Seamount Flexure

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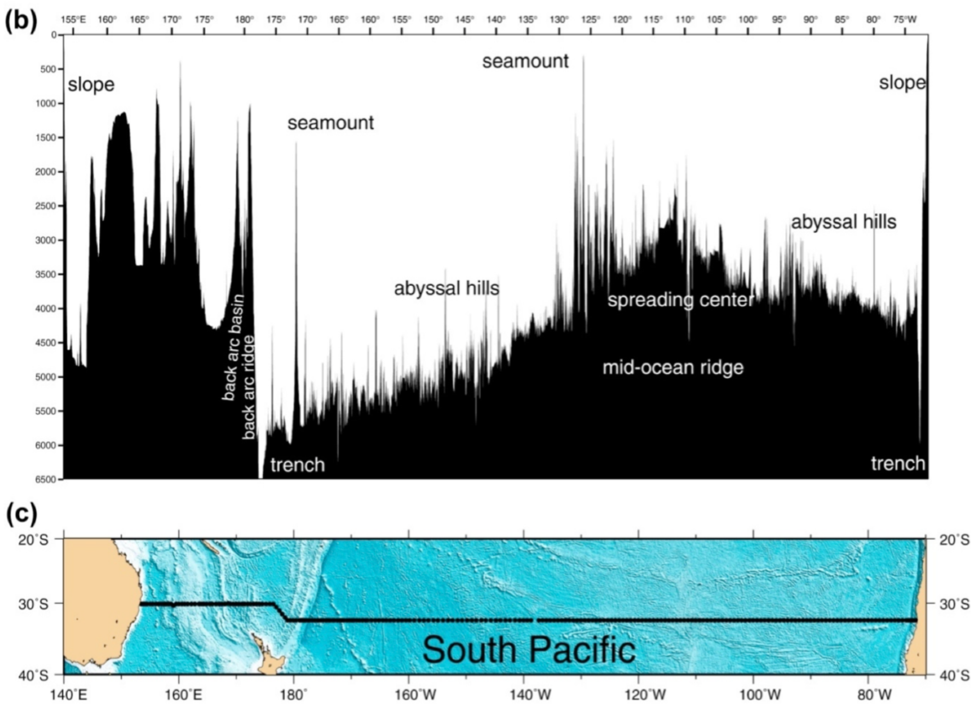
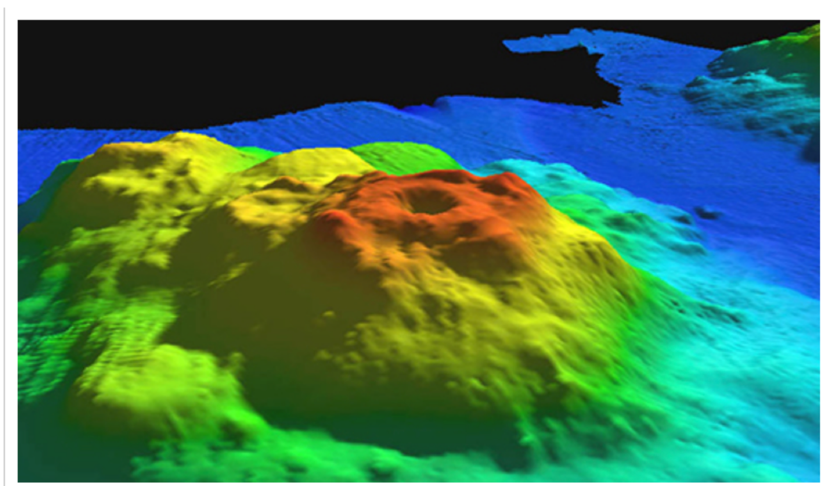
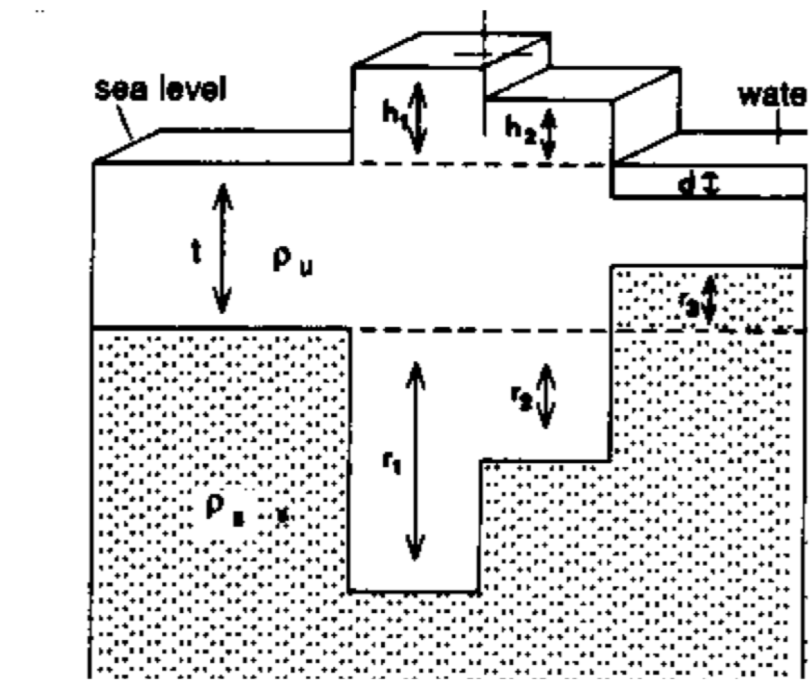


