CHAPTER 7

1

We can calculate the effective areas from the directivities using equations (7.5) and (7.9):

$$A_e = \frac{\lambda^2 D}{4\pi} \quad . \qquad \text{(a)}$$

For the rectangular aperture the directivity in dB is given by

$$11 + 10\log_{10}(ab/\lambda^2)$$

so the directivity is

$$D = 10^{1.1}\frac{ab}{\lambda^2} = \frac{12.6\,ab}{\lambda^2}$$

and, from (a), the effective area is 1.00 *ab*, equal to the geometrical area.

For the circular paraboloidal antenna the directivity is given by

$$10\frac{d^2}{\lambda^2}$$

so, from (a), the effective area is 0.80 d^2. The cross-sectional area of the dish is $(\pi/4)d^2 = 0.79\ d^2$, so the areas are again very similar.

2

The discussion in section 7.1.3 shows that the phase shift between adjacent antenna elements is

$$k\,d\sin\theta_0$$

where k is the wavenumber of the radiation, d is the separation of the elements and θ_0 is the beam steering angle. Since the addition of integer multiples of 2π to the phase makes no physical difference to it, it follows that the antenna will also respond in the direction θ, where

$$\sin\theta=\sin\theta_0+\frac{n\lambda}{d}$$

where n is any integer.

(a) In this case, $\lambda/d = 1.5$, so provided that

$$\sin\theta_0>0.5$$

the antenna will have a second response corresponding to $n = -1$.

(b) In order to avoid these multiple responses, λ/d must be at least 2. In this case, when $\sin\theta_0$ has its largest possible value of +1, the next largest value of $\sin\theta$ is less than -1 and thus cannot occur.

3

The observed brightness temperature will be an area-weighted average of the different species present in the antenna's footprint. (This follows from the Rayleigh-Jeans approximation.) If we write f_w, f_f and f_m for the proportions of open water, first-year ice and multi-year ice present in the footprint, we obtain the following three equations:

$$f_w + f_f + f_m = 1$$

(we assume that no other species are present in the footprint)

$$80 f_w + 252 f_f + 200 f_m = 180$$

and

$$119 f_w + 253 f_f + 168 f_m = 180$$

These equations are straightforward to solve. Here the solution is calculated using Octave:

```
>>> A=[1 1 1;80 252 200;119 253 168]
A =

     1     1     1
    80   252   200
   119   253   168

>>> v=[1; 180; 180]
v =

     1
   180
   180

>>> x=A\v
x =

   0.30371
   0.31626
   0.38003

>>>
```

This shows the fractions to be 0.304 for water. 0.316 for first-year ice and 0.380 for multi-year ice.

4

If the fraction of the footprint occupied by ice is *f*, the antenna temperature, in kelvin, is

$$253\,f + 119(1-f) = 119 - 134\,f \quad .$$

Setting this equal to 119-0.9, to represent the smallest detectable difference from the brightness temperature due to water alone, we find that *f* = 0.00672.

