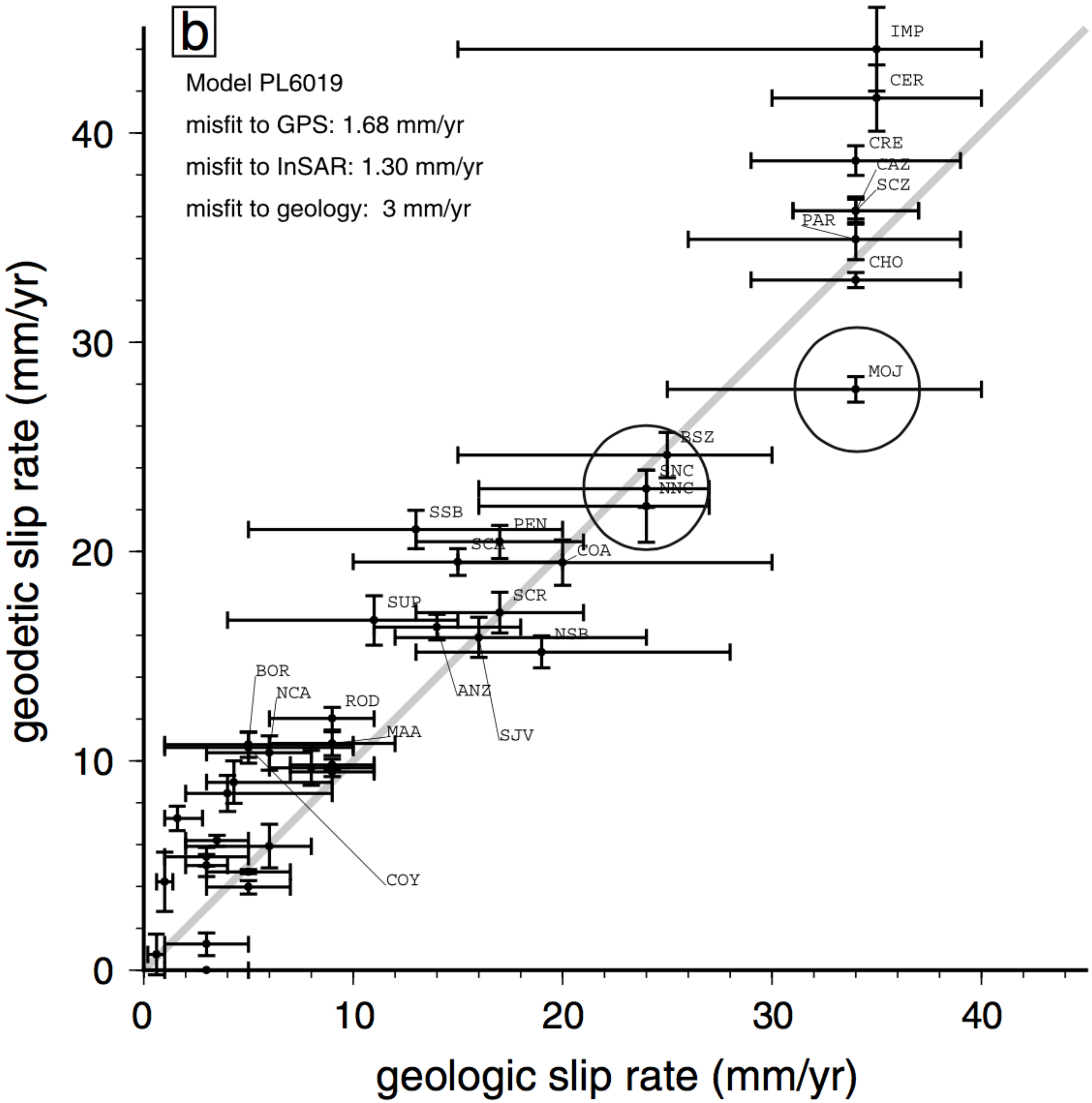
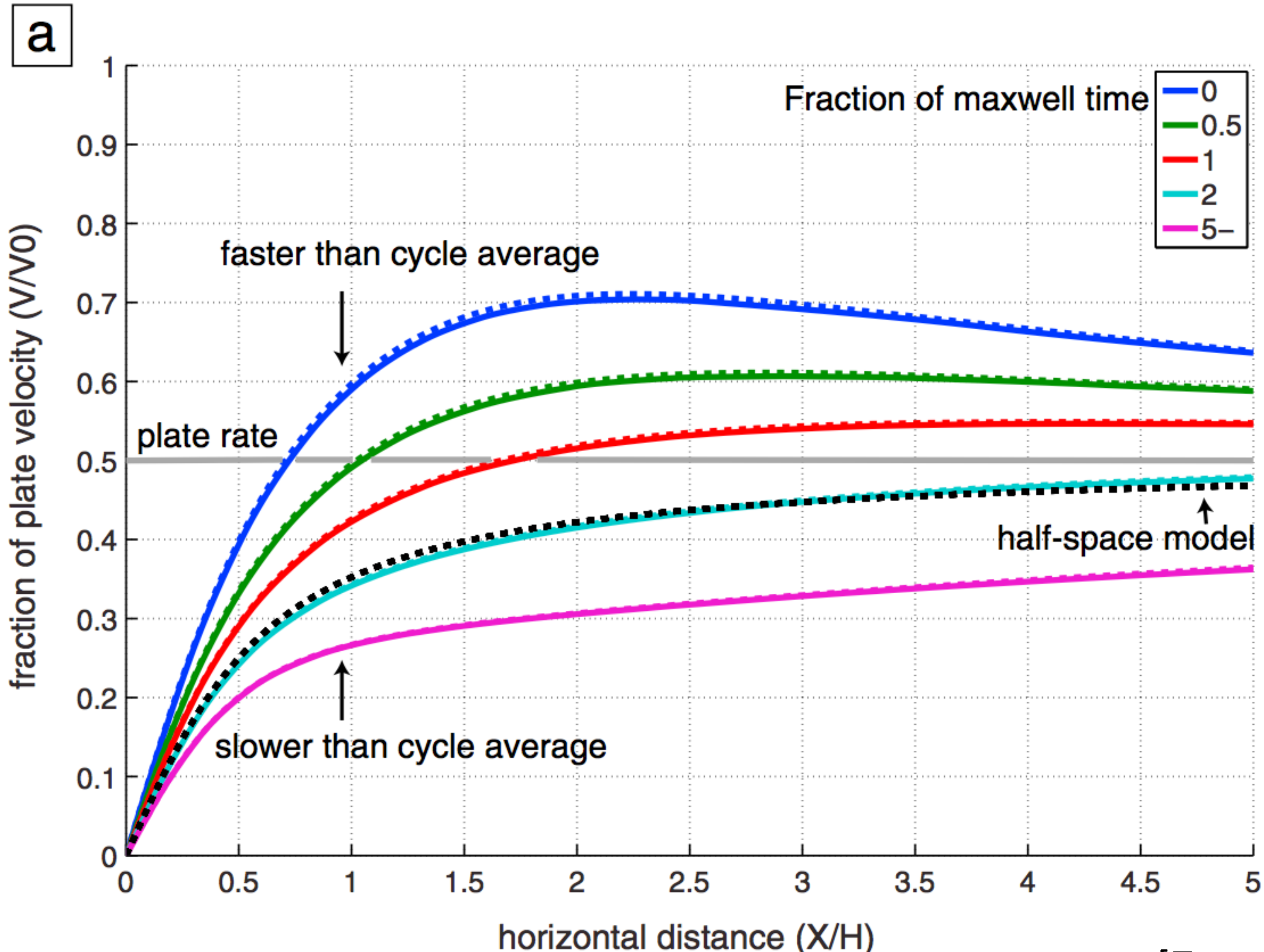


