RADAR ALTIMETRY

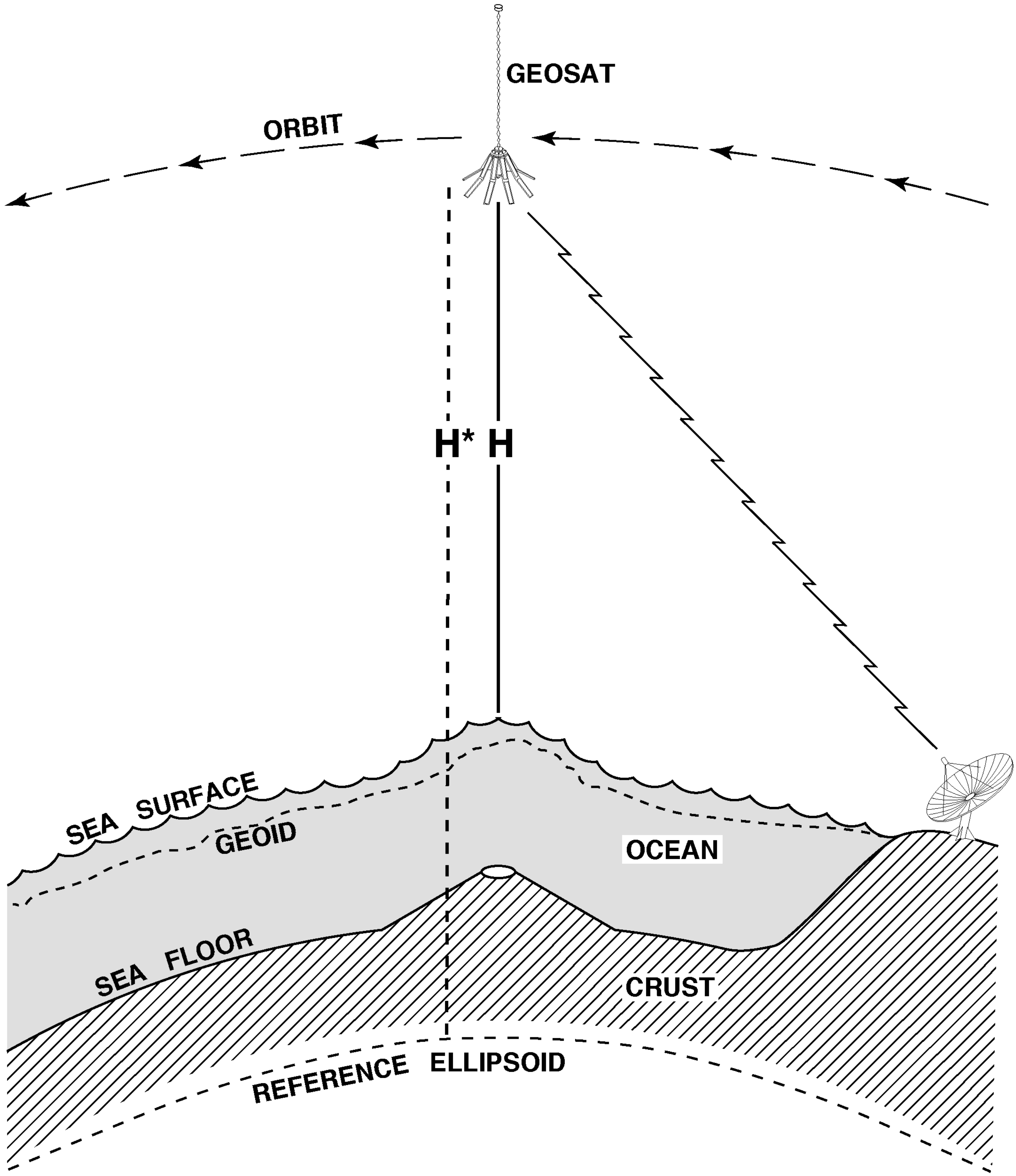
(Rees Chapter 8 - Sandwell's Notes)

Diverse Applications The primary objective of radar altimetry from satellites is to measure the topography of the ocean surface. In the next class I'll cover two applications of radar altimetry. The technical discussion, presented here, is motivated by the precision and accuracy requirements of the most common applications.

