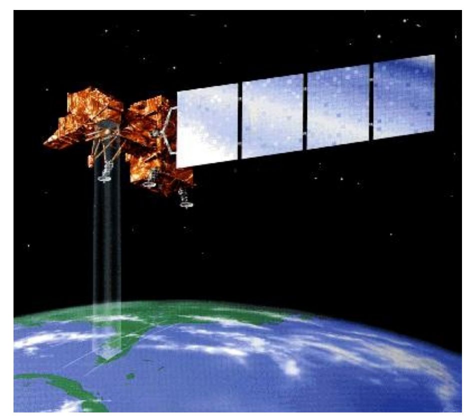



Satellite Remote Sensing
SIO 135/SIO 236

Electromagnetic Radiation and Polarization



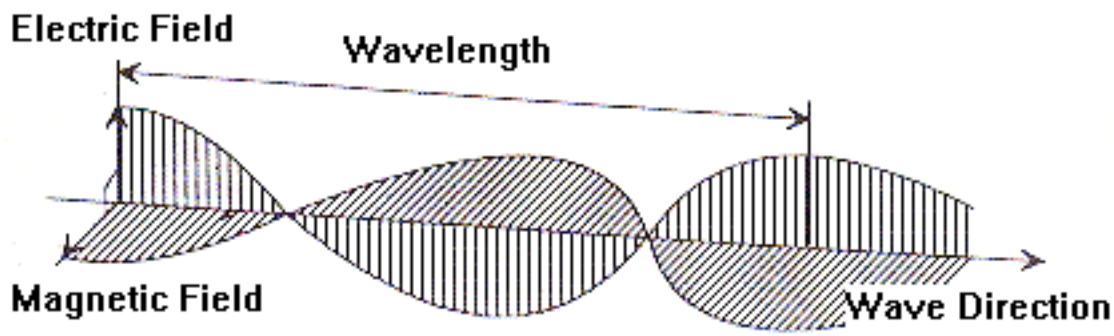
Electromagnetic Radiation Models

- ★ To understand the interaction that the EMR undergoes before it reaches the sensor, we need to understand the nature of EMR

- ★ To understand how EMR is created, how it propagates through space, and how it interacts with other matter, it is useful to describe the processes using two different models:

  - the *wave* model, and
  - the *particle* model.

Electromagnetic waves



- ★ EM waves are energy transported through space in the form of periodic disturbances of electric and magnetic fields

- ★ EM waves travel through space at the same speed, c = 2.99792458 x 10^8 m/s, commonly known as the speed of light

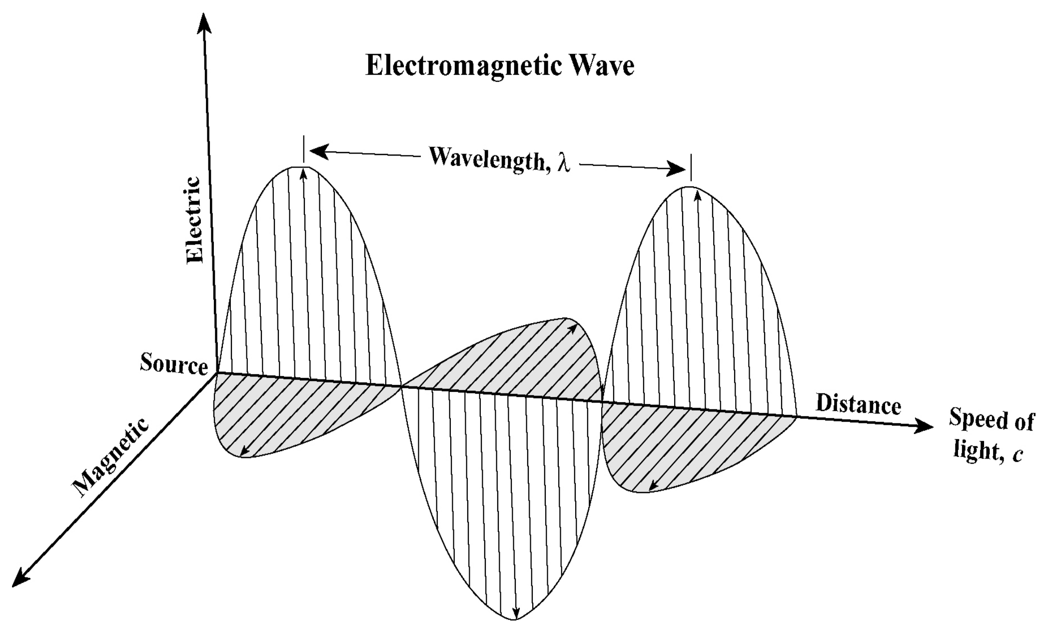
- ★ An EM wave is characterized by a **frequency** and a **wavelength**

- ★ These two quantities are related to the speed of light by the equation **speed of light = frequency x wavelength**

Wave Model of Electromagnetic Radiation

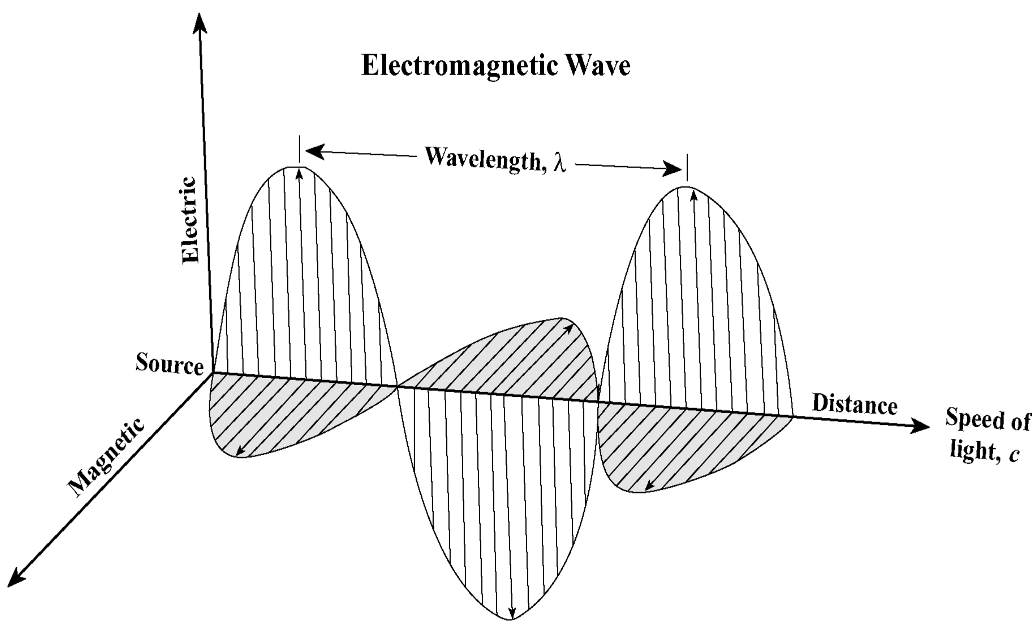
The EM wave consists of two fluctuating fields—one electric (E) and the other magnetic (B).

The two vectors are in phase and are at right angles (orthogonal) to one another, and both are perpendicular to the direction of travel.



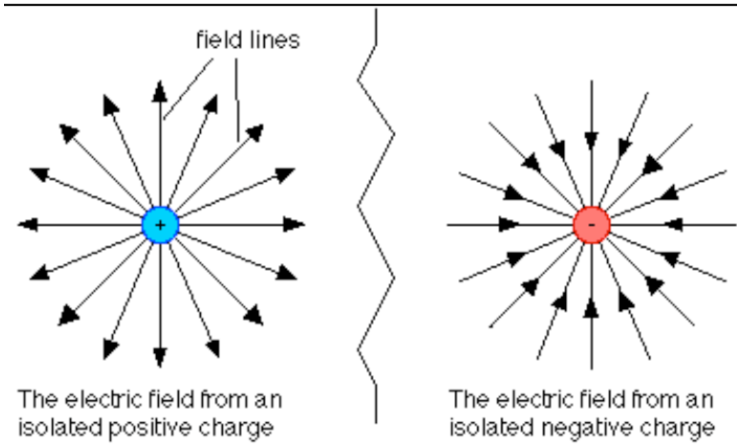
Wave Model of Electromagnetic Radiation

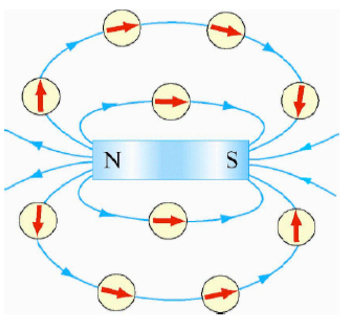
E is perpendicular to direction of propagation
B is perpendicular to direction of propagation
E and B are in phase
E is perpendicular to B
E x B is in direction of propagation
|B| = |E|/c



Electromagnetic (EM) Theory

Representation of the electric and magnetic field





A convenient way to represent the magnetic field is with a drawing of "magnetic field lines."
The direction of the lines shows the direction of the magnetic field, while the density of lines shows the strength of the field.

Electric Field (E)

E is the effect produced by the existence of an electric charge, e.g. an electron, ion, or proton, in the volume of space or medium that surrounds it.

E=F/q F = is the electric force experienced by the particle

q= particle charge

E= is the electric field where the particle is located

Magnetic Field (B)

B is the effect produced by a change in velocity of an electric charge q

In a major intellectual breakthroughs in the history of physics (in the 1800s), James Clerk Maxwell came up with the four equations which described all EM phenomena:

➔ MAXWELL'S EQUATIONS

Maxwell's Equations

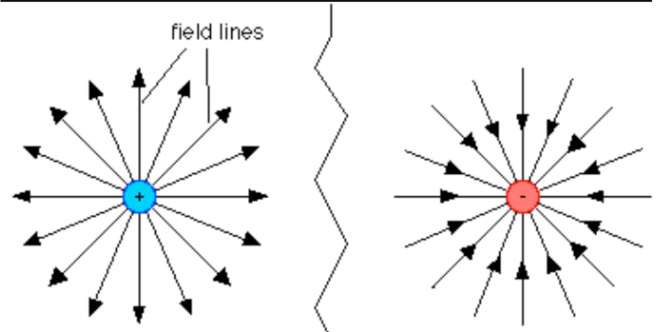
Gauss's law $\oint \mathbf{E}\cdot d\mathbf{A} = q/\varepsilon_0$

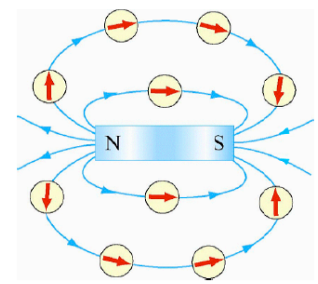
Gauss's law for magnetism: $\oint \mathbf{B}\cdot d\mathbf{A} = 0$

Maxwell's Faraday rotation: $\oint \mathbf{E}\cdot d\mathbf{S} = -d\Phi_{\mathbf{B}}/dt$

Ampere's circuital law: $\oint \mathbf{B}\cdot d\mathbf{S} = \mu_0 i + \mu_0\varepsilon_0 d\Phi_{\mathbf{E}}/dt$

Representation of the electric and magnetic field





A convenient way to represent the magnetic field is with a drawing of "magnetic field lines."
The direction of the lines shows the direction of the magnetic field, while the density of lines shows the strength of the field.

E = electric field (vector)
B = magnetic field (vector)
q = electric charge density,
μ_o = magnetic permeability of free space,
ε_o = electric permittivity of free space (dielectric constant),
i = electric current,
c =1 /sqrt($\mu_o\varepsilon_o$) ~3×10^8 ms^{-1} (speed of light)

Maxwell's Equations

In the absence of charges or currents:

1 Gauss's law: $\oint \mathbf{E} \cdot d\mathbf{A} = q / \varepsilon_0$

2 Gauss's law for magnetism: $\oint \mathbf{B} \cdot d\mathbf{A} = 0$

3 Maxwell's Faraday rotation: $\oint \mathbf{E} \cdot d\mathbf{S} = -d\Phi_{\mathrm{B}} / dt$

4 Ampere's circuital law: $\oint \mathbf{B} \cdot d\mathbf{S} = \mu_0 i + \mu_0 \varepsilon_0 d\Phi_{\mathrm{E}} / dt$

1: Charges create E, in specific "patterns". Or also that E field lines "emanate" from charges

2: B field lines aren't created (there are no magnetic monopoles), but they form loops, with no start or stop.

3: Changing B makes E

4: Electric currents create magnetic fields; changing electric fields create magnetic fields.