plate model

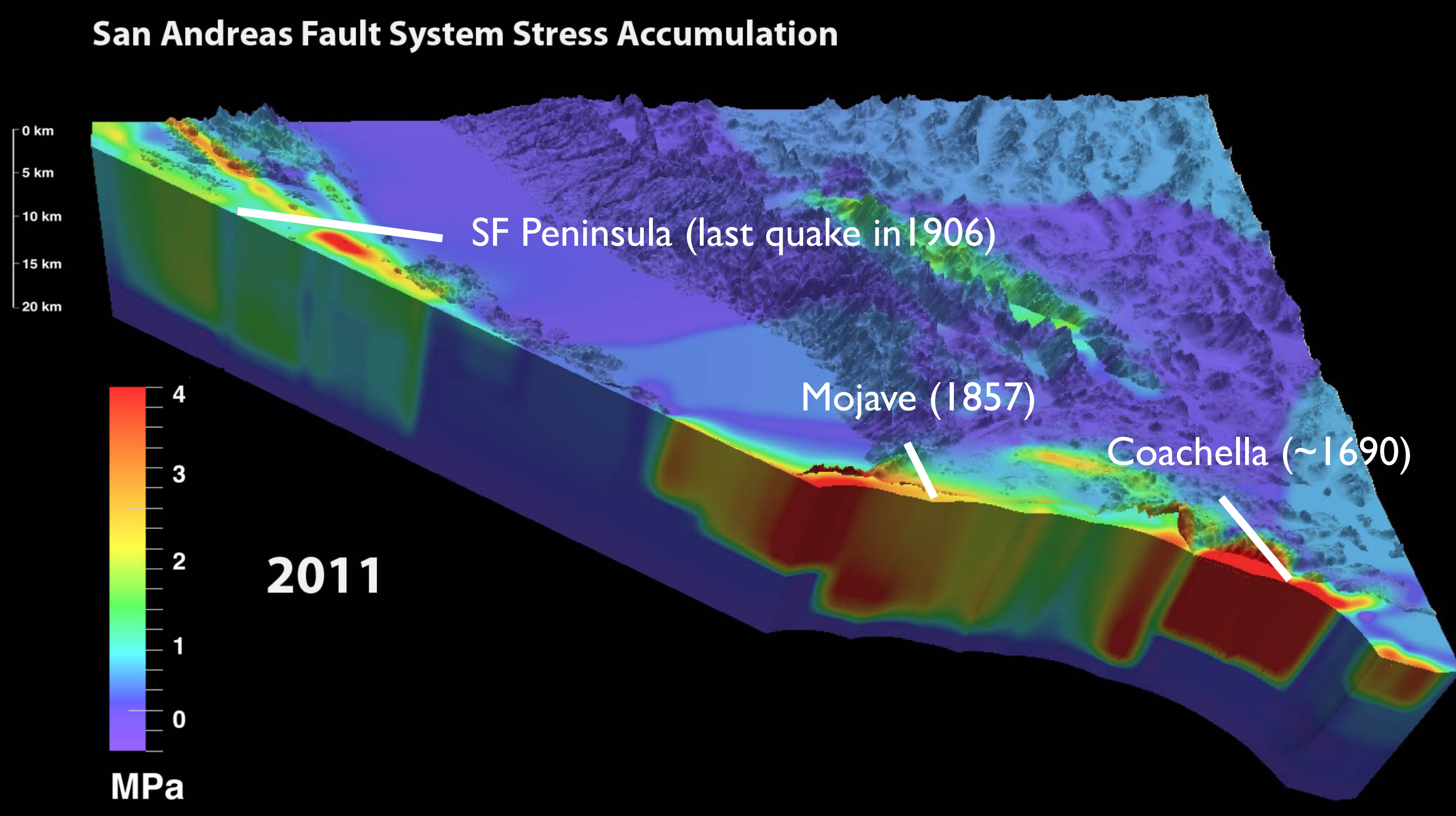
[*Tong et al.*, 2014]

Geodetic rate is within the error bounds of the geologic rate when a viscoelastic model is used. Agreement at Mojave segment was shown previously by *Chuang and Johnson*, [2011] and *Hearn et al.*, [2013].



