





Satellite Remote Sensing SIO 135/SIO 236

Satellite Radar and Laser Altimetry





Sea-ice thickness



Altimeters provide basin-scale observations over the Arctic Ocean



SIO 115 Ice in the Climate System – Helen A Fricker

What is the geoid?

The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level

h=H+N



What is the geoid?

The equipotential surface of the Earth's gravity field which best fits, in a least squares sense, global mean sea level





GeoSat Mission:

The U.S. Navy Geosat radar altimeter satellite delivered data from April 1985 to September 1989 covering the earth's surface between +/- 72 deg latitude.

SeaSat Mission:

The Seasat spacecraft was launched in late June 1978, and during its brief 110-day lifetime, collected 90 days of nearly continuous radar altimeter data between the latitudes of 72 deg S and 72 deg N.

GFO Mission:

The Navy's Geosat Follow-On (GFO) satellite was launched on February 1998 with the aim of continuing ocean observation started by the highly successfull Geosat mission.

ERS-1 Mission:

The European Space Agency's ERS-1 radar altimeter satellite was launched in July of 1991 and provided data from August 1991 to July 1996 in an orbit that extended coverage over the ice sheets to +/- 81.5 degrees.

ERS-2 Mission:

The European Space Agency's ERS-2 Radar Altimeter Satellite was launched in April of 1995 in an orbit that extended coverage over the ice sheets to +/-81.5 degrees. In April 2005, the mission reached its 10-year operation.

Envisat Mission:

The European Space Agency's Envisat satellite was launched into orbit on March 2002, with the aim of continuing data measurements of the ERS (ESA) Remote Sensing Satellites. Envisat is still in operations.

Cryosat Mission:

The European Space Agency developed the CryoSat mission. It will be a three-year radar altimetry mission, scheduled for launch on 15 September 2005, with the objective of determining variations in the thickness of the Earth's continental ice sheets and marice ice cover.



























- CryoSat breaks the ice with ocean currents
- Ice data at your fingertips
- ChuoSat ico catallito ridos non manos

CryoSat App CryoSatApp (iPad)

The British scientist who proposed the mission, Prof Duncan Wingham, is calling for the spacecraft to be re-built.

Altimeter satellite coverage



ERS radar altimeters

- ★ ERS-1 launched on 17 July 1991; ERS-2 on 21 April 1995
- Altitude 782 km, with sun-synchronous, near-polar orbit, inclination 98.52° to the equator, providing almost global coverage, between latitudes 82°S and 82°N
- ★ ERS altimeters more suited to ice sheet mapping than their predecessors (Geosat and Seasat) for the following reasons:
 - »they cover much more of the Antarctic ice sheet, since Geosat and Seasat only extended to 72.12°S.
 - »higher accuracy orbits are available (from Delft University)
 - »incorporate a specialized 'ice mode' for improved tracking over ice

ERS-1 phases and repeat periods (T)

Phase	Purpose	T (days)	Dates
A	Commissioning Phase	3	25 Jul 91 - 10 Dec 91
В	1 st Ice Phase	3	28 Dec 91 - 30 Mar 92
-	Roll Tilt Mode Campaign	-	4 Apr 92 - 13 Apr 92
С	1 st Multidisciplinary Phase	35	14 Apr 92 - 21 Dec 93
D	2 nd Ice Phase	3	23 Dec 93 – 10 Apr 94
Е	1 st Geodetic Phase	168	10 Apr 94 – 28 Sep 94
F	2 nd Geodetic Phase	168	28 Sep 94 – 21 Mar 95
G	2 nd Multidisciplinary Phase	35	21 Mar 95 – 5 Jun 96

On 29 April 1995, ERS-2 joined ERS-1, to start the 'Tandem Mission'.

