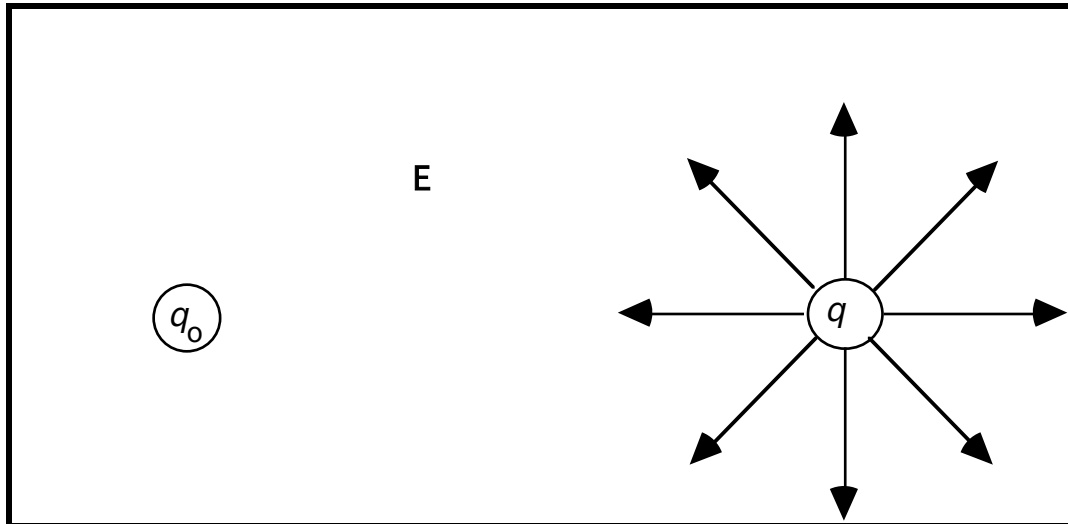


ELECTROMAGNETISM SUMMARY

(Rees Chapters 2 and 3)

Review of E and B

The electric field **E** is a vector function.



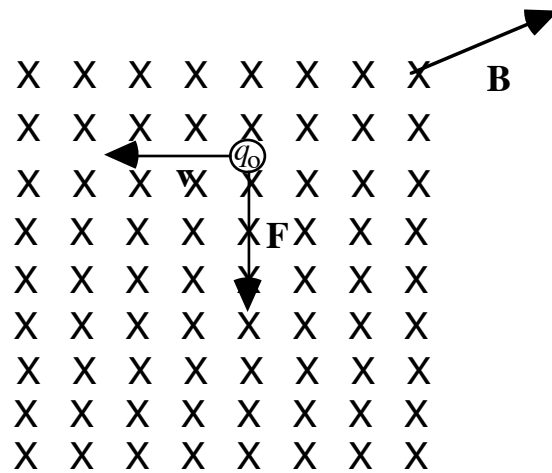
If we place a second test charged q_o in the electric field of the charge q , the two spheres will repel each other. The strength of the radial force F_r is proportional to the product of the charges and inversely proportional to the distance between them squared.

$$F_r = \frac{1}{4\pi\epsilon_o} \frac{q_o q}{r^2} \quad \text{Coulomb's Law}$$

where ϵ_o is the electric permittivity (8.85×10^{-12} F/m) . The radial component of the electric field is the force divided by the strength of the test charge q_o .

$$E_r = \frac{1}{4\pi\epsilon_o} \frac{q}{r^2} \quad \frac{\text{force}}{\text{charge}} \quad - \quad \frac{\text{ML}}{\text{T}^2\text{Q}}$$

The magnetic field **B** is also a vector function. Magnetic fields are generated by charges in motion. For example, a current traveling through a wire will produce a magnetic field. A charge moving through a uniform magnetic field will experience a force that is proportional to the component of velocity perpendicular to the magnetic field. The direction of the force is perpendicular to both **v** and **B**.



$$\mathbf{F} = q_o \mathbf{v} \otimes \mathbf{B}$$

The particle will move along a curved path that balances the inward-directed force F_{\perp} and the outward-directed centrifugal force which, of course, depends on the mass of the particle. The magnetic field strength B is

$$B = \frac{F_{\perp}}{q_o v} = \frac{\text{force}}{\text{charge} \times \text{velocity}} = \frac{M}{TQ}$$

The ratio of magnetic field to electric field has units of velocity.

$$\frac{E}{B} = \frac{\frac{ML}{T^2Q}}{\frac{M}{TQ}} = \frac{L}{T}$$

Electromagnetic radiation (Rees, pages 9-11, 34-41)

Chapters 2 and 3 in Rees contain a brief review of the results of E&M theory that are needed to understand the physical principles of remote sensing; most of this material is covered in a 3-quarter course in freshman physics. This review will take at least two lectures. If you have not seen this material before, you should read the chapter several times to pick up the terminology and important results. Even if you have covered this material before, you may have to go back to your old text books and review some basic physics.

