Lithospheric Dynamics:
Types of topics covered in this class

• obvious signals
  - heat flow, depth, and geoid height versus age
  - does hydrothermal circulation really transport 10 TW?

• inferred signals
  - lithospheric thickness and strength versus age
  - swell-push force and global stress from the geoid

• mysterious signals
  - details of 3-D plate shrinkage
  - are gravity lineaments and volcanic ridges due to lithospheric shrinkage?
  - are transform faults thermal contraction cracks?
global heat budget

- 25-15 TW
- 44 TW
- 7.5 TW
- 3-13 TW

- conduction
- convection
- mantle
- lithosphere
- core
oceanic lithosphere dominates mantle convection

largest surface area

greatest temperature drop across TBL = largest density contrast

> 1/2 of heat escapes in young oceanic lithosphere
thermal expansion

volumetric expansion

\[ \frac{\Delta V}{V} = \alpha \Delta T \quad \text{or} \quad \frac{\Delta \rho}{\rho} = -\alpha \Delta T \]

\( \alpha \) - thermal expansion coefficient \( \sim 3 \times 10^{-5} \, ^\circ C^{-1} \)

linear expansion

\[ \frac{\Delta l}{l} = \alpha_l \Delta T \]

\( \alpha_l \approx \frac{\alpha}{3} \)

thermal stress develops when

\[ \nabla (\Delta T) \neq 0 \]
obvious signals

- depth versus age
- heat flow versus age
- geoid height versus age
depth vs age $\implies d(t) = \frac{-\alpha \rho_m}{\rho_m - \rho_w} \int_0^L Tdz \implies d(t) \approx 2500 + 350t^{1/2}$

Fig. 1. Plot of mean depth in the North Pacific versus the square root of age. Numbers at the bottom of the figure denote selected Cenozoic and Mesozoic magnetic anomalies [from Parsons and Sclater, 1977].
heat flow vs age \[ \Rightarrow \quad q(t) = k \frac{\partial T}{\partial z} \quad \Rightarrow \quad q(t) \approx 480t^{-1/2} \]
What is the global heat output of the Earth?

How do we interpret this discrepancy?

A) The other 10 TW is transferred by hydrothermal circulation [Lister, 1972; Williams et al., 1974; Sleep and Wolery, 1978, Anderson and Hobart, 1976; Stein, 1995]

B) The other 10 TW does not exist and the total heat output from the Earth is < 34 TW [Hofmeister and Criss, 2005].
conservation of energy

\[ \rho_m C_P \mathbf{v} \cdot \nabla T = \nabla \cdot \mathbf{q} \]

thermal isostasy

\[ d(t) = \frac{-\alpha \rho_m}{\rho_m - \rho_w} \int_0^L T dz \]

\[
(q_b - q_u) = \frac{(\rho_m - \rho_w)C_p}{\alpha} (\mathbf{v} \cdot \nabla d)
\]

heat = constant \times scalar subsidence rate
heat flow related to subsidence rate

\[(q_b - q_u) = \frac{(\rho_m - \rho_w)C_p}{\alpha} \nabla A \cdot \nabla d \frac{\nabla A \cdot \nabla d}{\nabla A \cdot \nabla A}\]
4 TW cannot be observed with this method because isostatic assumption fails at the ridge axis.

Largest uncertainty related thermal expansion coefficient is $4.2-2.9 \times 10^{-5}$.

Possible range is 42-51 TW (total includes the 4TW not observed).
Mueller, personal communication 2006
obvious signals - summary

heat flow versus age

- surface temperature gradient
- noisy, observations $\ll$ model

\[ q_s(t) = k \frac{\partial T}{\partial z} \]

depth versus age

- integrated temperature
- observations = model

\[ d(t) = \frac{-\rho_m}{\rho_m - \rho_w} \int_0^L \alpha T \, dz \]

geoid height versus age

- first moment of temperature
- dominated by mantle geoid, observations $\sim$ model

\[ N(t) = \frac{-2\pi G \rho_m}{g} \int_0^L \alpha T z \, dz \]
Inferred signals

- lithospheric strength versus age (see Watts, 2001)

- swell-push force and global stress from the geoid
Hawaiian-Emperor seamount chain

Plate kinematics

Plate Mechanics (flexure)

Sandwell & Smith 1997 (offshore) + Woollard et al 1966 (onshore)
Gravity anomalies and crustal structure at Oahu/Molokai

Watts & ten Brink (1989)
Estimating $T_e$

$T_e$ can be estimated by comparing the amplitude and wavelength of the observed gravity anomaly to the predicted anomaly based on an elastic plate model. The minimum in the RMS difference between observed and calculated gravity anomaly indicates a ‘best fit’ $T_e \sim 30$ km.
Topography seaward of the Kuril Trench

Distance to bulge ~ 120-140 km
$T_e ~ 30$ km
Relationship between oceanic $T_c$ and plate and load age

Inferred signals

- lithospheric strength versus age (see Watts, 2001)
- swell-push force and global stress from the geoid
Plate Driving Forces on Earth

\[ F_S - \text{swell push} = -(g^2/2\pi G) N_s \]

\[ F_D - \text{drag} \]

\[ F_T - \text{trench pull} \]

Forsyth and Uyeda, GJRAS, 1975

[Parsons and Richter, 1980; Dahlen, 1981; Fleitout and Froidevaux, 1982; 1983]

trench pull \(\approx 3 \times\) ridge push?
swell-push force

\[ F_s = \int_0^L \Delta P(z) \, dz \]
swell push = geoid height

- Assume: isostatic compensation and $\lambda >> 2\pi L$

- swell push
  \[ F_s = \int_o^L \Delta P(z)dz = [\Delta P(z)]_o^L - \int_o^L z \frac{\partial \Delta P}{\partial z} dz = g \int_o^L \Delta \rho \, z \, dz \]