Feature	**Amplitude**	**Horizontal Scale**	**Timescale**
geoid	30 m	10,000 km	
dynamic topography	1 m	10,000 km	
climate changes	0.01 m	10,000 km	10 - 100,000 yr
tides	0.2-2 m	100 - 10,000 km	lunar and solar freq.
El Nino	0.1 m	6,000 km	~5 yr
fronts and eddies	0.3 m	100 - 1000 km	~1 mo
seamounts	1 m	50 km	
ridge axes	0.02 m	10 km	

You see that these applications span a wide range of measurement requirements. The most stringent applications are *climate change* and small-scale gravity features such as *ridge axes*. The gravity applications require a point-to-point precision of 0.02 m which is very difficult to achieve when 1 m tall ocean waves are present. The climate change application requires an 0.01 m accurate altimeter over a much longer horizontal scale. In addition to a problem with E/M bias due to waves, this application also requires a 0.01 m knowledge of the absolute spacecraft position over a 10 year plus timescale. All of the applications requiring high accuracy at long wavelength also require an accurate knowledge of the delay of the radar echo as it passed through the ionosphere, the dry part of the atmosphere, and the wet (variable) part of the troposphere. It is also apparent that one person's signal is another person's noise so, for example, most applications require removal of the tidal signal to correct the data although there are scientists who use altimeter data to observe the tidal signal.

Beam-Limited Footprint As discussed earlier in the course, since the radar altimeter operates in the microwave part of the spectrum it has the following attributes: the atmosphere is very transparent at 13 GHz; there is little stray radiation coming from the Earth; and the illumination pattern on the surface of the ocean is very broad for reasonable sized antennas.


GEOSAT
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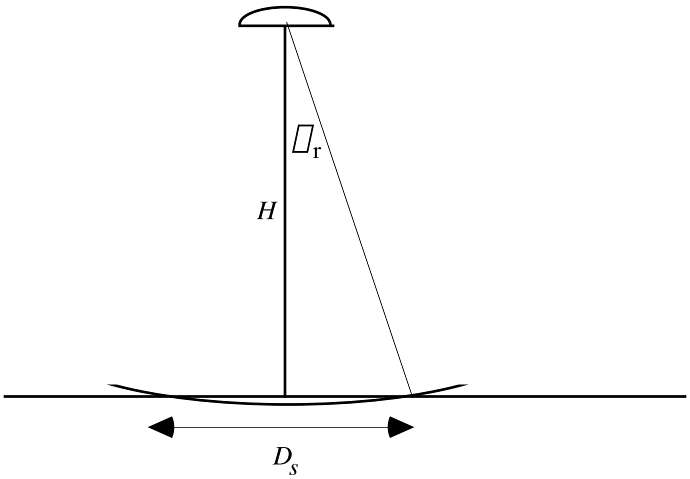


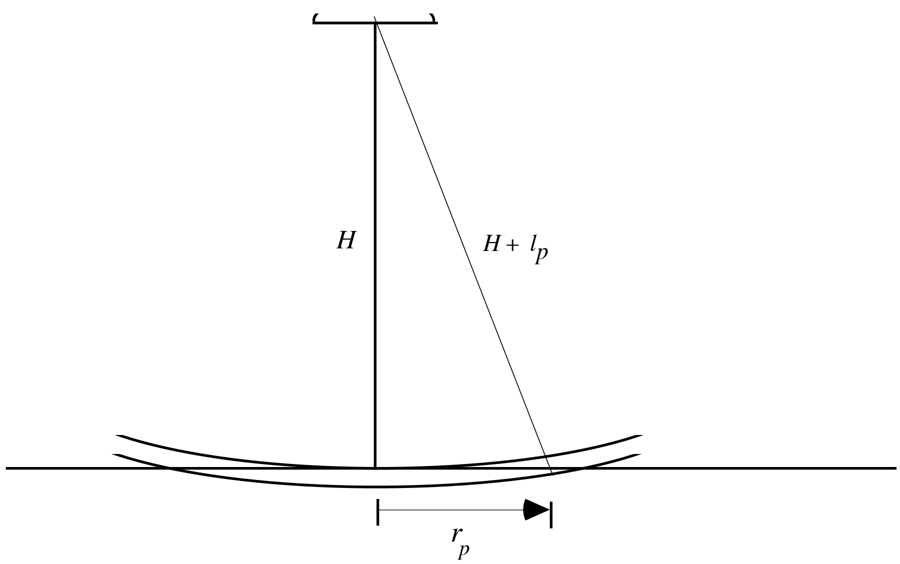
As derived previously, the angular resolution θ_r of a circular aperture having radius D_a is given by $\sin \theta_r = 1.22 \frac{\lambda}{D_a}$ where λ is the wavelength of the radar. Suppose we have a 1 m diameter radar operating at a wavelength of 22 mm (Ku-band) and this is mounted on a satellite orbiting at an altitude of 800 km. What is the diameter of the illumination pattern on the ocean surface?

