Group B
Heat Flow Across the San Andreas Fault

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Outline

- Background information on the problem:
  - Heating due to frictional sliding
  - Shear stress at depth
  - Average stress drop along fault
- Solution of heat equation in 2D
  - Fourier series
  - Method of images
- Compare with Lachenbruch and Sass 1980
- Consider hydrothermal heating to explain heat flow anomaly

Earthquake Energy Release

- Energy released during an earthquake should be related to:
  - Size of fault
  - Amount of stress on the fault
  - Displacement of fault

$$E = A\tau\Delta$$

- Majority of energy is converted into heat
- Assuming this heat can be averaged over many cycles, can express heat flow as a function of plate velocity

$$E = \tau V$$

Fault Heat Production

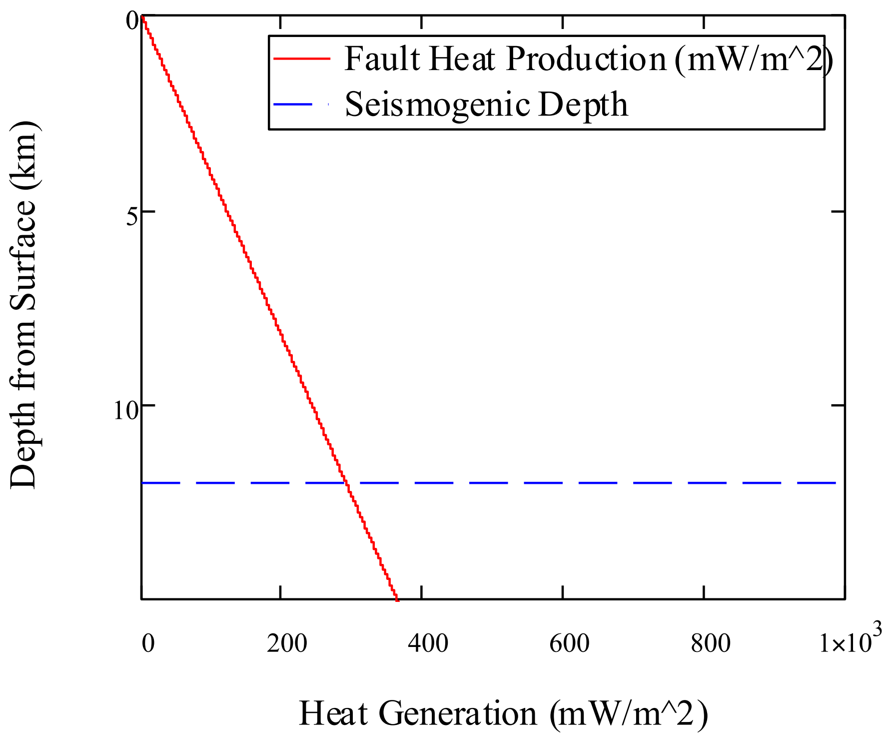
- Plot of fault heat production

$q(z)=\tau v$

f=0.60

v=5cm/yr

$q(12km)=290\ mW/m^2$



Shear Stress on Fault

- Typical stress drop during a major earthquake is about 3-5MPa (30-50 bar)
- About two orders of magnitude lower than shear stresses at the bottom of the seismogenic zone
  - Fault is not restored to a stress-free state even after very large events
  - Fault is under significant shear stress at all times

Shear Stress on Fault

- Seismogenic zone: surface to 12 km depth
- Shear stress is a large portion of the overburden stress