Maxwell's Equations

Differential form in the absence of magnetic or polarizable media:

1 Gauss's law: $$\nabla \cdot E = \frac{\rho}{\varepsilon_0} = 4\pi k \rho$$

2 Gauss's law for magnetism: $$\nabla \cdot B = 0$$

3 Maxwell's Faraday rotation: $$\nabla x\, E = -\frac{\partial B}{\partial t}$$

4 Ampere's circuital law: $$\nabla\, x\, B = \frac{4\pi k}{c^2} J + \frac{1}{c^2}\frac{\partial E}{\partial t} = \frac{J}{\varepsilon_0 c^2} + \frac{1}{c^2}\frac{\partial E}{\partial t}$$

$$k = \frac{1}{4\pi\varepsilon_0} = \textit{Coulomb's constant} \qquad c^2 = \frac{1}{\mu_0 \varepsilon_0}$$

Maxwell's Equations

Differential form in free space:

1 Gauss's law: $\nabla \cdot E = 0$

2 Gauss's law for magnetism: $\nabla \cdot B = 0$

3 Maxwell's Faraday rotation: $\nabla \times E = -\partial B/\partial t$

4 Ampere's circuital law: $\nabla \times B = \mu_o \varepsilon_o \, \partial E/\partial t$

Solution to Maxwell's equations

The harmonic plane wave $E_x = E_o \cos(\omega t - kz)$; $E_y = 0$; $E_z = 0$

$B_x = 0$; $B_y = E_o/c \cos(\omega t - kz)$; $B_z = 0$

satisfies Maxwell's equations

wave speed $c = \omega/k = 1/\mathrm{sqrt}(\varepsilon_o \mu_o)$

ω is the angular frequency, k is the wave number

$\omega = 2\pi f$

$k = 2\pi/\lambda$

Wave Model of Electromagnetic Energy

- EM waves propagate at the speed of light, c, and consists of an electric field E and a magnetic field B.
- E varies in magnitude in the direction perpendicular to the traveling direction; B is perpendicular to E.
- E is characterized by: frequency (wavelength), amplitude, polarization, phase.

Wave equation:

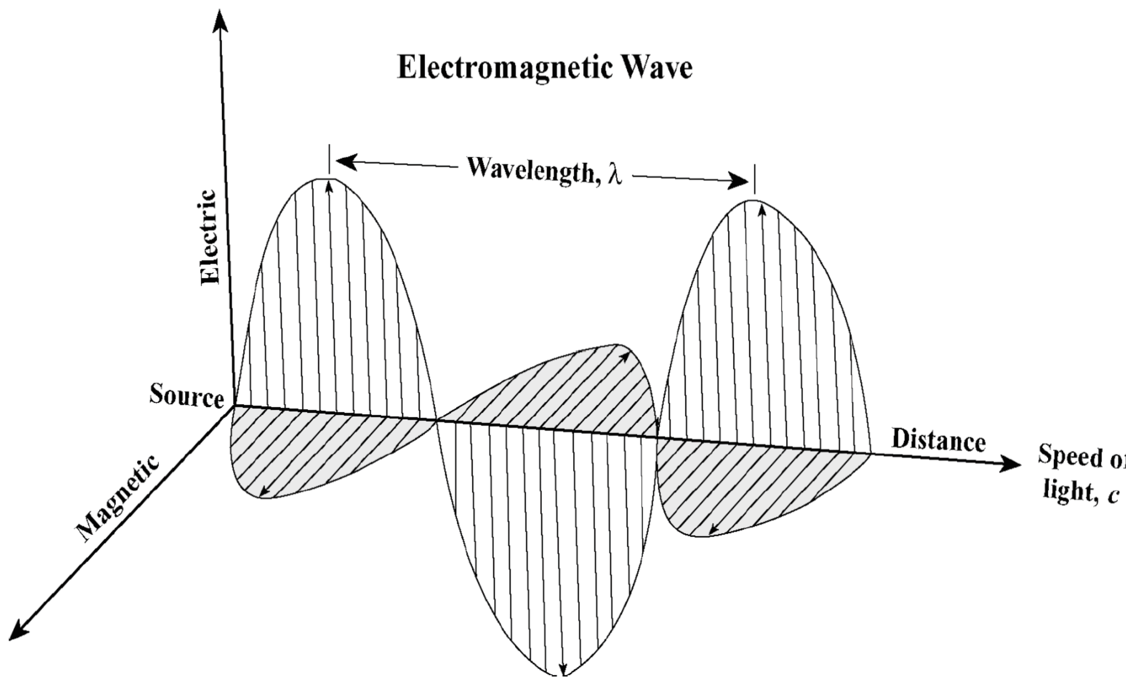
$$E_x = E_o \exp i(\omega t - kz)$$

$k = 2\pi/\lambda$ = wave number

$\omega = 2\pi f$ =angular frequency

f = frequency

Φ =phase



Light is a traveling EM wave

So…Maxwell's equations tell us that the velocity of EM wave is equal to the speed of light ⇨ i.e. light travels as an EM wave.

$\lambda = c / f$

λ = wavelength (m)

c = speed of light (m/s)

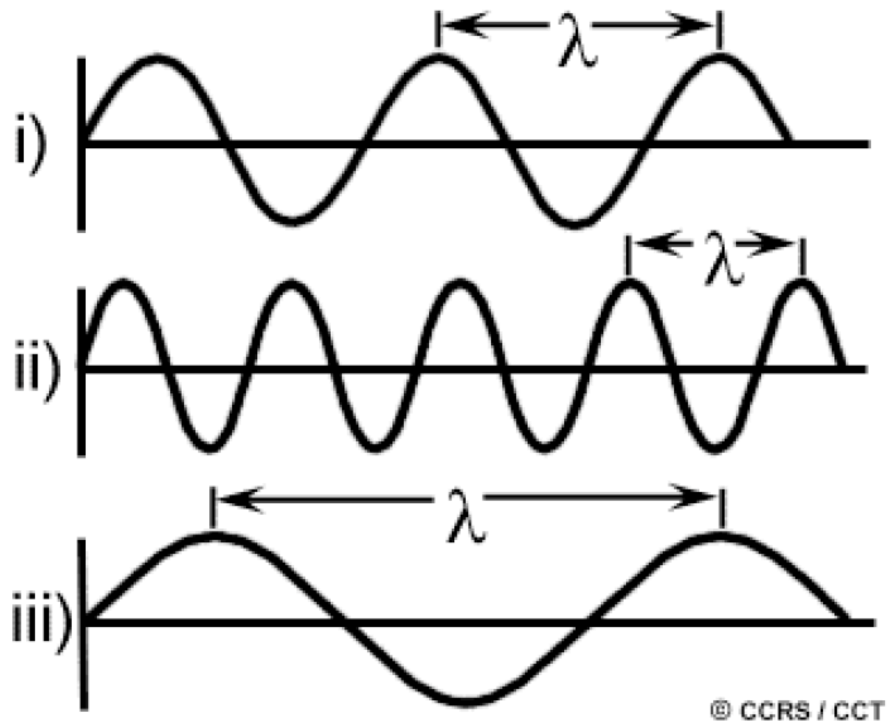
f = frequency (hz or s-1)

c = 300,000 km/s

f = 5.6 GHz; λ = 5.6 cm

f = 1.2 GHz; λ = 24 cm.

λ = 0.4 mm; f = 750 GHz.



Light is a traveling EM wave

Maxwell's equations also tell us that EM waves don't carry any material with them. They only transport **energy**:

$E = h\,f = h\,c\,/\,\lambda$

c = speed of light (m/s)

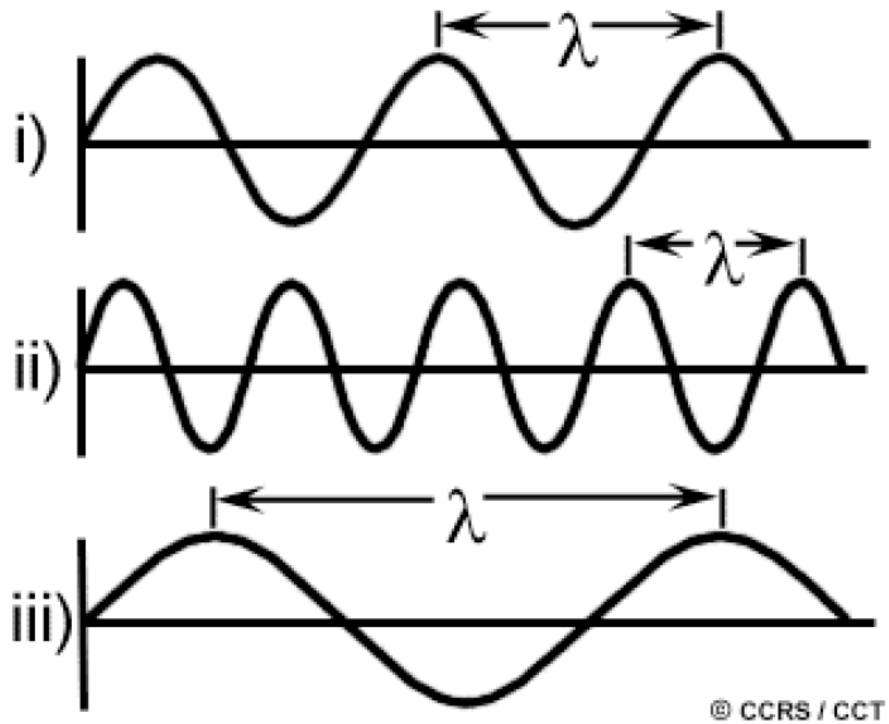
h = Planck's constant.

f = frequency (hz or s^{-1})

λ=wavelength

High-frequency electromagnetic waves have a short wavelength and high energy;

Low-frequency waves have a long wavelength and low energy



Wave Model of Electromagnetic Radiation

- EMR is generated when an electrical charge is accelerated.

- The wavelength of EMR (λ) depends upon the length of time that the charged particle is accelerated and its frequency (ν) depends on the number of accelerations per second.

- Wavelength is the mean distance between maxima (or minima) of a roughly periodic pattern and is normally measured in micrometers (mm) or nanometers (nm).

- Frequency is the number of wavelengths that pass a point per unit time. A wave that sends one crest by every second (completing one cycle) is said to have a frequency of one cycle per second or one hertz, abbreviated 1 Hz.

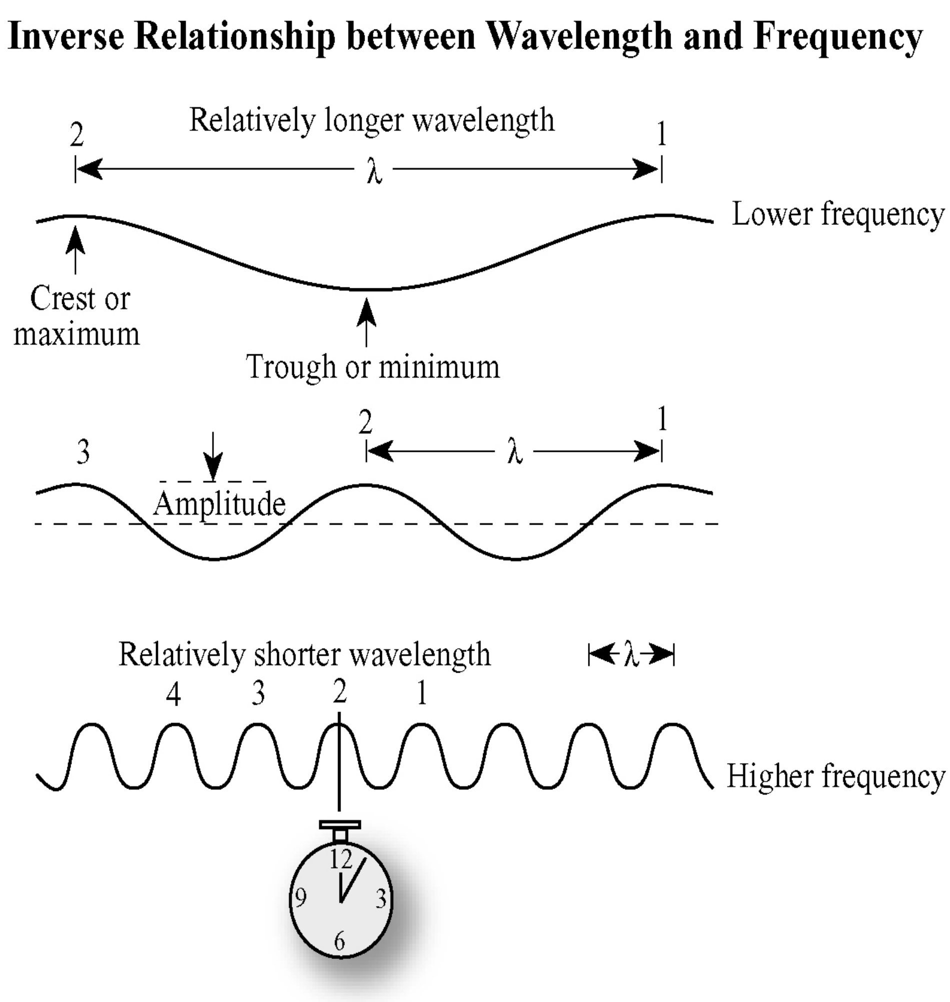
Wave Model of Electromagnetic Energy

The relationship between the wavelength, λ, and frequency, ν, of EMR is based on the following formula, where c is the speed of light:

$$c = \lambda \cdot \nu \qquad \nu = \frac{c}{\lambda} \qquad \lambda = \frac{\nu}{c}$$

Note that frequency, ν, is inversely proportional to wavelength, λ. The longer the wavelength, the lower the frequency, and vice-versa.