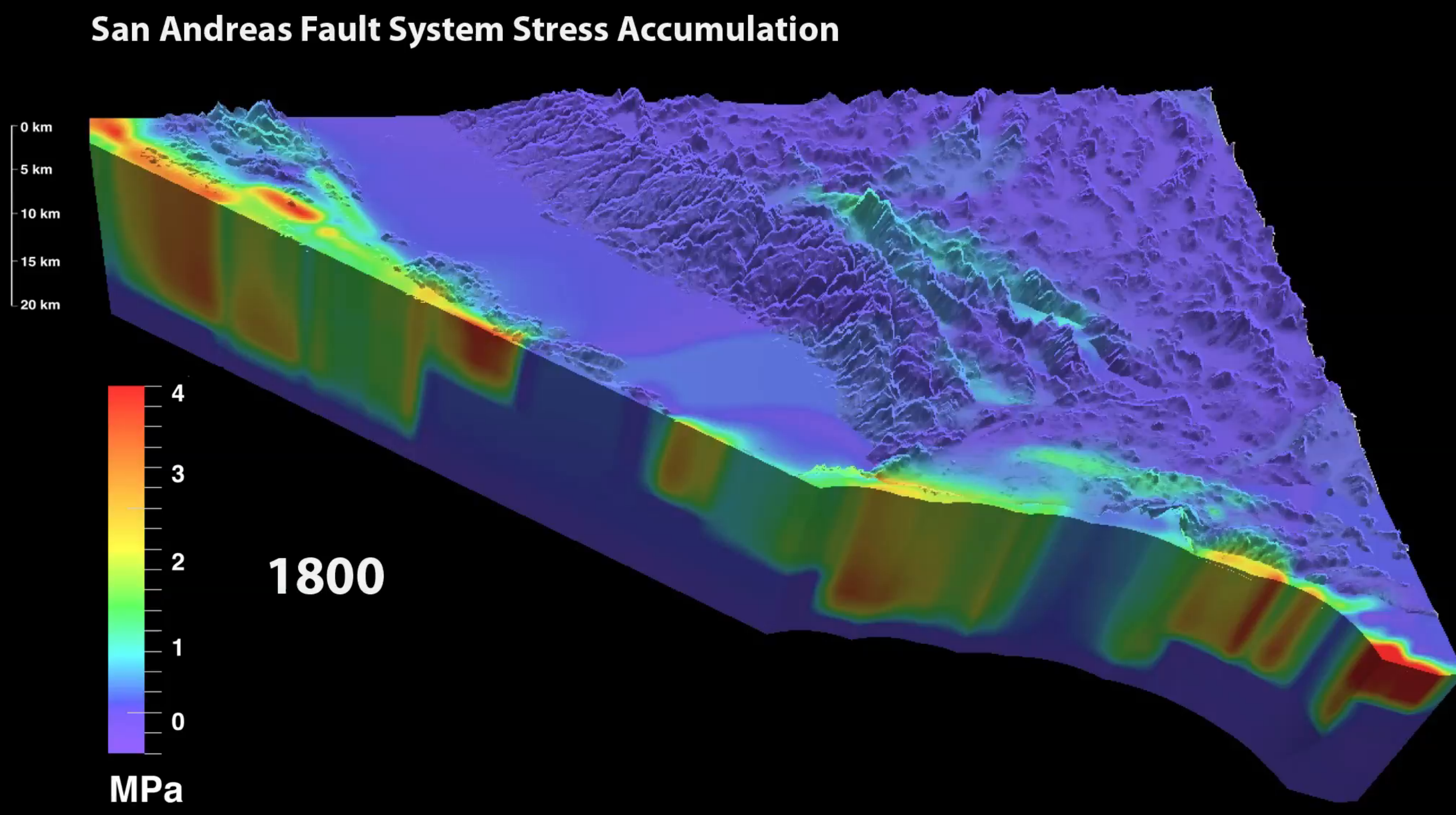
[*Tong et al.*, 2014]

4-D stress



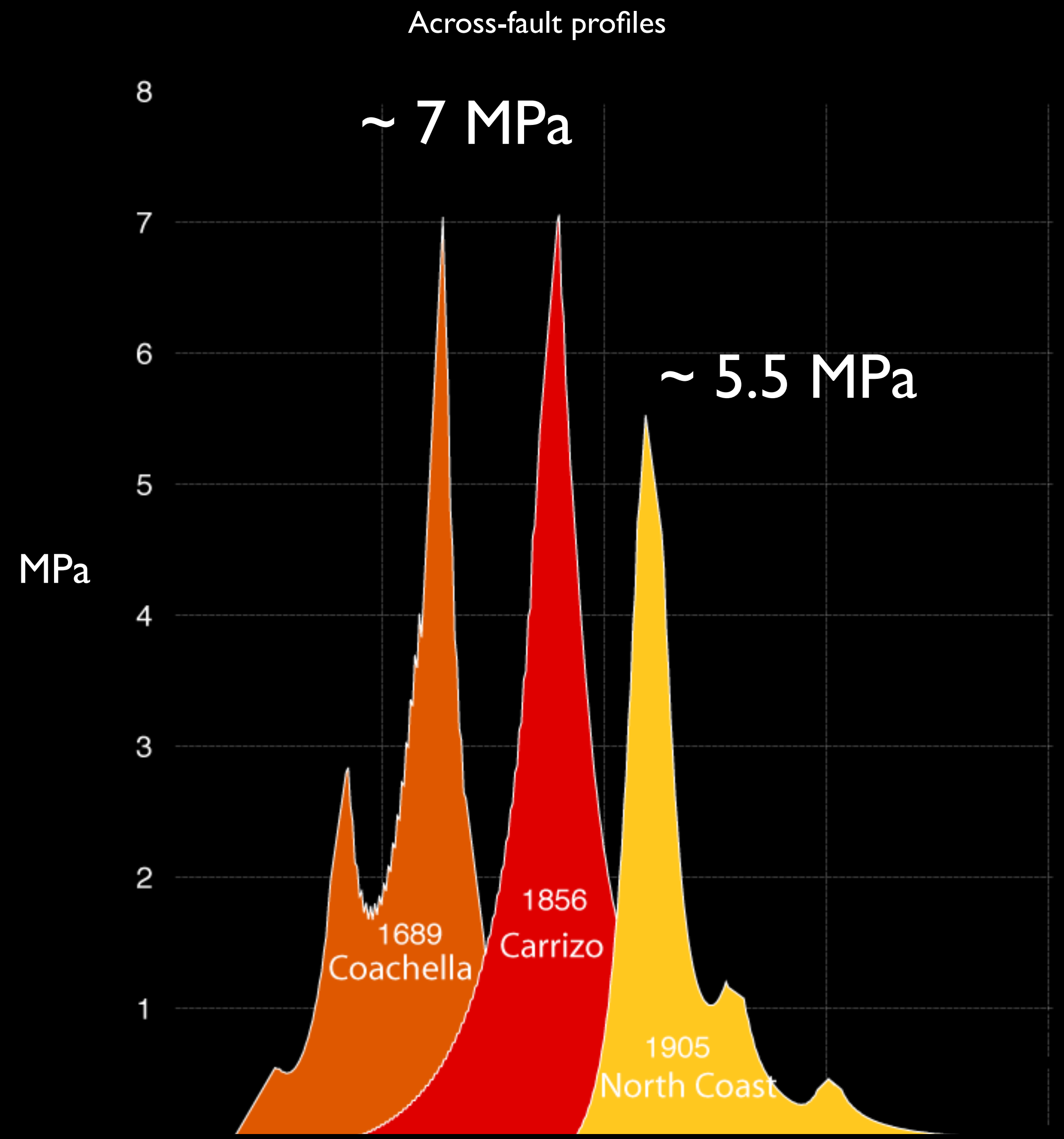
[Smith-Konter, AGU 2011]

