Lake/Ocean Loading Flexure

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SIO 234 Geodynamics

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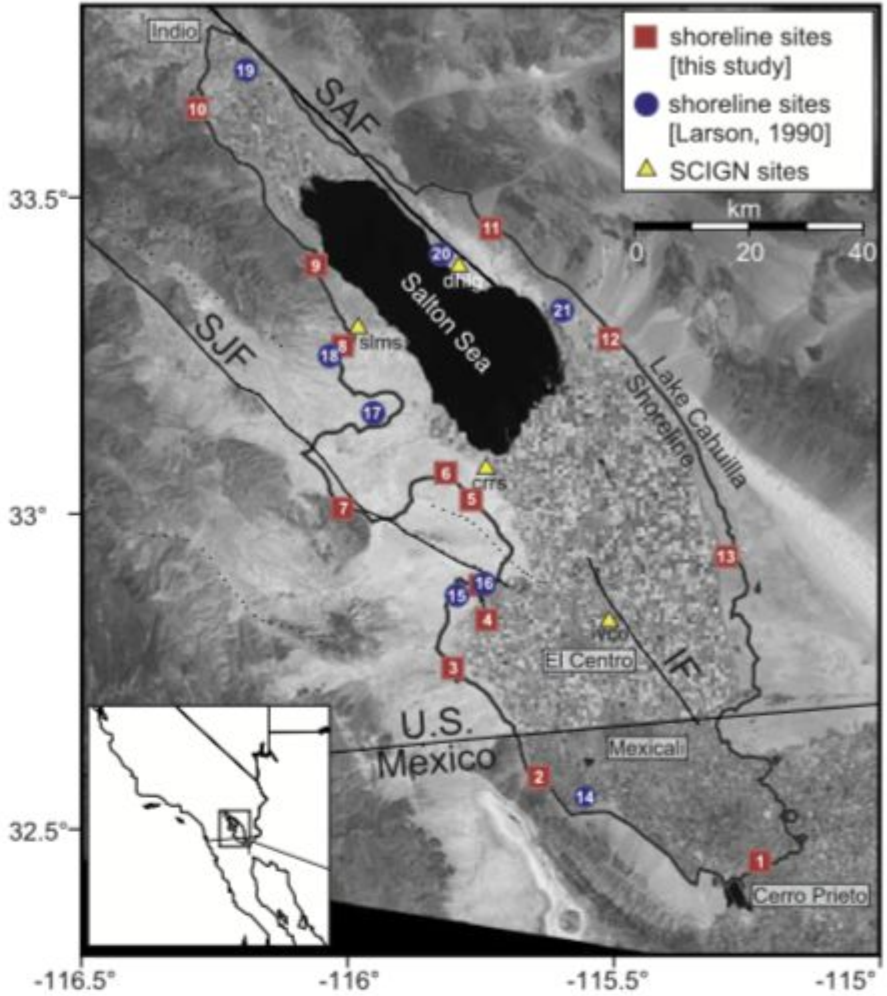
- **Ancient Lake Cahuilla (Ka wee Uh) region**
- Ancient shoreline (13 m above sea level)
- GPS to measure elevation
- Recurrence interval of filling/draining of Lake
- Cahuilla is similar to the recurrence interval of
- major ruptures on the SAF
- Last 4 lake highstands over the last 1300
- years



Luttrell et al. 2007

Background / Tasks

-Immediately after lake Cahuilla fills the Earth will respond to the weight of the surface load approximately as an elastic half-space.

Thick plate over 3-D viscoelastic half-space problem solved by [Smith and Sandwell 2003, 2004]

Checks solution against 2-D thin-elastic plate approximation.

This should be a good approximation if we are considering a thin 2-D elastic plate at infinite time and 3-D thick elastic plate overlying viscoelastic half-space model at 0 time.

Thus our task is to derive the thin 2-D elastic plate approximation...