R. J. Banks, R. L. Parker & S. P. Huestis, 1977
D. Sandwell, Appendix, Chapter 17
Turcotte & Schubert, Chapter 12

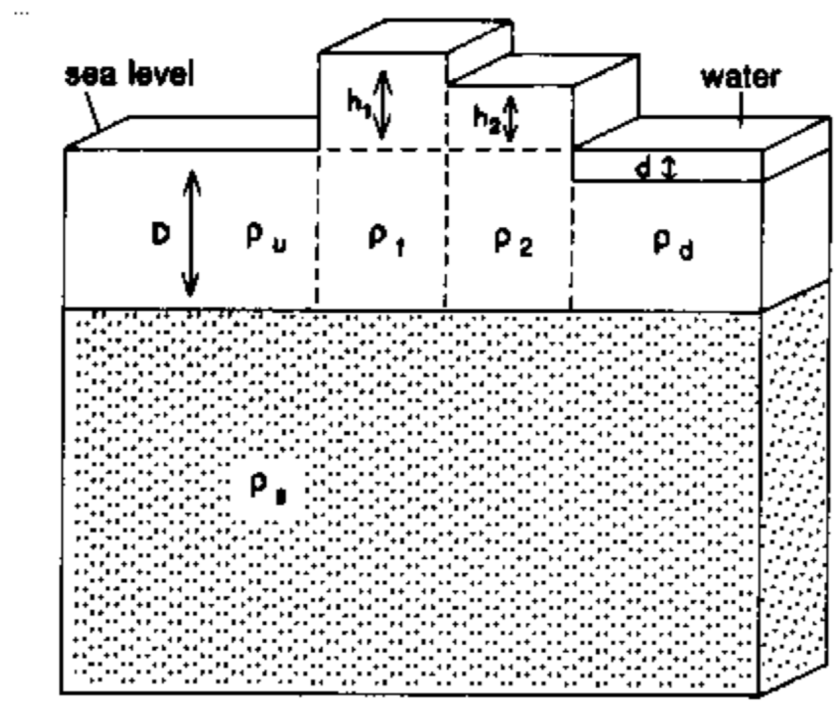


Isostasy – local models

Airy model



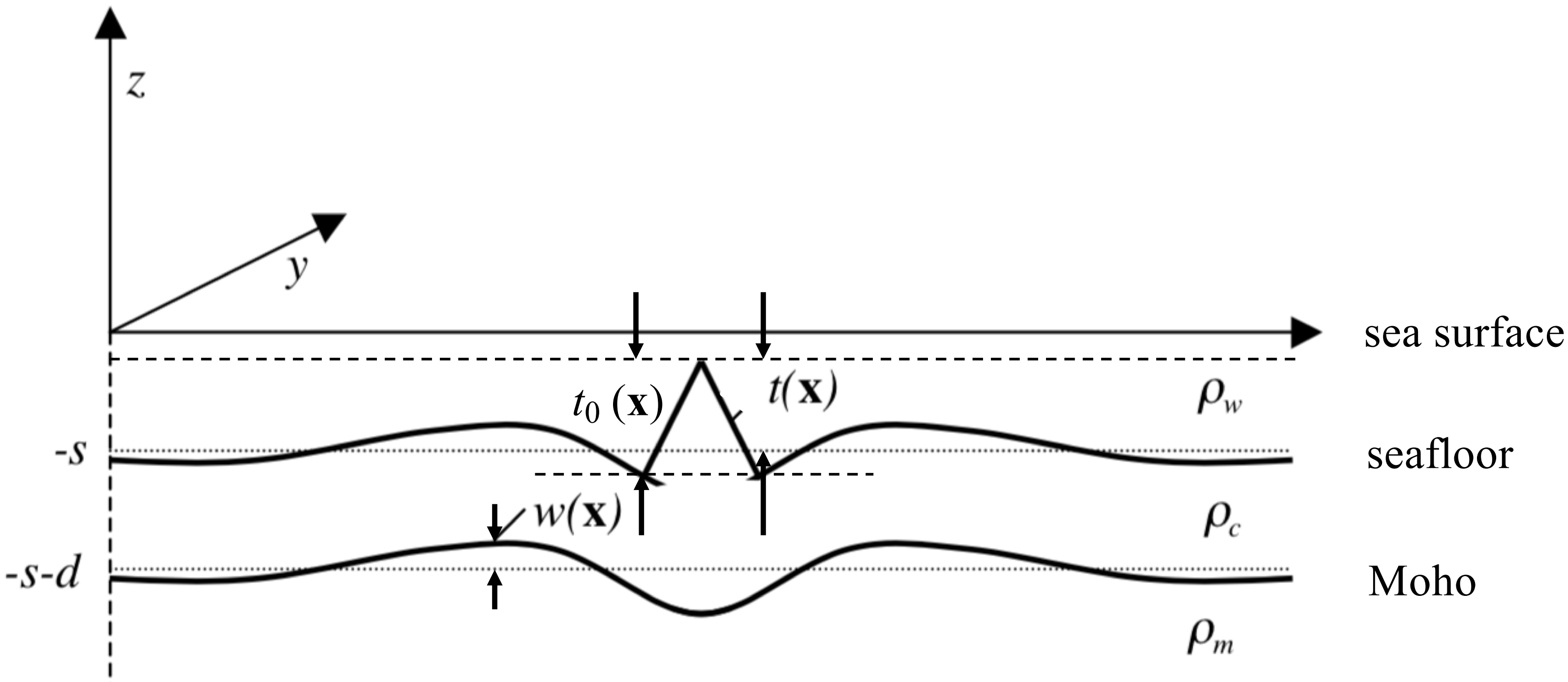
Pratt model



Difficulties with such local models: crust has no strength at all for vertical loads.

Isostasy – regional compensation model

The outer shell of the earth is treated as a thin elastic plate, floating on the surface of a liquid.



$$t(\mathbf{x}) = t_o(\mathbf{x}) + w(\mathbf{x})$$

s: mean ocean depth – 4 km in this study

d: crust thickness – 6 km in this study

Plate deformation in response to topographic load

$$D\nabla^4 w(\mathbf{x}) \ = q(\mathbf{x})$$

