CHAPTER 10

1

Since we require only to calculate the velocity increments, without consideration of how these are achieved, we may consider a body (rocket, satellite etc) of fixed mass m. To prove equation (10.2), consider the total energy (kinetic plus potential) of the body at A, just after it has been given velocity Δv_1. The kinetic energy is

$$\frac{1}{2} m \Delta v_1^2$$

and the gravitational potential energy is

$$-\frac{G M m}{R_E} \quad .$$

The body comes to rest at B, so its kinetic energy is zero and its potential energy is

$$-\frac{G M m}{R} \quad .$$

Equating the total energies at A and B gives the required result:

$$\Delta v_1^2 = 2 G m \left(\frac{1}{R_E} - \frac{1}{R} \right) \quad .$$

To prove equation (10.3), we need only to calculate the speed v of a body in a circular orbit of radius R. This is easily done by equating the gravitational force

$$\frac{G M m}{R^2}$$

to the centripetal force

$$\frac{m v^2}{R} \quad .$$

Thus

$$\Delta v_2^2 = \frac{G M}{R}$$

as required.

To prove equation (10.4) and (10.5) we need to find the elliptical transfer orbit between the Earth's surface and the desired circular orbit of radius R. If the velocity of the body when it is grazing the Earth's surface is Δv_1 the total energy is

$$\frac{1}{2} m \Delta v_1^2 - \frac{G M m}{R_E}$$

as before. When the body reaches B, conservation of momentum dictates that its speed must be given by

$$\frac{R_E \Delta v_1}{R}$$

so its total energy is given by

$$\frac{m R_E^2 \Delta v_1^2}{2 R^2} - \frac{G M m}{R} \quad .$$

Equating the two energies gives

$$\Delta v_1^2 = \frac{2 G R M}{R_E (R + R_E)}$$

as required for equation (10.4).

The speed of the body in the circular orbit of radius R is

$$\sqrt{\frac{GM}{R}}$$

and the speed in the elliptical orbit at B is

$$\frac{R_E}{R} \Delta v_1 = \sqrt{\frac{2 G R_E M}{R (R + R_E)}} \quad ,$$

so the increment needed at B is

$$\Delta v_2 = \sqrt{\frac{GM}{R}} - \sqrt{\frac{2 G R_E M}{R (R + R_E)}}$$

as required.

2

Equation (10.10) shows that the largest angular error will be approximately $2e$. This proves the result.

3

We are considering the descending (southbound) part of the satellite's orbit. For definiteness, let us assume that the orbit crosses the Prime (Greenwich) meridian in the southbound direction at 09:30 UT. We need to find when and where it crosses latitude 52° north, again travelling southwards. Equation (10.11.1) shows that

$$\sin\phi = \frac{\sin 52^\circ}{\sin 98.2^\circ}$$

so φ = 127.236°. (We resolve the ambiguity in taking the inverse sine by noting that the satellite must have made between a quarter and a half of an orbit around the earth since it passed through the ascending node.) Thus the satellite crosses latitude 52° earlier than it crosses the equator by a fraction

$$\frac{180-127.236}{360} = 0.14657$$

of an orbit. The orbital period is 16/233 days so this time interval is equal to

$$\frac{0.14657\times 16}{233}\,\text{days} = 14.493\,\text{minutes} \quad .$$

The time at which the satellite crosses latitude 52° is thus 09:30 – 14.493 minutes = 09:15:30 UT. However, we need to calculate the local time so we need to know the longitude of the satellite too. This is given by equation (10.11.2) as

$$l = l_0 + \operatorname{atan2}\left(\frac{\tan 52^\circ}{\tan 98.2^\circ}, \frac{\cos 127.236^\circ}{\cos 52^\circ}\right) = l_0 + \operatorname{atan2}(-0.18444, -0.98284) = l_0 - 169.371^\circ \quad .$$