ERS-1 operating principles



• Altimeter measures the time delay (*T*) taken for a radar pulse to travel to the surface and back again

• Range (*R*) from the satellite to the surface is determined from the time delay from the equation R = cT/2 where *c* is the propagation velocity of e/m waves in free space

Radar altimeter height measurement



If satellite altitude (A) known, surface height (h) can be determined from range (R)

h
$$(\varphi, \lambda, t) = A(\varphi, \lambda, t) - R(\varphi, \lambda, t)$$

Radar altimeter height measurement



If satellite altitude (A) known, surface height (h) can be determined from range (R) $h(\varphi, \lambda, t) = A(\varphi, \lambda, t) - R(\varphi, \lambda, t)$

Radar altimeter waveforms (ideal surface)



 $0 < t < t_0$ pulse propagates as part of expanding spherical shell. Area of sphere that can be received on ground defined by antenna beamwidth (θ_A)

 $t = t_0$ when incident radar pulse first meets surface it illuminates a point, & a reflected signal begins to return to the satellite

 $t_0 < t < t_1$ point becomes centre of disc (area increases with time)

 $t = t_{1}$ back of shell reaches centre of circle & annulus is formed, whose radius increases whilst maintaining constant area, until it reaches edge of the radar beam

Radar altimeter waveforms (ideal surface)



PLF (pulse limited footprint) size

• Pulse-limited operation: leading edge of pulse has not extended to limit of antenna beam when back of pulse has reached ground.

Beam-limited operation: antenna beam width determines footprint size

• Period $t_0 < t < t_1$ is critical, as it contains waveform leading edge, corresponding to initial interaction of pulse with surface

• Maximum area of the surface corresponding to the leading edge of the waveform is the 'pulse-limited footprint'. Area & diameter of the pulse-limited footprint on the ground are determined by the pulse-width (τ). For the ideal surface, the pulse-limited footprint has an area (A_{t}) and diameter (D_{t}) of:

$$A_f = \pi R c \tau \qquad \qquad D_f = 2\sqrt{R c \tau}$$

• For ERS altimeters, τ = 3.03 ns and R ~800 km, so the area & diameter of the ERS pulse-limited footprint over an ideal surface are ~2.3 km² and 1.7 km respectively

Real altimeter waveforms

- Waveforms from real surfaces differ considerably than those from ideal surfaces
- When microwave pulse intercepts surface, some energy will be scattered, some transmitted through surface layer & remainder reflected back in direction of radar
- Varying magnitudes of these components governed by surface geometry (roughness) & physical properties of the surface
- First reflections returned from scattering elements closest to spacecraft & those from elements at more distant locations are received successively later
- Information about surface is therefore contained in envelope of returned energy intensity, i.e. the waveform of the returned pulse
- Leading edge of waveform corresponds to initial interaction with surface
- In theory, several surface geophysical parameters can be deconvolved from the waveform

Waveforms over ocean (1)

• Over the ocean, scattering is mainly from surface

• When surface roughness is present, such as ocean waves, scattering from wave crests precedes that from troughs as pulse-front propagates downward - this increases effective area over which scattering elements are simultaneously illuminated, broadening rise time of return, and widening waveform leading edge

• Width of the leading edge related to height of ocean waves; pulse-limited footprint diameter D_f becomes:

$$D_f = 2\sqrt{c\tau' h}$$
 where $\tau' = \left\{\tau^2 + \left(4\frac{\sigma_s}{c}\ln 2\right)^2\right\}^{1/2}$ where σ_s is RMS wave-height

 For ERS altimeters, if RMS wave-height increases from zero to 5 m, diameter of PLF increases from 1.7 to 7 km

Slope of waveform leading edge used to estimate significant wave-height (h_{1/3})

• Mean range to PLF obtained by measuring to range position corresponding to point on leading edge of waveform that has 50% of total received power

Waveforms over ocean (2)

• Approximation for ocean altimeter return can be made, by assuming that mean surface is horizontal & planar, with a large number of scattering elements distributed normally about the mean surface

• For near-vertical incidence, the *mean* pulse-limited altimeter return power as a function of delay time is described by the convolution of two terms.

$$P_R(t) = P_T(t) * P_S(t)$$

where $P_{R}(t)$ is the power received back at the altimeter

 $P_{T}(t)$ is the transmitted pulse profile

 $P_{s}(t)$ is a function that includes effect of antenna gain, range distribution & backscattering properties of surface scattering elements

Brown model for ocean returns

• Scattering elements assumed to return equal power, so $P_{\rm S}(t)$ can be written as convolution of 2 terms: 1st describing 'average flat surface impulse response' & 2nd representing range distribution of scatterers

• Expression for the returned power becomes a triple convolution:

 $P_{R}(t) = P_{T}(t) * q(t) * P_{fs}(t)$

where $P_{fs}(t)$ is 'average flat surface impulse response' i.e. average back-scattered power from a mean flat surface which has a low surface roughness but same backscattering cross section per unit scattering area (σ_0) as true surface

q(t) is the delay time distribution of the surface scattering elements with range distribution Q(R) so that $q(t) = \frac{2 \cdot Q(R)}{q(t)}$

• For pulse-limited altimeter, $P_{fs}(t)$ is a step function with an exponential decay that expresses attenuation of return pulse shape by antenna



ERS waveforms over ice sheets



 Response of altimeter over ice-covered surface considerably more complex than over ocean

 Surface topography varies on wide range of spatial scales (including those close to PLF) & Gaussian distribution cannot be used to approx. height distribution of surface scatterers

• Presence of topographic feature in the pulse-limited footprint will distort leading edge

 Amount of returned power from each element depends on its 'backscattering coefficient' & angle to incident beam

• Total reflected power received by radar depends on total backscattering coefficient of the illuminated surface (σ_0). Variations in σ_0 can arise from changes in surface properties (e.g. moisture content & surface roughness)

• Variations in so and surface topography across footprint result in a complex leading edge shape, which is often difficult to interpret

Waveform averaging

• ERS altimeters have a Pulse Repetition Frequency of 1020 Hz (transmit approx. one pulse per millisecond)

 Individual altimeter return-pulse echoes affected greatly by speckle, or Rayleigh noise, due to random interference of coherent transmitted radiation across footprint

 To improve SNR ratio (so required range precision criteria is met) ERS altimeter sums 50 individual waveforms, to form average recorded waveform at ~20 Hz

Waveform recording

 Only finite sample of each averaged return waveform is recorded, over a finite time interval called the RANGE WINDOW, equivalent to a range interval of 28.6 m, which is quantised into 64 equal range intervals, or 'range bins'

• Quantity recorded in the waveform is equivalent to histogram of average return pulse energy as function of time

Waveform tracking

Tracking: altimeter keeps track of the return waveform by positioning the range window so that it captures it.

• ERS tracker programmed to position the range window such that it contains the leading edge of the waveform, by predicting arrival time of waveform based on previous measurements

• In ocean-mode, measurements are made assuming the shape of the average return waveforms conform to the Brown surface-scattering model

• ERS tracker uses a Sub-optimal Maximum Likelihood Estimator (SMLE) algorithm which uses a simplified version of the Brown model

• Tracking point is 50% power point on waveform leading edge, & tracking device positions the centre of its range window at the predicted position from SMLE

• Over the ocean, tracker will position the centre of the range window at the half peak power point on the leading edge

• Range telemetered by the satellite is measured to centre of range window - over much of the ocean surface, this is the true range – however this is not true over ice

Problems with ERS tracking over ice

• When ERS altimeters operate in ocean mode over ice, following situations occur:

- shapes of the received pulses do not conform to the Brown model
- surface topography changes rapidly

 Both these problems make it difficult for ERS tracker to predict trend in surface slope based on the most recent few measurements, making it difficult to position range window correctly

•Over ice, waveform leading edge will either be offset from tracking point, or not sampled at all

First situation is corrected for by a post-processing procedure called 'retracking'

 Second situation is known as 'loss-of-lock'. Gaps in altimeter data collected over the ice sheets are caused by the tracking loop failing to keep up with changes in time delay caused by some of the rapid changes in surface height

•To improve altimeter tracking over ice, ERS altimeters incorporate a second tracking mode, 'ice mode', in addition to ocean mode

ERS "ice mode" tracker

Ice mode tracker differs from the ocean mode tracker in two ways

➤ waveforms tracked using algorithm which ensures that centre-of-gravity (COG) of the recorded portion of the waveform is contained in range window

increased dynamic range - increases the duration of the range window by four

• COG position is unique, therefore there is no ambiguity in definition of tracking point, and also it is independent of waveform shape

• COG tracker will usually maintain leading edge in range window. However, this tracker will introduce a bias depending on waveform shape, because waveform COG has no physical meaning, waveform data therefore *must* be retracked

• Wider range window permits greater rate of range change to be tracked successfully, but results in a coarser resolution

• ERS waveforms have 64 range bins of width 0.455 m in ocean mode and 1.82 m in ice mode - range window width is 28.8 m & 116.48 m for ocean & ice mode respectively.