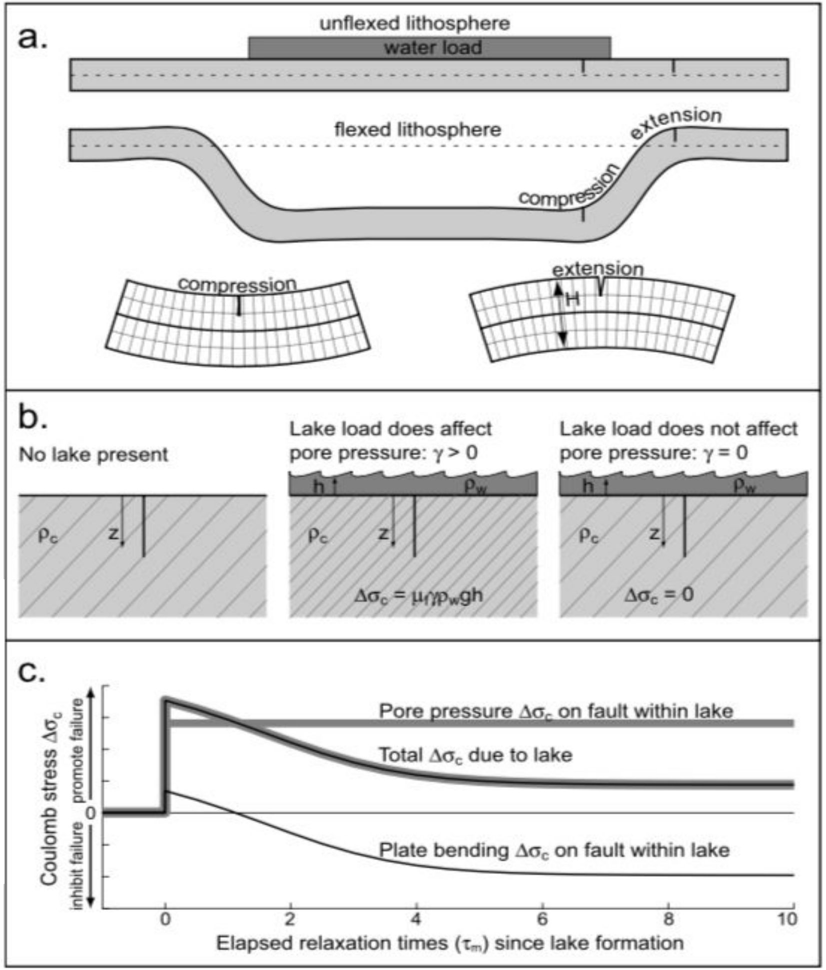
Recall:

Thin 2-D Elastic Plate Model

..but we've seen this before..

$$C_f = \tau_B + \mu(\sigma_B + P)$$

$$\sigma_c = \sigma_s - \mu_f \sigma_n$$



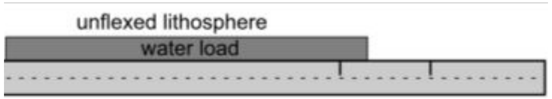
Luttrell et al. 2007

Derivation

Deflection of
A Plate
(T&S) 3.103

$$D\frac{d^4 w(x)}{dx^4} + (\rho_m - \rho_w) g w(x) = q(x)$$

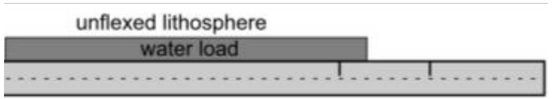
w(x) * [step function (load)]

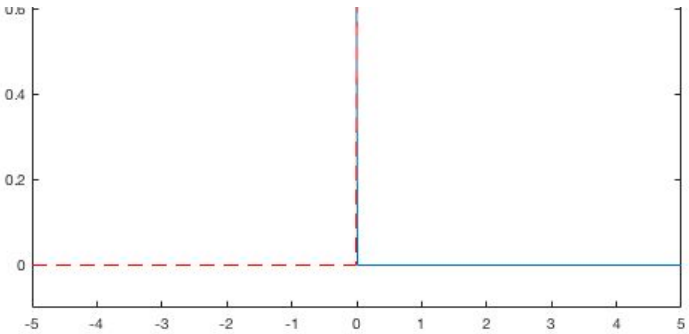


$$W(x) = \int_{-\infty}^{\infty} w(x')[1 - H(x - x')]dx' \quad x'))]dx'$$

$$H(x - x')]$$

$$[1 - H(x - x')] = \int_{-\infty}^{\infty} w(x')[1 - H(x - x')]dx'$$





$$W(x) = \int_{-\infty}^{\infty} w(x')[1 - (\frac{1}{2}(1 + sgn(x - x'))]dx'$$

$$W(x) = \int_{-\infty}^{\infty} w(x')[1 - H(x - x')]dx'$$

$[1\text{-H}(\text{x-x}')] = 1$ if $\text{x} < \text{x}'$
$[1\text{-H}(\text{x-x}')] = 0$ if $\text{x} \geq \text{x}'$

