THERMAL RADIATION SUMMARY

(Rees Chapter 2)

Planck's Law describes the amplitude of radiation emitted (i.e., spectral radiance) from a black body. It is generally provided in one of two forms; $L_\lambda(\lambda)$ is the radiance per unit wavelength as a function of wavelength λ and $L_\nu(\nu)$ is the radiance per unit frequency as a function of frequency ν. The first form is

$$L_\lambda(\lambda) = \frac{2hc^2}{\lambda^5}\left[exp\ \frac{hc}{\lambda kT} - 1\right]^{-1} \quad \text{where}$$

T	-	temperature	
c	-	speed of light	2.99 x 10^{-8} m s^{-1}
h	-	Planck's constant	6.63 x 10^{-34} J s
k	-	Boltzmann's constant	1.38x10^{-23} J °K^{-1}
L_λ	-	spectral radiance	W m^{-3} sr^{-1}
L_ν	-	spectral radiance	W m^{-2} Hz^{-1} sr^{-1}

To relate the two forms and establish L_ν one takes the derivative of L with respect to ν using the chain rule $\frac{\partial L}{\partial \nu} = -\frac{\partial L}{\partial \lambda}\frac{\partial \lambda}{\partial \nu}$. Note that $\lambda = c/\nu$ so that $\frac{\partial \lambda}{\partial \nu} = -\frac{c}{\nu^2}$ and finally

$$L_\nu(\nu) = \frac{2h\nu^3}{c^2}\left[exp\ \frac{h\nu}{kT} - 1\right]^{-1}$$

The **Stefan-Boltzmann Law** gives the total black body irradiance as a function of the temperature T. One can derive this law by integrating the spectral radiance over the entire spectrum. This is left to the reader as an exercise.

$$L = \int_0^\infty L_\lambda d\lambda = \frac{2\pi^4 k^4}{15c^2h^3}T^4$$

or $M=\pi L = \sigma T^4$ where σ is the Stefan-Boltzmann constant (5.67 x 10^{-8} W m^{-2} °K^{-4}).

Wein's Law provides the wavelength (or frequency) where the spectral radiance has maximum value. This can be found by taking the derivative of L_λ with respect to wavelength and determining where this function is zero. This is another excellent exercise; after some algebra you should arrive at the following transcendental equation

$$1 - e^{-\gamma} = \frac{\gamma}{5} \;\Rightarrow\; \gamma = 4.965$$

where

$$\gamma = \frac{hc}{kT\lambda_{max}}.$$

The more common form is $\lambda_{max} = C_w/T$ where $C_w = 2.898\ x\ 10^{-3}$ °Km. Note that one could perform an experiment to measure the total radiance from a black body and establish the Stefan-Boltzmann constant σ. Similarly one could determine the wavelength for maximum black body output to estimate Wein's constant C_W. Then with a knowledge of these two constants one could estimate Planck's constant h and Boltzmann's constant k without every doing any quantum measurements!

The **Rayleigh-Jeans Approximation** provides a convenient and accurate description for spectral radiance when for wavelengths much greater than the wavelength of the peak in the black body radiation formula. To derive the Rayleigh-Jeans approximation, expand the exponential in the denominator of Planck's Law in a Taylor series about zero argument; this is a good appropriation when $\lambda >> \lambda_{\text{max}}$. This is a third exercise left to the reader. The approximate formula is

$$L_\lambda = \frac{2kcT}{\lambda^4} \;\; or \;\; L_\nu = \frac{2kT\nu^2}{c^2}.$$

This approximation is better than 1% when $\lambda T > 0.77$ m K. For example, for a body at 300˚K, the approximation is valid when $\lambda > 2.57$ mm; in other words this approximation is good when viewing thermal emissions from the Earth over the microwave band. Microwave radiometers can measure the power received L_λ at an antenna. This is sometimes called the brightness temperature and it is linearly related to the physical temperature of the surface T_p. The Rayleigh-Jeans approximation provides a simple linear relationship between measured spectral radiance and surface temperature as long as the emissivity ε of the surface is known or, in the case of sea ice, one knows the temperature of the surface so the emissivity of the ice can be estimated.