•Measurement precision over ice is 0.28 m in ocean mode & 0.49 m in ice mode

Error sources in altimetry over ice

Bias/error	Magnitude (m)	Residual after correction (m)		
a) 'Standard' biases/errors for all surfaces				
Instrumental errors	~1.25	<0.01		
Satellite corrections	~4.8	<0.08		
Propagation delays (Doppler, c of g))	~2.3	~0.05		
Surface corrections (ocean and Earth tides)	1-2	<0.11		
Radial orbit error	<u>+</u> 0.3	<0.09 ^H		
b) 'Ice' biases/errors for ice sheets and ice shelves				
Non-acceptable waveform shape	N/A	waveforms removed through nested filtering		
Tracker error	<u>+</u> ~20	<i>up to 0.4</i>		
Surface bias	0 to 0.25	cannot be corrected for		
Surface penetration	< 2*	unknown		
Slope-induced error	30 m for 0.5° slope	depends on local data density and surface topography		
Seasonal variation	< 0.30 (Yi et al. 1997)	unknown		
Tide (ice shelves only)	1-2	depends on model used and response of ice shelf		

Volume scattering



 RA pulse penetrates the surface firn layer, leading to 'volume scattering' from within layer & reflections from subsurface layers & ice lenses

 Increases the path length of the reflected radiation & the travel-time back to the satellite, resulting in the waveform having a different shape to that if only surface scattering occurred

 Surface penetration can cause height errors of ~3.3 m

Slope-induced error





 Altimeter range measurement made to closest point on surface within its 'beam-limited' footprint (BLF)

• If reflecting surface tilted wrt ellipsoid, point will not be at sub-satellite point (nadir), displaced up-slope from nadir along direction of maximum slope

• Range measurement provided in altimeter data records does not correspond to nadir position & offset must be applied either to range measurement or to coordinates provided in ERS-1 data set, or both

• Difference in position of nearest point compared to that at nadir proportional to magnitude of slope & is displaced along the direction of the maximum gradient within BLF

• Over sloping regions of Antarctica, this is largest error encountered in altimetry e.g. 0.1° slope known to 5% accuracy results in a δ H of 1.2 m \pm 12 cm

Slope-induced error correction

a) Direct method

b) Intermediate method

c) Relocation method



(a) treats slope error as range error - calculates corrected range to nadir position using estimate of the surface slope between the reflecting point & nadir. Assumes const. slope between near-range point & nadir point

(b) treats slope error as combination of range error & position error - calculates position on surface at which measured range is correct (constant slope assumed)

(c) treats slope error as position error - estimates location of near-range point corresponding to range measurement & calculates correct elevation for that point from measured range using slope value at near-range point

Waveform retracking



• Range measurement in ERS records does not correspond to reflecting surface

• Estimation of true range to reflecting surface can be obtained by determining offset (in range bins) of leading edge from mid-point of range window, on a waveform by waveform basis

Process known as 'retracking'

 Retracking corrections over the ice sheets can be up to several metres

• Failure to retrack can result in false surface features & can overlook real features

• For retracking algorithm to be useful for mass balance studies, it must produce a repeatable surface height measurement

β-parameter retracking (GSFC)



•Used for 'simple' waveform shapes with single leading edge •One of parameters (β_3) defines the position of waveform leading edge, which is used to correct altimeter range measurement

β-parameter retracking (GSFC)



 Used for 'complex' waveforms, with two leading edges - occurs when two main reflecting surfaces at different ranges contribute to return power

• Two parameters ($\beta_3 \& \beta_6$) define leading edges, providing two corrections

Offset Center of Gravity (OCOG) retracker



• Waveform amplitude (A) determined

• Threshold value, $T_{x\%}$, is pre-defined % of *A* - 1st bin on leading edge with value exceeding $T_{x\%}$ determined. Linear interpolation between this bin & preceding bin provides location of retrack point on leading edge [x = 10, 25 or 50%]

Surface & volume scattering retracker



- Attempts to account for effect of surface penetration of radar pulse theoretical model for return power envelope
- Total power is combination of surface + volume component
- $\mbox{ }^{\bullet}$ Between t_{o} & $t_{1},$ both components increase with time & waveform gradient is steep
- Surface scattering becomes constant at t₁, but volume scattering continues to increase produces a 'notch' or point of inflection in the waveform, which can be used to locate mean surface using model
- Also returns other parameters assoc. with scattering material