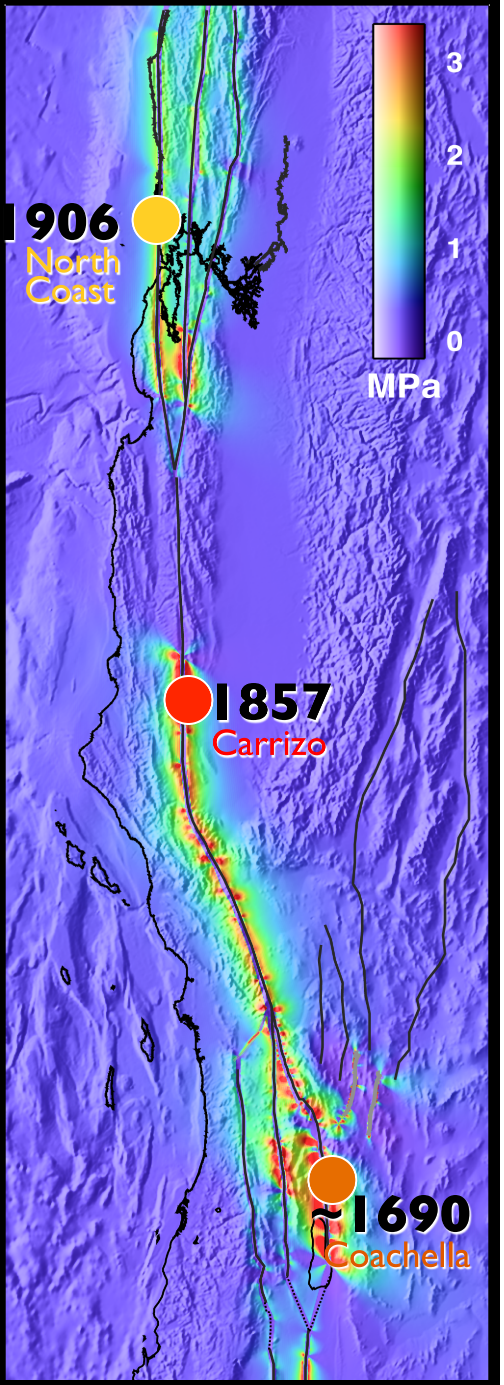
Model of 4-D Stress



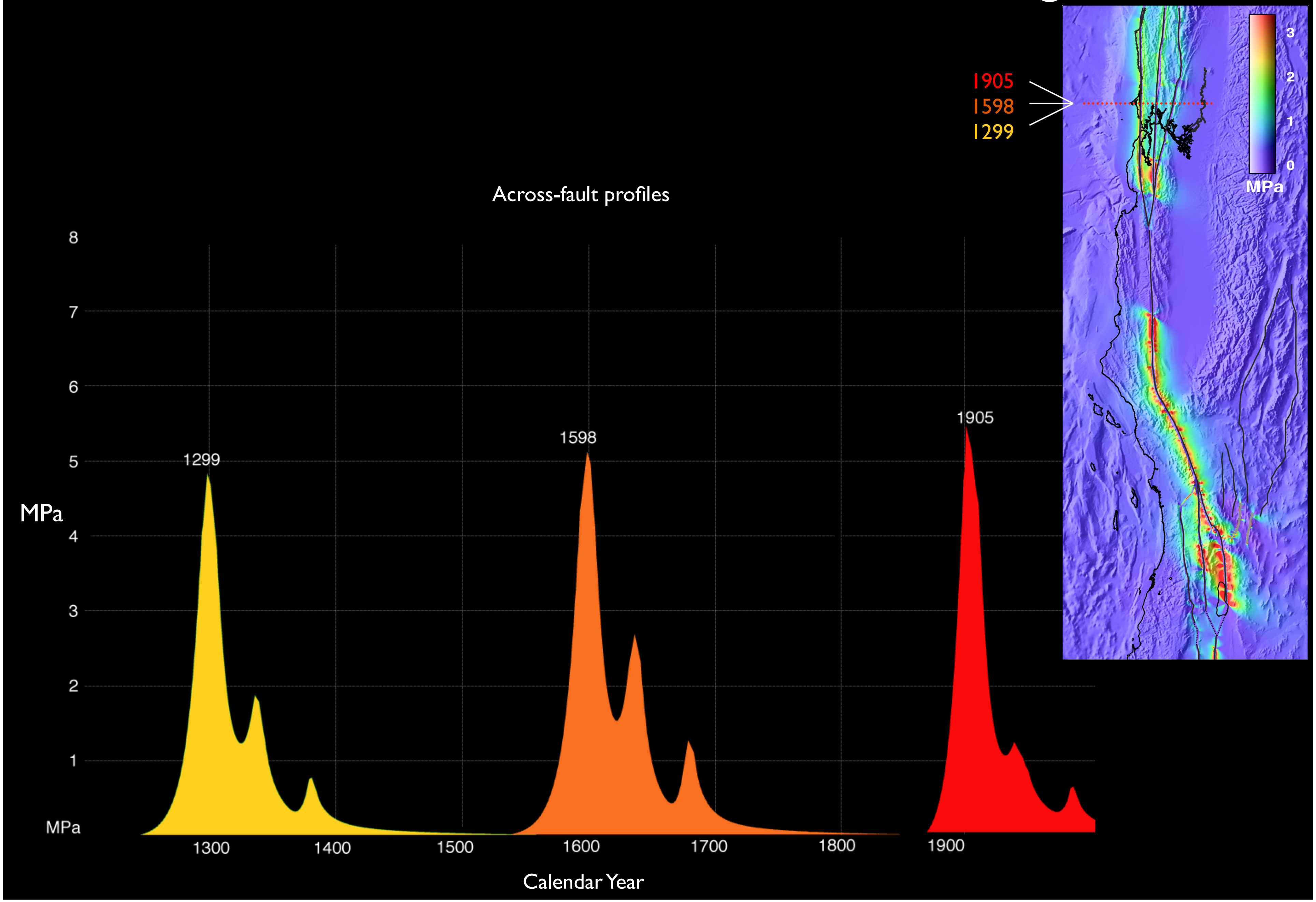
[Smith-Konter, AGU 2011]