The topics that you will need to understand for later in the course include: refractive index - n , flux density - F , the electromagnetic spectrum, absorption and absorption length, reflection and dispersion, polarization and Doppler shift.

A second set of topics treated later include: diffraction and resolution, Snell's law, thermal radiation, radiance, and irradiance. I'll spend some extra time on thermal radiation because this is an important topic for detection and interpretation of infrared and microwave radiation. Moreover, it is interesting that quantum physics was developed to understand the spectrum of black-body radiation. Today, some of the most challenging problems in climate remote sensing are related to developing stable calibration for thermal infrared sensors.

Maxwell's Equations - We'll start with Maxwell's equations under the assumptions that there are no free charges $\nabla \cdot \mathbf{E} = 0$, no currents $\nabla \cdot \mathbf{B} = 0$, and all materials are non-magnetic $\mu = 1$. In this case Maxwell's equations reduce to vector wave equations and the solutions will be propagating waves

$$\nabla^2 \mathbf{E} - \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{E}}{\partial t^2} = 0$$

$$\nabla^2 \mathbf{B} - \frac{\epsilon}{c^2} \frac{\partial^2 \mathbf{B}}{\partial t^2} = 0$$

where c is the speed of light ($\sim 3 \times 10^8$ m/s) and ϵ is the relative electric permittivity which is also known as the dielectric constant. The speed of light is related to the electric permittivity of free space ϵ_0 and magnetic permeability of free space μ_0 .

$$c = (\epsilon_0 \mu_0)^{-1/2}$$

ϵ_0 - electric permittivity of free space - 8.85×10^{-12} F/m

μ_0 - magnetic permeability of free space - $4\pi \times 10^{-7}$ H/m

Note that μ_0 has a defined value and ϵ_0 is implicitly measured when one measures the speed of light. For our purposes, the important quantity is ϵ , the dielectric constant which contain the variations in permittivity with respect to the free-space value; ϵ , can be either a real value or a complex value. If ϵ is complex, then the waves will be attenuated as they propagate through the medium.

Since we have assumed there are no charges or currents, both \mathbf{E} and \mathbf{B} are perpendicular to the direction of the propagation of the wave and they are also perpendicular to each other. Consider a wave propagating in the negative z direction (use negative to follow the development in Rees) with a frequency ω and a wavenumber k . The x -component of the electric field is

$$E_x = E_o \exp i(\omega t - kz)$$

$$\omega = \frac{2\pi}{T}$$

$$k = \frac{2\pi}{\lambda}$$

where E_o is the amplitude, T is the period and λ is the wavelength. Plugging this solution into the x -differential equation for the electric field yields a relationship between the frequency and wavenumber.

$$\frac{\partial^2 E_x}{\partial z^2} - \frac{\epsilon}{c^2} \frac{\partial^2 E_x}{\partial t^2} = 0$$

$$(-ik)^2 e^{i(\omega t - kz)} - \frac{\epsilon}{c^2} (i\omega)^2 e^{i(\omega t - kz)} = 0$$

$$-k^2 + \frac{\epsilon}{c^2} \omega^2 = 0$$

$$\omega = c k \epsilon^{-1/2}$$

The phase velocity of the wave is

$$v \equiv \frac{\omega}{k} = c \epsilon^{-1/2} \equiv \frac{c}{n}$$

where n is the index of refraction, $n = \epsilon^{1/2}$. Also note that $k = n \frac{\omega}{c}$ and we'll use this later to eliminate the wavenumber from the equations.

