CHAPTER 11

1

Putting $\theta = 10^\circ$ and $h = 35800$ km into equation (11.1) gives $\varphi = 71.43^\circ$. This is the angular radius of the small circle, centred on the sub-satellite point, within which the line of sight to the satellite has an elevation of at least 10°. If we place the sub-satellite point on the equator at longitude zero, simple spherical geometry shows that the small circle will pass through latitude 66.5° at longitudes 37° east and west of the prime meridian. Thus the longitudinal coverage of this satellite is 74°, so at least 5 satellites will be needed to cover the whole of the specified area.

2

A printed page of A4 paper might contain up to around 1000 English words. Since the average word in is around 5 letters long, and each letter needs around one byte to specify it (this allows for upper and lower case letters, and a reasonable number of symbols), we might estimate the data storage of a single page as around 5 kilobytes.

Estimating a large book to contain as much paper as in a ream of 500 sheets, we find the data storage of the book to be around 5 MB (each sheet has two sides). The volume of this much paper is around 0.003 cubic metres, so the volume needed to store a terabyte would be of the order of 10^3 m^3.

3

The satellite data given in the box on pages 353-4 show that, in band 6.1, a digital number of 1 corresponds to a spectral radiance of 0 W m^{-2} sr^{-1} μm^{-1} while a digital number of 255 corresponds to a spectral radiance of 17.040 W m^{-2} sr^{-1} μm^{-1}. Thus a digital number of 132 corresponds to a spectral radiance of

$$0+\frac{132-1}{255-1}(17.04-0)=8.788\,\mathrm{W\,m^{-2}\,sr^{-1}\,\mu m^{-1}} \quad .$$

From the formula given in the problem, the brightness temperature is 295.5 K.

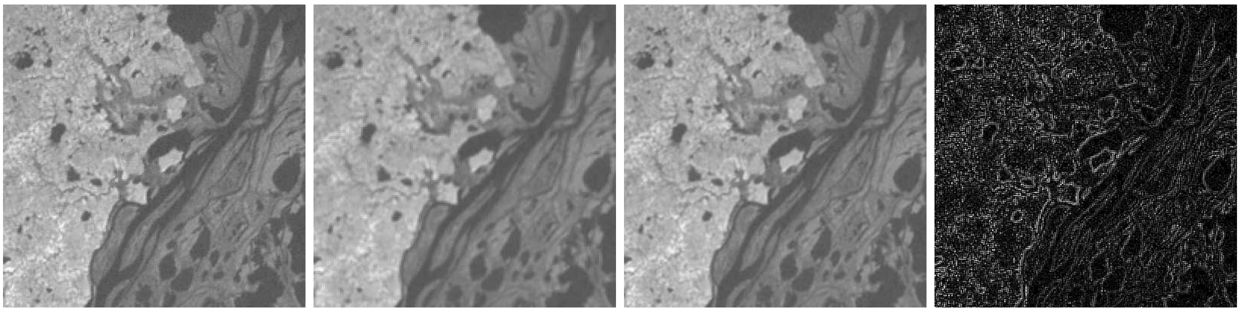
4

All the filters are isotropic, i.e. they do not have any directionality. The change of sign in (a) shows that it will reduce low spatial frequencies relative to higher frequencies. The sum of the weights is not zero (it is 8), so it is an edge-enhancing (sharpening) filter rather than one that will remove uniform areas of the image. It is not normalised, so unless the output from this filter is divided by 8 it will increase the image brightness.

Filter (b) is an averaging (smoothing, or low-pass) filter, since all the weights are positive. Again, the filter is not normalised (the sum of the weights is 22) so it will increase the image brightness unless the output is divided by 22. The weights taper with increasing distance from the centre and in fact this filter is an approximation to a Gaussian smoothing filter.

Filter (c), like (a), increases high spatial frequencies relative to lower frequencies. In this case, however, the sum of the weights is zero, so the filter will entirely suppress uniform areas. It is therefore an edge-detection (high-pass) filter, and it cannot be normalised.

