CHAPTER 4

1

At z=10000 m the temperature T = 223 K and the pressure p = 261 Pa. From equation (4.3), the density is found to be 4.08×10^{-3} kg m^{-3}.

2

We model the variation of the attenuation coefficient with height z as

$$\gamma_a = \gamma_{a,0} e^{-z/z_0}$$

where $\gamma_{a,0}$ is the sea-level attenuation coefficient and z_0 is the scale height. The total optical thickness of a vertical path through the atmosphere is

$$\tau = \int_0^\infty \gamma_a \, dz = \gamma_{a,0} z_0 \quad .$$

Taking $\gamma_{a,0}$ = 0.1 km^{-1} and τ = 0.2 thus gives z_0 = 2 km.

3

If we assume that the water droplets are all spherical with radius a and scattering cross-section πa^2, and that their number density is n, the density of the fog is given by

$$\rho = \frac{4}{3} \pi a^3 n \rho_w$$

where ρ_w is the density of liquid water, and the scattering coefficient is

$$\gamma_s = n \pi a^2 \quad .$$

Since the visibility V is defined as the distance that gives an optical thickness of 4, we must have

$$n = \frac{4}{\pi a^2 V}$$

and hence

$$a = \frac{3 \rho V}{16 \rho_w} \quad .$$

Substituting the data given in the question, we obtain a = 19 µm.

4

Assume that the water droplets are spherical with radius a. The absorption cross-section of each droplet is given by equation (4.12) and the relationship between the number density n of the droplets and the mass density ρ of the cloud is as given in the solution to the previous question. Combining these two results gives the absorption coefficient as

$$\gamma_a = \frac{9k\rho\omega\tau\epsilon_p}{\rho_w\left(\left[(\epsilon_\infty+2)(1+\omega^2\tau^2)+\epsilon_p\right]^2+\omega^2\tau^2\epsilon_p^2\right)} \quad .$$

At sufficiently low frequencies, such that $\omega\tau \ll 1$, this can be approximated as

$$\gamma_a = \frac{9k\rho\omega\tau\epsilon_p}{\rho_w\left(\epsilon_\infty+\epsilon_p+2\right)^2} \quad .$$

Now we can write $\omega = 2\pi f$ and $k = 2\pi f/c$, so the expression for the absorption coefficient becomes

$$\gamma_a = \frac{36\pi^2\tau\epsilon_p}{c\rho_w\left(\epsilon_\infty+\epsilon_p+2\right)^2}\rho f^2 \quad .$$

This shows that the absorption coefficient is proportional to the mass density and to the squate of the frequency. We need to find the constant of proportionality. Substituting the data given in the problem, we find the absorption coefficient at a frequency of 1 GHz for a mass density of 1 kg m^{-3} is 1.22×10^{-4} m^{-1}, or 0.53 dB km^{-1}. This proves the result given in the question.

To discuss the transparency or otherwise of clouds to microwave radiation, consider for example radiation at a frequency of 10 GHz incident on a dense cumulonimbus cloud. From the data given in the problem, the absorption coefficient is around 0.5 dB km^{-1}, so such a dense cloud would need to be several kilometres thick to introduce significant attenuation. The cloud would however be opaque at much higher frequencies. Much less dense clouds such as fog and haze will be transparent at all frequencies in the microwave spectrum.

5

For spherical raindrops of radius a, number density n, scattering cross-section πa^2 and sedimentation speed v, the mass density is given by

$$\rho = \frac{4}{3}\pi a^3 n \rho_w \qquad \text{(a)}$$

the scattering coefficient is given by

$$\gamma_s = \pi a^2 n \qquad \text{(b)}$$

and the rain rate is given by

$$R = \frac{4}{3}\pi a^3 n v \qquad \text{(c)}$$

Thus

$$a = \frac{3\rho}{4\rho_w \gamma_s}$$

$$n = \frac{\gamma_s}{\pi a^2}$$

$$v = \frac{3R}{4\gamma_s a}$$

Substituting the data given in the problem yields the following results:

R = 1 mm/hr: a = 0.51 mm, n = 61 m^{-3}, v = 8.4 m s^{-1}.
R = 100 mm/hr: a = 1.18 mm, n = 320 m^{-3}, v = 12.6 m s^{-1}.