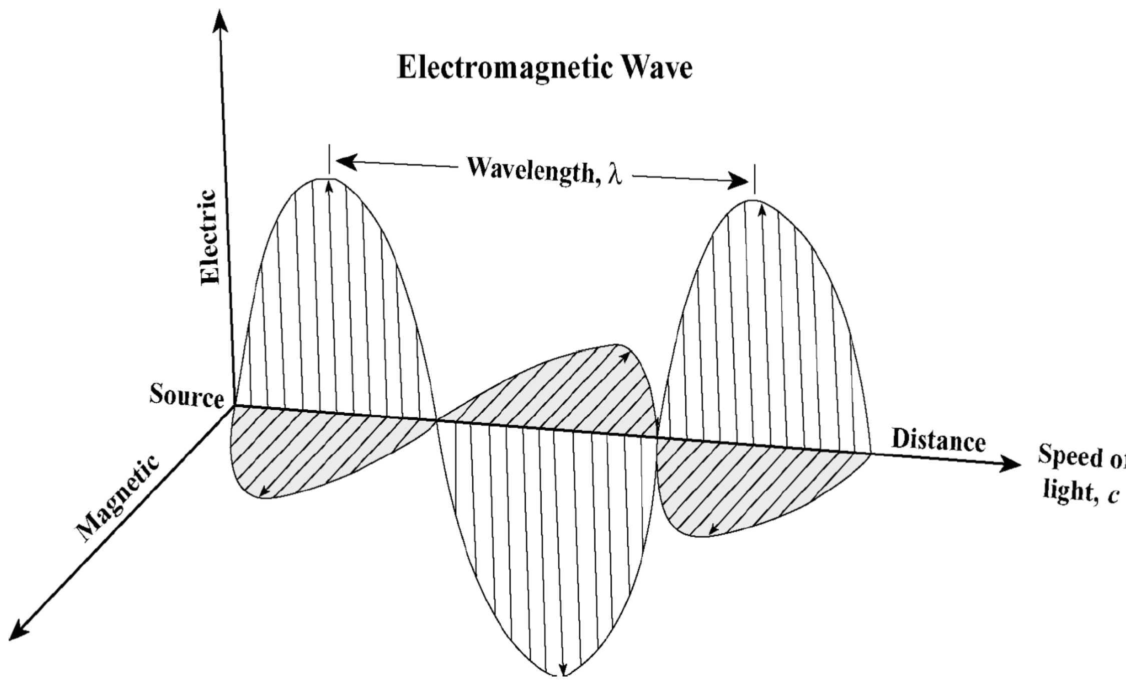
Wave Model of Electromagnetic Energy



- This cross-section of an EM wave illustrates the inverse relationship between wavelength (λ) & frequency (ν). The longer the wavelength the lower the frequency; the shorter the wavelength, the higher the frequency.
- The amplitude of an EM wave is the height of the wave crest above the undisturbed position.
- Frequency is measured in cycles per second, or hertz (Hz).

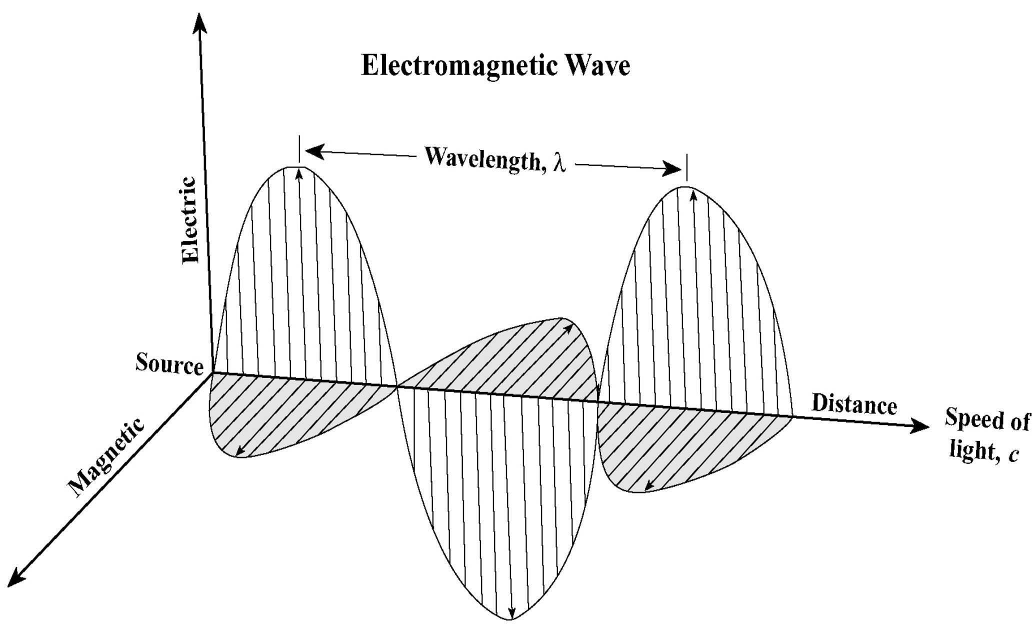
Wave Model of EM Energy

- EMR is carried by a series of continuous waves that are equally and repetitively spaced in time (harmonic waves);
- EMR consists of two fluctuating fields: an electrical field (E) which varies in magnitude in a direction perpendicular to the direction in which the radiation is traveling, and a magnetic field (M) oriented at right angles to the electrical field. Both these fields travel at the speed of light (c).
- Paired fields are perpendicular to each other, and both are perpendicular to direction of wave propagation (transverse waves). Each has a sinusoidal shape.
- Wave nature of EMR is characterized by wavelength and frequency.



Wave Model of Electromagnetic Radiation

Maxwell theory tells us that EMR is an EM wave that travels through space at the speed of light, *c*, which is 3 x 10^8 meters per second (hereafter referred to as m s^{-1}) or 186,282.03 miles s^{-1} (1 foot per nanosecond).

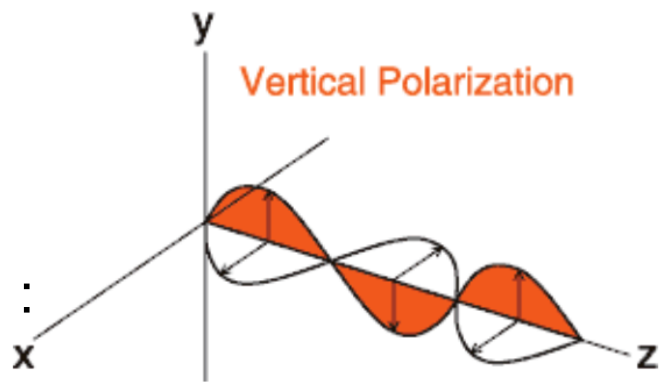


Polarization

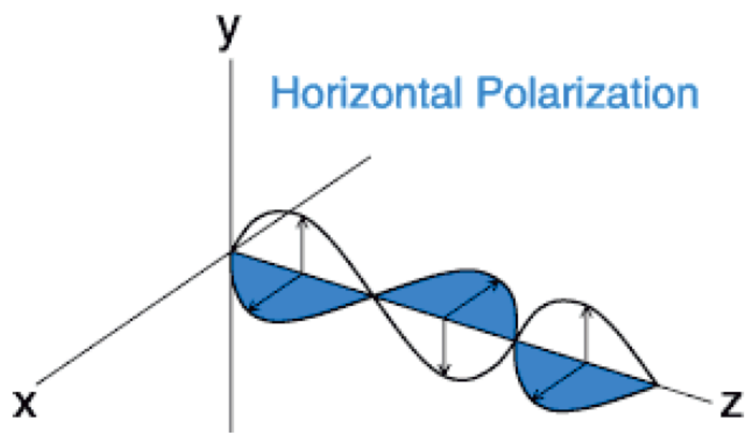
Refers to orientation of the electric field **E**

If both **E** and **B** remain in their respective planes, the radiation is called "**plane or linearly polarised**" :

- vertically polarized (E is parallel to the plane of incidence)
- horizontally polarized (E is perpendicular to the plane of incidence)



E

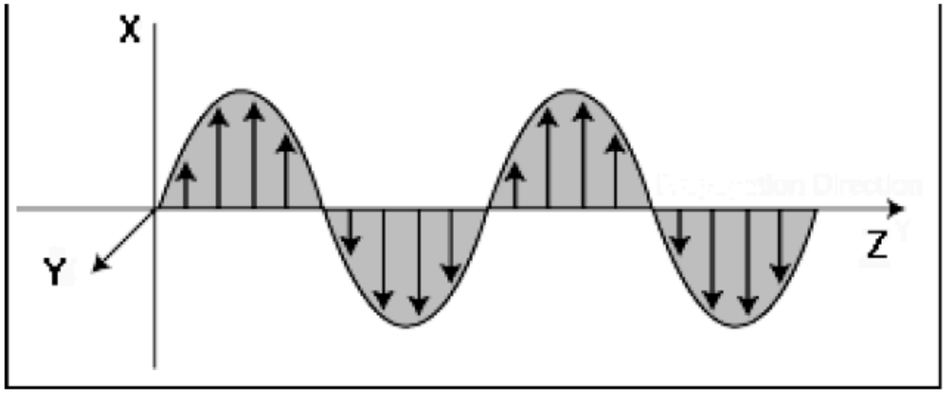


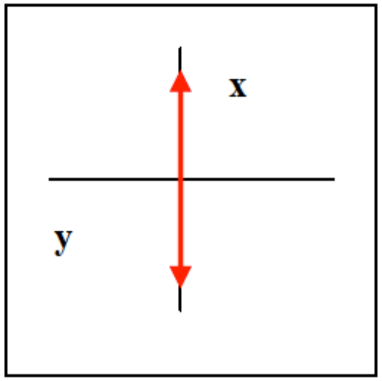
E

Plane of incidence = the plane defined by the vertical and the direction of propagation

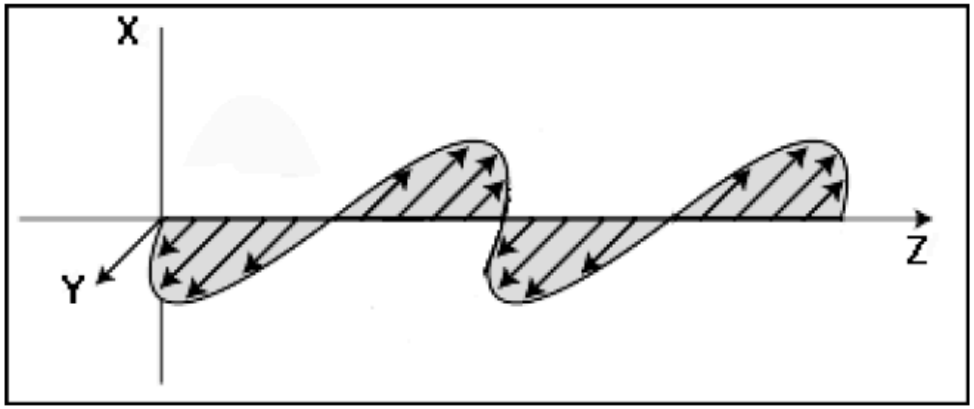
Polarization

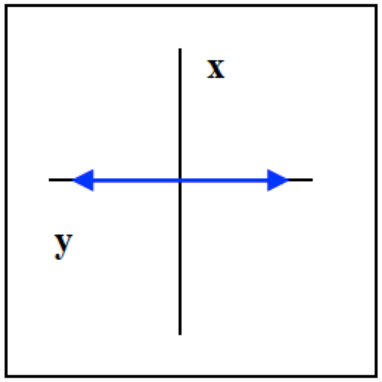
Vertically polarized wave is one for which the electric field lies only in the x-z plane.





Horizontally polarized wave is one for which the electric field lies only in the y-z plane.