The beam solid angle is given by

$$\Omega_A = \frac{\lambda^2}{A_e} = 2.19\times10^{-4}\,\mathrm{sr}$$

so the area of the footprint on the earth's surface is

$$800^2\times2.19\times10^{-4}\,\mathrm{km}^2 = 140\,\mathrm{km}^2 \quad .$$

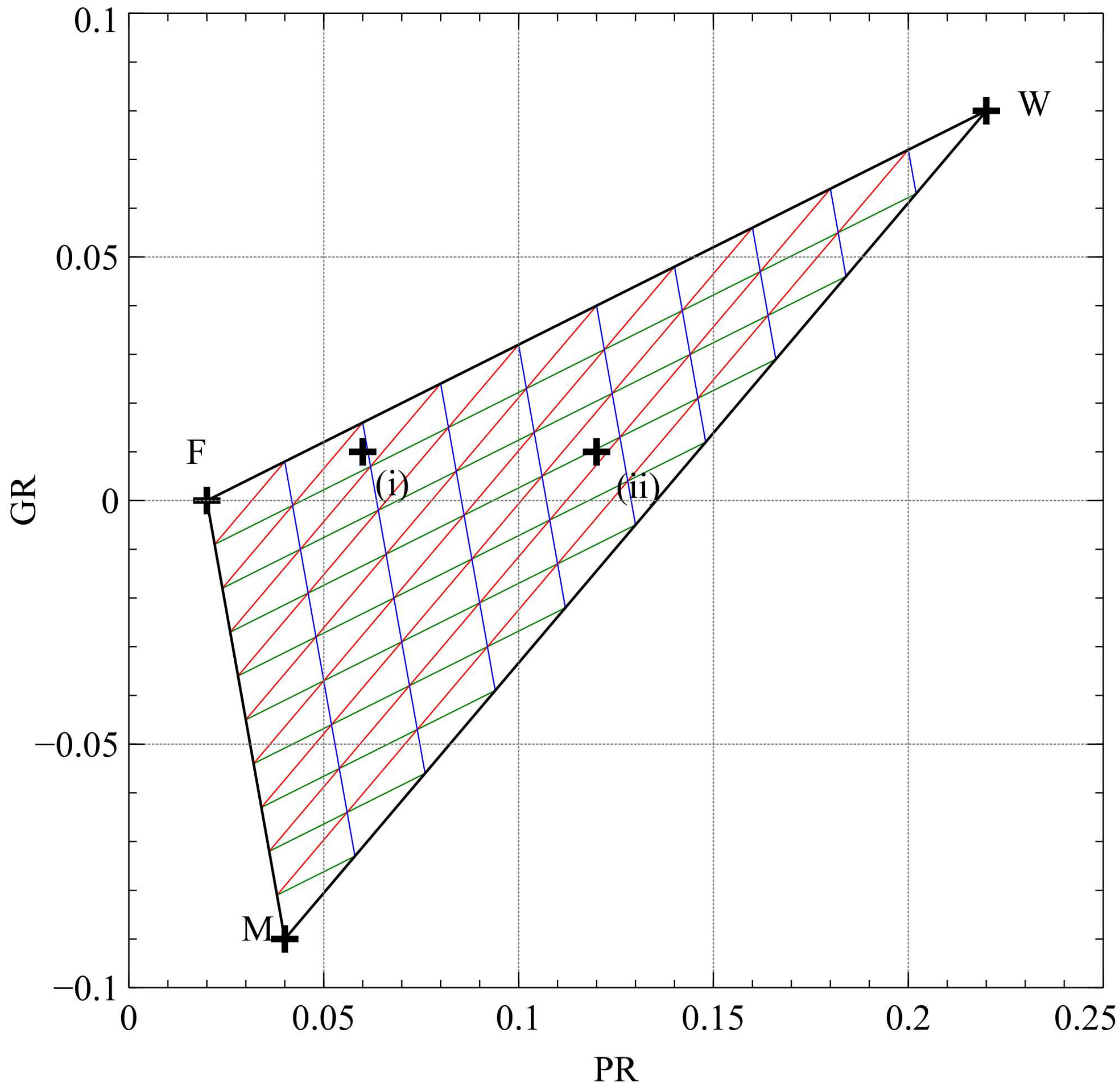
Thus the area of the smallest detectable floe is

$$0.00672\times140\,\mathrm{km}^2 = 0.94\,\mathrm{km}^2 \quad .$$

5

First, plot the points W, M and F corresponding to water, first-year and multi-year ice and connect these points by straight lines (shown in black). These lines represent 0% concentration of the corresponding material: for example, the line connecting M and W represents 0% first-year ice.

Next, draw regularly spaced lines parallel to these 0% contours. In the diagram here, the blue lines show the water concentration in 10% steps, the red lines show the first-year ice concentration and the green lines the concentration of multi-year ice, again both in steps of 10%.



Now plot the points (i) and (ii) corresponding to the two cases described in the problem. By measuring from the graph, and comparing with the concentration contours, we find the following concentrations:

(i) 19% water, 75% first-year ice, 6% multi-year ice
(ii) 46% water, 22% first-year ice, 31% multi-year ice

A low value of PR implies that the emissivities of the two kinds of ice are similar in both polarisations, as can be seen in figure 7.9. A low value of GR implies that the emissivity of first-year ice is similar for 19V and 37V radiation. Again, this can be seen in figure 7.9.

6

Since we are considering microwave radiation, we can assume the Rayleigh-Jeans approximation to be valid in which case we can use equation (3.96). The optical thickness between the surface and some altitude *z* is

$$\tau' = \int_0^z \gamma \, dz = \tau \left(1 - e^{-\beta z}\right)$$

and if we substitute this expression into (3.96) we can identify the weighting function $a(z)$ as

$$a(z) = \tau \beta e^{-\beta z - \tau e^{-\beta z}} \quad .$$

This is slightly easier to deal with if we take logarithms:

$$\ln(a(z)) = \ln \tau + \ln \beta - \beta z - \tau e^{-\beta z} \quad .$$

To find the value of *z* at which $a(z)$ is maximum, we differentiate this expression with respect to *z*:

$$\frac{\partial \ln(a(z))}{\partial z} = -\beta + \tau \beta e^{-\beta z} \quad .$$

Setting this equal to zero gives

$$z = \frac{\ln \tau}{\beta}$$

as required.

To show that the effect of changing τ (at fixed β) is to shift $a(z)$ along the *z*axis without change of scale, we need to show that

$$a(z - z_{0,} \tau^*) = a(z, \tau)$$

for all values of τ. Thus we need to show that the equation

$$\ln \tau + \ln \beta - \beta z - \tau e^{-\beta z} = \ln \tau^* + \ln \beta - \beta(z - z_0) - \tau^* e^{-\beta(z - z_0)}$$

can be satisfied for all values of τ. It is straightforward to demonstrate that this is valid provided that

$$\tau^* e^{\beta z_0} = \tau$$

This result implies that, provided our simple model of the height-dependence of the absorption coefficient is valid, the vertical resolution of the sounder is independent of the height being sounded.