$$D_s = 2 H \sin \theta_r = 2.44 H \frac{\lambda}{D_a}$$

The illumination diameter or *beam width* of the radar is quite large (43 km). Using this configuration, it will be impossible to achieve the 10 km horizontal resolution required for the gravity anomaly applications. However, one benefit of this wide illumination pattern is that small (~1 degree) pointing errors away from *nadir* are not a problem.

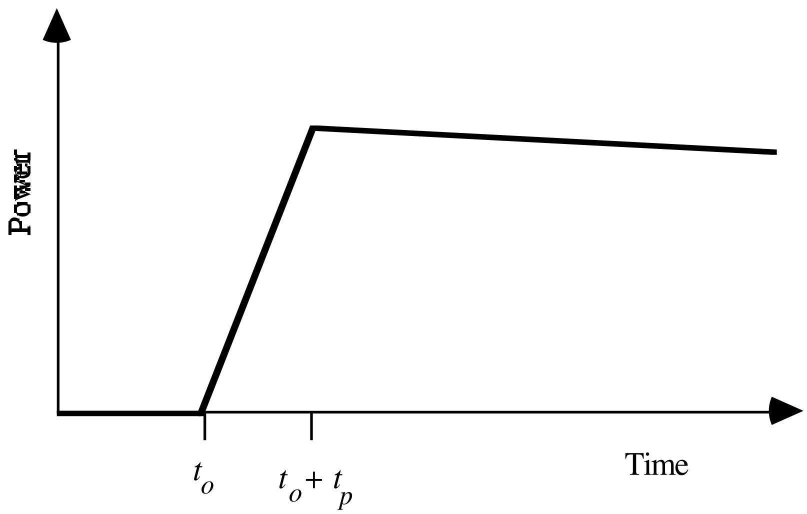
To achieve the 0.02 m range resolution needed for several of the above applications, one must measure the travel time of the radar echo to an accuracy of $\Delta t = \frac{2 \Delta h}{c} = \frac{0.04}{3 x 10^8} = 1.3 x 10^{-10} s$. This can be translated into the bandwidth of the radiation needed to form such a sharp pulse $\Delta \nu = \frac{1}{\Delta t}$. In this case a 8 GHz bandwidth is needed. Note that the carrier frequency of our radar altimeter is only 13 GHz so the pulse must span most of the electromagnetic spectrum. Can you imagine all of the televisions that would be effected when this radar passed over Los Angeles at 7 PM!!. Obviously we can't use the entire EM spectrum so we'll have to live with a bandwith of only 0.3 GHz. However, it turns out that ocean waves effectively limit the accuracy of the travel time measurement so a 0.3 GHz bandwith is adequate. We'll just have to do a lot of averaging to reduce the noise.

Pulse-limited Footprint Assume for the moment that the ocean surface is perfectly flat (actually ellipsoidal) but has point scatters to reflect the energy back to the antenna. The radar forms a sharp pulse having a length of about 3 nanoseconds corresponding to the 0.3 GHz bandwidth. (In practice, to reduce the peak output requirement of the transmitter, the radar emits a frequency-modulated chirp having a much lower amplitude but extending over a longer period of time. The chirped radar signal reflects from the ocean surface and returns to the antenna where it is convolved with a *matched filter* to regenerate the desired pulse. This is a common signal processing trick used in all radar systems. After the matched filter one can treat the measurement as a pulse.) The diagram below illustrates how the pulse interacts with a flat sea surface.



H	-	satellite altitude	800 km
t_p	-	pulse length	3 ns
c	-	speed of light	3 x 10^8 m/s
$l_p = c\, t_p$	-	length of pulse	1 m
r_p	-	radius of pulse on ocean surface	

