Algorithm for Estimating $\dot{R}$ and $\ddot{R}$ - (David Sandwell, SIO, August 4, 2006)

Azimith compression involves the alignment of successive echoes to be focused on a point target. Let s be the slow time along the satellite track. If the trajectory of the satellite was a straight line then the range to the point target would vary as a hyperbola. However, because the aperture (< 30 km) is short compared with the nominal range (~850 km), a parabolic approximation is commonly used

$$R(s) = R_o + \dot{R}_o(s - s_o) + \frac{\ddot{R}_o}{2}(s - s_o)^2 + \ldots$$

where R_o is the closest approach of the spacecraft to the target and s_o is the time of closest approach. We showed above that the critical parameters for focusing the SAR image, the Doppler centroid f_{DC} and the Doppler frequency rate f_R, can be related to the coefficients of this polynomial. The relationships are:

$$f_{DC} = \frac{-2\dot{R}}{\lambda} \quad \text{and} \quad f_R = \frac{2\ddot{R}}{\lambda}.$$

where λ is the wavelength of the radar. In addition, if one assumes a linear trajectory of the spacacraft V relative to the target then the Doppler centroid and Doppler rate can be approximately related to the velocity and closest range as

$$f_{DC} = \frac{-2V}{\lambda}\frac{\left(x - s_o V\right)}{R_o} \quad \text{and} \quad f_R = \frac{2V^2}{\lambda R_o}.$$

We will not consider the Doppler centroid further because it is accurately estimated from the raw signal data [*Madsen*, 1989]. For C-band SARs such as ERS-1 and Envisat, the above formula for the f_R provides adequate focus. However, for L-band SARs such as ALOS, the aperture is much longer so other factors must be considered such as the curvature and ellipticity of the orbit as well as the rotation rate of the Earth.

Curlander and McDonough [1991] discuss the estimation of the Doppler rate and there are two main approaches. The *Autofocus* approach uses the crudely-focussed imagery to improve on the estimate of the Doppler rate. The orbit approach uses the more precise geometry of the elliptical orbit about a rotating elliptical Earth to provide a more exact estimate of $\ddot{R}$.

Here we consider a new, and more direct approach, to estimating $\ddot{R}$. Consider the following three vectors which form a triangle;

$\vec{R}_s$ – the vector position of the satellite in the Earth-fixed coordinate system;

$\vec{R}_e$ – the vector position of a point scatterer on the Earth and somewhere in the SAR scene;

$\vec{R}_o$ – the line-of-sight vector between the satellite and the point scatterer.

The three vectors form a triangle such that $\vec{R}_e = \vec{R}_s + \vec{R}_o$. The scalar range, which is a function of slow time, is given by

$$R(s) = \left|\vec{R}_s(s) - \vec{R}_e\right| \cong R_o + \dot{R}_o(s - s_o) + \frac{\ddot{R}_o}{2}(s - s_o)^2 + ...$$

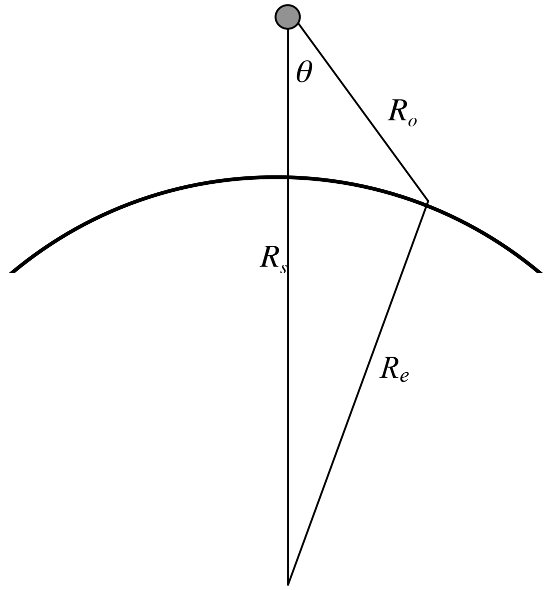
Measurements of scalar range versus slow time can be used to estimate the coefficients off the parabolic approximation. The algorithm is to:

(1) Use the precise orbit to calculate the position vector of the satellite and compute a time series $R(s)$ over the length of the aperture.