$$q(\mathbf{x}) = -(\rho_c - \rho_w) g t(\mathbf{x}) - (\rho_m - \rho_c) g w(\mathbf{x})$$

$\Big\downarrow$ *Fourier transform*

$$D(2\pi|\mathbf{k}|)^4 W(\mathbf{k}) + (\rho_m - \rho_c) g W(\mathbf{k}) = -(\rho_c - \rho_w) g T(\mathbf{k})$$

$\Big\downarrow$ *rearrange*

$$W(\mathbf{k}) = \frac{-(\rho_c - \rho_w)}{(\rho_m - \rho_c)}\left[1 + \frac{D(2\pi|\mathbf{k}|)^4}{g(\rho_m - \rho_c)}\right]^{-1} T(\mathbf{k})$$

$$W(\mathbf{k}) = \frac{-(\rho_c - \rho_w)}{(\rho_m - \rho_c)} \left[1 + \frac{D(2\pi|\mathbf{k}|)^4}{g(\rho_m - \rho_c)} \right]^{-1} T(\mathbf{k})$$

$$\sim \frac{\lambda_f^4}{\lambda_x^4}$$

$$\lambda_f = 2\pi \left[\frac{D}{g(\rho_m - \rho_c)} \right]^{1/4} = \sqrt{2}\pi\alpha$$