- geoid height
  \[ N = -\frac{2\pi G}{g} \int_o^L \Delta \rho(k, z) \frac{e^{-2\pi |k| z}}{2\pi |k|} \, dz \approx -\frac{2\pi G}{g} \int_o^L \Delta \rho \, z \, dz \]

\[ F_s = \frac{-g^2}{2\pi G} N \quad \text{and} \quad \mathbf{f} = \frac{-g^2}{2\pi G} \nabla N \]
Swell-push force is independent of compensation mechanism!!

assumptions
local compensation
long wavelength
($\lambda > 2\pi L$)

$$\vec{f}_s = \frac{-\nu}{(1 - \nu)} \frac{g^2}{2\pi G L} \Delta N$$

body force in thin elastic plate or shell
stress in a spherical shell
(modified from Banerdt, JGR, 1986)

\[ \vec{f} = \frac{-\nu}{(1-\nu)} \frac{g^2}{2\pi GL} \nabla N \quad \text{poloidal body force in thin shell} \]

\[ \tau_{\theta\theta} + \tau_{\phi\phi} - 2\tau_{rr} = \frac{2\nu}{(1-\nu)} \frac{g^2}{2\pi GL} \left[ \frac{l(l+1)}{l(l+1)-2} \right] N_l^m \quad \text{differential stress} \]

\[ \tau_{\theta\theta} + \tau_{\phi\phi} - 2\tau_{rr} \cong \frac{2\nu}{(1-\nu)} \frac{g^2}{2\pi GL} N \]

\[ N=120 \text{ m produces 315 MPa in a 50 km thick lithosphere} \]
Geoid Height (EGM96 - Lemoine et al., 1998)
stress from geoid (EGM96)
failed experiment - give up!
\[ \text{N} = \text{N}_{\text{swell}} + \text{N}_{\text{convection}} \]

- Earth \( N_{\text{convection}} > N_{\text{swell}} \)

- Assume:
  - \( N_{2,0} = 0 \);
  - For degrees 2-8, \( N_{\text{swell}} \) is correlated with the topography (4m/km);
  - For degrees > 8, \( N \) unchanged.

- Assume ridges are weak so deviatoric stress should be small and slightly extensional (15 MPa over 15km thick plate).

- Fit a harmonic spline model to residual geoid at ridges to enforce the weak-ridge boundary condition.
$N_{swell}$ - coherent with topography degrees 2-8
-15 MPa extensional stress at ridges
inferred signals - summary

• swell-push signal in geoid is contaminated by mantle convection signal

• global stress = slab pull + swell push + drag

• geoid height provides a lower bounds on stress in the lithosphere and crust  ➔ stress > 75 MPa in 50 km thick plate

• Can plate driving forces and 3-D crustal stress be estimated from?
  global geoid
  locations of ridges and transform faults - oceans
  short λ, global topography
  World Stress Map - continents
mysterious signals

- details of 3-D plate shrinkage

- are gravity lineaments and volcanic ridges due to non-uniform lithospheric shrinkage?

- are transform faults thermal contraction cracks?
thermoelastic stress

linear expansion

\[ \frac{\Delta l}{l} = \alpha_i \Delta T \]
\[ \alpha_i \approx \frac{\alpha}{3} \]

thermal stress

\[ \sigma = \alpha_i E \Delta T \approx 300 \text{ MPa} \]
\[ \Delta T = 450 \, ^\circ\text{C} \]
\[ \alpha_i = 10^{-5} \, ^\circ\text{C}^{-1} \]
\[ E = 65 \text{ GPa} \]
thermal bending moment in a cooling plate
[Parmentier and Haxby, 1986; Wessel, 1992]

**A cooling, growing halfspace**

Fig. 3. In a cooling, growing plate the thermal contraction will always increase with depth. This is easily shown by approximating the thickening of the mechanically strong lithosphere by adding thin layers to the base of the plate. Each new layer is emplaced at the temperature $T_i$ (which controls the transition from brittle to ductile deformation) and will eventually cool off to some lower temperature $T_0$. The first layer emplaced at time $t = 0$ will have cooled off somewhat by time $t = 1$ when

**Static Plate Cooled from Above**

$\frac{\partial T}{\partial t} = \frac{T - T_0}{\alpha} + \frac{1}{1 - v} \frac{\partial \sigma}{\partial x}$

$M_{th} = \frac{1}{2} \rho c a \left( T_0 - T \right)$

**Growing Plate Cooled from Above**

$\frac{\partial T}{\partial t} = \frac{T - T_0}{\alpha} + \frac{1}{1 - v} \frac{\partial \sigma}{\partial x}$

$M_{th} = \frac{1}{2} \rho c a \left( T_0 - T \right)$

Fig. 5. (a) The upper panel shows the development of thermal stresses in a cooling slab. As it cools from above, thermal stresses will accumulate as shown in the upper left diagram, leading to bending stresses that will force the plate to flex upward. (b) The lithosphere differs in that its thickness increases with time. The lower panel portrays the development of thermal stresses in a growing, cooling plate. Here the material added to the bottom will always cool most rapidly, setting up stresses that will flex the plate downward.
thermal bending moment in finite-strength lithosphere

[Wessel, 1992]
• gravity lineations are common on the Pacific plate

• volcanic ridges are in troughs of gravity lineations

• thermoelastic model predicts amplitude and spacing of gravity lineations versus plate age

• gravity lineations and volcanic ridges are warps and cracks in the plate due to thermal contraction of the lithosphere