Hindcast Stress Estimates: 1690, 1857, 1906

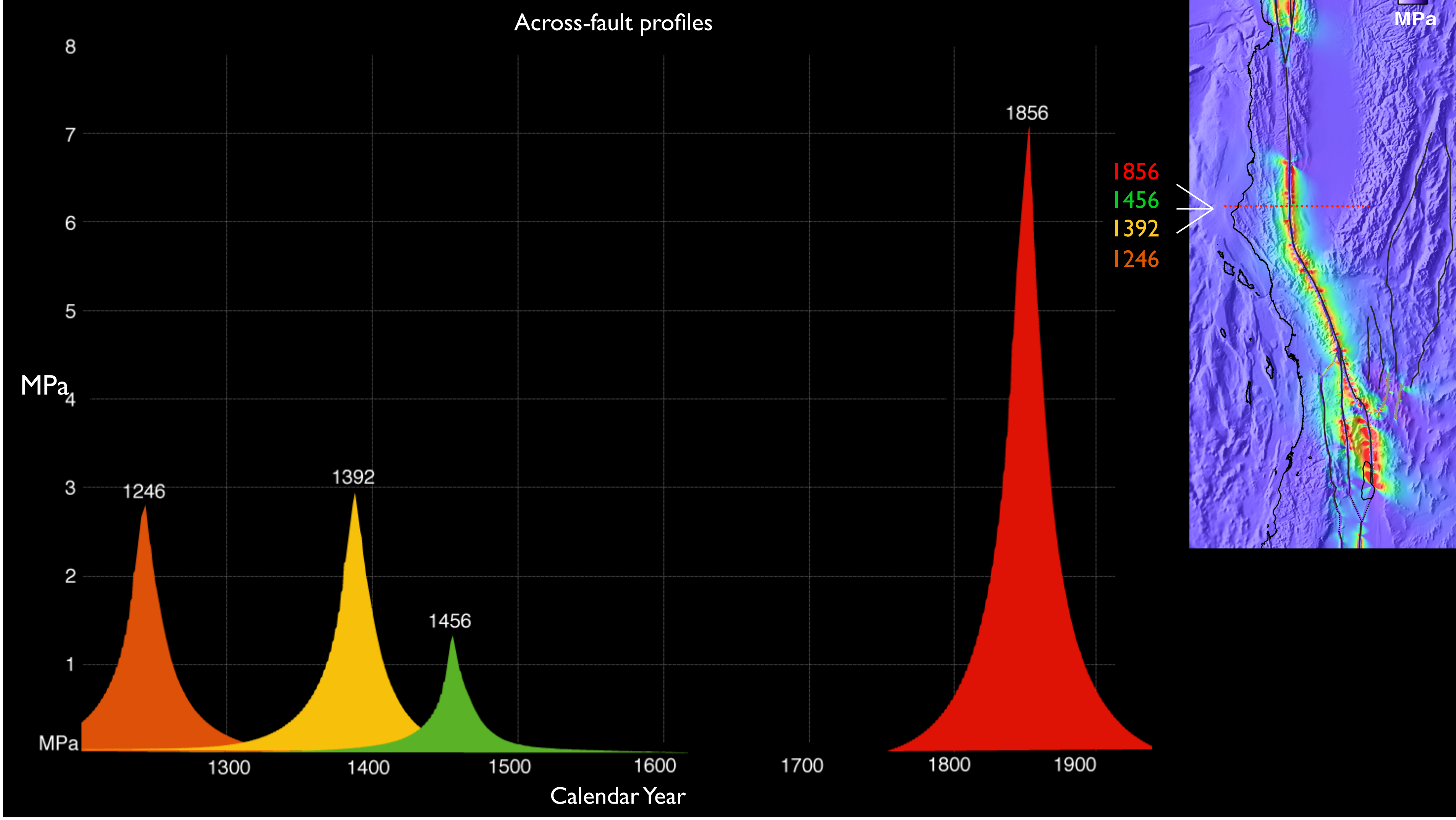




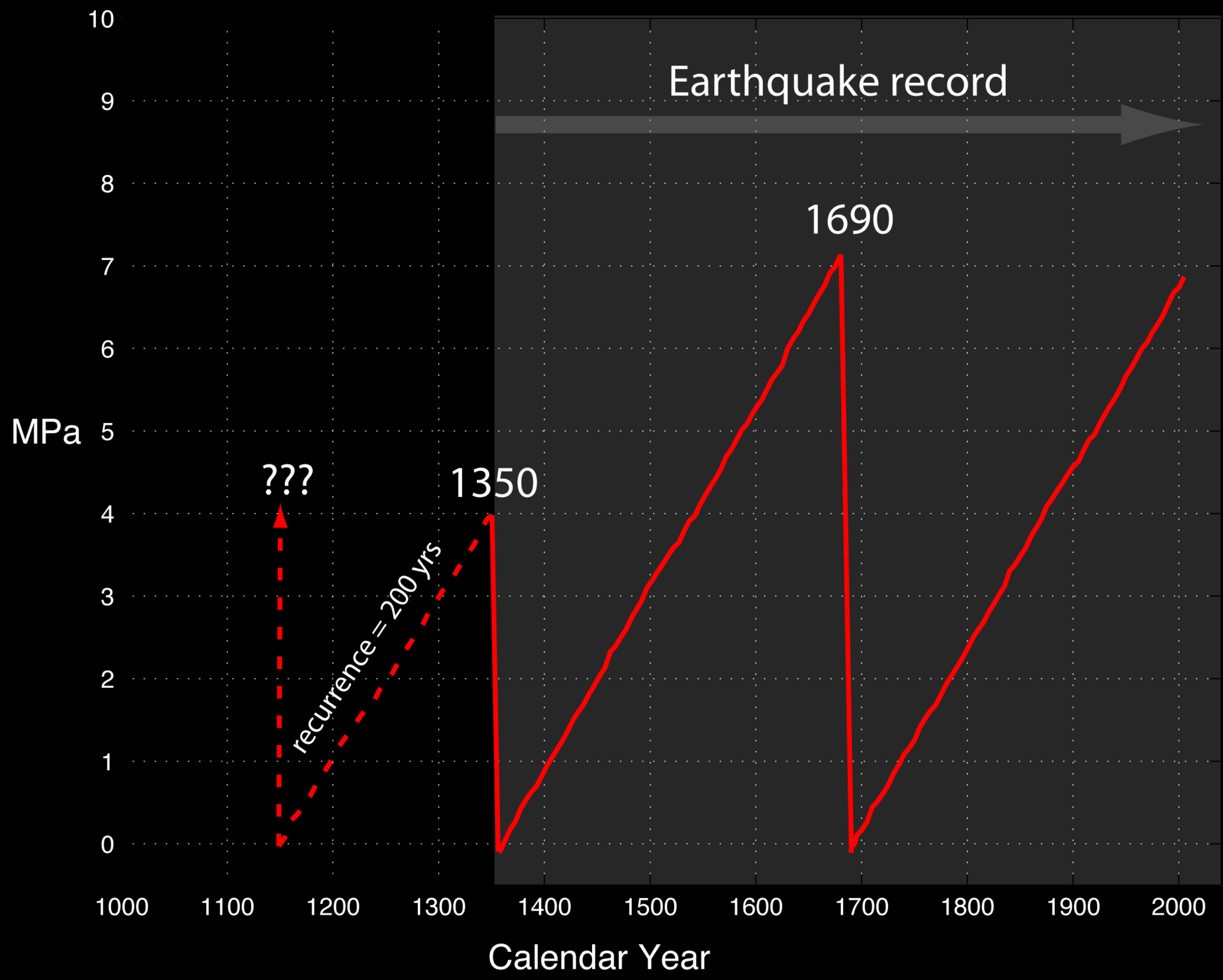
Hindcast Stress Estimates: North Coast Segment