-- flexural wavelength

$$D \equiv \frac{Eh^3}{12(1-\nu^2)}.$$

-- flexural rigidity

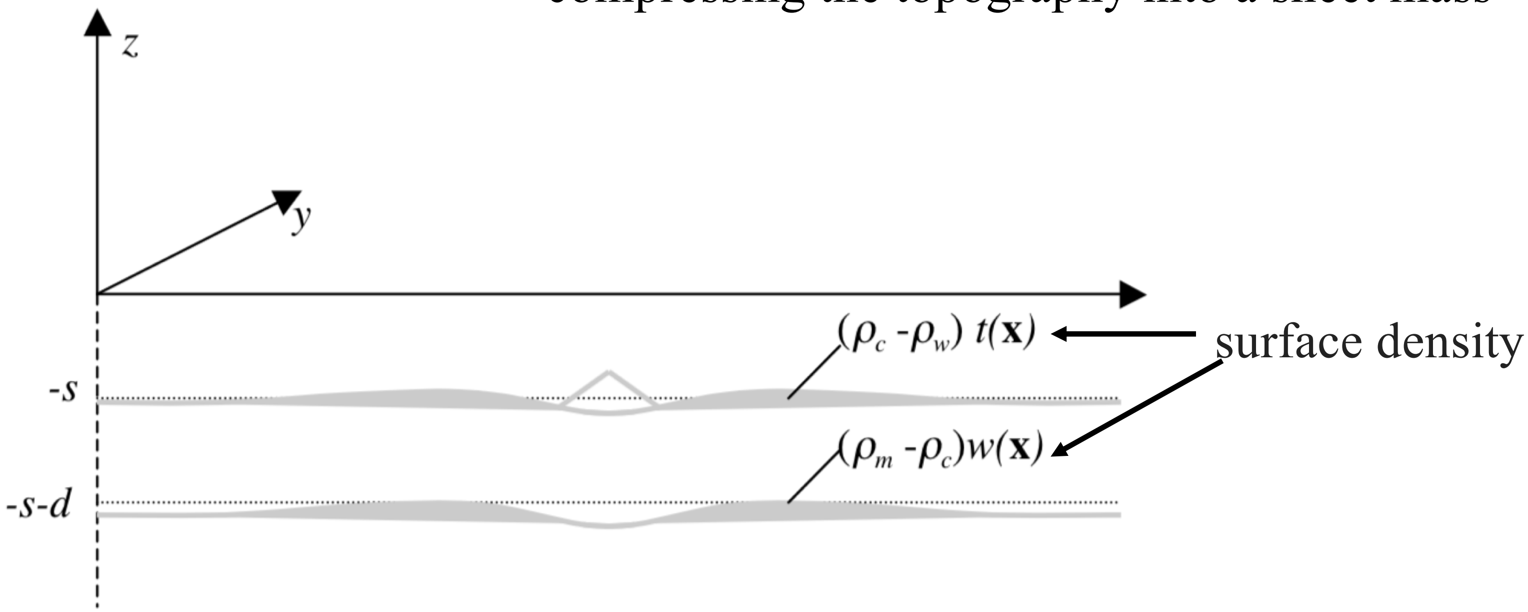
$\lambda_f << \lambda_x$ Airy-compensation model, compensated topography

$\lambda_f >> \lambda_x$ $w = 0$, uncompensated topography

elastic thickness (h) of the lithosphere is the **thickness** of an **elastic** layer that would respond to applied loads in the same way as the heterogeneous lithospheric plate.

Gravity due to topographic load

--- compressing the topography into a sheet mass



$$\Delta g(\mathbf{k}, z) = -\frac{\partial \Phi}{\partial z} = 2\pi G \sigma(\mathbf{k}) e^{-2\pi|\mathbf{k}|(z-z_o)}$$ — Solution to Poisson's equation

$z = 0$ (gravity anomaly at sea surface)

$$\Delta g(\mathbf{k}) = 2\pi G\,(\rho_c - \rho_w)\, e^{-2\pi|\mathbf{k}|s}\, T(\mathbf{k}) + 2\pi G\,(\rho_m - \rho_c)\, e^{-2\pi|\mathbf{k}|(s+d)}\, W(\mathbf{k})$$