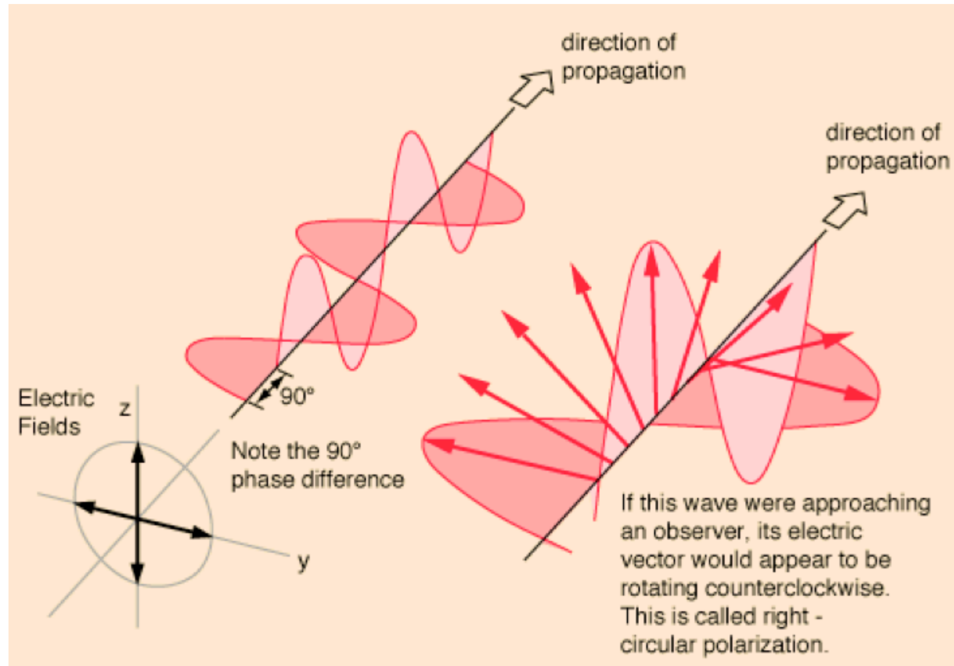




- Horizontal and vertical polarizations are an example of **linear polarization**.

Polarization

If instead of being confined to fixed direction, E rotates in the x-y plane with constant amplitude, it is said to be circularly polarised (either right- or left-hand circular (clockwise/anti-clockwise respectively)



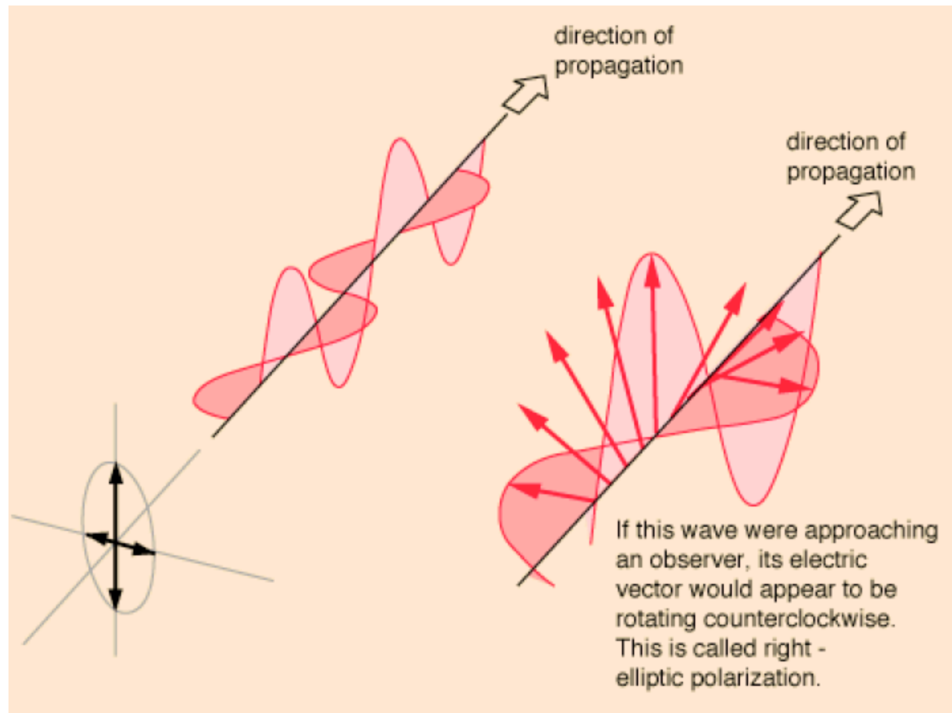
Circularly polarised light consists of two perpendicular EM plane waves of equal amplitude and 90° difference in phase. The light illustrated is right-hand circularly polarised

Radiation from the sun is unpolarised (at random angles)

Polarization

Elliptically polarised light consists of two perpendicular waves of unequal amplitude which differ in phase by 90°. The illustration shows right-hand elliptically polarised light.

If the thumb of your right hand were pointing in the direction of propagation of the light, the electric vector would be rotating in the direction of your fingers.



Stokes parameters

Set of four values that describe the polarisation state of EMR

- intensity **I**,
- the degree of polarization **Q**,
- the plane of polarization **U**,
- the ellipticity **V.**

- **Notation [I,Q,U,V] = Stokes vector**

Stokes parameters

Stokes parameters can be expressed via the amplitudes and the phase shift of the parallel & perpendicular components of the electric field vector

$$I = E_{ro}^2 + E_{lo}^2$$

$$Q = E_{ro}^2 - E_{lo}^2$$

$$U = 2E_{ro}E_{lo}\cos(\Delta\varphi)$$

$$V = 2E_{ro}E_{lo}\sin(\Delta\varphi)$$

Some common Stokes vectors

- Stokes vector for the vertical polarization:
- For this case $\boldsymbol{E_l} = 0$

$$\begin{pmatrix} I \\ Q \\ U \\ V \end{pmatrix} = \begin{pmatrix} E_{ro}^{2} \\ E_{r0}^{2} \\ 0 \\ 0 \end{pmatrix} = E_{ro}^{2} \begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$$

Some common Stokes vectors

Unpolarized Light (1, 0, 0, 0)

Linear horizontal (1, 1, 0, 0)

Linear vertical (1, -1, 0, 0)

Linear at 45 degrees (1, 0, 1, 0)

Linear at -45 degrees (1, 0, -1, 0)

Right circular (1, 0, 0, 1)

Left circular (1, 0, 0, -1)

Stokes parameters

For **unpolarized** light:

$$Q = U = V = 0 \tag{3.7}$$

The **degree of polarization *P*** of a light beam is defined as

$$P = (Q^2 + U^2 + V^2)^{1/2} / I \tag{3.8}$$

The **degree of linear polarization *LP*** of a light beam is defined by neglecting U and V

$$LP = -\frac{Q}{I} \tag{3.9}$$

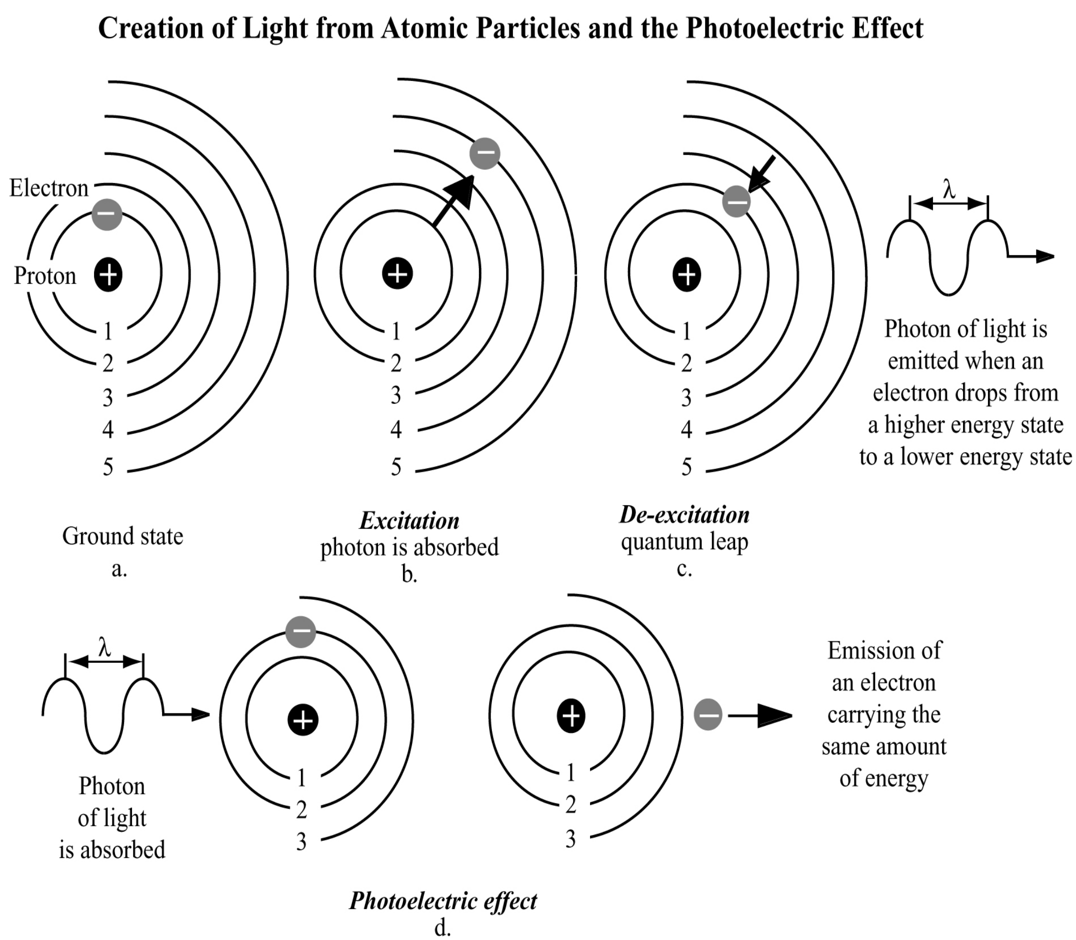
Measurements of polarization are actively used in remote sensing in the solar and microwave regions.
Polarization in the microwave – mainly due to reflection from the surface.
Polarization in the solar – reflection from the surface and scattering by molecules and particulates.
Active remote sensing (e.g., radar) commonly uses polarized radiation.

Quantum Theory of Electromagnetic Energy

- Maxwell's equations show us that light is a smooth and continuous wave, and we often describe EMR in terms of its wave-like properties. .
- Albert Einstein (1905) found that when light interacts with electrons, it has a different character -- it behaves as though it is composed of many individual bodies called *photons*, which carry such particle-like properties as energy and momentum.
- So when the energy interacts with matter it is useful to describe it as discrete packets of energy, or *quanta*.
- According to quantum physics, the energy of an EM wave is quantized, i.e. it can only exist in discrete amount.
- The basic component of energy for an EM wave is called a **photon**. The energy E of a photon is proportional to the frequency of radiation f

$\boldsymbol{E = h f}$ where h is **Planck's Constant** = 6.626×10^{-34} J s.

