**HOMEWORK 2** – Electromagnetic waves - due April 18

1) Propagation of an E/M wave traveling in free space is described by the following 6 equations.

$$E_x = E_o \cos(\omega t - kz)$$
$$E_y = E_z = 0$$

$$B_x = B_z = 0$$
$$B_y = \frac{E_0}{c}\cos(\omega t - kz)$$

a) Show that these equations satisfy $\nabla \bullet \vec{E} = 0$ and $\nabla \bullet \vec{B} = 0$.

b) Show that $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$ and $\nabla \times \vec{B} = \frac{1}{c^2}\frac{\partial E}{\partial t}$.

2) The equation for the electric field of a wave that enters a real material of complex index of refraction $n = m - i\kappa$ is given by

$$E = E_o e^{\frac{-\omega\kappa z}{c}} e^{i\omega\left(t - \frac{mz}{c}\right)}$$

where $t$ is time, $z$ is distance into the material, $\omega$ is frequency, and $c$ is the speed of light.

(a) What is the phase velocity of the wave?

(b) What is the absorption length where the intensity (amplitude squared) is reduced by a factor of 1/e?

(c) What is the wavelength of the wave?

3) Write the formulas for a right-hand and left hand circularly polarized electromagnetic wave traveling in the $z$-direction having frequency $\omega$. Assume the waves are traveling in free space. If these two waves are superimposed, what is the resulting polarization?

4) An aircraft is flying directly overhead at a velocity of $V$ at an altitude of $H$. The aircraft emits an electromagnetic wave having a constant frequency $f_o$. Calculate the frequency of the signal versus time as recorded by an observer on the ground. Assume $V << c$.