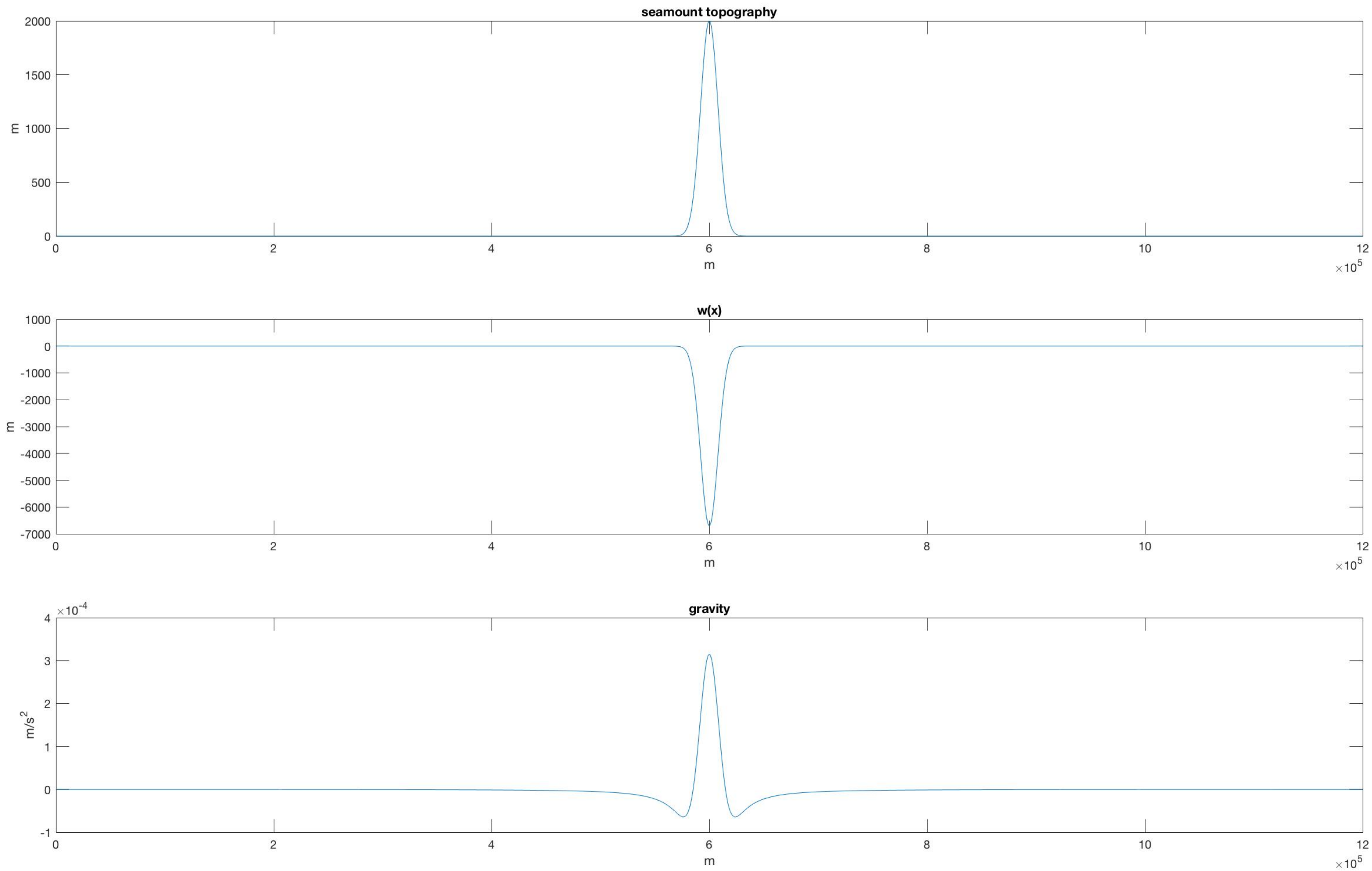
$$\Delta g(\mathbf{k}) = 2\pi G\,(\rho_c - \rho_w)\, e^{-2\pi|\mathbf{k}|s} \left\{1 - \left[1 + \frac{D\,(2\pi\,|\mathbf{k}|)^4}{g\,(\rho_m - \rho_c)}\right]^{-1} e^{-2\pi|\mathbf{k}|d}\right\} T(\mathbf{k})$$