(2) Perform a least-squares parabolic fit to this time series to estimate R_o and $\ddot{R}_o$.

(3) Compute the effective speed as $V_e^2 = R_o\ddot{R}_o$.

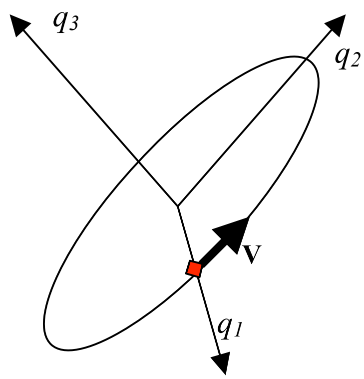
The only remaining step is to select the position of the point target $\vec{R}_e = \vec{R}_s + \vec{R}_o$. Actually this can be any point in the image so the selection criteria is that the point lie at the proper radius of the surface of the Earth $R_e = \left|\vec{R}_e\right|$ and the $\vec{R}_o$vector is perpendicular to the velocity vector of the satellite $\vec{V}$. If we prescribe the length of the R_o-vector then the angle between the satellite position vector and the line-of-sight vector is given by the law of cosines as shown in the diagram below.



This diagram has the satellite at the gray dot with the velocity vector pointing into the page. The cosine of the look angle is

$$\cos\theta = \frac{\left(R_s^2 + R_o^2 - R_e^2\right)}{2R_sR_o}.$$

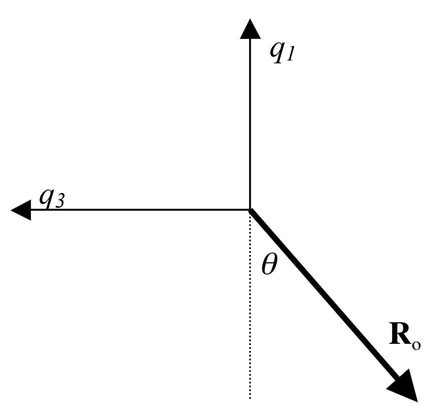
Next consider a local co-ordinate system where the q_2-axis is aligned with the velocity vector of the satellite as shown in the following diagram



In this primed co-ordinate system, the q_1-axis is parallel to $\vec{R}_s$, the q_2 axis is parallel to $\vec{V}$, and the q_3 axis is perpendicular to both and is given by their cross product

$$q_3 = \frac{\vec{R}_s}{|R_s|} \times \frac{\vec{V}}{|\vec{V}|}$$

The following diagram has the velocity vector of the satellite going into the page and aligned with the q_2 coordinate. The q_1 vector is the radial vector from the center of the earth. The line-of-sight right-look vector is in the q_1- q_3 plane as shown in the diagram.