Creation of Light from Atomic Particles

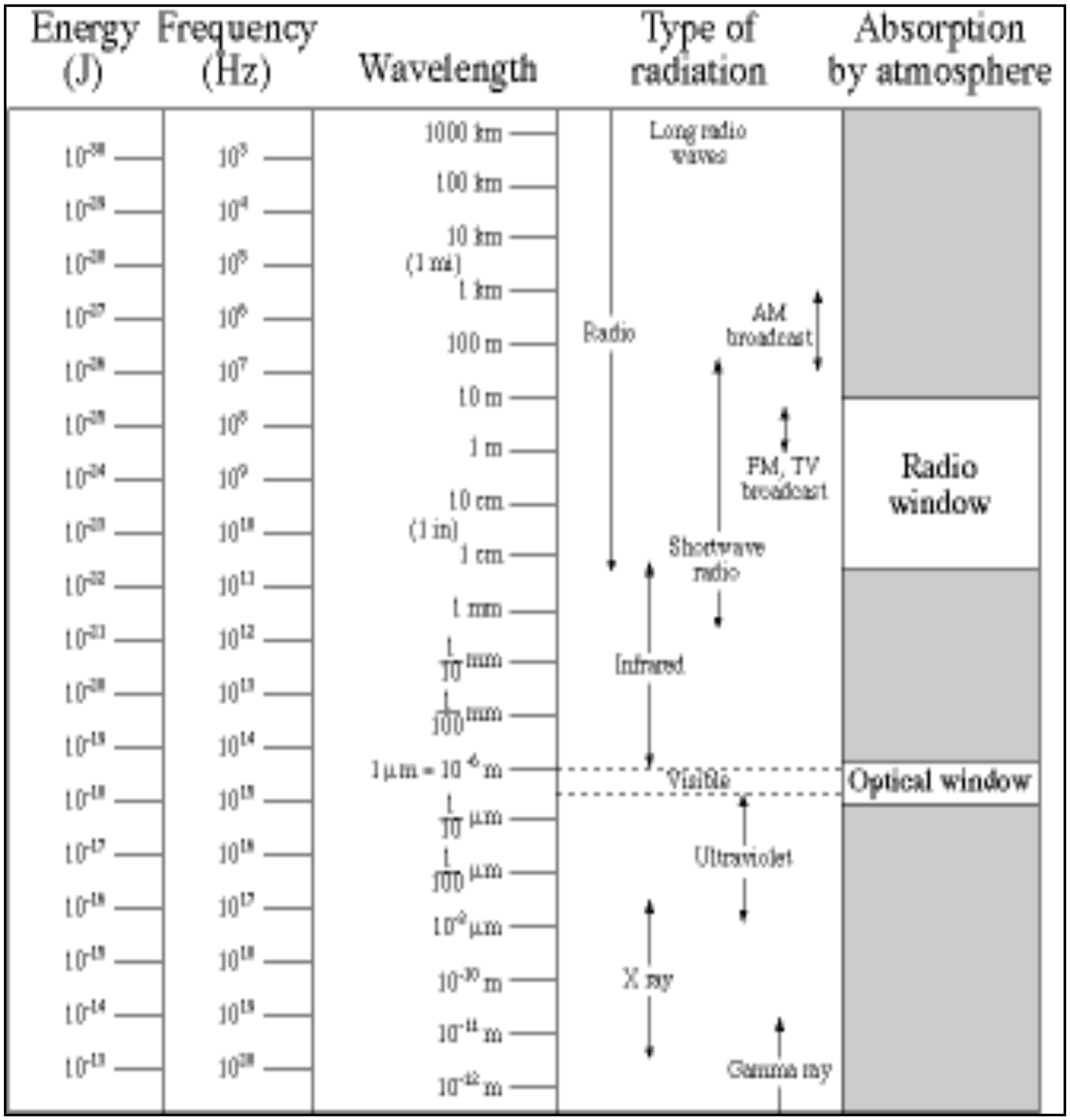


A **photon** of EM energy is emitted when an electron in an atom or molecule drops from a higher-energy state to a lower-energy state. The light emitted (i.e., its wavelength) is a function of the changes in the energy levels of the outer, valence electro, e.g., yellow light may be produced from a sodium vapor lamp.

Matter can also be subjected to such high temperatures that electrons, which normally move in captured, non-radiating orbits, are broken free. When this happens, the atom is left with a positive charge equal to the negatively charged electron that escaped.

The electron becomes a free electron, and the atom is now an ion. If another free electron fills the vacant energy level created by the free electron, then radiation from all wavelengths is produced, i.e., a continuous spectrum of energy. The intense heat at the surface of the Sun produces a *continuous spectrum* in this manner.

Energy of Quanta (Photons)



The energy of quanta (photons) ranging from gamma rays to radio waves in the EM spectrum.

Electromagnetic Radiation (EMR)

- The first requirement for remote sensing is to have an **energy source to illuminate the target**. This energy for remote sensing instruments is in the form of electromagnetic radiation (EMR).
- Remote sensing is concerned with the measurement of EMR returned by Earth surface features that first receive energy from (i) the sun or (ii) an artificial source e.g. a radar transmitter.
- Different objects return different types and amounts of EMR.
- Objective of remote sensing is to detect these differences with the appropriate instruments.
- Differences make it possible to identify and assess a broad range of surface features and their conditions

Electromagnetic Spectrum

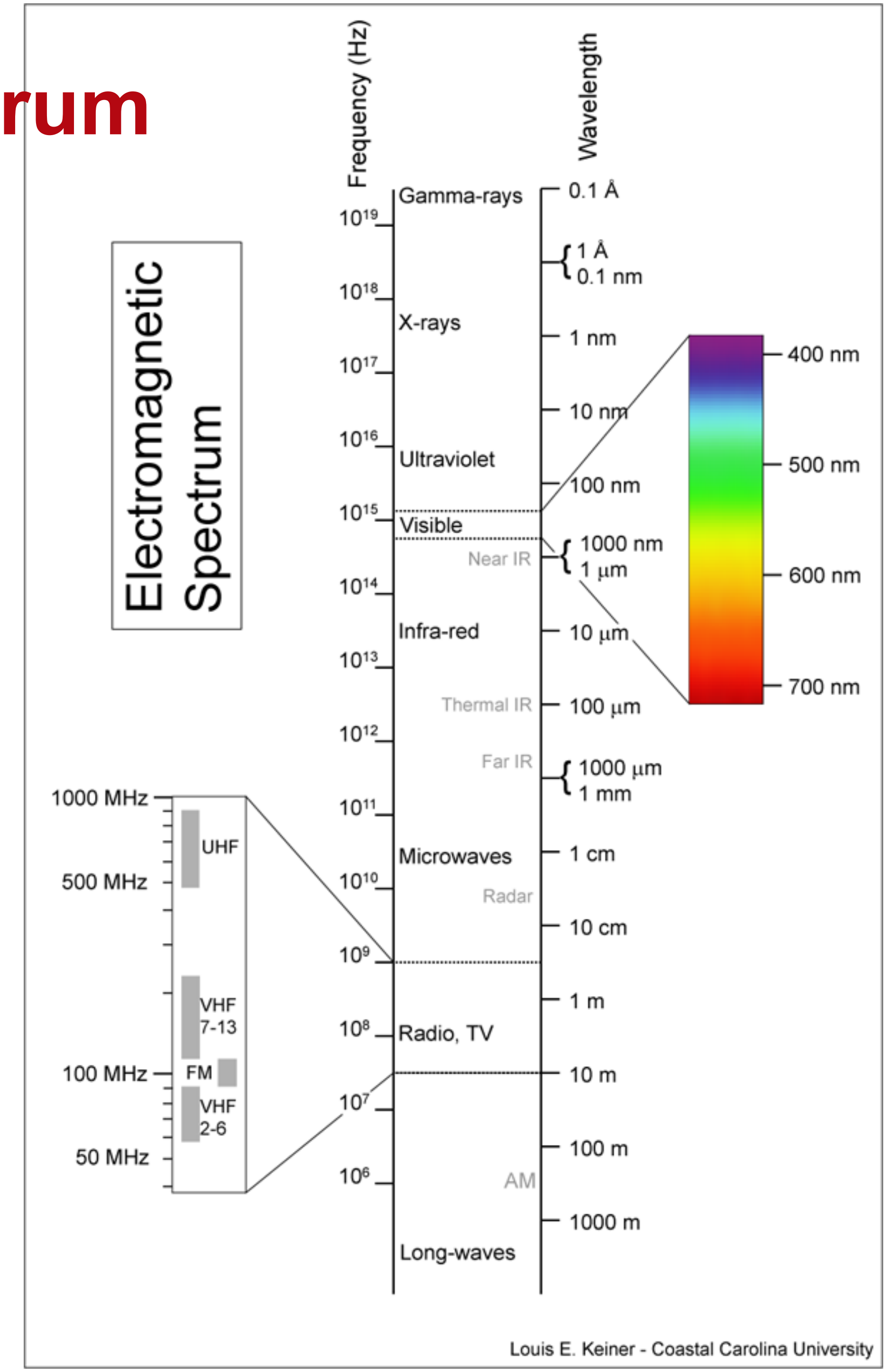
- ★ Frequency (or wavelength) of an EM wave depends on its source.
- ★ There is a wide range of frequency encountered in our physical world, ranging from the low frequency of the electric waves generated by the power transmission lines to the very high frequency of the gamma rays originating from the atomic nuclei.
- ★ This wide frequency range of electromagnetic waves make up the **Electromagnetic Spectrum**.

Electromagnetic Spectrum

- Represents the continuum of electromagnetic energy from extremely short wavelengths (cosmic and gamma rays) to extremely long wavelengths (microwaves).

- No natural breaks in the EMS -- it is artificially separated and named as various spectral bands (divisions) for the description convenience.

- Common bands in remote sensing are visible, infra-red & microwave.

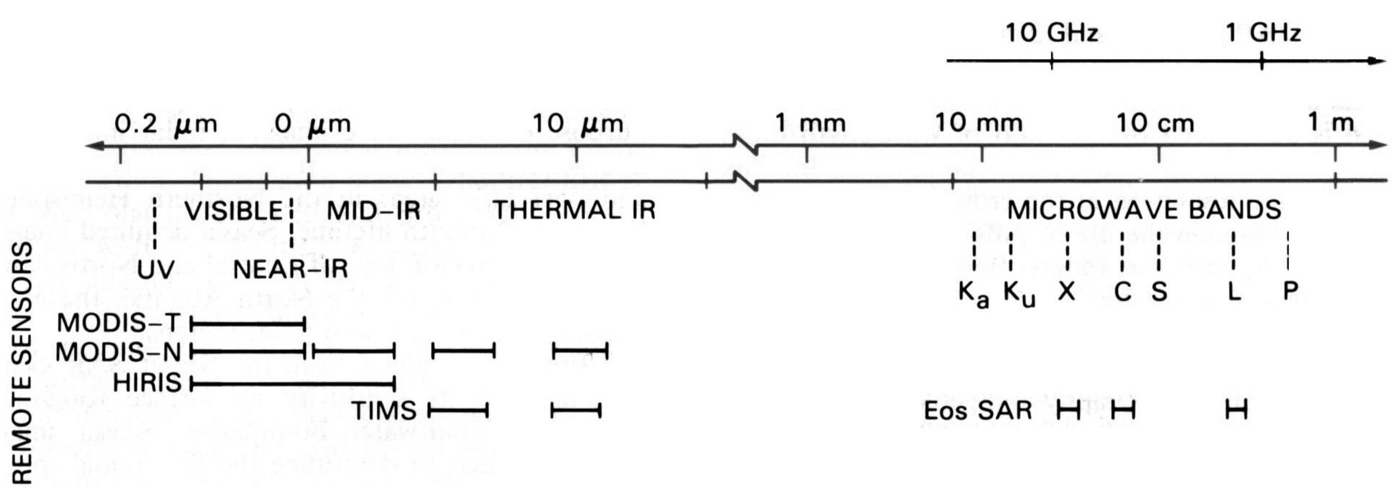


Spectral bands

Three important spectral bands in remote sensing:

- visible light
- infrared radiation
- microwave radiation

Image from NASA 1987. SAR: Synthetic Aperture Radar. Earth Observing System, Vol. IIf.



Also see Figure 2.2 in Rees

Electromagnetic Radiation (EMR)

- EM energy (radiation) is one of many forms of energy. It can be generated by changes in the energy levels of electrons, acceleration of electrical charges, decay of radioactive substances, and the thermal motion of atoms and molecules.

- All natural and synthetic substances above absolute zero (0 Kelvin, -273°C) emit a range of electromagnetic energy.

- Most remote sensing systems are passive sensors, i.e. they relying on the sun to generate all the required EM energy.