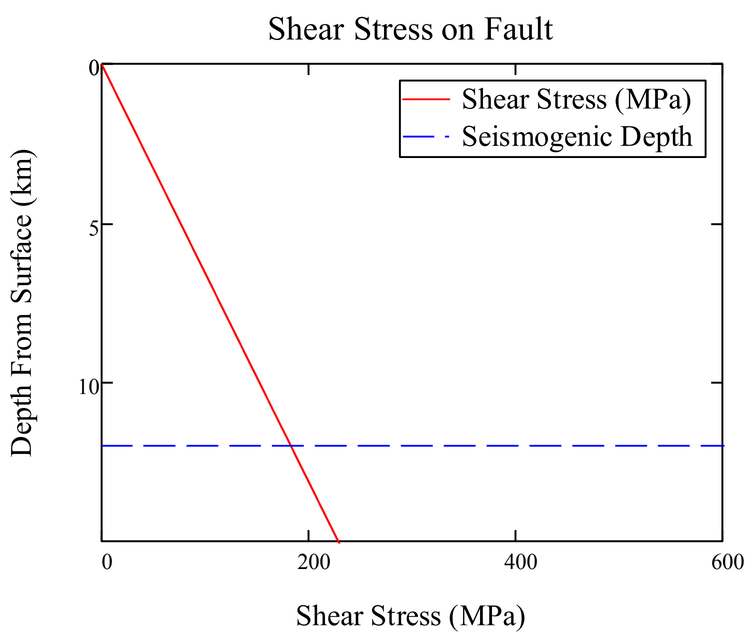
$\tau(z)=f\rho_c gz$

$f=0.60$

$\rho_c=2600kg\text{-}m^{-3}$

$g=9.81m\text{-}s^{-2}$

$\tau(12km)=183$ MPa

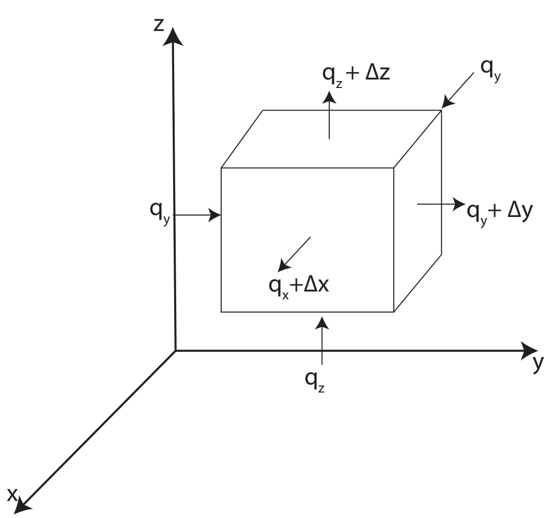


Derivation of Heat equation

- Starting with the first law of thermodynamics:

$$dU = \delta Q + \delta W$$ (Conservation of Energy)

- Consider the control element:



Derivation of the Heat equation

Rewrite using the heat flux through the control element:

$$\rho c_p (\Delta x \Delta y \Delta z) \frac{dT}{dt} = q_x - q_{x+\Delta x} + q_y - q_{y+\Delta y} + q_z - q_{z+\Delta z} + Q(x,y,z,t)$$

Substitute in Fourier's Law:

$$\rho c_p (\Delta x \Delta y \Delta z) \frac{dT}{dt} = \left\{ -k(\Delta y \Delta z)\frac{\partial T}{\partial x} - \left[-k(\Delta y \Delta z)\frac{\partial T}{\partial x} + \frac{\partial}{\partial x} - k(\Delta y \Delta z)\frac{\partial T}{\partial x}\Delta x \right] \right\}$$

$$+ \left\{ -k(\Delta x \Delta z)\frac{\partial T}{\partial y} - \left[-k(\Delta x \Delta z)\frac{\partial T}{\partial y} + \frac{\partial}{\partial y} - k(\Delta x \Delta z)\frac{\partial T}{\partial y}\Delta y \right] \right\}$$

$$+ \left\{ -k(\Delta x \Delta y)\frac{\partial T}{\partial z} - \left[-k(\Delta x \Delta y)\frac{\partial T}{\partial z} + \frac{\partial}{\partial z} - k(\Delta x \Delta y)\frac{\partial T}{\partial z}\Delta z \right] \right\}$$