A similar solution holds for the magnetic field \mathbf{B} . Since \mathbf{B} is perpendicular to \mathbf{E} we have for this case

$$B_y = B_o e^{i(\omega t - kz)}$$

E_o and B_o are the amplitudes of the electric and magnetic fields and in free space, their ratio is a constant.

$$\frac{E_o}{B_o} = \frac{Z_o}{\mu_o}$$

$Z_o = 377 \Omega$ – impedance of free space

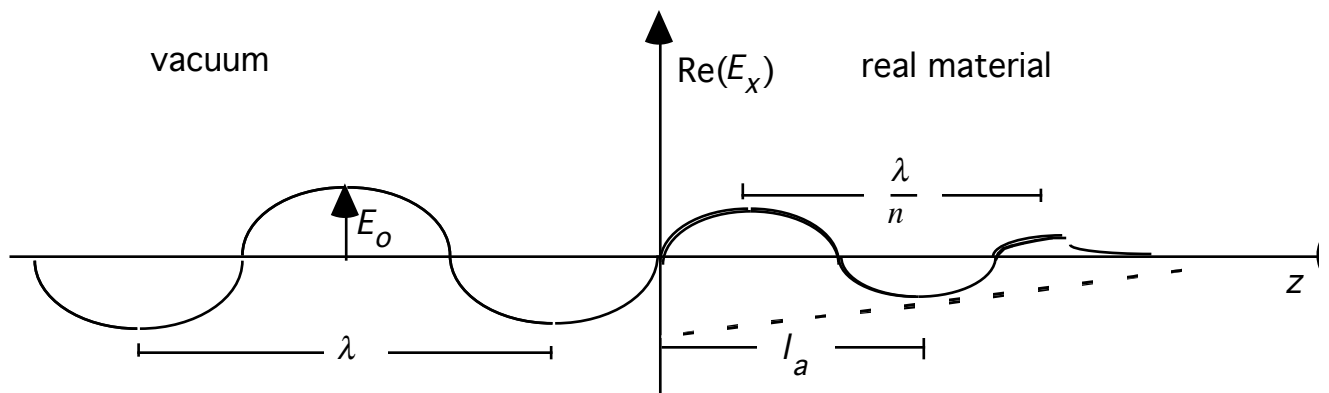
The flux density F has units of watts per meter squared and is thus a measure of the power of the radiation.

$$F = \frac{E_o^2}{2Z}$$

(***** Show Figure 2.1 - electromagnetic spectrum)

Slow Lossy Medium - Next consider the changes in the electromagnetic wave when it enters a real material which is slow and lossy.

$$E_x = E_o e^{i(\omega t - n\frac{\omega}{c}z)}$$



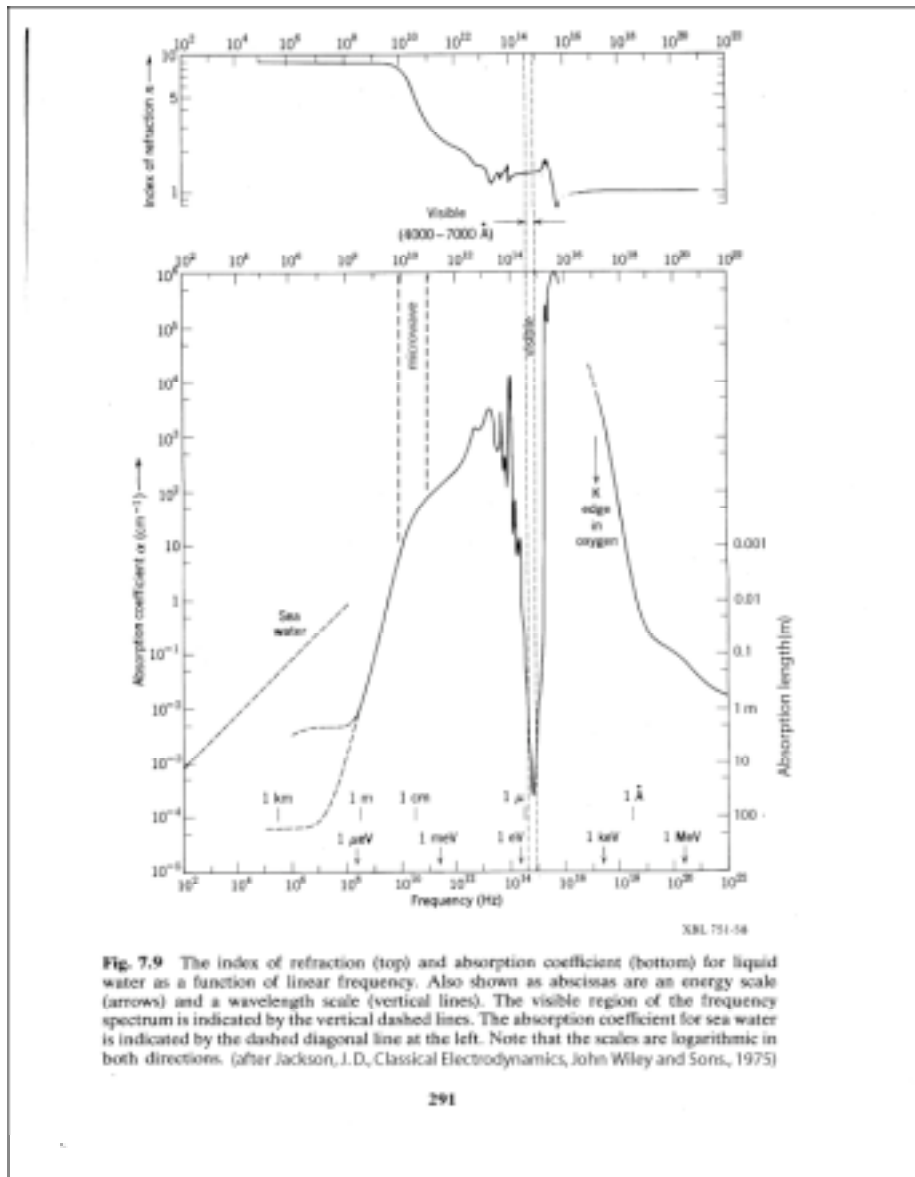
Vacuum - The wave travels at the speed of light so $n = 1$ and $E_x = E_o e^{i \omega(t - \frac{z}{c})}$.