Filters (a) and (c) have been derived from the approximate Gaussian filter (b). The effects of all three filters are shown here:



(Left to right: original image, then effects of filters a, b and c respectively.)

5

(a) Consider the behaviour of each type of filter in the spatial frequency domain:

	q=0	q large
I	1	1
A	1	0
S	1	>1
E	0	$\neq 1$

(i) the filter $a\mathbf{I} + (1-a)\mathbf{A}$ gives a response of zero at q=0 and a at large q, so provided $a > 0$ it is a sharpening filter.
(ii) the filter $b(\mathbf{I}-\mathbf{S})$ gives a response of zero at $q = 0$ and a negative response at large q if b is positive, so provided $b \neq 0$ it is an edge-detection filter. A practical implementation of this filter applied to 8-bit data (i.e. positive integers only) would also need to add an offset term to avoid producing negative output values.

(b) The response of this filter at $q = 0$ is found from the sum of the weights to be zero. It is thus an edge-detection filter. We can find sharpening filters from

$$\mathbf{S} = \mathbf{I} - \mathbf{E}/b.$$

Thus, for example, $b = 1/9$ and $b = 2/9$ give, respectively,

$$\frac{\begin{bmatrix} 1 & 0 & 1 \\ 0 & 5 & 0 \\ 1 & 0 & 1 \end{bmatrix}}{9} \text{ and } \frac{\begin{bmatrix} 1 & -1 & 1 \\ -1 & 9 & -1 \\ 1 & -1 & 1 \end{bmatrix}}{9}.$$

The second of these provides more sharpening than the first.

Smoothing (averaging) filters can be obtained from the sharpening filters using

$$\mathbf{A} = \frac{a\,\mathbf{I} - \mathbf{S}}{a-1}$$

where a>1. For example, applying $a = 2$ to both of the above filters would give

$$\frac{\begin{bmatrix} 1 & 2 & 1 \\ 2 & -3 & 2 \\ 1 & 2 & 1 \end{bmatrix}}{9} \text{ and } \frac{\begin{bmatrix} 1 & 3 & 1 \\ 3 & -7 & 3 \\ 1 & 3 & 1 \end{bmatrix}}{9}$$

These are unconventional but useable averaging filters.

6

The image histogram is approximately

$$n_x = A e^{\frac{-(x-x_0)^2}{2\sigma^2}} \quad ,$$

where A is a constant. The total number of pixels in the image is obtained by integrating the histogram:

$$N = \int_{-\infty}^{\infty} n_x dx = A \int_{-\infty}^{\infty} e^{\frac{-(x-x_0)^2}{2\sigma^2}} dx = A \sigma \sqrt{2\pi} \quad .$$

In order to apply equation (11.31) we also need to evaluate

$$\int_{-\infty}^{\infty} n_x \log_2 n_x \, dx = \frac{\int_{-\infty}^{\infty} n_x \ln n_x \, dx}{\ln 2} \quad .$$

The right-hand side can be expanded as

$$\frac{A \ln A \int_{-\infty}^{\infty} e^{\frac{-(x-x_0)^2}{2\sigma^2}} dx - \frac{A}{2\sigma^2} \int_{-\infty}^{\infty} (x-x_0)^2 e^{\frac{-(x-x_0)^2}{2\sigma^2}} dx}{\ln 2} = \frac{A \ln A \sigma \sqrt{2\pi}}{\ln 2} - \frac{A \sigma^3 \sqrt{2\pi}}{2\sigma^2 \ln 2}$$

Thus, from equation (11.31), the total information content of the image is

$$\frac{A\sigma\sqrt{2\pi}}{\ln 2} \ln\left(\sigma\sqrt{2\pi}\right) + \frac{A\sigma\sqrt{2\pi}}{2\ln 2} \text{bits} \quad .$$

Dividing this expression by the total number of pixels gives the average number of bits per pixel as

$$\frac{\ln\sigma}{\ln 2} + \frac{\ln\sqrt{2\pi}}{\ln 2} + \frac{1}{2\ln 2} = \log_2\sigma + 2.05 \text{ bits}$$

as required.