CHAPTER 8

1

Figure 8.7 shows a dry-atmosphere zenith correction of about 2.35 m for a wavelength of 1 µm, so for a path that passes through 1-0.26 = 0.74 atmospheres at an angle of 45˚, the estimated correction is

$$\frac{2.35\times0.74}{\cos 45^{\circ}} = 2.46\ \mathrm{m}\quad .$$

Figure 8.8 shows a water vapour correction of about 0.35 m per metre of precipitable water, or about 0.018 m for a vertical path through the entire atmosphere. If we assume that all of the water will be found below 10000 m, the only correction needed is to multiply by 1/cos(45˚) to account for the oblique path, so the correction needed for water vapour is approximately 0.025 m.

2

The time taken for one complete back-and-forward scan of the mirror is $1/f_s$, during which the aircraft moves forward a distance v/f_s. This is the *maximum* distance between adjacent samples in the along-track direction, so the *average* distance is $v/2f_s$.

$$s_a = \frac{v}{2f_s}$$

The time taken to make one sweep of the swath is $1/2f_s$, so the number of samples collected across the swath is $f_0/2f_s$. The swath width w is $2H\theta$ (we are using the small-angle approximation), so the cross-track spacing of the samples is

$$s_c = \frac{2H\theta}{f_0/2f_s} = \frac{4H\theta f_s}{f_0} \quad .$$

(i) The condition on the across-track sampling can be rewritten as

$$s_c = \frac{2wf_s}{f_0} < 1\,\text{m}$$

so the swath width w is given by

$$w < \frac{16650\,\text{m/s}}{f_s} \qquad \text{(a)}$$

where we have used the fact that f_0 = 33300 Hz. Thus the condition that w should be as large as possible is equivalent to requiring the scan frequency f_s is as small as possible.

The condition on the along-track sampling interval, s_a < 1 m, can be rewritten using our earlier result, and the fact that the speed v = 70 m, as

$$f_s > 35\,\text{Hz} \quad . \qquad \text{(b)}$$

This shows that the lowest consistent value of the scan frequency is 35 Hz, and from (a) the maximum swath width w is 476 m.

(ii) There are two constraints on the value of the scan angle θ:

$$\theta < 0.35\,\text{radian}$$

and

$$\theta < \frac{14\,\text{Hz}}{f_s} = 0.4\,\text{radian} \quad .$$

Thus the maximum value of the scan angle is 0.35 radians. Since the value of $H\theta$ is known to be 237.9 m, this gives the minimum value of H as 680 m.

It is preferable to minimise the flying height in order to maximise the signal-to-noise ratio of the return signal, which will tend to increase the precision of the measurements.

3

The answer to the first part of the question follows the derivation given in section 8.2.1.

At a pulse repetition frequency p and platform speed v, the number of pulses within the footprint is

$$N = \frac{2\left(c H t_p\right)^{1/2} p}{v} \quad .$$

The range accuracy of a single pulse can be taken as

$$\frac{c t_p}{2}$$

so the accuracy obtained by averaging the N pulses across the footprint is given by

$$\frac{c t_p}{2 N^{1/2}} = \left(\frac{c^3 t_p^3 v^2}{64 H p^2} \right)^{1/4}$$

as required.

Using the values given in the problem, and taking $v \approx 7$ km/s and $H \approx 800$ km, we can estimate the best vertical resolution as around 2 cm.

4

From equation (8.16), the time-differential of the return pulse is the convolution of the transmitted pulse with the surface height distribution converted to a time distribution through the factor $c/2$. Thus we have two Gaussians, with widths to the $1/e$ points of 3.00 ns and 6.67 ns. The convolution of these is another Gaussian, with a width to the $1/e$ points of $(3.00^2 + 6.67^2)^{1/2} = 7.31$ ns. Thus the rise time of the return pulse from 8% to 92% of the final value is 7.31 ns.

5

Suppose the radar is located a height H above the mean surface, and consider a scatterer a radial distance r from the nadir point and height h above the mean surface. Its distance from the radar is approximately

$$H+\frac{r^2}{2H}-h$$

so a signal received from the scatterer at time t must have been emitted at time-differ

$$t-\frac{2}{c}\left(H+\frac{r^2}{2H}-h\right) \quad .$$

If we can write the number of scatterers between r and $r + dr$ and between h and dh as

$$k\, r\, f(h)\, dr\, dh$$

(the $r\,dr$ part accounts for the area of surface and the $f(h)\,dh$ part for the distribution of scatterers with height), the received power at time t is given by

$$P_r=k\int_{r=0}^{\infty}\int_{k=-\infty}^{\infty} P_t\left(t-\frac{2H}{c}-\frac{r^2}{Hc}+\frac{2h}{c}\right) r\, f(r)\, dr\, dh \quad .$$

Change the r variable to

$$s=t-\frac{2H}{c}-\frac{r^2}{Hc}$$

so that the expression for the received power becomes

$$P_r=\frac{kHc}{2}\int_{s=-\infty}^{t-2H/c}\int_{h=-\infty}^{\infty} P_t\left(s+\frac{2h}{c}\right) f(h)\, ds\, dh \quad .$$

Differentiating this expression with respect to s gives the required result.

