CHAPTER 2

1

(a) The direction of propagation is along the positive *x*-axis (this can be seen from the phase term)

(b) The radiation is elliptically polarised. Equations (2.12) and (2.13) become

$$E_y = E_{0\mathrm{y}} \cos(\omega t - k x - \phi_y)$$

and

$$E_z = E_{0\mathrm{z}} \cos(\omega t - k x - \phi_z)$$

respectively in a rotated coordinate system, so we have $E_{0\mathrm{y}} = E$, $E_{0\mathrm{z}} = 2\mathrm{E}$, $\phi_y = 0$ and $\phi_z = \pi/2$. Thus $\phi_z - \phi_y = \pi/2$ and the radiation is right-hand polarised (in the convention used for radio waves).

(c) The magnetic field **B** is given by

$$B_y = -\frac{E_z}{c}$$

and

$$B_z = \frac{E_y}{c}$$

This gives the correct ratio of the magnetic field amplitude to the electric field amplitude, and also ensures that the cross product $\mathbf{E} \times \mathbf{B}$ points in the propagation direction, i.e. along the positive *x*-axis.

(d) From (2.15), the flux density of the radiation is $5E^2/(2Z_0)$ = 6.6 kW m^{-2}.

2

For example, using the Stokes vectors given in section 2.2, we find the following values of the detected power

polarisation state	Stokes vector	Detected power
random	[1 0 0 0]	1
x-linear	[1 1 0 0]	2
y-linear	[1 -1 0 0]	0
+45° linear	[1 0 1 0]	1
-45° linear	[1 0 -1 0]	1
RHC	[1 0 0 1]	1
LHC	[1 0 0 -1]	1
RHE	[1 0.6 0 0.8]	1.6

3

Using the hint, we see that the term $e^{hf/kT}-1$ that occurs in the denominator of the Planck formula can be approximated as hf/kT.

4 The formula for the ratio is

$$\frac{e^{hf/kT_2}-1}{e^{hf/kT_1}-1}=\frac{e^{hc/\lambda kT_2}-1}{e^{hc/\lambda kT_1}-1}$$

where T_1 = 6000 K and T_2 = 300 K.

(a) at a wavelength of 0.1 μm the ratio is $\dfrac{e^{480.1}-1}{e^{24.01}-1} \approx 10^{198}$.

(b) at a wavelength of 1 μm the ratio is $\dfrac{e^{48.01}-1}{e^{2.401}-1} \approx 10^{20}$.

(c) at a frequency of 1000 GHz the ratio is $\dfrac{e^{0.1600}-1}{e^{0.00800}-1} \approx 22$.

(d) at a frequency of 1 GHz the ratio is $\dfrac{e^{1.600\times10^{-4}}-1}{e^{8.00\times10^{-6}}-1} = 20$.

All of these expressions can be evaluated directly using Octave, although most electronic calculators will have some difficulty in evaluating them. The answers to (a) and (b) show that thermal emission can almost always be ignored with respect to reflected solar radiation, where present, at near-infrared and shorter wavelengths. The answer to (d) shows that the Rayleigh-Jeans approximation is valid at this frequency.

5

From (2.18) we can write the Fourier transform as

$$a(\omega)=\frac{1}{2\pi}\int_{-\infty}^{\infty}\exp\left(-\frac{(t-t_0)^2}{2\sigma^2}-i\omega t\right)dt$$

Substitute

$$z=\frac{t-t_0}{\sigma\sqrt{2}}+\frac{i\omega\sigma}{\sqrt{2}}$$

to obtain

$$a(\omega)=\frac{\sigma\sqrt{2}}{2\pi}\int_{-\infty}^{\infty}\exp(-z^2)\exp\left(-\frac{i\omega t_0}{2}-\frac{\omega^2\sigma^2}{2}\right)dz \quad .$$

This can be rearranged as

$$a(\omega)=\left[\frac{\sigma\sqrt{2}}{2\pi}\int_{-\infty}^{\infty}\exp(-z^2)\,dz\right]\exp\left(-\frac{i\omega t_0}{2}-\frac{\omega^2\sigma^2}{2}\right)$$

where everything inside the square brackets evaluates to a constant (although we don't need to know this to answer the question, its value is $\sigma/\sqrt{2\pi}$) and the part outside the brackets is a Gaussian function of ω. The width of this Gaussian function is $1/\sigma$, and the phase factor exp $(-i\omega t_0/2)$ arises from the fact that the original function was centred not on t=0 but on t=t_0.