$$L_{measured} = \varepsilon \; \frac{2kcT_p}{\lambda^4}$$

Radiation from the Sun and Earth

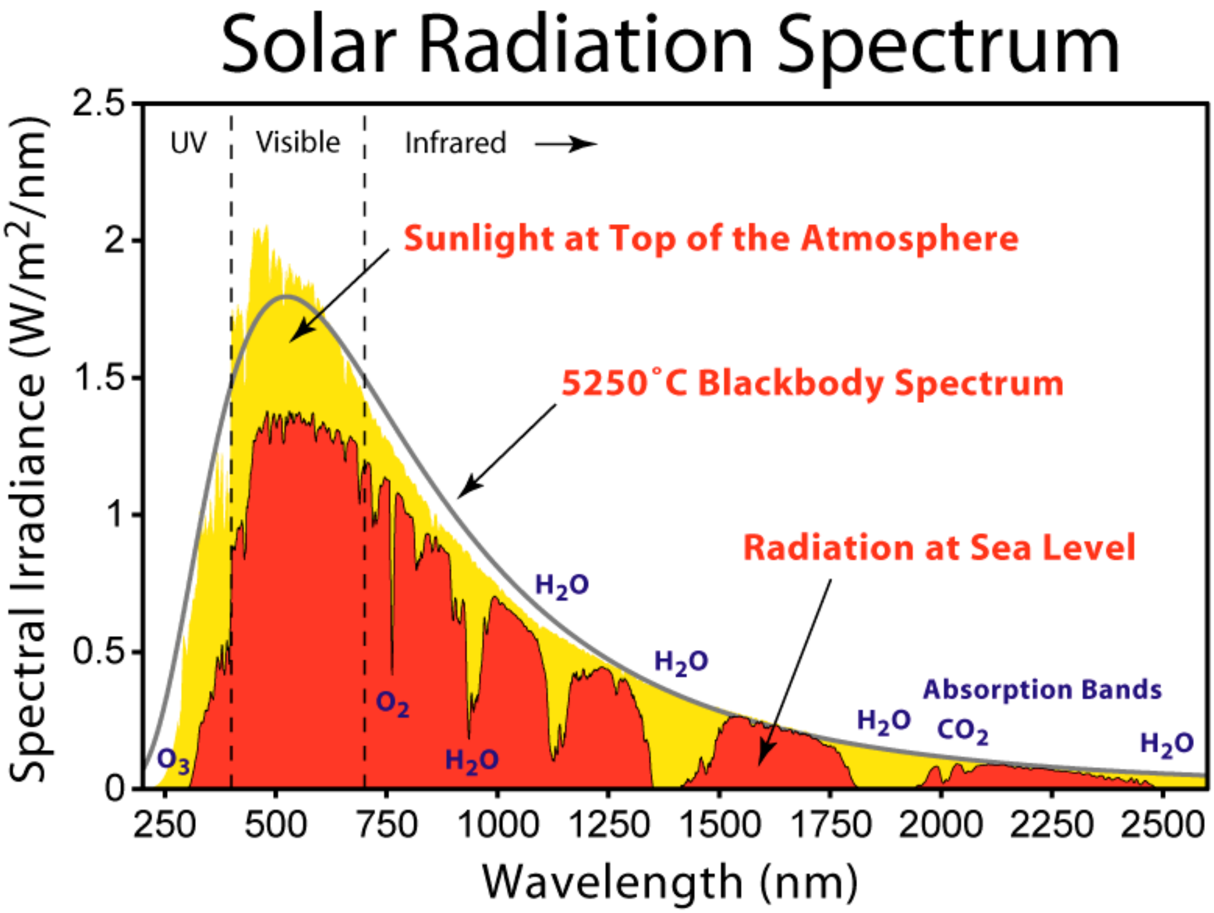


Figure shows the theoretical blackbody radiation curve for 5250˚C (black curve). Solar radiation at the top of the atmosphere is well approximated by a blackbody spectrum (yellow). The atmosphere absorbs and reflects radiation so the spectral radiance at sea level is lower and has bands of low incident radiation. Note that the peak in the spectral radiance from the sun occurs in the visible part of the spectrum.

