CHAPTER 9

1

We can rearrange equation (9.4) to give

$$G^2 = \frac{(4\pi)^3 \eta R^4 P_r}{\lambda^2 \sigma^0 A P_t} = \frac{(4\pi)^3 \times 1 \times 800000^4 \times 10^{-16}}{0.05^2 \times 10^{-2.5} \times 10^3 \times 4000} = 2.57 \times 10^9$$

so the necessary gain $G = 5.07 \times 10^4$. The antenna's effective area is given by

$$A_e = \frac{\lambda^2 G}{4\pi} = 10.1\,\mathrm{m}^2 \quad .$$

2

If the wind speed is v (metres per second) and its direction is φ measured from north to east, the data in the question give

$$0.8\,v-30+(3.5-0.1\,v)\cos 2\phi=-22.9$$

for the north-looking observation and

$$0.8\,v-30+(3.5-0.1\,v)\cos(\pi-2\phi)=-21.1$$

for the east-looking observation. The second equation can be rewritten as

$$0.8\,v-30-(3.5-0.1\,v)\cos 2\phi=-21.1 \quad .$$

Solving the two equations gives

$$v=10$$

and

$$\cos 2\phi=-0.36$$

so the speed is 10 m s^{-1} and the value of φ is 55.5˚, 235.5˚, 124.5˚ or 304.5˚, namely two possible directions with a 180˚ ambiguity in each. A third observation could remove one of these two directions, though not the 180˚ ambiguity which is present in the model (though not in the more sophisticated model of equation 9.6).

To demonstrate this, suppose that φ in fact has the value 55.5˚ and a third measurement is made looking north-east. In this case the value of σ^0 will be

$$0.8\times 10-30+(3.5-0.1\times 10)\cos(111^\circ-90^\circ)=-19.7\,\mathrm{dB}$$

and the same value would be obtained if φ were equal to 235.5˚. However, if φ were equal to 124.5˚, the value of σ^0 would be

$$0.8\times 10-30+(3.5-0.1\times 10)\cos(249^\circ-90^\circ)=-24.3\,\mathrm{dB} \quad .$$

3

(i) If the along-track resolution is r_a, and the distance from the antenna to the ground is R, the angular resolution of the antenna is

$$\frac{r_a}{R}$$

and this implies that the length of the synthetic aperture is

$$\frac{\lambda R}{r_a} \quad .$$

If the antenna is moving at speed v, the time taken to travel the length of the synthetic aperture is thus

$$\frac{\lambda R}{r_a v}$$

and the coherence time of the transmitted radiation must be at least as long as this. Inserting the value given in the question (and taking R = 775/cos 23° = 842 km), we find the minimum coherence time to be 0.24 seconds.

(ii) The across-track distance to the centre of the swath is 775 tan 23° = 329 km, so the across-track distances to the edges of the swath are 279 km and 379 km. The distance from the antenna to the near edge of the swath is thus

$$\sqrt{775^2+279^2}=823.69\ \text{km}$$

so the two-way travel time for a pulse of radiation travelling at the speed of light is 5.50 ms. Repeating this calculation for the far edge of the swath gives a time of 5.76 ms. The difference between these times is 0.26 ms.

The time taken for the antenna to move forward a distance of 30 m is 4.46 ms. Thus, in order to obtain an along-track resolution of 30 m, the pulses must be transmitted at least as often as once every 4.46 ms. Provided this interval is not shorter than 0.26 ms, no ambiguity arises in deciding which return pulse corresponds to which transmitted pulse.

4

First we consider a single component, with amplitude a and phase φ. Writing this in complex exponential notation and then taking the real part x, we have

$$x = a \cos\phi \quad .$$

If a is fixed and φ is drawn from a uniform distribution between 0 and 2π, the expectation value of x is given by

$$x = a \langle \cos\phi \rangle = 0$$

and similarly

$$x^2 = a^2 \langle \cos^2\phi \rangle = \frac{a^2}{2} \quad .$$

Now we add a large number N of such components. The Central Limit Theorem shows that the probability distribution of the value of X, the sum of all the xs, will be Gaussian with zero mean and variance $Na^2/2$. In other words, we have

$$p(X) = \frac{1}{a\sqrt{N\pi}} e^{-\frac{X^2}{Na^2}} \quad .$$

Clearly, an identical argument applies to the imaginary parts y, giving

$$p(X) = \frac{1}{a\sqrt{N\pi}} e^{-\frac{Y^2}{Na^2}} \quad .$$

The amplitude R of the signal is given by

$$R^2 = X^2 + Y^2$$

so we must have

$$p(R) = \frac{2\pi R}{a^2 N\pi} e^{-\frac{R^2}{Na^2}} \quad .$$

This is a Rayleigh distribution, as required.

5

(a) The range from M to A is

$$r_{MA}=\left((x-x_M)^2+(y-y_M)^2\right)^{1/2}$$

where (x_M, y_M) are the coordinates of M. Thus

$$\frac{\partial r_{MA}}{\partial x}=\frac{x-x_M}{r_{MA}} \quad .$$

Setting x_M = -400000 m and r_{MA} = (400000^2 + 800000^2)$^{1/2}$ m = 894427 m, and taking x = 0, this gives

$$\frac{\partial r_{MA}}{\partial x}=0.4472136$$

at (x, y)=0

Similarly, we find that

$$\frac{\partial r_{MA}}{\partial y}=-0.8944272$$

at (x, y)=0, so we can write

$$r_{MA}=0.4472136\,x-0.8944272\,y+\text{constant}$$

as required.

Repeating this analysis for the point S, we find

$$\frac{\partial r_{SA}}{\partial x}=0.4473030$$

and

$$\frac{\partial r_{SA}}{\partial y}=-0.8943825 \quad .$$

Thus,

$$\frac{\partial(r_{SA}-r_{MA})}{\partial x}=0.0000894$$

and

$$\frac{\partial(r_{SA}-r_{MA})}{\partial y}=0.0000447$$

which gives

$$r_{SA}-r_{MA}=0.0000894\,x+0.0000447\,y+\text{constant}$$

as required.

(b) Denoting the origin by O, if the distance MA = the distance MO we must have

$$0.4472136\,x - 0.8944272\,y = 0 \qquad \text{(a)}$$

(This is more simply expressed as $x = 2y$).

The wavelength is 6 cm, so (SA – MA) – (SO – MO) = 3 cm and thus

$$0.0000894\,x + 0.0000447\,y = 0.03\,\mathrm{m} \qquad \text{(b)}$$

Solving these two equations gives $x = 268$ m, $y = 134$ m.

(c) The point A is one fringe away from the origin. Thus if we divide its coordinates by 50, we find the coordinates of the point that is just resolvable from the origin. Hence we see that the height (y) resolution of this configuration is 2.7 m.