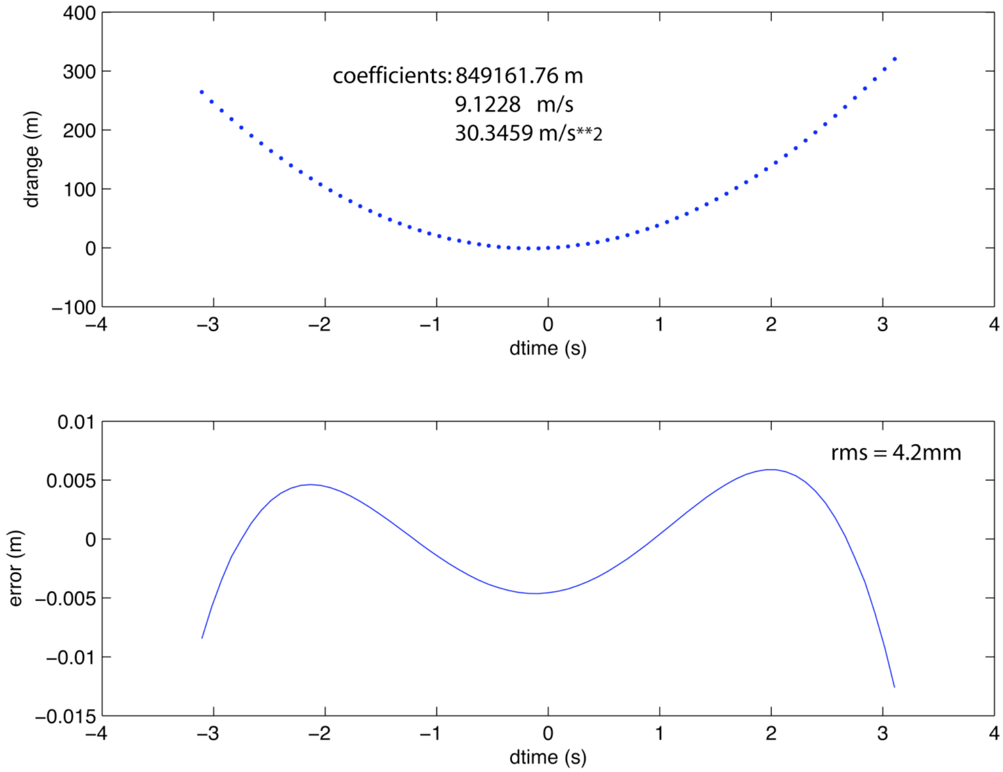


The line of sight vector is $\vec{R}_o = R_o\left(-\cos\theta\,\hat{q}_1 \quad 0\,\hat{q}_2 \quad -\sin\theta\,\hat{q}_3\right)$.

Example with ALOS L-Band Orbit and SAR Data

Now that we have an algorithm for finding a point on the surface of the Earth that is within the SAR scene, we can compute the time-evolution of the range to that point as the satellite orbits above the rotating Earth. We consider data from a descending orbit over Koga Japan where three radar reflectors have been deployed (FBS343_RSP058_20060427). The image is a fine-beam, single polarization having a nominal look angle of 34.3 degrees. A Hermite polynomial interpolation was used to calculate the *x-y-z* position of the satellite from 28 position and velocity vectors spaced at 60-second intervals. Thus the entire arc is 28 minutes or a quarter on an orbit. The accuracy of the Hermite interpolator was checked by omitting a central point and performing an interpolation using 6 surrounding points. The accuracy of the interpolation was found to be better than 0.2 mm suggesting that the 60-s interval provides an accurate representation of the orbital arc.

Next we compute the range to the ground point as a function of time before and after the perpendicular LOS vector $\vec{R}_o$. We used a before/after time interval of 3 seconds which is about twice the length of the synthetic aperture for ALOS. A second-order polynomial was fit to the range versus time function and the three coefficients provide estimates of R_o, $\dot{R}_o$, and $\ddot{R}_o/2$. The range versus time, as well as the residual of the fit, are shown below. One can learn a great deal from this exercise.



1) The first thing learned is that the parabolic approximation to a hyperbola used in the SAR processor has a maximum error of about 1 cm at a time offset of 3 seconds. This corresponds to a small fraction of the 23-cm wavelength. Note that the actual aperture length for ALOS is only +/- 1.5 seconds so this approximation is justified.

2) The range at zero offset has an error of -0.24 m. This is due to approximating the shape of the earth as a sphere having a local radius given by the WGS84 ellipsoid formula. In other words there is a small error in the LOS vector because it intersects the surface of the Earth at a latitude that is slightly different from the spacecraft latitude so the earth radii will differ slightly.

3) The Doppler shift when the satellite is perpendicular to the target can be calculated from the range rate coefficient. This value seems too small because for this pass the yaw angle was 3 degrees which would produce a Doppler shift of 375 m/s. I don't yet understand this.

4) Finally the range acceleration can be used to calculate the effective speed of the satellite $V_e^2 = R_o \ddot{R}_o$. For this example we arrive at a speed of 7174 m/s. By trial and error we found the optimal focus parameter corresponds to a speed between 7173 and 7183 so this new estimate is accurate. The simple Cartesian ground speed approximation provided an estimate of 7208 m/s which provided poor focus.