$$W(x) = \int_{-\infty}^{\infty} w(x')dx'$$

$$W(x) = \int_{-\infty}^{\infty} w(x')dx'$$

$$= \int_{-\infty}^{\infty} \frac{V_o\alpha^3}{8D} \exp(\frac{-x'}{\alpha})(cos(\frac{x'}{\alpha}) + sin(\frac{x'}{\alpha}))dx'$$

$$= A[\alpha \exp(\frac{-x'}{\alpha})sin(\frac{x'}{\alpha}) - \int_{-\infty}^{\infty} sin(\frac{x'}{\alpha}) \exp(\frac{-x'}{\alpha})]$$

$$= A[\alpha \exp(\frac{-x'}{\alpha})sin(\frac{x'}{\alpha}) - \alpha \exp(\frac{-x'}{\alpha})[cos(\frac{x'}{\alpha}) + sin(\frac{x'}{\alpha})]$$

$$= A[-\alpha \exp(\frac{-x}{\alpha})cos(\frac{x'}{\alpha})]_{-\infty}^{\infty}$$

$$= A[-\alpha \exp(\frac{-x}{\alpha})cos(\frac{x'}{\alpha})]_{0}^{x}$$

$$= \frac{V_o\alpha^4}{8D}[1 - \exp(\frac{-x}{\alpha})cos(\frac{x}{\alpha})]$$

$$W(x) = \frac{V_o\alpha^4}{8D}[1 - \exp(\frac{-|x|}{\alpha})cos(\frac{|x|}{\alpha})]sgn(x)$$

$$W(x) = \frac{V_o\alpha^4}{8D}[1 - \exp(\frac{-x}{\alpha})cos(\frac{x}{\alpha})]$$

$$\frac{dW}{dx} = \frac{V_o\alpha^4}{8D}[\exp(\frac{-x}{\alpha})sin(\frac{x}{\alpha}) + \exp(\frac{-x}{\alpha})cos(\frac{x}{\alpha})]$$

$$\frac{d^2W}{dx^2} = \frac{V_o\alpha^3}{8D}[\frac{-1}{\alpha}\exp(\frac{-x}{\alpha})sin(\frac{x}{\alpha})+(\frac{1}{\alpha})\exp(\frac{-x}{\alpha})cos(\frac{x}{\alpha})+\frac{-1}{\alpha}\exp(\frac{-x}{\alpha})cos(\frac{x}{\alpha})+$$