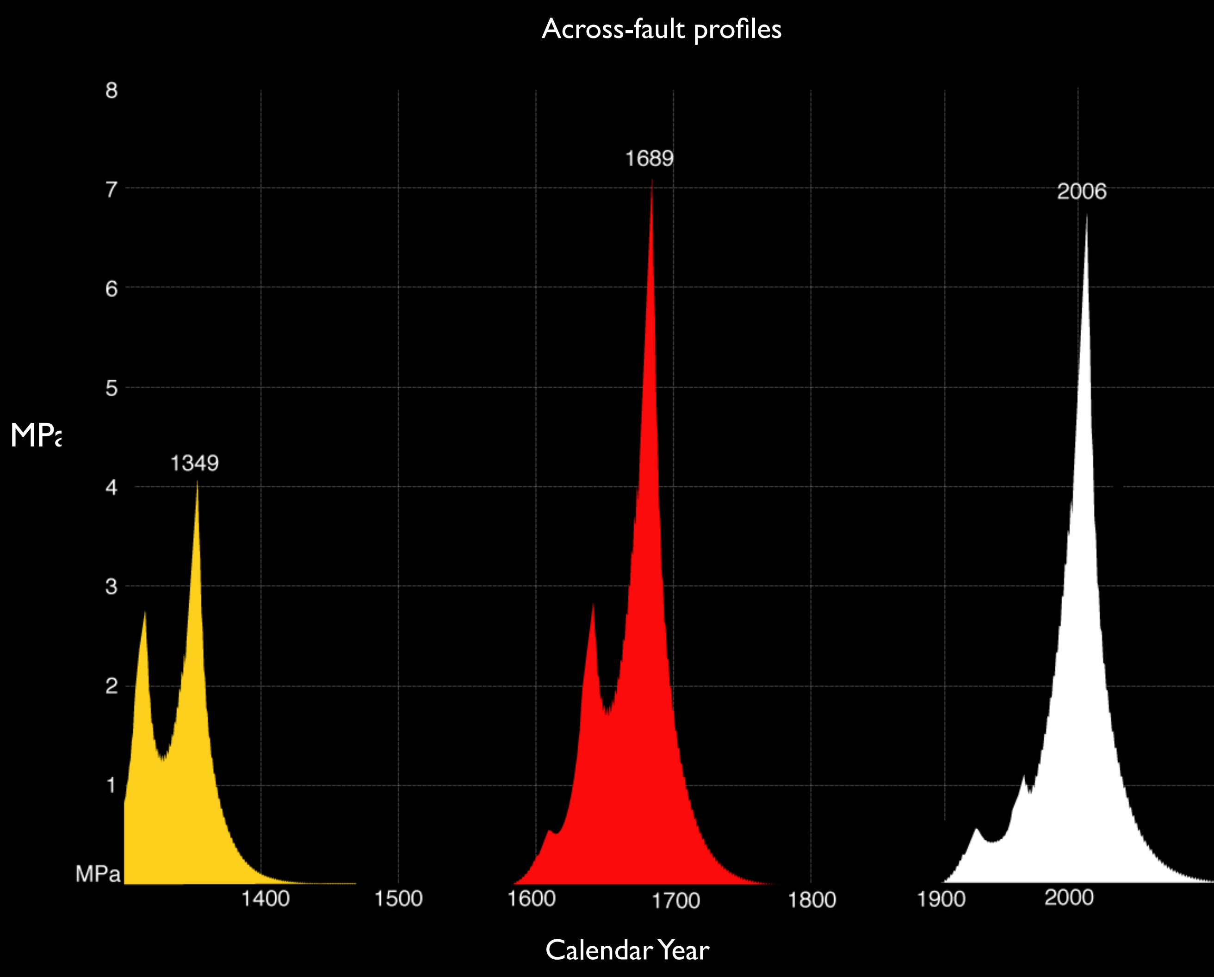
Hindcast Stress Estimates: Carrizo Segment