Major limitations in measuring ice sheet & land elevations using radar altimetry

- Large footprint diameter on ground (10-20 km over ice, depending on surface roughness & surface slope)
- Measurement represents an average elevation over this footprint, therefore features appear broadened
- Radar altimeter detects topographic highs early, as soon as they enter footprint
- Slope-induced error is significant over relatively low slopes; difficult to relocate the measurement without apriori knowledge of surface topography

Major limitations in measuring ice sheet and land elevations using radar altimetry


Major limitations in measuring ice sheet and land elevations using radar altimetry

- Radar altimeter cannot maintain lock over surfaces with slopes greater than 1°, or rough surfaces, so the margins of the ice sheets, ice shelves and outlet glaciers are poorly sampled
- Radar altimeter pulse (frequency 13.8 GHz) penetrates the surface of the ice, leading to volume scattering within the snow-pack. Effect increases in the dry snow zone and high accumulation areas
- Radar altimeter prone to 'snagging' over certain bright ice sheet features such as meltstreams & melt ponds

Laser altimetry for ice sheet mapping

- Airborne laser altimeters have demonstrated their suitability to ice sheet mapping since 1991 in Greenland (ATM, AOL etc)
- First satellite-borne laser altimeter instrument dedicated to ice sheet mapping - Geoscience Laser Altimeter System (GLAS) - launched on board the Ice Cloud and land Elevation Satellite (ICESat) in January 2003
- Precise height measurements (< 10cm) on ice

Radar and laser altimeter waveform shapes over ice sheets

- Radar altimeter return pulses severely distorted by local slope and surface roughness; shapes also vary widely over ice sheets due to varying proportions of surface & volume scattering
- Laser altimeter waveforms generally much "cleaner" waveforms contain surface scattering from a much smaller number of scattering surfaces
- Laser altimeter return pulses become broadened as the surface slope and roughness increases -Gaussian return with a larger sigma

ICESat instrument characteristics [1]

- Beam divergence of 0.1 mrad produces spot size with diameter 70 + 10 m (600 km nominal altitude)
- Dual wavelength Nd:YAG laser 1064 nm (for surface elevation) and 532 nm (for cloud detection)
- Pulse width 4 ns and pulse frequency 40 Hz will produce spots 175 m apart (centre-to-centre)
- Pointing accuracy 20 arcsec; post-processing 1.5 arcsec
- Collects 4,500,000 1 ns samples for each return pulse and processes these on-board to retrieve desired data

ICESat instrument characteristics [2]

- On-board algorithm developed to maximise chance that ground return will be recorded - searches backwards through all samples to retrieve data - only records 544 (81.6m) samples for land & 200 (30m) for sea ice & ocean
- No acquisition or tracking phase: instead an onboard DEM determines which part of the echo should be searched for ground return

Effect of slope and roughness on laser



altimeter waveforms

- Returned pulse broadened by the distribution of surface heights in the footprint
- Return waveform is convolution of transmitted pulse with height distribution function
- Knowledge of transmitted pulse is critical - can retrieve through deconvolution if roll angle is large





Ice, Cloud and Iand Elevation Satellite (ICESat): Earth's first polar orbiting satellite laser altimeter



ICESat mission and launch





- Ice Cloud and land Elevation
 Satellite (ICESat) launched 12 Jan
 2003 from Vandenberg, CA
- Primary goal of ICESat mission is ice sheet change detection

ICESat laser altimeter



Schematic of single profile science data measurements, compiled through time to build elevation data sets

Surface elevations along a ground track determined from laser time of flight, combined with precise orbit and pointing information

Laser-beam pointing determined from startrackers and internal angle system

GLAS had 3 lasers

ICESat laser altimeter



Schematic of single profile science data measurements, compiled through time to build elevation data sets

ALTIMETER

- Measures distance to ice, land, water, or clouds from laser pulse "time of flight" (d = r x t)
- Digitizes the transmitted and received 1064-nm pulse waveforms
- Laser-beam attitude from star-trackers and internal angle system

ATMOSPHERIC LIDAR

- Measures laser backscattering from clouds and aerosols
- Data from 532 nm and 1064 nm pulses
- Photon counting/along-track signal integration
- Simultaneous operation with altimeter



Emitted Laser Pulse (532 nm)