parameters

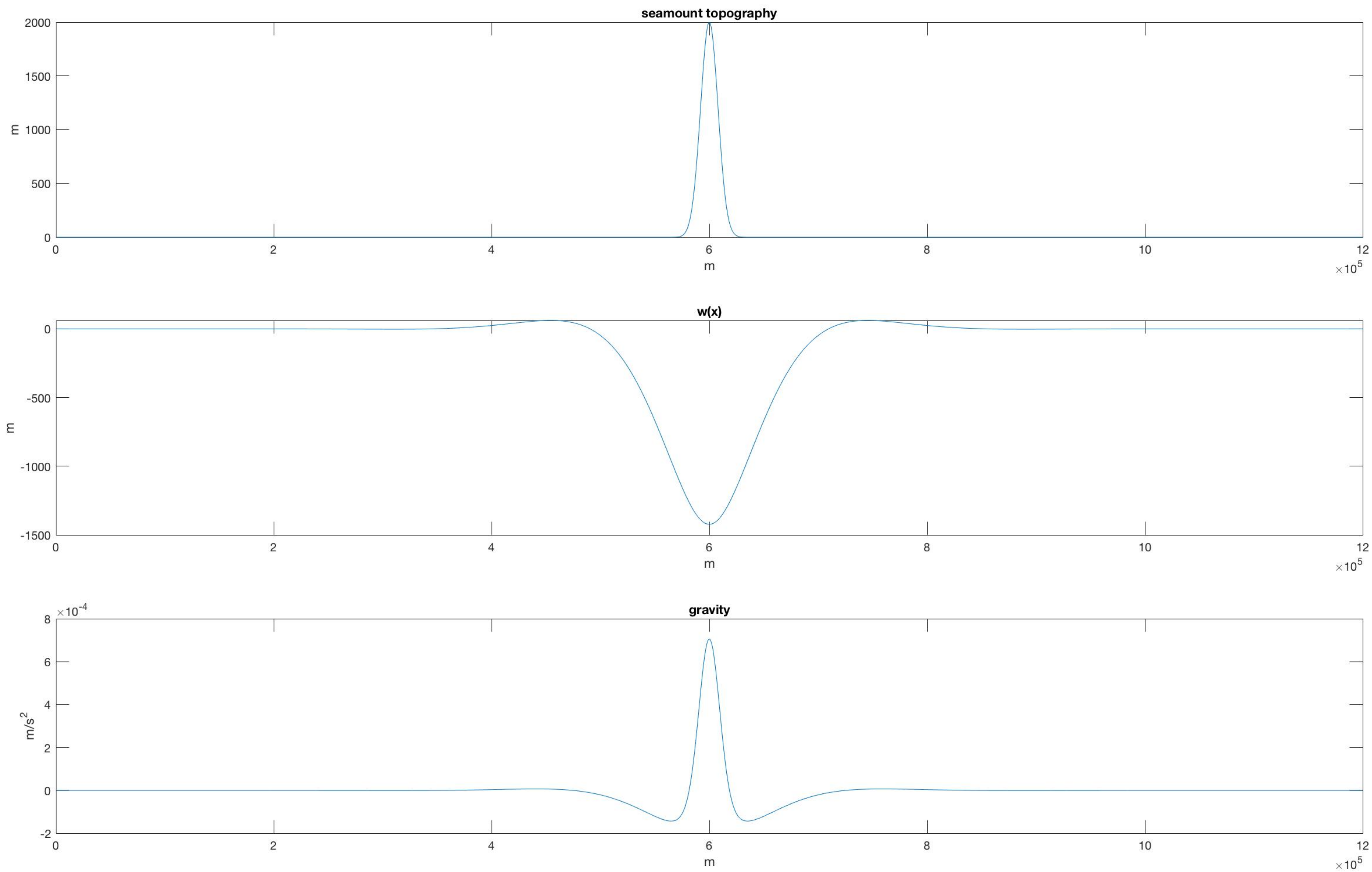
Parameter	Definition	Value/Unit
$w(\mathbf{x})$	deflection of plate (positive up)	m
$D = \dfrac{Eh^3}{12(1-\nu^2)}$	flexural rigidity	N m
h	elastic plate thickness	m
ρ_w	seawater density	1025 kg m^{-3}
ρ_c	seawater density	2800 kg m^{-3}
ρ_m	mantle density	3330 kg m^{-3}
g	acceleration of gravity	9.82 m s^{-2}
E	Young's modulus	6.5 x 10^{10} Pa
ν	Poisson's ratio	0.25