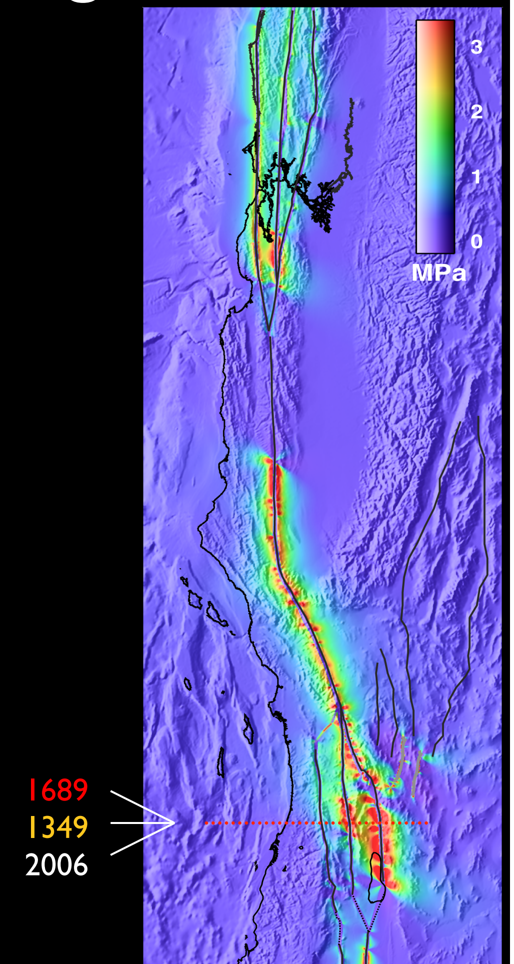


Stress Accumulation Time Series: Coachella Segment

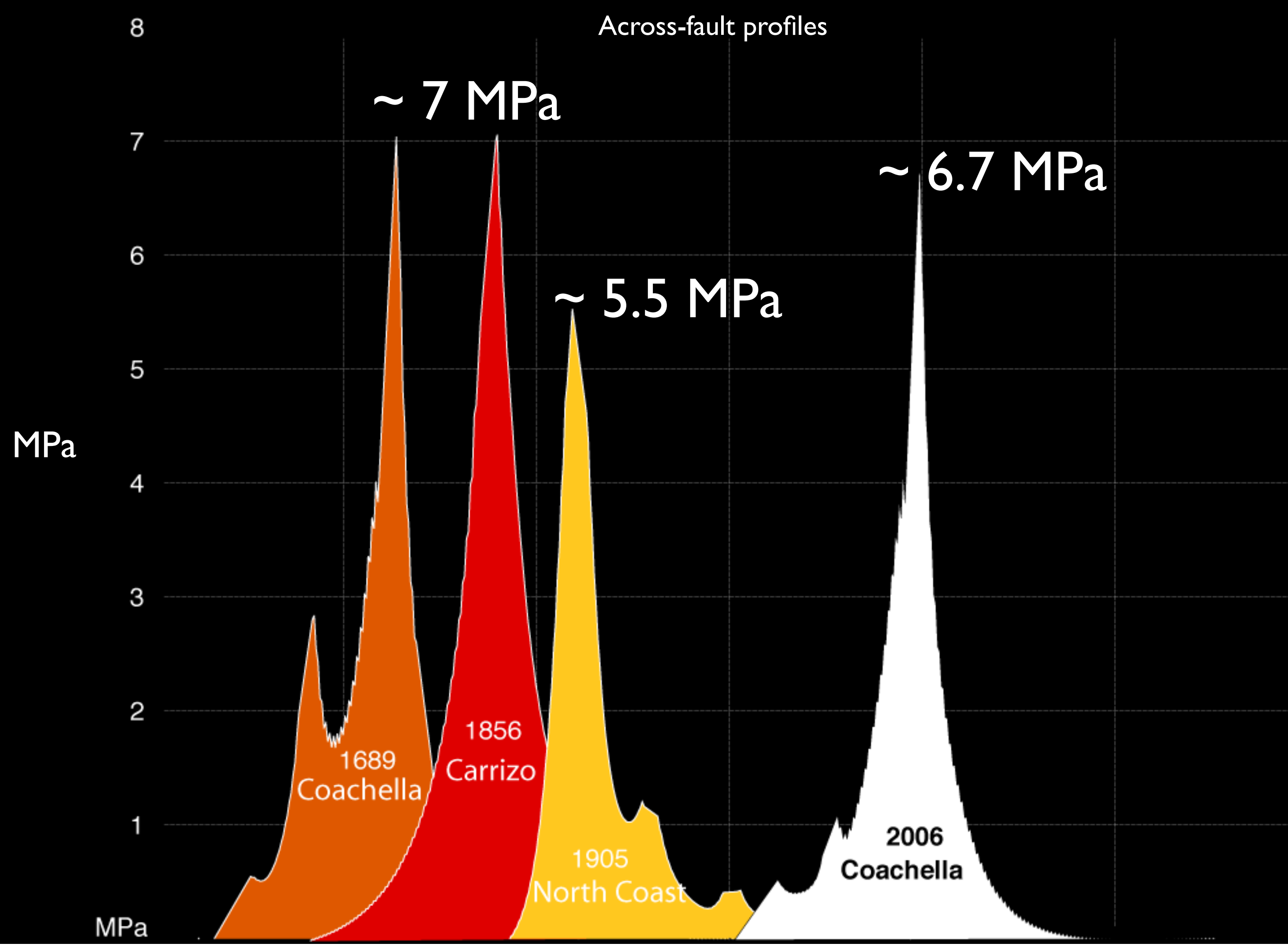


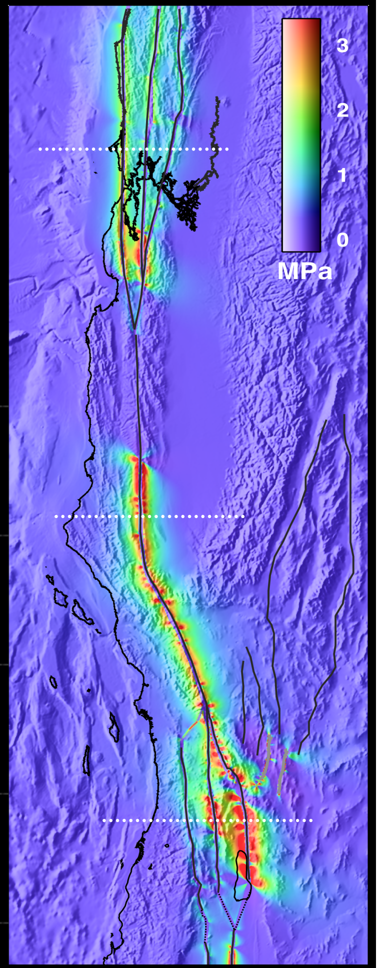
Hindcast Stress Estimates: Coachella Segment