$$\frac{-1}{\alpha}\exp\frac{-x}{\alpha}sin(\frac{x}{\alpha})]$$

$$\epsilon_{xx} = -y\frac{d^2W}{dx^2}$$

$$D = \frac{Eh^3}{12(1-v^2)}$$

$$\sigma_{xx} = \frac{E}{1-v^2}\epsilon_{xx}$$

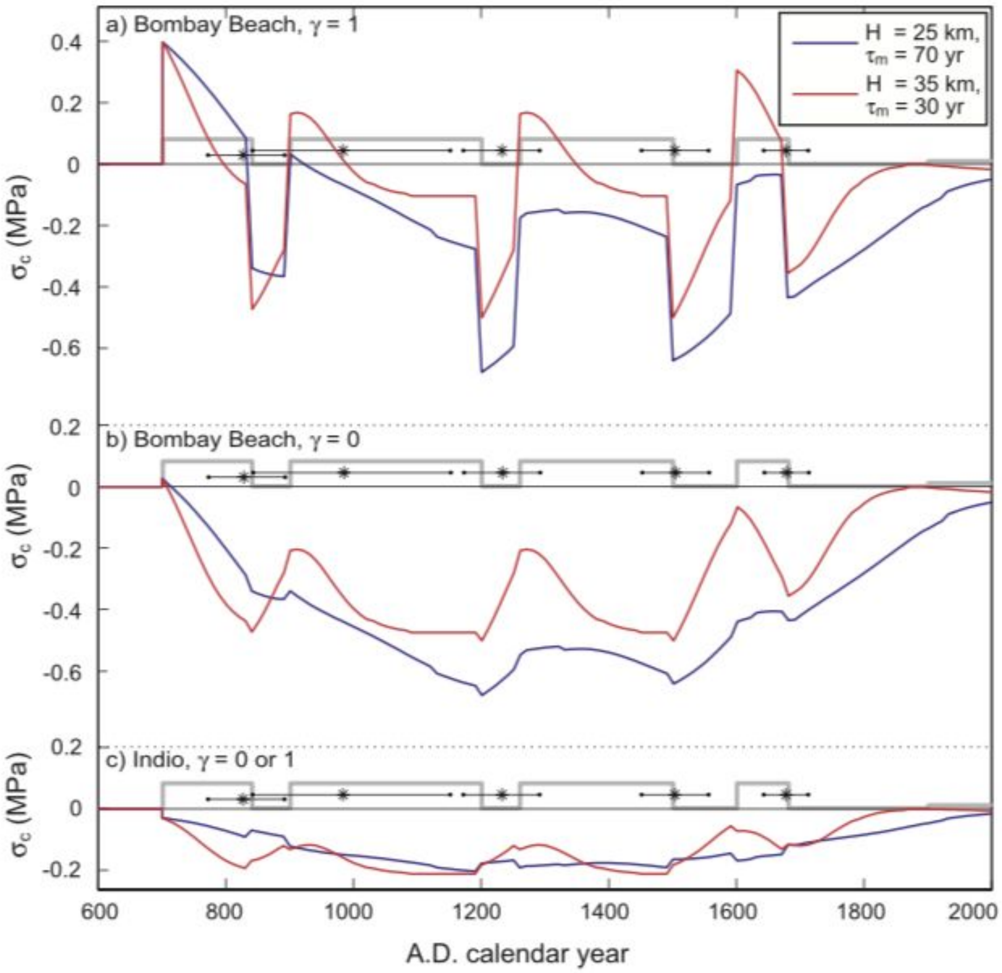
$$\epsilon_{xx} = -y\frac{V_o\alpha^2}{4D}[[exp(\frac{-x}{\alpha})sin(\frac{x}{\alpha})]$$

$$\sigma_{xx} = \frac{E}{1-v^2} - y\frac{V_o\alpha^2}{4D}[\exp\frac{-x}{\alpha}sin(\frac{x}{\alpha})]$$

$$\sigma_{xx} = \frac{E}{1-v^2} - y\frac{V_o\alpha^2 12(1-v^2)}{4Eh^3}[\exp\frac{-x}{\alpha}sin(\frac{x}{\alpha})]$$

$$\sigma_{xx} = \frac{3V_o\alpha^2}{h^2}\frac{y}{h}\exp\frac{-|x|}{\alpha}sin(\frac{|x|}{\alpha})sgn(x)$$

- Modeled at maximum pore pressure effect and no pore pressure effect
- 0.2-0.6 MPa within lake
- 0.1-0.2 MPa outside lake
- An order of magnitude lower than tectonic loading
    - Therefore likely only modulates for critically stressed faults
- 4 of last 5 events show reasonable timing with lake level changes.