Thus the satellite moved (180-169.371) = 10.629 degrees west, relative to a fixed orientation, during these 14.493 minutes. However, the Earth also rotated through

$$\frac{14.493\times 360}{1440} = 3.625$$

degrees east during the same period, so relative to the Earth's surface the satellite moved 14.254° west. Thus, at 09:15:30 UT the satellite was at longitude 14.254° east, where the local time was

$$09:15:30 + \frac{14.254}{15}\,\text{hours} = 10:12:31.$$

4

(i) The angular frequency of precession is

$$\Omega_p = -\frac{2.01280\times10^{-6}\cos 98.52\,^\circ}{1.12251^{7/2}}\,\mathrm{s}^{-1} = 1.990\times10^{-7}\,\mathrm{s}^{-1}$$

which is very close to the Earth's angular speed around the Sun. Thus the orbit is at least close to being Sun-synchronous.

(ii) The nodal period of the satellite is

$$P_N = 5069.3\times1.12251^{3/2}\left(1+\frac{1.62395\times10^{-3}(1-4\cos^2 98.52\,^\circ)}{1.12251^2}\right)\mathrm{s} = 6035.9\,\mathrm{s} \quad .$$

Thus, using equation (10.18), the number of orbits per repeat cycle is given by

$$\frac{35\times86400}{6035.9} = 501.0$$

and we conclude that the exactly repeating orbit makes 501 revolutions every 35 days.

(iii) In exactly one day, the track makes

$$\frac{501}{35} = 14.314$$

orbits. The nearest integer to this is 14, and the time taken to make exactly 14 orbits is

$$\frac{14\times35}{501} = 0.978\,\mathrm{days} \quad .$$

Thus after 14 orbits the Earth has rotated through 0.978 × 360˚ = 352.10˚ to the east, so the satellite has moved 352.10˚ west relative to the Earth's surface. This is equivalent to having moved 7.90 degrees east. Since the value of Δl is 360/501 = 0.719˚, we see that the satellite's longitude is indeed +11 Δl after one day.

Repeating the calculation for three days, we find that the number of orbits is 42.94 after exactly thee days so the nearest integer is 43. The time taken to make exactly 43 orbits is 3.00399 days, so the eastward motion of the satellite in this time is -0.00399 × 360 = -1.44˚

5

(i) We can use equations (10.19) to (10.21) to calculate the velocity of the subsatellite point relative to the Earth's surface. We also need to know the satellite's orbital period and precession rate. These are given by equations (10.13) and (10.14) as

$$P_N = 6746\,\mathrm{s}$$

and

$$\Omega_p = -4.21 \times 10^{-7}\,\mathrm{s}^{-1}$$

respectively. For simplicity, we will calculate the velocity for a point on the equator. From (10.19), we have

$$\frac{db}{d\phi} = 0.9135\,.$$

From (10.20), we find

$$v_N = 5427\,\mathrm{m\,s}^{-1}$$

and from (10.21),

$$v_E = -468 + 2417 = 1949\,\mathrm{m\,s}^{-1} \quad .$$

Thus the ascending path at the equator makes an angle of about 20˚ east of north. The descending path therefore makes an angle of about 160˚ east of north, so the acute angle between the tracks is about 40˚. This is not 'perfect' for an altimetric orbit (which would give an angle of 90˚) but is reasonable.

(ii) We need to find whether the orbital path repeats itself exactly. Equation (10.17) shows how to do this. We have P_N = 6746 s, Ω_p = -4.21 × 10^{-7} s^{-1} and Ω_e = 2π/81864 = 7.2921 × 10^{-5} s^{-1} so

$$\frac{n_1}{n_2} = \frac{P_N(\Omega_e - \Omega_p)}{2\pi} = 0.07874\,.$$

To find values of n_1 and n_2 that satisfy this equation we can use the Octave function rat, which is used to find a rational approximation:

```
>>> [n,d]=rat(0.07874,0.00001)
n =  10
d =  127
>>>
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This shows that 0.07874 ≈ 10/127, within a tolerance of 0.0001, so we conclude that the orbit repeats every ten days and the observation frequency is 0.1 measurements per day. This is not ideal for observing tidal phenomena since (for example) the 12-hour tidal period will be aliased to a frequency of zero (i.e. every observation at a particular location will be at the same phase of the tide.)