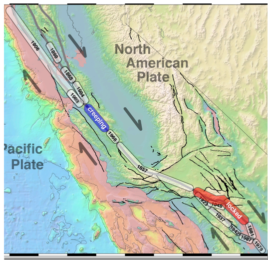




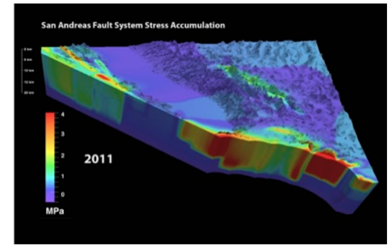
Past & Present Stress Estimates



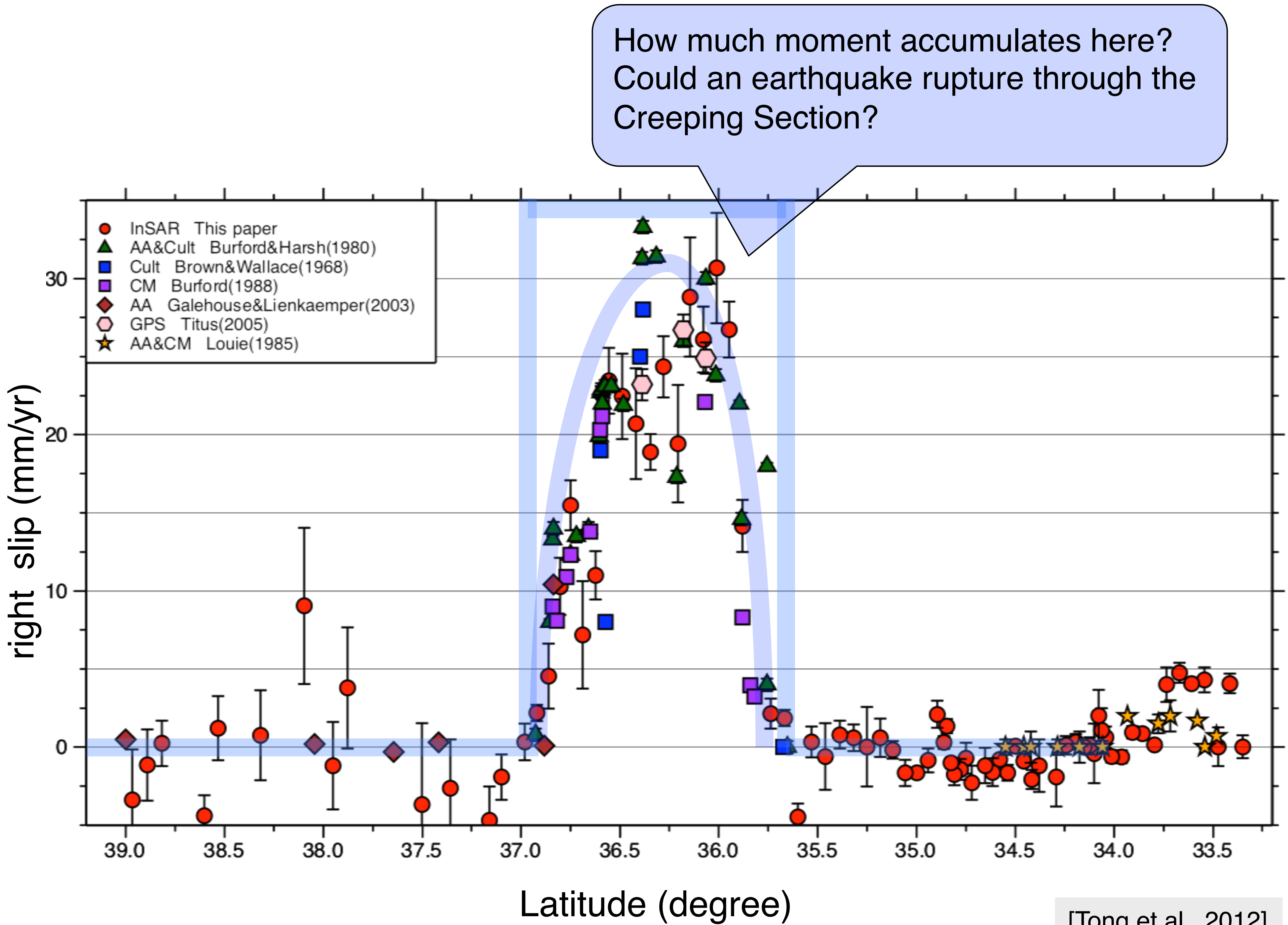




Conclusions

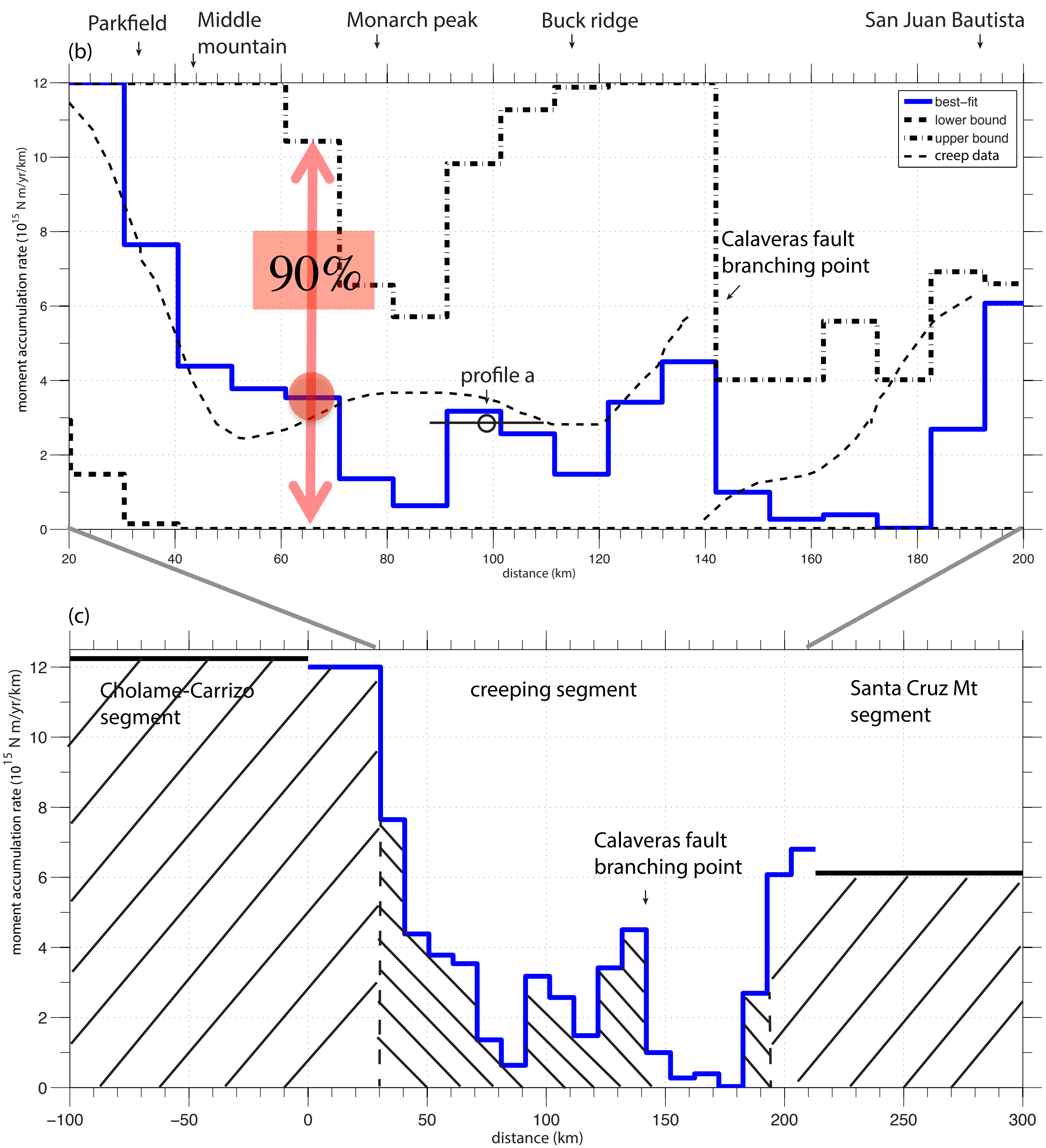


- Combined GPS (far-field) and InSAR (near field) provide a direct measure of moment accumulation rate.
- Block models do not include earthquake cycle effects and provide incorrect vertical motions.
- We have developed a semi-analytic 4-D earthquake cycle model that is fast enough for large-scale inversions with 2000 year simulations.
- Geodetic and geologic slip rates agree when earthquake cycle model is used.
- In the past earthquakes have occurred when the Coulomb stress reached ~7 MPa.
- ALOS-2 and Sentinel-1 will provide a second InSAR look direction for tighter bounds on moment accumulation rate.
- code available at http://topex.ucsd.edu/body_force



[Tong et al., 2012]

bounds on moment accumulation rate in the "creeping" segment



[*Tong et al.*, 2015]