ICESat-GLAS Integration





ICESat as Delta II Payload



Ball and Boeing graphics of ICESat - Delta II launch vehicle



ICESat Components On-Orbit



Graphics Courtesy of GLAS Instrument Team



ICESat Solar Array Articulation



ICESat range measurement



- Output pulse recorded
- 32 gates
- pulse width 4ns
- Digitised in 1ns (15cm) range bins
- 544 range bins recorded over ice & 200 over ocean
- Range measurement derived from travel time of laser pulse, using waveform centroid as reference point
- Centroid valid over most ice surfaces where single Gauss fit is appropriate, but not for double peaked waveforms resulting from two surfaces in footprint



Calculation of Range: Simple Surface





Calculation of Range: Complex Surface





Movie courtesy of NASA GSFC Conceptual Image Lab

 Surface elevations along a ground track determined from laser time of flight, combined with precise orbit and pointing information

- Laser-beam pointing determined from startrackers and internal angle system
- GLAS has 3 lasers

Primary goal of ICESat mission is ice sheet change detection



s on ICESat waveforms

- Returned pulse broadened by the distribution of surface heights in the footprint
- Return waveform is convolution of transmitted pulse with height distribution function
- Knowledge of transmitted pulse is critical - can retrieve through deconvolution if roll angle is large

GLAS Measurement of Echo pulse from Trees



ICESat elevation profile across two Antarctic ice streams



Movies and graphics courtesy of NASA GSFC Conceptual Image Lab



A first day track across Antarctica showing ice sheet elevations in red and atmospheric phenomena in varying colors from light blue for thin clouds to white for opaque layers







ICESat data acquisition

- Due to problems discovered with the lasers during Laser 1 (L1) operations, Laser 2 (L2) and Laser 3 (L3) have been operated in a quasi-continuous fashion between Sept 2003 and Mar 2007
- Last 33 days of Laser 2a (L2a) have been repeated in subsequent campaigns

chart courtesy of Jim Abshire



ICESat global data coverage

Lots of great data for many science applications

ICESat Laser 2a Elevations - 9/25 to 11/19/03



S. Hemisphere Spring and N. Hemisphere Fall (whiter areas indicate data gaps, in part due to clouds)

Polar Elevation Data - Laser 2a



S. Hemisphere Spring and N. Hemisphere Fall



ICESat Calibration/Validation



- Post-launch instrument verification and data product validation plans use ground truth sites
 - > On-orbit verification of prelaunch calibrations
 - » Validate GLAS data products
- Example: dry lake beds (Bonneville Salt Flats shown) suitable calibration surfaces:
 - » Emulate ice sheet
 - » Accessible for ground truth measurements

ICESat-2 Observation Strategy





Example: MABEL



Transmit Pattern


Receiver Fields of View



Volume scattering



 RA pulse penetrates the surface firn layer, leading to 'volume scattering' from within layer & reflections from subsurface layers & ice lenses

 Increases the path length of the reflected radiation & the travel-time back to the satellite, resulting in the waveform having a different shape to that if only surface scattering occurred

 Surface penetration can cause height errors of ~3.3 m

Slope-induced error



 $\delta H = \frac{H\alpha^2}{2}$

• Altimeter range measurement made to closest point on surface within its 'beam-limited' footprint (BLF)

• If reflecting surface tilted wrt ellipsoid, point will not be at sub-satellite point (nadir), displaced up-slope from nadir along direction of maximum slope

 Range measurement provided in altimeter data records does not correspond to nadir position & offset must be applied either to range measurement or to coordinates provided in ERS-1 data set, or both

• Difference in position of nearest point compared to that at nadir proportional to magnitude of slope & is displaced along the direction of the maximum gradient within BLF

• Over sloping regions of Antarctica, this is largest error encountered in altimetry e.g. 0.1° slope known to 5% accuracy results in a δ H of 1.2 m \pm 12 cm

Slope-induced error correction

a) Direct method

b) Intermediate method

c) Relocation method



(a) treats slope error as range error - calculates corrected range to nadir position using estimate of the surface slope between the reflecting point & nadir. Assumes const. slope between near-range point & nadir point

(b) treats slope error as combination of range error & position error - calculates position on surface at which measured range is correct (constant slope assumed)

(c) treats slope error as position error - estimates location of near-range point corresponding to range measurement & calculates correct elevation for that point from measured range using slope value at near-range point