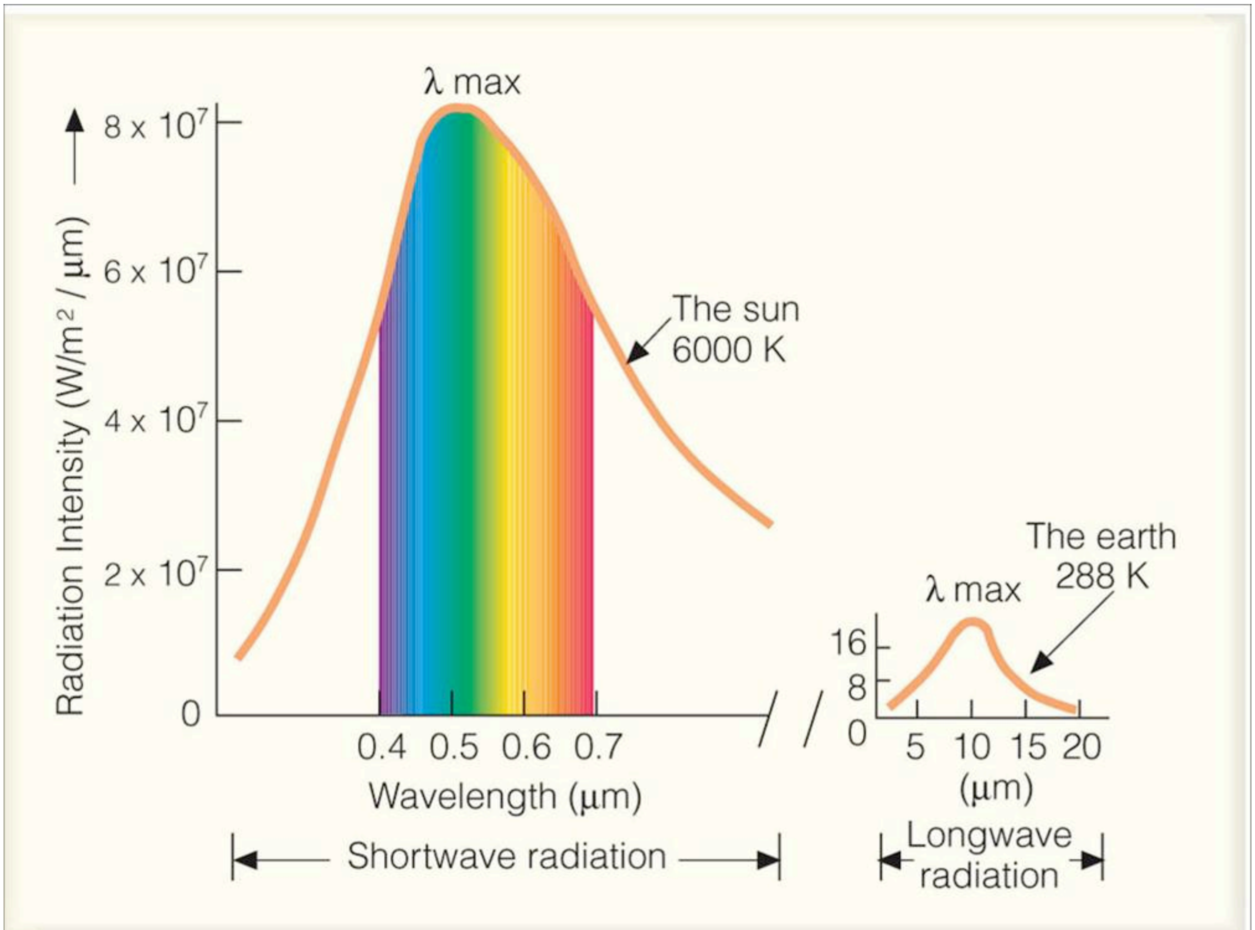
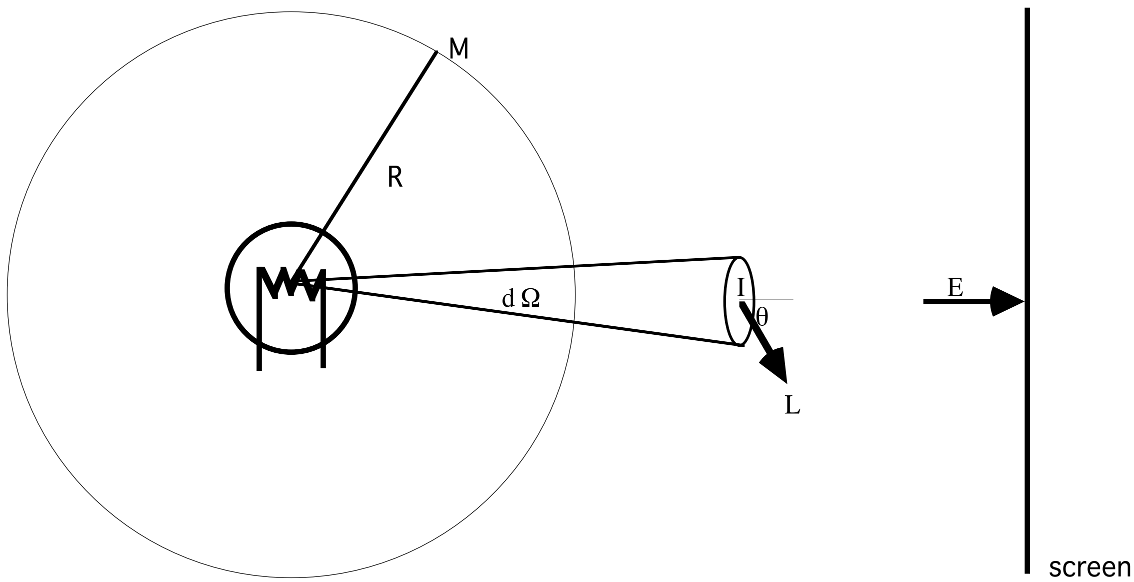