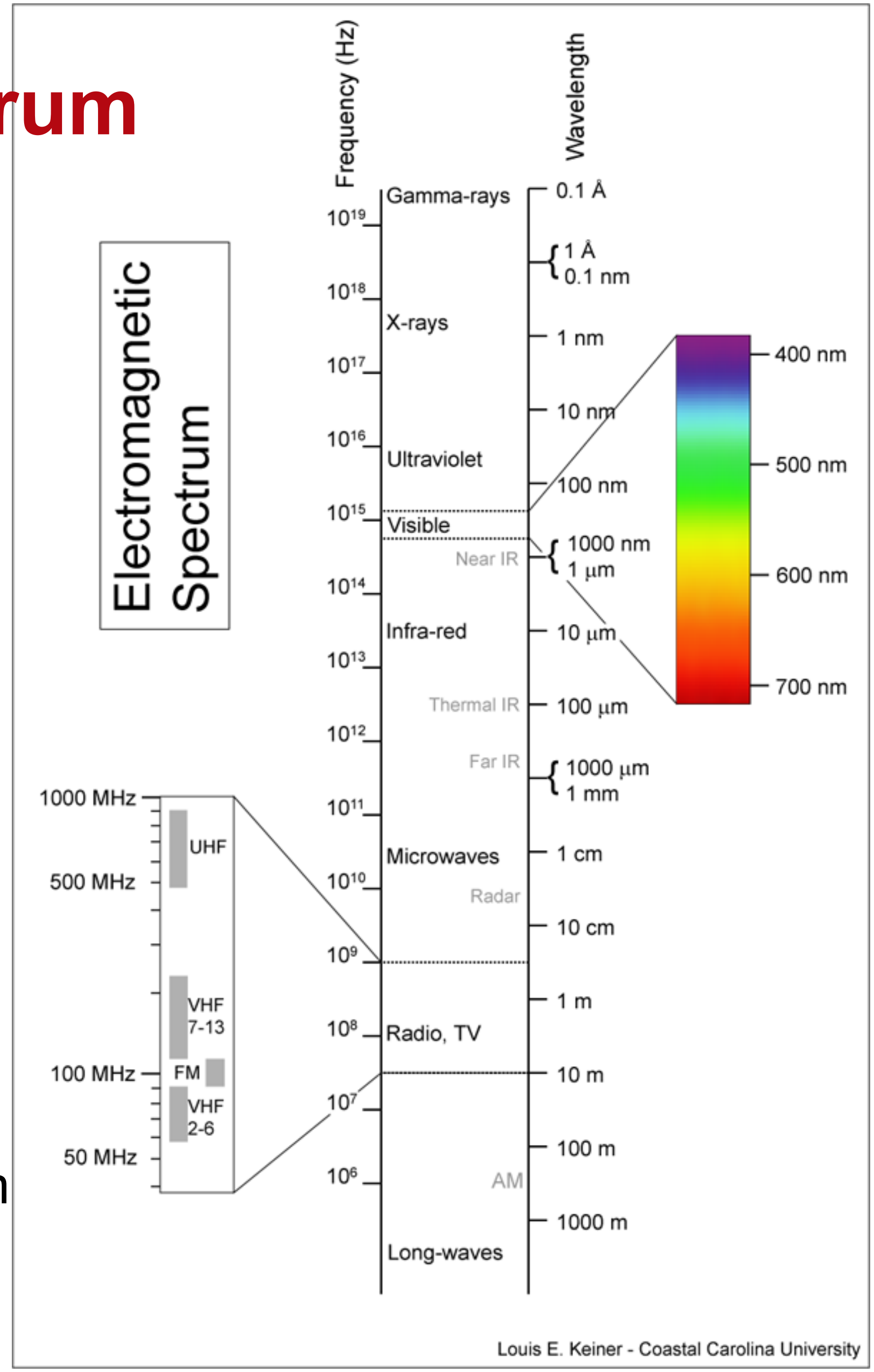
- Active sensors (like radar) transmit energy in a certain direction and records the portion reflected back by features within the signal path.

Electromagnetic Spectrum

Visible: Small portion of the EMS that humans are sensitive to:
blue (0.4-0.5 μm); green (0.5-0.6 μm); red (0.6-0.73 μm)

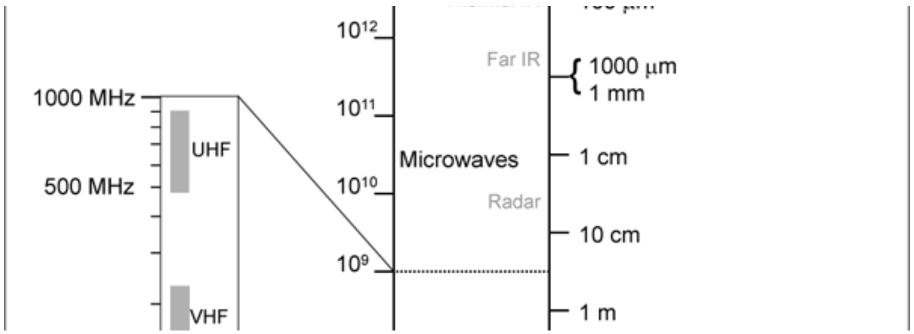
Infrared: Three logical zones:
1. Near IR: reflected, can be recorded on film emulsions (0.7 - 1.3μm).
2. Mid infrared: reflected, can be detected using electro-optical sensors (1.3 - 3.0μm).
3. Thermal infrared: emitted, can only be detected using electro-optical sensors (3.0 - 5.0 and 8 - 14 μm).

Microwave
Radar sensors, wavelengths range from 1mm - 1m (K_a, K_u, X, C, S, L & P)



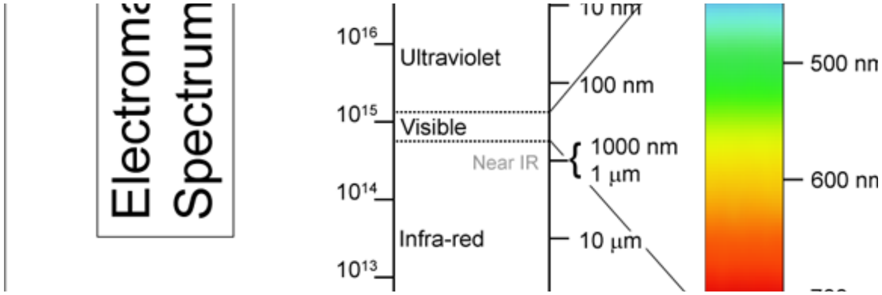
Wavelengths of microwave

- **Microwaves**: 1 mm to 1 m wavelength.
- Further divided into different frequency bands:**(1 GHz = 10^9 Hz)**
- **P band**: 0.3 - 1 GHz (30 - 100 cm)
- **L band**: 1 - 2 GHz (15 - 30 cm)
- **S band**: 2 - 4 GHz (7.5 - 15 cm)
- **C band**: 4 - 8 GHz (3.8 - 7.5 cm)
- **X band**: 8 - 12.5 GHz (2.4 - 3.8 cm)
- **Ku band**: 12.5 - 18 GHz (1.7 - 2.4 cm)
- **K band**: 18 - 26.5 GHz (1.1 - 1.7 cm)
- **Ka band**: 26.5 - 40 GHz (0.75 - 1.1 cm)



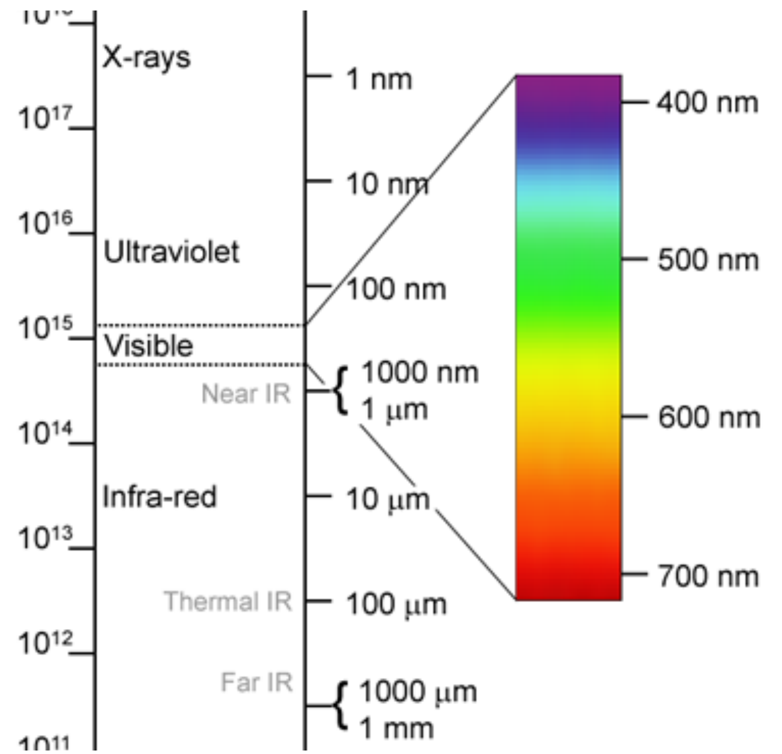
Wavelengths of Infrared

- **Infrared**: 0.7 to 300 μm wavelength. This region is further divided into the following bands:
- **Near Infrared (NIR)**: 0.7 to 1.5 μm.
- **Short Wavelength Infrared (SWIR)**: 1.5 to 3 μm.
- **Mid Wavelength Infrared (MWIR)**: 3 to 8 μm.
- **Long Wavelength Infrared (LWIR)**: 8 to 15 μm.
- **Far Infrared (FIR)**: longer than 15 μm.
- The NIR and SWIR are also known as the **Reflected Infrared**, referring to the main infrared component of the solar radiation reflected from the earth's surface. The MWIR and LWIR are the **Thermal Infrared**.

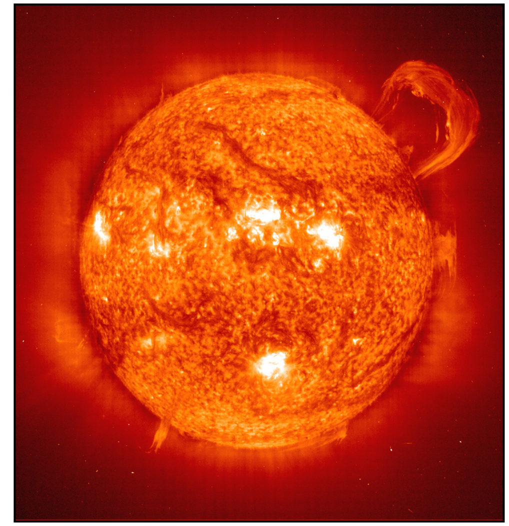


Wavelengths of visible light

- **Red**: 610 - 700 nm
- **Orange**: 590 - 610 nm
- **Yellow**: 570 - 590 nm
- **Green**: 500 - 570 nm
- **Blue**: 450 - 500 nm
- **Indigo**: 430 - 450 nm
- **Violet**: 400 - 430 nm

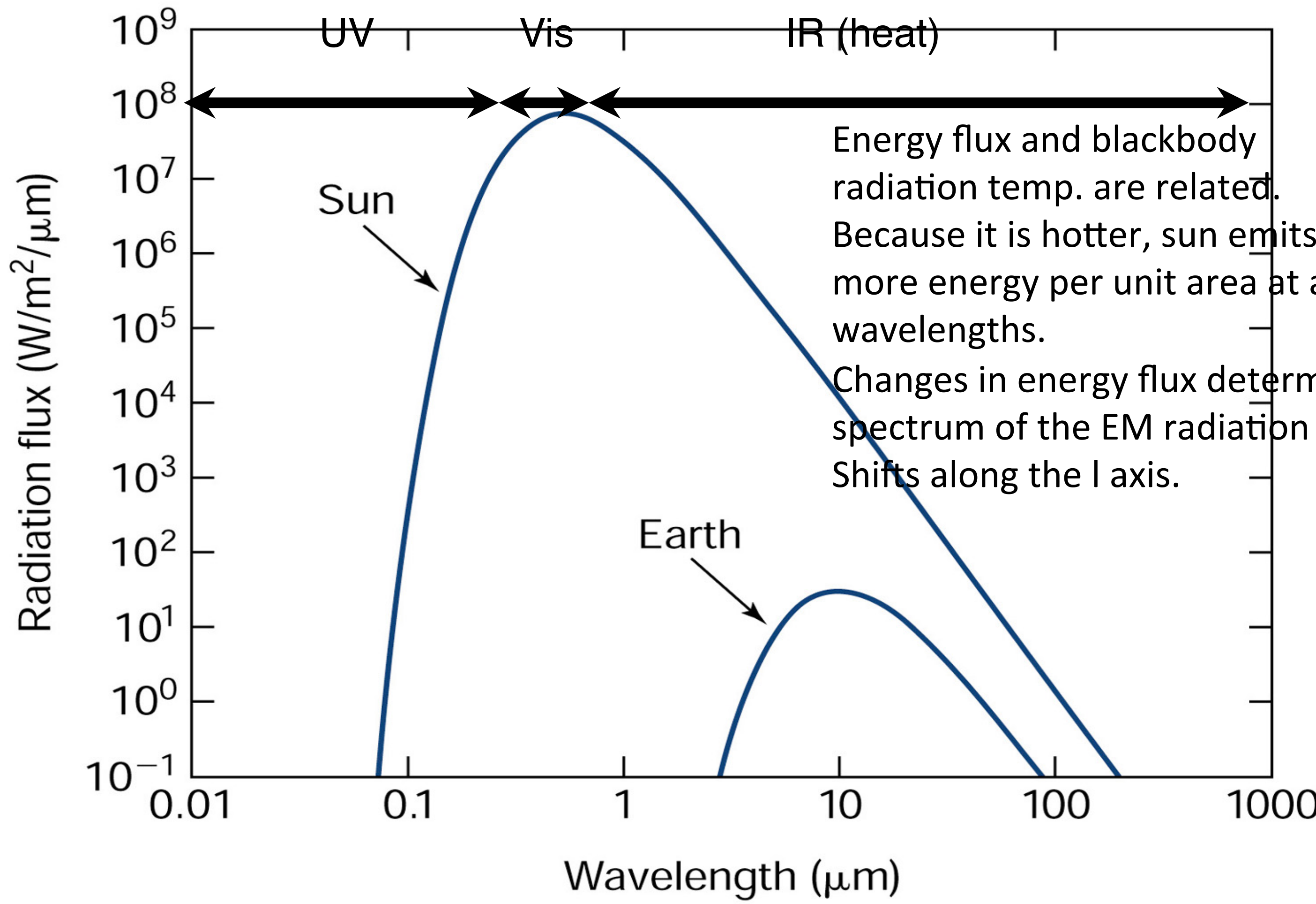


EM energy from the Sun



- EMR from the Sun travels in eight minutes across the intervening 93 million miles (150 million km) of space to the Earth.
- The Sun produces a continuous spectrum of EMR ranging from very short, extremely high frequency gamma and cosmic waves to long, very low frequency radio waves
- The Earth approximates a 300 K (27˚C) blackbody and has a dominant wavelength at approximately 9.7 mm.