Luttrell et al. 2007

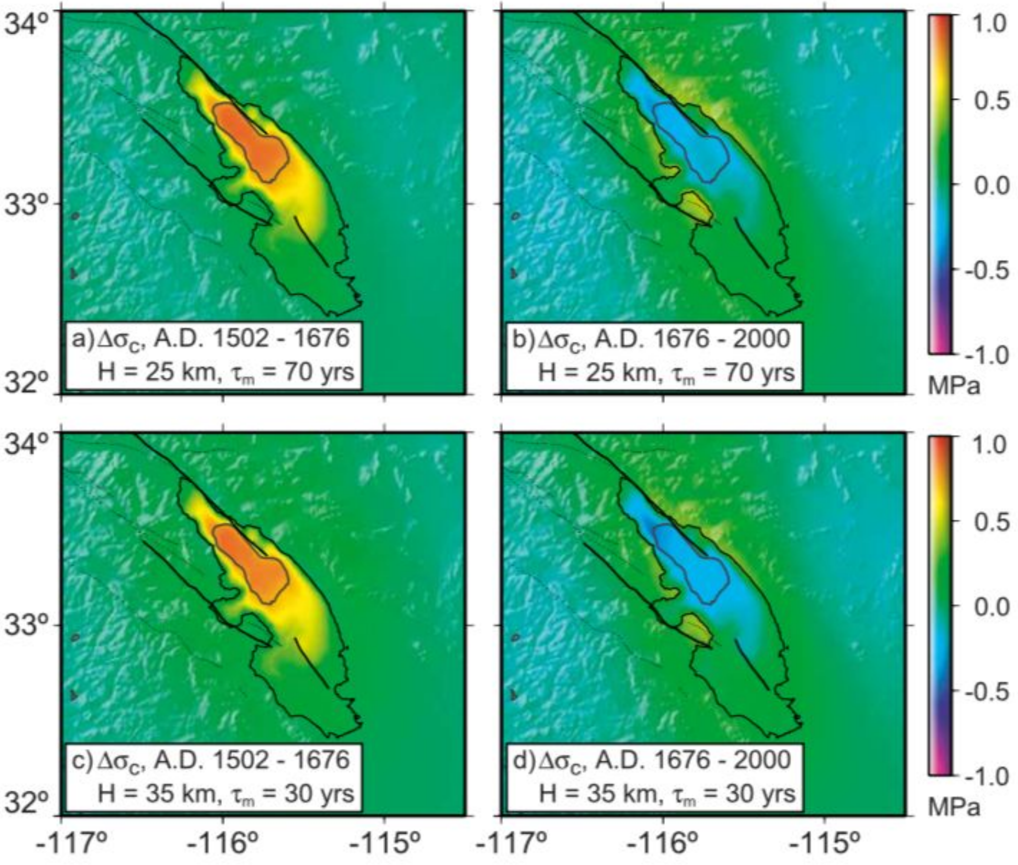
Constrained best fit 3-D models between the last 2 San Andreas ruptures

H = 25 km, τ = 70 years

(thinner, more viscous)

H = 35 km, τ = 30 years

(thicker, less viscous)



Luttrell et al. 2007

2-D Modeling

We use a 2D model with a *thin elastic plate* to check the 3-D response of a thick elastic plate overlying a viscoelastic halfspace