- Seamount width : 40 km
- Seamount height : 2 km

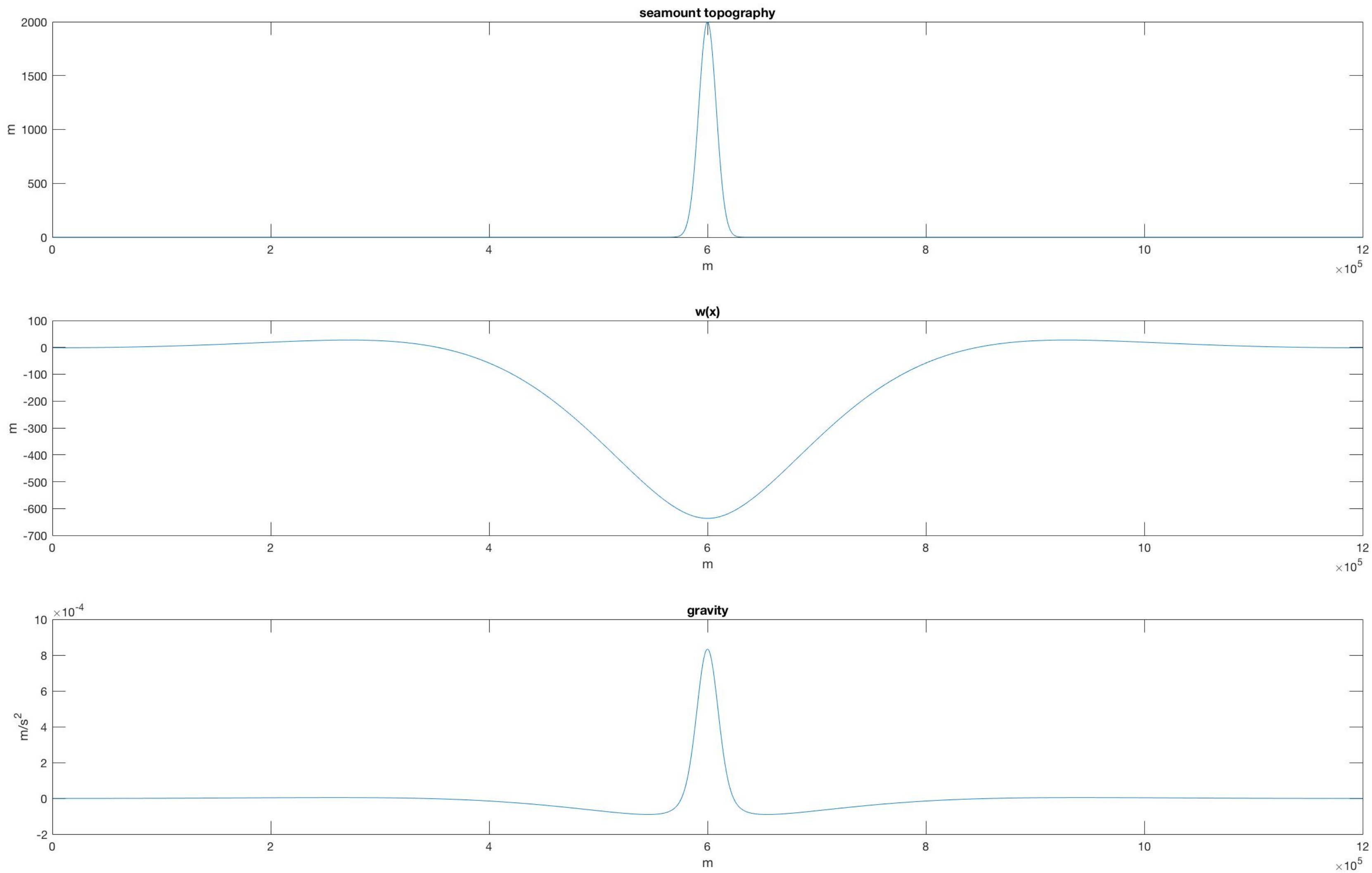
h = 0 km
Seafloor depth = 4 km



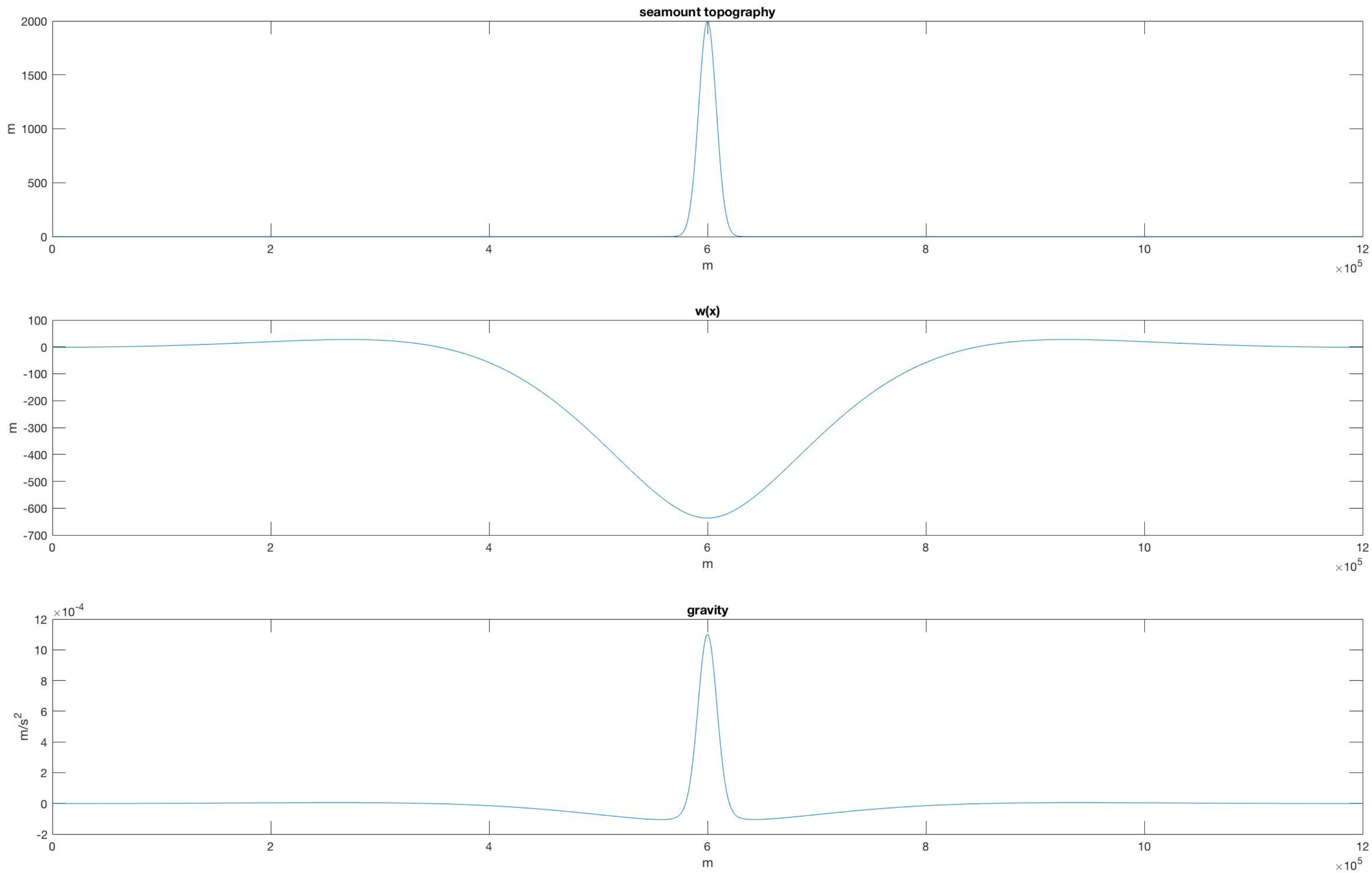
h = 10 km
Seafloor depth = 4 km



h = 30 km
Seafloor depth = 4 km



h = 30 km
Seafloor depth = 2 km



h = 0 km
Seafloor depth = 4 km

$$\Delta g(\mathbf{k}) = 2\pi G\,(\rho_c - \rho_w)\; e^{-2\pi|\mathbf{k}|s}\; T(\mathbf{k}) + 2\pi G\,(\rho_m - \rho_c)\; e^{-2\pi|\mathbf{k}|(s+d)}\; W(\mathbf{k})$$