Earth’s energy budget



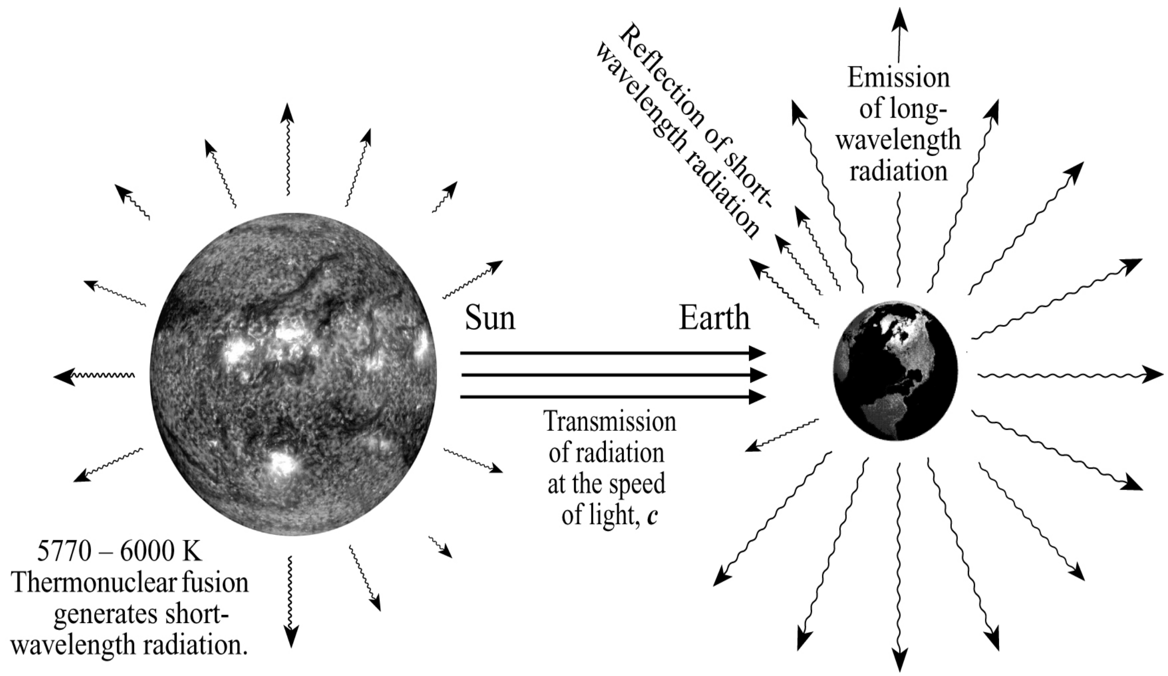
Energy flux and blackbody radiation temp. are related. Because it is hotter, sun emits more energy per unit area at all wavelengths.
Changes in energy flux determine the spectrum of the EM radiation emitted. Shifts along the l axis.

Blackbody emission curves for sun and earth – note higher energy shorter wavelength emissions for the sun.

Energy output of the sun

- ★ Predicts a maximum of radiation emission in the visible region (0.4 – 0.7 mm)
- ★ Large component of high energy UV (remember this is damaging)
- ★ Assume Earth is a blackbody radiator
- ★ The Earth System responds to both the amount of solar radiation and its EM spectrum
- ★ Earth energy balance: If temperature is constant, the planet has to be in radiative balance

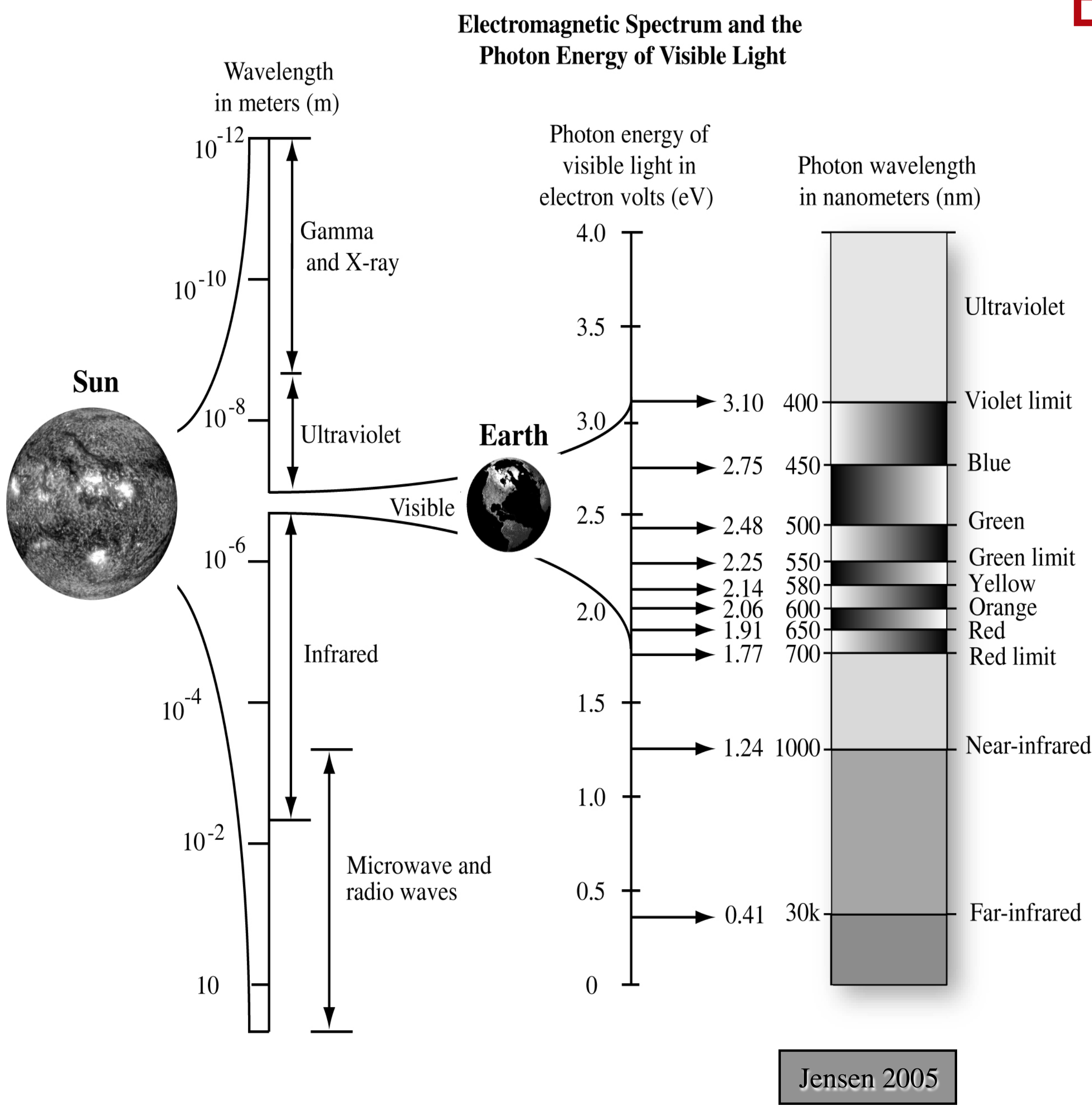
Sources of EM Energy



Jensen 2005

Thermonuclear fusion taking place on the surface of the Sun (at temp 5770 – 6000 kelvin (K)) yields a continuous spectrum of EM energy. Produces a large amount of relatively short wavelength energy that travels through the vacuum of space at the speed of light. Some of this energy is intercepted by the Earth, where it interacts with the atmosphere and surface materials. The Earth reflects some of the energy directly back out to space or it may absorb the short wavelength energy and then re-emit it at a longer wavelength

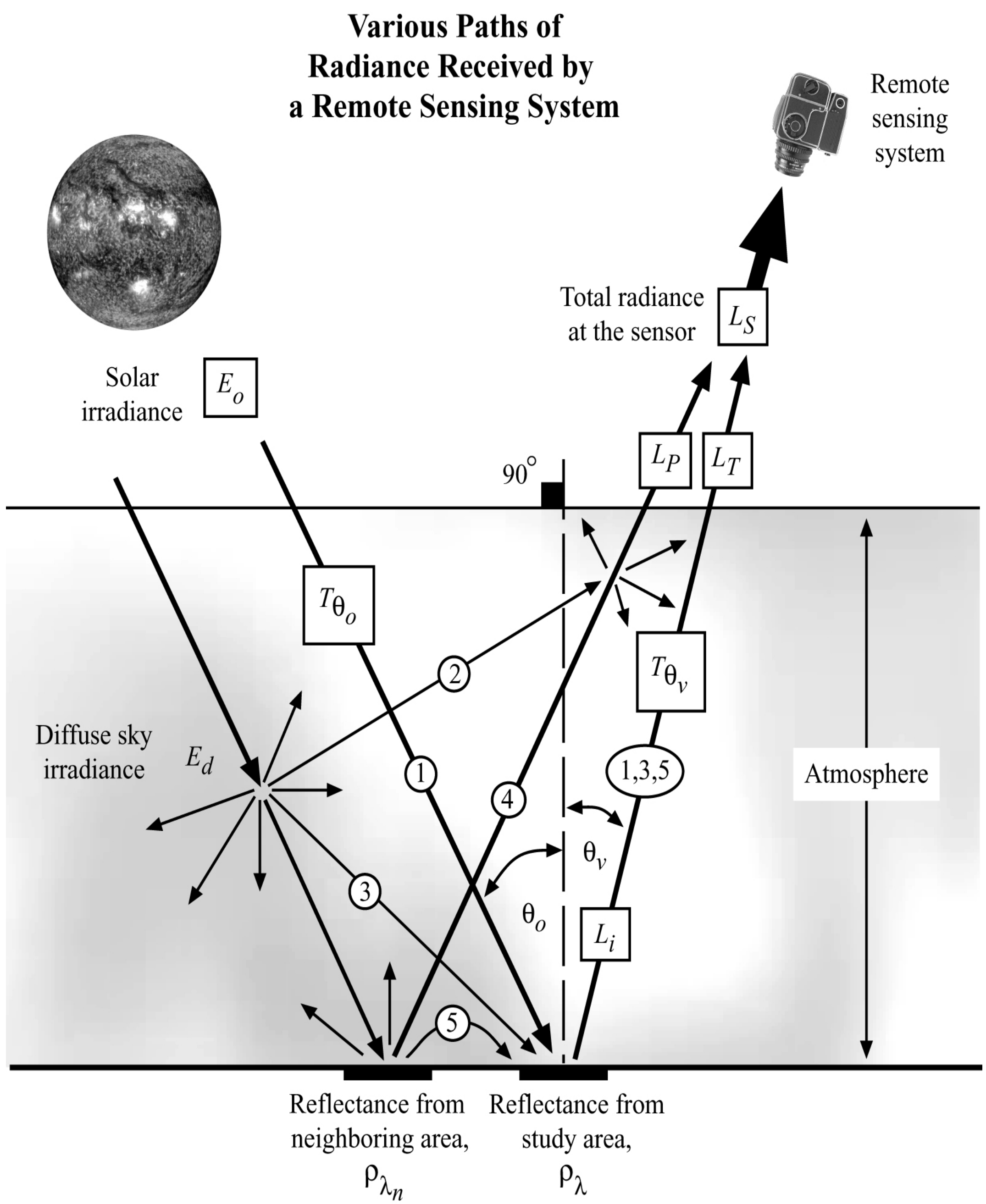
Electromagnetic Spectrum



The Sun produces a *continuous spectrum* of energy from gamma rays to radio waves that continually bathe the Earth in energy. The visible portion of the spectrum may be measured using wavelength (measured in micrometers or nanometers, i.e., mm or nm) or electron volts (eV). All units are interchangeable.