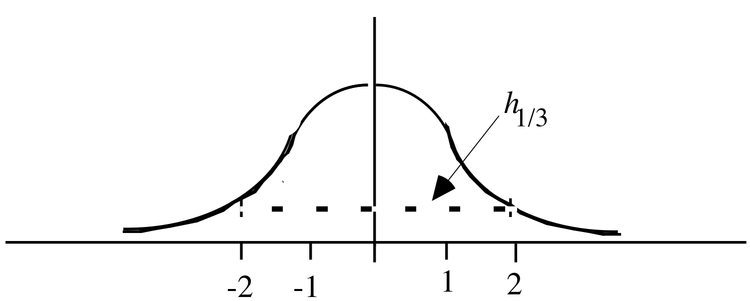
The radius of the leading edge of the pulse is derived as follows.
$H^2 + r_p^2 = (H + l_p)^2 = H^2 + l_p^2 + 2\,Hl_p$. The H^2's cancel and we can assume l_p^2 is very small compared with the other terms so the pulse radius is $r_p = (2Hl_p)^{1/2} = (2Hct_p)^{1/2}$. For a 3 ns pulse length, the pulse radius is 1.2 km so the diameter or *footprint* of the radar is 2.4 km. Since this footprint is much less than the beam width, the power that is returned to the radar will be a ramp function.



The power begins to ramp-up at time $t_o = \frac{2H}{c}$ and the ramp extends for the duration of the pulse. At times greater than $t_o + t_p$, the diameter of the radar pulse continues to grow and energy continues to return to the radar. The amplitude of this energy decreases gradually according to the illumination pattern of the radar on the ocean surface. Because of the finite pulse width, the bottom and top of the ramp will be rounded.

Ocean Waves Of course the actual ocean surface has roughness due to ocean waves and swells. This ramp-like return power will be convolved with the height distribution of the waves within the footprint to further smooth the return pulse and make the estimate of the arrival time of the leading edge of the pulse less certain. The diagram on the next page [*Walsh et al.,* 1978] shows the effect of increasing wave height on the shape of the return pulse. We can investigate the effects of wave height on both return pulse length and footprint diameter using a Gaussian model for the height distribution of ocean waves. This model provides and excellent match to observed wave height distributions as shown on the diagram on the following page [*Stewart, 1985*].

$$G(h) = \frac{1}{\sqrt{2\pi}\,\sigma_h}\ exp\left(-\frac{h^2}{2\,\sigma_h^2}\right)$$



Approximately 1/3 of the waves will have height greater than σh while 2/3 will have height less than σh. An observer on a ship can accurately report the peak-to-trough amplitude of the highest 1/3 of the waves. Normally these waves will be $2\sigma h$ from the zero level; this is called the significant wave height and it is $h_{1/3} = h_{swh} = 4\sigma h$.

Now suppose we fly a beam-limited **laser** altimeter over this surface and observe the distribution of travel times. Assume that the beam-width of the laser is narrow enough to observe the topography of the wave field and we can map this into a distribution of two-way travel time.

$$h = \frac{c\,\tau}{2} \qquad G(\tau) = exp\left(\frac{-c^2\tau^2}{8\,\sigma h^2}\right)$$

Suppose we send a δ-function pulse. The probability distribution of the reflected pulse *O(t)* is the input pulse convolved with the Gaussian wave height model.

$$O(t) = \int_{-\infty}^{\infty} \delta(t)\, G(t-\tau)\, d\tau = G(t)$$

Now we can equate this to a wide-beam radar pulse as it reflects from entire wave field within the footprint. There will be many waves within the > 2.4 km footprint so we can regard the radar return pulse as the average of all of the laser returns over the wave field. The radar return pulse width t_w is measured as the full width of the pulse where the power is 1/2.

$$\frac{1}{2} = exp\left(\frac{-2c^2\left(\frac{t_w}{2}\right)^2}{8\,\sigma h^2}\right) \quad \text{so} \quad t_w^2 = 16\,\frac{\sigma h^2}{c^2}\, ln\, 2.$$

Since we were unable to form a very sharp radar pulse because the radar bandwidth is limited to 0.3 GHz, the total width of the return pulse will be established by convolving the outgoing pulse with the Gaussian wave model. If the outgoing pulse can also be modeled by an Gaussian function having a total pulse width of t_p, then the total width of the return pulse is given by $t^2 = t_p^2 + \frac{h_{swh}^2}{c^2}\, ln\, 2$. This provides and expression for the pulse width as a function of significant wave height (SWH). Similarly the diameter of the pulse as a function of significant wave height is $d = 2\,(cHt)^{1/2}$. Both functions are shown on the following page for SWH ranging from 1 to 10 m.

It is clear that the quality of the altimeter measurement will decrease with increasing SWH. In practice we have found that Geosat, ERS, and Topex data are unreliable when SWH exceeds about 6 m.