6

(i) The propagation delay in each case is proportional to the column-integral of the density of the molecular species responsible for it. In the case of the dry atmosphere component, this is proportional to the pressure difference between the top and bottom of the path; for a path that originates above the atmosphere it is thus simply proportional to the pressure at the bottom of the path. Water vapour, on the other hand, is not well mixed in the atmosphere so that the density of the vapour along the path is not necessarily proportional to the atmospheric density at that point. In such a case, the column-integral of the density can be usefully expressed as the depth of precipitable water.

(ii) The proof that the ionospheric delay is given by

$$\frac{e^2 N_t}{8\pi^2 \epsilon_0 m_e f^2}$$

is given in section 8.2.6 and earlier.

(iii) For the conditions given in the question, the dry-atmosphere delay is

$$2.33 \times 0.970 = 2.260\,\mathrm{m}$$

and the water vapour delay is

$$7.10 \times 0.15 = 1.065\,\mathrm{m} \quad .$$

The ionospheric delay can be rewritten as

$$4.031 \left(\frac{N_t}{10^{17}\,\mathrm{m}^{-3}} \right) \left(\frac{f}{\mathrm{GHz}} \right)^{-2} \mathrm{m} \quad .$$

Thus the measurement at 3.2 GHz is consistent with a true range R to the surface of

$$R = 793125.20 \pm 0.10 - 2.260 - 1.065 - 0.394\,N_t = 793121.88 \pm 0.10 - 0.394\,N_t$$

and the measurement at 13.6 GHz is consistent with

$$R = 793123.35 \pm 0.05 - 2.260 - 1.065 - 0.022\,N_t = 793120.03 \pm 0.05 - 0.022\,N_t$$

where in both equations R is measured in metres and N_t is measured in units of 10^{17} m^{-3}. To find R, we eliminate N_t to give

$$R = 793119.92 \pm 0.05\,\mathrm{m} \quad .$$

To find N_t we equate the two expressions to give

$$N_t = (5.0 \pm 0.3) \times 10^{17}\,\mathrm{m}^{-3} \quad .$$

7

First, find the Fourier transform of the original chirp signal:

$$a(\omega)=\frac{1}{2\pi}\int_{-T/2}^{T/2} e^{i\left(\omega_0 t'+\frac{\Delta\omega}{2T}t'^2\right)} e^{-i\omega t'}\,dt' \quad .$$

The signal that emerges from the delay-line is therefore, in frequency representation,

$$a'(\omega)=\frac{1}{2\pi}\int_{-T/2}^{T/2} e^{i\left(\omega_0 t'+\frac{\Delta\omega}{2T}t'^2-\frac{\omega_0 T}{\Delta\omega}\omega-\frac{T}{2\Delta\omega}\omega^2\right)} e^{-i\omega t'}\,dt' \quad .$$

To retransform into the time domain will require the following operation:

$$f(t)=\int_{-\infty}^{\infty} a'(\omega)e^{i\omega t}\,d\omega \quad .$$

In order to evaluate this integral we first rearrange it:

$$f(t)=\frac{1}{2\pi}\int_{-T/2}^{T/2} e^{i\left(\omega_0 t'+\frac{\Delta\omega}{2T}t'2\right)}\int_{-\infty}^{\infty} e^{i\left(\left[t-t'-\frac{\omega_0 T}{\Delta\omega}\right]\omega-\frac{T}{2\Delta\omega}\omega^2\right)}\,d\omega \quad .$$

The second integral, over *ω*, can be evaluated by completing the square. It is

$$\sqrt{\frac{2\pi i\Delta\omega}{T}}\,e^{-i\frac{\Delta\omega}{2T}\left(t-t'-\frac{\omega_0 T}{\Delta\omega}\right)^2}$$

so inserting this result into the first integral and simplifying it gives

$$f(t)=\frac{1}{2\pi}\sqrt{\frac{2\pi i\Delta\omega}{T}}\,e^{i\left(\frac{-\omega_0^2 T}{2\Delta\omega}+\omega_0 t-\frac{\Delta\omega}{2T}t^2\right)}\int_{-T/2}^{T/2} e^{i\frac{\Delta\omega}{T}t'}\,dt' \quad .$$

The integral over t' is just

$$T\,\mathrm{sinc}\left(\frac{t\Delta\omega}{2}\right) \quad ,$$

so this is the modulating function. It is centred at *t*=0 and falls to zero when

$$t=\pm\frac{2\pi}{\Delta\omega} \quad .$$

The only one of the phase terms (before the integral) that is not negligible is the term

$$e^{i\omega_0 t}$$

which is the carrier wave term.