Electromagnetic Energy Interactions

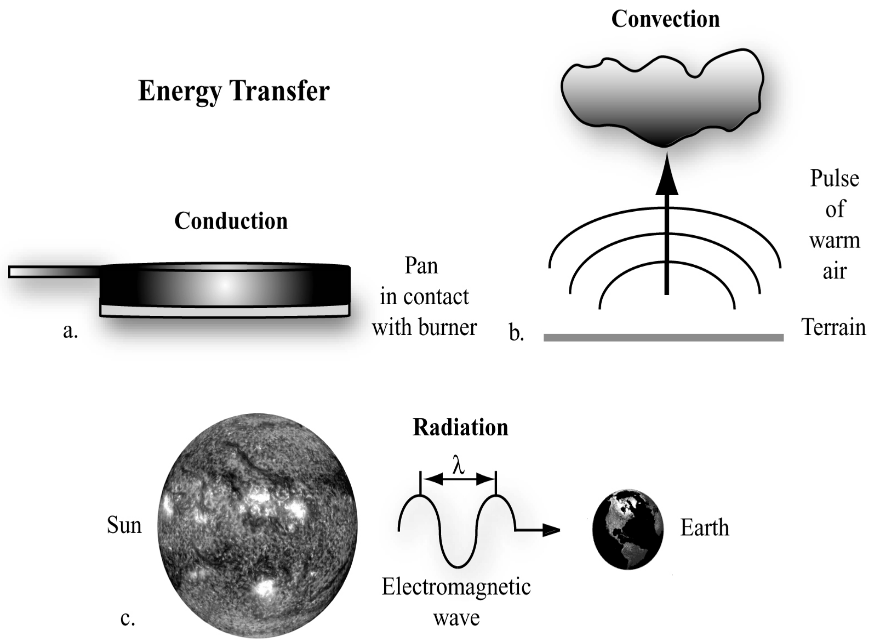
- EM energy is the means by which information is transmitted from an object to a sensor
- Energy recorded by remote sensing systems undergoes fundamental interactions that must be understood to properly interpret the remotely sensed data. e.g., if the energy being remotely sensed comes from the Sun, the energy:
    - is radiated by atomic particles at the source (the Sun)
    - propagates through the vacuum of space at the speed of light
    - interacts with the Earth's atmosphere
    - interacts with the Earth's surface,interacts with the Earth's atmosphere once again
    - finally reaches the remote sensor where it interacts with various optical systems, filters, emulsions, or detectors.



Energy-matter interactions

- atmosphere
- study area
- detector

How is Energy Transferred?

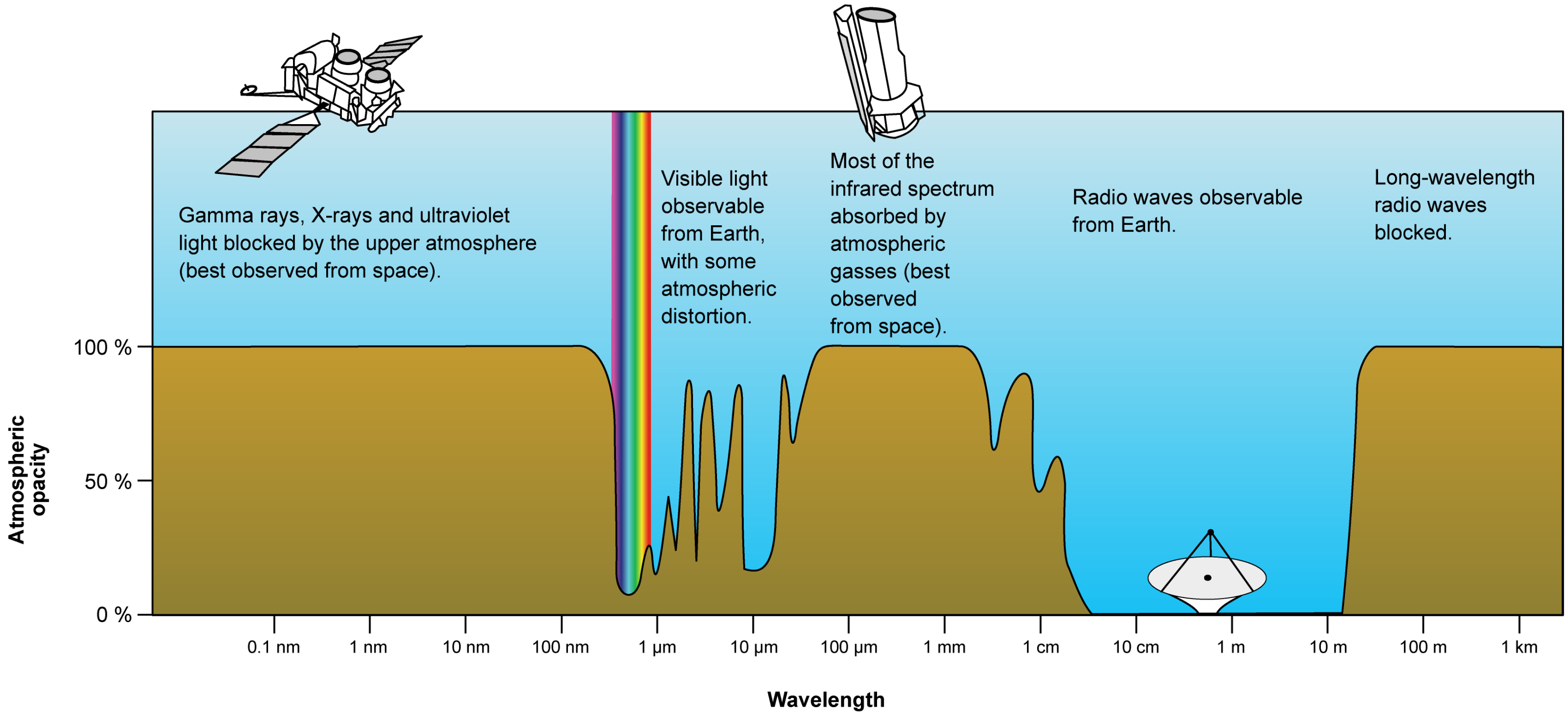


Jensen 2005

Energy is transferred three ways: *conduction*, *convection*, and *radiation*.

1) Energy may be conducted directly from one object to another as when a pan is in direct physical contact with a hot burner
2) The Sun bathes the Earth's surface with radiant energy causing the air near the ground to increase in temperature. The less dense air rises, creating convectional currents in the atmosphere.
3) EM energy in the form of EM waves may be transmitted through the vacuum of space from the Sun to the Earth.

Absorption of EMR by atmosphere

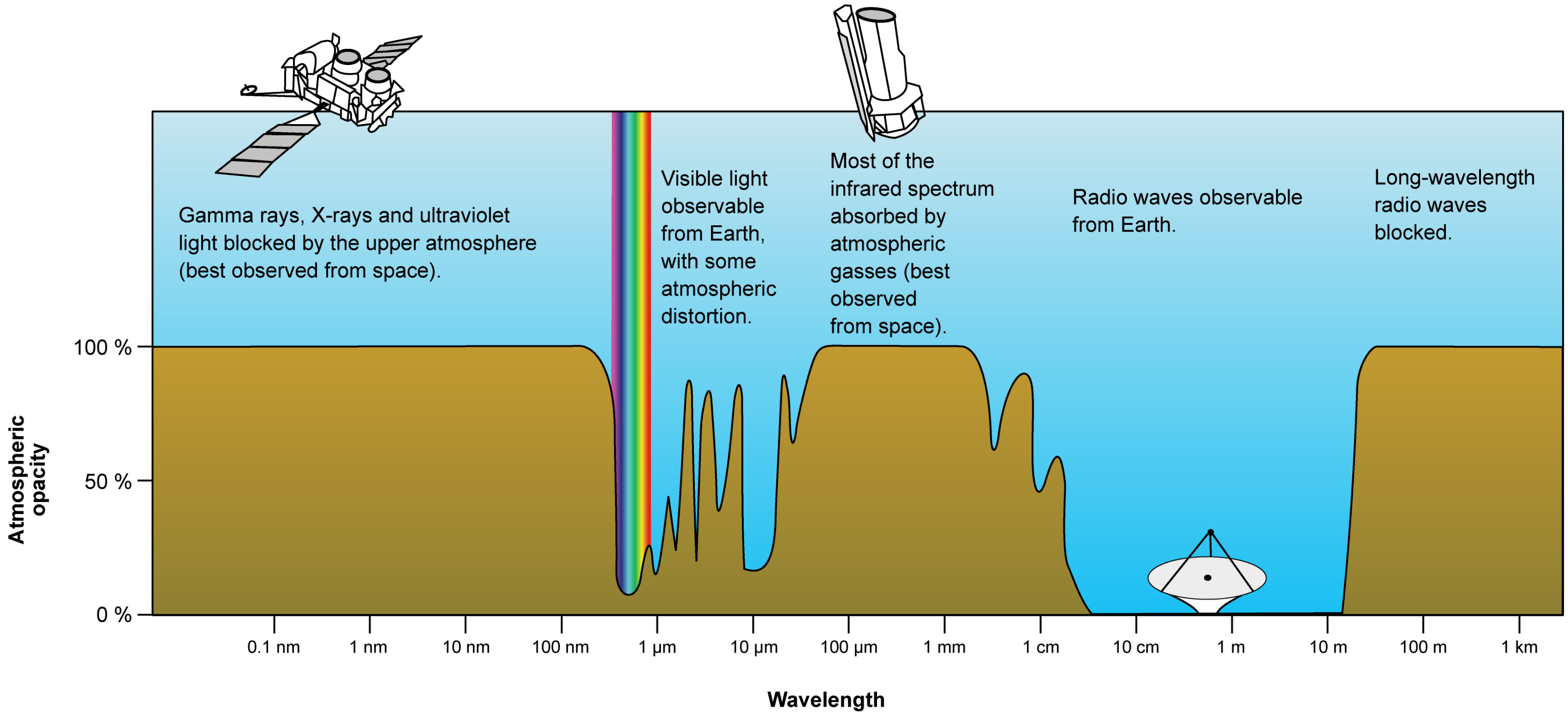


Different molecules absorb different wavelengths of radiation:

- O_2 and O_3 absorb almost all wavelengths shorter than 300 nm.
- Water (H_2O) absorbs many wavelengths above 700 nm, but this depends on the amount of water vapor in the atmosphere.

When a molecule absorbs a photon, it increases the energy of the molecule. We can think of this as heating the atmosphere, but the atmosphere also cools by emitting radiation.

Absorption of EMR by atmosphere



- When you combine the absorption spectra of the gasses in the atmosphere, you are left with "windows" of low opacity, allowing the transmission of only certain EMR.
- Optical window runs from around 300 nm (UV-C) up the visible spectrum (commonly called light), at roughly 400–700 nm and continues to the infrared to around 1100 nm.
- There are also infrared and radio windows.

Interaction of EMR with matter

Radiative properties of natural surfaces

- Radiation incident upon a surface must either be *transmitted* (τ) through it, *reflected* (α) from the surface, or be *absorbed* (ξ).
- For solar radiation α is referred to as the surface *albedo*
- If we consider only part of the EM spectrum α is referred to as spectral reflectance

Transmissivity (τ) + Reflectivity (α) + Absorptivity (ξ) = 1

Interaction of EMR with matter

Propagation of EMR in a uniform homogenous material depends on two properties of medium:

1) *relative electric permittivity ε_r or dielectric constant $\varepsilon_r = \varepsilon/\varepsilon_0$*

2) *relative magnetic permeability $\mu_r = \mu/\mu_0$*

with no absorption, ε_r and μ_r are real, dimensionless numbers

$E_x = E_o \cos(\omega t\text{-}kz)$ and $B_y = E_o/c \cos(\omega t\text{-}kz)$

wave speed (phase velocity) $\omega/k = c/\mathrm{sqrt}(\varepsilon_r\, \mu_r)$

The ***refractive index $n = (\varepsilon_r\, \mu_r)^{1/2}$*** is a measure of how much the speed of EMR is reduced inside the medium [for free space, n=1]

Complex dielectric constant

For most medium we shall need to consider, $\mu_r = 1$ (non magnetic materials)

If the medium absorbs energy from the wave, the dielectric constant becomes complex (real + imaginery)

$\varepsilon_r = \varepsilon' - i\varepsilon''$ or $\varepsilon_r = \varepsilon'(1 - \tan\delta)$

loss tangent

See page 36 of Rees, arrive at the following wave equation:

➔ $E_x = E_o \exp(-\omega kz/c) \exp(i[\omega t - \omega mz/c])$

Simple harmonic wave whose amplitude decreases exponentially with z
Flux density $F = F_0 \exp(-2\,\omega kz/c)$

➔ Absorption length $l_a = c/2\omega k$