Significant Wave height is typically 2 meters so the radar footprint is typically 3.5 km and the pulse-width increases from 3 ns to 8 ns. Now we see that our original plan of having a very narrow pulse of 60 picoseconds to resolve 0.02 m height variations was doomed because the ocean surface is usually rough; a 3 ns pulse is all that could be resolved anyway. But how do we achieve the 0.02 m resolution needed for our applications when typically we can only resolve 1.2 m? The way to improve the accuracy by a factor of 10^2 is to average 10^4 measurements and hope the noise is completely random.

The speed of light provides an interesting limitation for space borne ranging systems. At a typical orbital altitude of 800 km it takes 5.2 milliseconds for the pulse to complete its round trip route. One can have several pulses en-route but because we actually send a long chirp rather than a pulse, a pulse repetition frequency is limited to about 1000 pulses per second; during this time the altimeter moves about 7000 m along its track. Thus in each second there are 1000 pulses available for averaging; this will reduce the noise from 1.2 m to 0.04 m. Further averaging can be done for many of the oceanographic applications where the horizontal length scale of the feature is > 50 km. Of course one should be careful to remove all of the known signals using the full resolution data and then smooth the residual data along the profile to achieve the 0.02 m accuracy. (Please avoid boxcar filtered because it produces terrible sidelobes.)

Because of these limitations, single altimeter profiles are unable to achieve the point-to-point accuracy of 0.02 m needed for high-resolution gravity field recovery. For this application one must rely on repeat or nearby profiles to gather the 10 samples needed to reduce the noise. We'll discuss this in a following lecture on marine gravity field recovery.

Modeling the Return Waveform There are a couple of other relevant engineering issues related to picking the travel time of the return pulse. First, after the return echo is passed through a matched filter to form the pulse, the pulse power is recorded at 64 times or *gates* in a window that is about 30 ns (~9 m) long. An adaptive tracker is used to keep the power ramp in the center of the window. The ocean surface it typically smooth at length scales greater than the footprint so keeping the pulse in the window is not a problem. However, over land or ice, it is not usually possible to keep the pulse within the window because 9 m variations in topography over several kilometers of horizontal distance are quite common. Geosat and Topex altimeters lose lock over land and must re-acquire the echo soon after moving back over the ocean. The ERS-1/2 altimeters widen the gate spacing over land and ice so they can measure land topography as well as ocean topography.

After recording the waveform of the return pulse, 100 echoes are averaged and an analytic function is fit to each waveform. The function has 3 parameters. 1) The position of the steepest part of the ramp provides the range estimate. 2) The width of the ramp provides and estimate of SWH. 3) The height of the ramp (called sigma-naught -σ_o) provides an estimate of surface roughness at the 20-30 mm length scale. This latter measurement can be related to surface wind speed since wind will roughen the ocean surface. Precise calibration is performed for each of the three measurements. Absolute range calibration is performed in the open ocean using an oil platform having a GPS receiver and accurate tide gauge. Both SWH and wind speed are calibrated using open-ocean shipboard measurements

Corrections

Sea State Bias - This is perhaps the most insidious problem for monitoring global sea level changes over long periods of time. While the topography of open ocean waves is quite symmetric, the crests of the waves preferentially scatter the radar waves outward away from nadir while the troughs of the waves focus the energy back toward radar. This skews the ramp of the radar return toward later times. This effect is sometines called the E/M bias and it is typically 5% of the SWH. This correction is highly uncertain and poorly understood. Wave height varies seasonally so this correction can easily introduce a 0.05 - 0.10 m bias in estimates of large scale dynamic topography.

Ionospheric Delay - We have already discussed how the electron plasma in the ionosphere slows the group velocity of the radar pulse. The homework problem in Rees, Ch3, #3, is to derive an expression relating the total electron content (TEC) in the nadir direction to the travel time difference between pulses at 2 and 5 GHz. The attached plot shows how electron density varies with altitude at different times of the day. The smallest ionospheric correction occurs at 6 AM while the largest correction is at 12 noon. The dual frequency correction scheme is quite accurate over large length scales (> 50 km) but at shorter wavelengths the correction can actually add noise to the range measurement; we don't apply this correction when computing gravity anomalies from satellite altimetry. Also note that the TEC has an eleven year cycle; the next peak is in 2002.