- The thin model at infinite time should match the as the 3-D model

- We will have no horizontal displacement U(x) for the 2-D model

Luttrell et al. 2007

2-D Modeling

Plate thickness H = 30 km

Young's Modulus E = 70 GPa

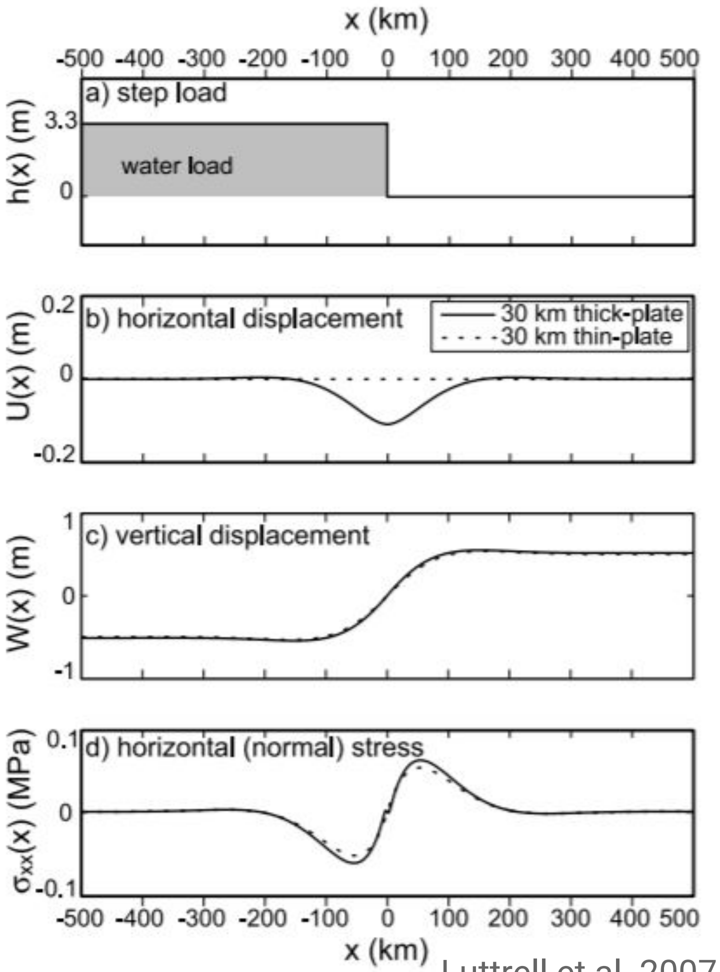
Poisson's Ratio ν = 0.25

Material density ρ_m = 3300 kg/m^3

Water density ρ_w = 1000 kg/m^3

Water depth h = 3.3 m

Seismogenic height z_o = 5 km



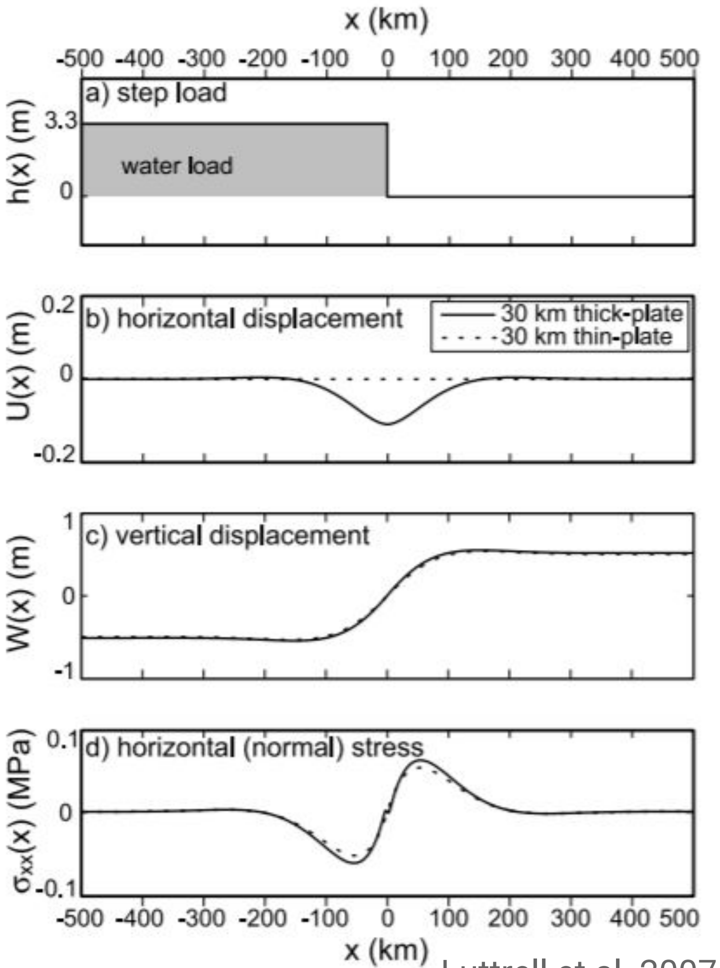
Luttrell et al. 2007

2-D Modeling

Magnitude of vertical load $V_o = \rho_w gh$

Flexural rigidity $D = EH^3/12(1-v^2)$