Real Material - In a real material the index of refraction is generally a complex number . In this case the real part of the index of refraction, which is greater than 1, causes the wave to slow down and thus the wavelength decreases. The imaginary part of the index of refraction causes an exponential attenuation of the wave.

$$E_x = E_0 e^{i\omega t} e^{-i\frac{\omega z}{c}(n' - ik)}$$

$$E_x = E_0 e^{-\frac{\omega \kappa z}{c}} e^{i\omega(t - \frac{n'}{c}z)}$$

The distance over which the amplitude of the wave decreases by a factor of e is called the absorption length $l_a = \frac{c}{\omega \kappa}$ or skin depth.



Dielectric Constant and Index of Refraction - From the above discussion it is clear that a complex value of index of refraction is needed to compute velocity and absorption length in real materials. However, experimentalists and theorists usually describe materials using a complex dielectric constant. This section contains the equations needed to convert between the two representations.

$$n = n' - i\kappa$$

$$\epsilon = \epsilon' - i\epsilon''$$

$$n^2 = \epsilon$$

$$n^2 = n'^2 - \kappa^2 - i2n'\kappa$$

$$n^2 = \epsilon' - i\epsilon''$$

Equate the real and imaginary parts

$$\epsilon' = n'^2 - \kappa^2$$

$$\epsilon'' = 2n'\kappa$$

$$\kappa = \frac{\epsilon''}{2n'}$$

As an exercise, show the following is true

$$2n'^2 = \epsilon' + (\epsilon'^2 + \epsilon''^2)^{1/2}$$

Dielectric Properties of Materials - Rees discusses 4 categories of materials based on properties of their dielectric constant:

non-polar material

ϵ' and ϵ'' are constant with ω .

polar material (water)

ϵ' and ϵ'' vary with ω following the Debye equation.

conductive (salt water, copper) $\epsilon'' = \frac{\sigma}{\epsilon_0\omega}$

plasma (ionosphere)

$$\epsilon = n^2 = 1 - \frac{Ne^2}{\epsilon_0 m \omega^2}$$

N - electron density

m - electron mass

e - electron charge

For a plasma if $n^2 > 0$ the waves are slowed as they travel through the ionosphere. However, if $n^2 < 0$, n is purely imaginary and all the energy is reflected off the ionosphere. Under typical ionospheric conditions, low-frequency radio waves reflect while higher frequency microwaves can propagate through. Since the ionosphere is dispersive (i.e. speed depends on ω) a dual frequency microwave instrument such as a radar altimeter or GPS can measure the total electron content of a column of ionosphere and can use this to correct for the delay along the path of one or both frequencies.