Figure shows blackbody radiation curves for the Sun at 6000˚K and the Earth at 288˚K. The solar radiation peaks in the visible part of the spectrum while the Earth has a peak in the thermal infrared part of the spectrum (~ 10 µm).

Terminology

Consider a 60 W light bulb. An electric current passes through the tungsten filament and heats it to about 3000˚K. Our bulb is perfect in the sense that it radiates all of this energy, perhaps as a gray body.



radiant flux (total)	ϕ		60	W
radiant intensity	I	$d\phi/d\Omega$	$60/4\pi$	W sr^{-1}
radiant exitance	M	$d\phi/dA$	$60/4\pi R^2$	W m^{-2}
radiance (brightness temp.)	L	$\cos\theta\ d^2\phi/(d\Omega dA)$		W sr^{-1} m^{-2}
irradiance	E	$d\phi/dA$		W m^{-2}

Radiance from the Sun and Earth

		Sun 6000˚ K	Antarctica 240˚K	ocean 280˚K	desert 315˚K
radiant exitance	$M = \varepsilon\sigma T^4$	$7.3\text{x}10^7 \text{Wm}^{-2}$ ε=0.99	231 ε=0.8 (8-12 μm)	472 ε=0.8 (8-12 μm)	772 ε=0.9 (8-12 μm)
total power	$4\pi R_s^2 M$	$3.9\text{x}10^{26}$ W	$1.2\text{x}10^{17}$ W	$2.2\text{x}10^{17}$ W	$3.9\text{x}10^{17}$ W
irradiance at Earth	$M\dfrac{R_s^2}{D^2}$	1370 Wm^{-2}			
power of reflected sunlight	$aM\dfrac{R_s^2}{D^2}$		822 Wm^{-2} a=0.6	274 Wm^{-2} a=0.2	411 Wm^{-2} a=0.3
spectral peak	$\lambda_{\max} = \dfrac{C_w}{T}$	0.48 μm	12 μm	10 μm	9 μm

D - Earth to Sun distance $1.49\text{x}10^{11}$ m
R_s - radius of Sun $6.96\text{x}10^{8}$ m
a - albedo 0.35