Flexural parameter $\alpha = ((4D)/(\rho_m g))^{1/4}$



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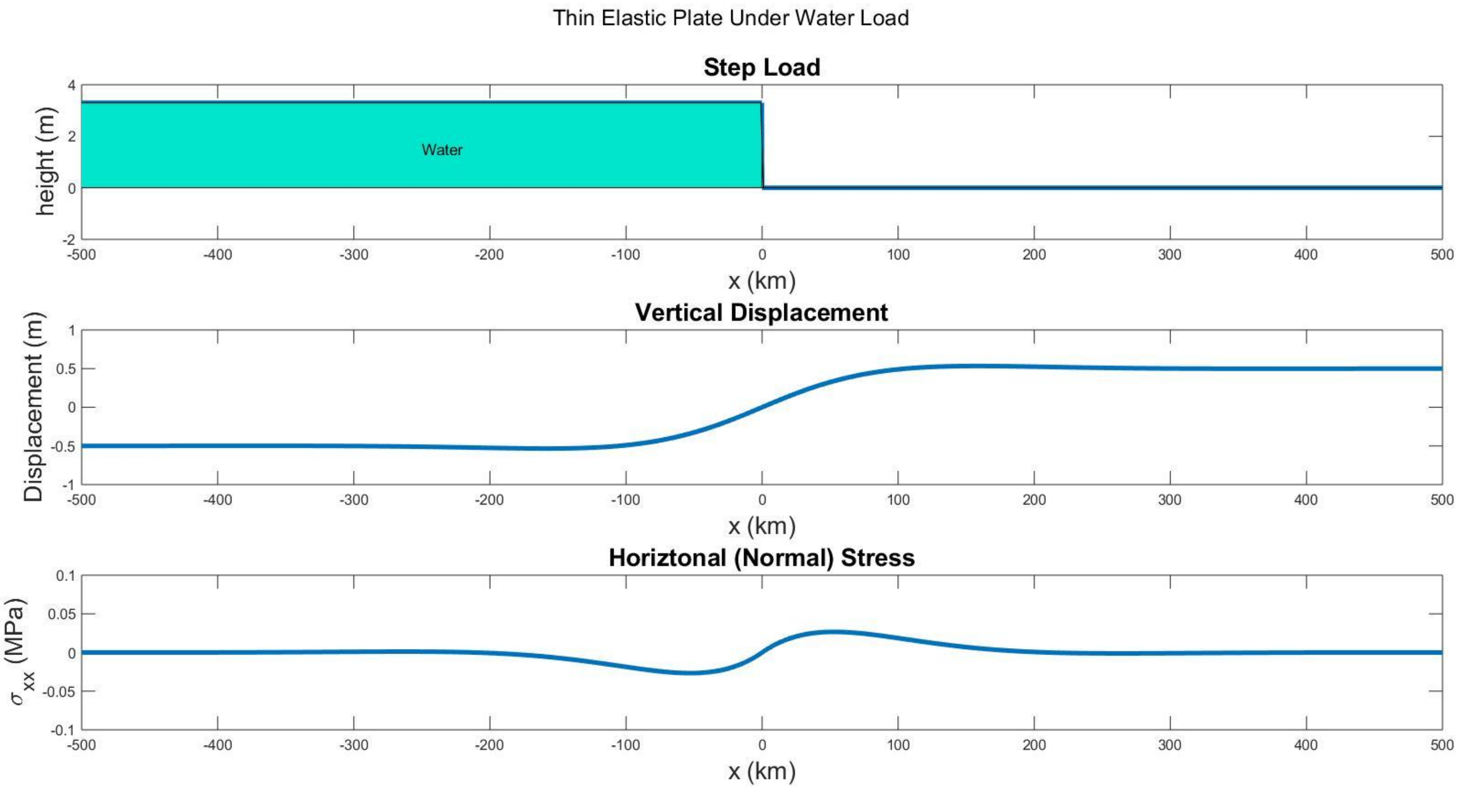
2-D Modeling

```
W=((Vo*a^4)/(8*D)).*(1-(exp(-abs(x)./a).*cos(abs(x)./a))).*sign(x);
sig_xx=(((3*Vo*a^2)/(H^2)).*(zo/H)).*exp(-abs(x)/a).*sin(abs(x)/a).*sign(x);
```

The vector x goes from 0 to an infinite distance from the lake.

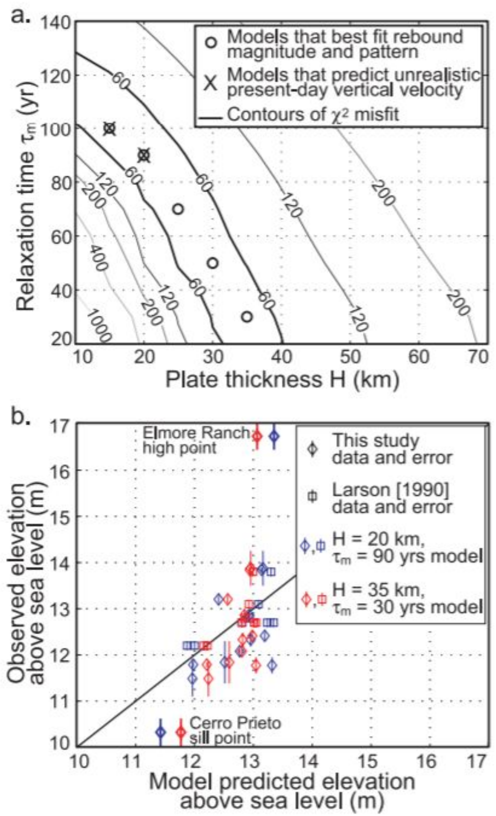
Since we cannot use a vector of infinite length in Matlab we make a large vector (500,000 km either direction from the lake)