Dry Atmosphere - The index of refraction of the dry atmosphere is simply related to the surface temperature and pressure. The water vapor causes and additional delay. The total correction is:

$\Delta h = 2.27 \times 10^{-5}\ Ps + 1.723\ W/T_a$ where

Ta - air temperature (˚K)

W - zenith water vapor (kg m^{-2})

Ps - surface pressure (Pa)

Typical values of dry tropospheric delay are 2.3 m while the wet delay can vary from 0.06 - 0.30 m

Orbit Error - Until recently, radial orbit error was the major limitation of satellite altimetry. Back in 1978 when Seasat was in orbit, the best radial orbit accuracy was about 1 m rms. Nowadays GPS tracking provides radial orbit error of 0.02m rms. Even altimeters without GPS tracking have radial orbital accuracy of about 0.07 m.

Dual Frequency Altimeter A dual frequency altimeter is used to measure the ionospheric delay and then apply the correction to the range measurement. Here is an example from problem 3 of chapter 4 in Rees:

A dual frequency altimeter emits short pulses at 2 GHz and 5 GHz. Beacuse the ionosphere is dispersive, the reflected pulses are separated in time by 15 ns. Calculate the total electron content of the ionosphere.

The total electron content N_T has units of electrons per area and thus it is the integral of the electron density N_e from the ocean surface to the height of the spacecraft H.

$$N_T = \int_0^H N_e(z)\, dz$$

The round trip travel time of the radar pulse T is

$$T = 2\int_0^H \frac{1}{v_g(z)} dz$$

where v_g is the velocity of the pulse. When the radar angular frequency ω is much greater than the plasma frequency, the index of refraction is approximately

$$n(\omega, z) \approx 1 - \frac{N_e(z)\, e^2}{2\, \varepsilon_0 m\, \omega^2}$$

N_e	-	electron density
e	-	electron charge
m	-	electron mass
ε_0	-	permittivity of free space
ω	-	radar angular frequency (radians/second)

The phase velocity of the radar wave is $v_p = \frac{c}{n(\omega)}$ and it exceeds the speed of light. However, the pulse travels at the group velocity $v_g = \frac{d\omega}{dk}$ which is less than the speed of light. With a little algebra one can arrive at the group velocity.

$$v_g = c\left[1 + \frac{N_e e^2}{2\varepsilon_0 m \omega^2}\right]^{-1}$$

The travel time difference Δt can be written as

$$\Delta t = T_2 - T_5 = \frac{2}{c}\int_o^H \left[\left(1 + \frac{N_e e^2}{2\varepsilon_0 m \omega_2^2}\right) - \left(1 + \frac{N_e e^2}{2\varepsilon_0 m \omega_5^2}\right)\right] dz$$

or

$$\Delta t = \frac{e^2}{mc\varepsilon_0}\left[\frac{1}{\omega_2^2} - \frac{1}{\omega_5^2}\right]\int_0^H N_e(z)\, dz$$

Note that the integral on the right side is the total electron content so solving for the TEC one finds

$$N_T = \Delta t\, \frac{mc\varepsilon_0}{e^2}\left[\frac{1}{\omega_2^2} - \frac{1}{\omega_5^2}\right]^{-1}$$

Plugging in the numbers one finds that for a 15 ns delay, the TEC is 2.52×10^{17} electrons m^{-2}. It is interesting to calculate the total delay of a microwave signal for as a function of frequency for this TEC.

frequency (GHz)	wavelength (mm)	band	2-way delay (m)
1	300	P	20
2	150	L	5
5	60	C	0.8

10	30	X	0.2
13	23	Ku	0.1

It is clear that ionospheric delay is a major source of error for all active microwave ranging systems, especially those operating at longer wavelength.

References

Stewart, R. H., Methods of Satellite Oceanography, University of California Press, Berkeley, CA, 1985.

Special Topex/Poseidon Issue of JGR, 1996?

Geosat Issue I, *J. Geophys. Res.*, v. 95, C3, p.2833-3448, 1990.

Geosat Issue II, *J. Geophys. Res.*, v. 95, C10, p.17865-18367, 1990.

Seasat Special Issue I: Geophysical Evaluation, J. Geophys. Res., v. 87, C5, p. 3173-3431, 1982.

Seasat Special Issue II: Scientific Results, J. Geophys. Res., v. 88, C3, p. 1529-1937, 1983.

The Navy GEOSAT Mission, Johns Hopkins APL Technical Digest, v.8, no. 2, 169-267, April-June, 1987.

Walsh, E. J., E. A. Uliana and B. S. Yaplee, Ocean wave heights measured by a high resolution pulse-limited radar altimeter, Boundary Layer Meterology, V. 13, p. 263-276, 1978.