2-D Modeling


Thin Elastic Plate Under Water Load
Step Load
height (m)
Water
x (km)
Vertical Displacement
Displacement (m)
x (km)
Horiztonal (Normal) Stress
σ_{xx} (MPa)
x (km)

3-D Modeling Constraints

- Used the current elevation of the observed ancient lake
- Plate thicknesses: 10-100 km
- Half-space relaxation time 20-200 years
- Compared expected deformation values to current elevations
- a) Plate thickness vs. relaxation time
- b) Model vs. observed
  - Two best fit after throwing away deformations above 5 mm/yr
    - H=25, relaxation time = 70 years
    - H=35, relation time = 30 years

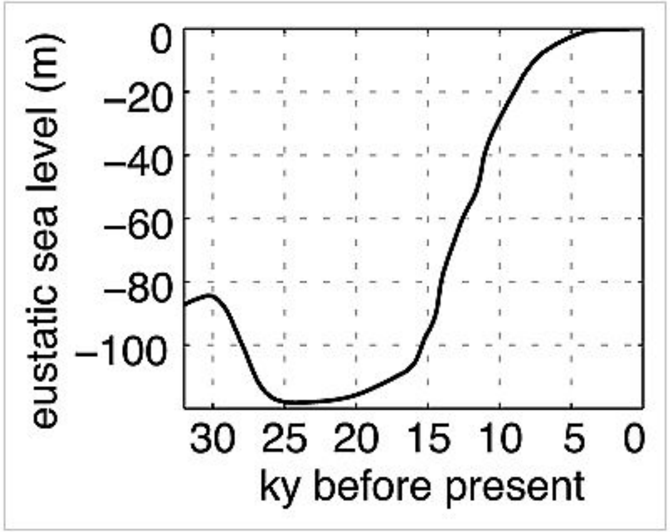


Conclusions

- 2-D thin plate model is a large simplification of situation in real earth, but the solution approximates the vertical displacement and horizontal normal stress quite well
- Last 1300 years - the fluctuation in lake level has resulted in an 0.2-0.6 MPa perturbation (in the lake) and 0.1-0.2 MPa (near shore)
    - An order of magnitude *lower* than the stress associated with tectonic loading
    - Faults near critical stress will be modulated by this
- We need more information (paleoseismic history) for the Salton Trough region to make better conclusions about the relation between the lake loading and ruptures

Ocean loading effects on stress at near shore plate boundary fault systems
Luttrell and Sandwell [2010]

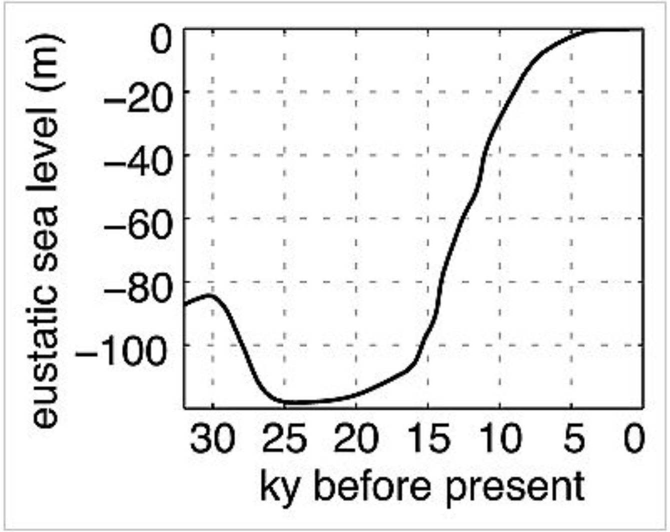
- Paper explores the possibility that coastal plate boundary faults are affected by rise in eustatic (global mean) sea level
- Changes in sea level alter the state of stress of the lithosphere at coastal regions
  - Due to the vertical load to ocean basins
- This stress perturbation will likely have a bigger effect on secondary onshore faults
  - These smaller faults have lower tectonic loading rate



Luttrell, Sandwell 2010

Ocean loading effects on stress at near shore plate boundary fault systems
Luttrell and Sandwell [2010]

- **Example**
  - The Black Sea rose more rapidly compared to the eustatic rate
  - Resulted in a normal stress perturbation of 1 MPa
    - Nearby strike slip North Anatolian fault had Coulomb stress changes of up to 2 kPa/year
    - Likely that this *decrease* in fault strength *increased* the frequency of earthquakes soon after the load was *increased*



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