Derivation of the Heat equation

Simplify:

$$\rho c_p \frac{dT}{dt} = -\frac{\partial}{\partial x}(-k_x \frac{\partial T}{\partial x}) - \frac{\partial}{\partial y}(-k_y \frac{\partial T}{\partial y}) - \frac{\partial}{\partial z}(-k_z \frac{\partial T}{\partial z}) + Q(x,z)$$

Assume steady-state and isotropic conditions.

$$Q(x,z)\frac{1}{k} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$

With infinitely long line-source:

$$\delta(x)\delta(z+a)\frac{1}{k} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$

Solving the Heat equation

Equation:

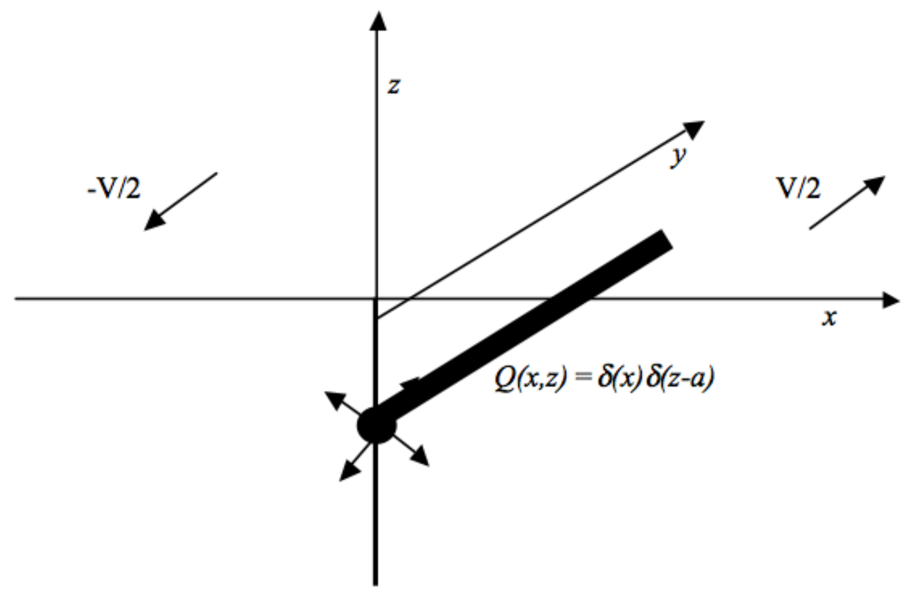
$$\delta(x)\delta(z+a)\frac{1}{k} = \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}$$

Boundary Conditions:

$$T(x,0) = 0$$

$$T(x,\infty) = 0$$

$$T(x,-\infty) = 0$$



Solving the heat equation

Take Fourier Transform of both sides:

$$\Im^2\left\{\delta(x)\delta(z+a)\frac{1}{k}\right\} = \Im^2\left\{\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial z^2}\right\}$$

$$T(\underline{k}) = \frac{-e^{i2\pi k_z a}}{(2\pi)^2(k_x^2 + k_z^2)}$$

Now we need to perform inverse transforms to get back to spatial coordinates:

Solving the heat equation

Take inverse transform with respect to k_z:

$$T(k_x, z) = -\frac{1}{(2\pi)^2}\int_{-\infty}^{\infty}\frac{e^{i2\pi k_z(z+a)}}{(k_x^2 + k_z^2)}dk_z$$

$$= \frac{e^{-2\pi|k_x|(z+a)}}{(2\pi)\left|k_x\right|}$$

Take inverse transform with respect to k_x:

$$T(x,z) = -\int_{-\infty}^{\infty}\frac{e^{-2\pi\left|k_x\right|(z+a)}}{2\pi\left|k_x\right|}e^{i2\pi k_x x}dk_x$$

$$= -\frac{1}{2\pi}\left\{\ln\left[x^2 + (z+a)^2\right]^{\frac{1}{2}}\right\}$$

Solving the heat equation

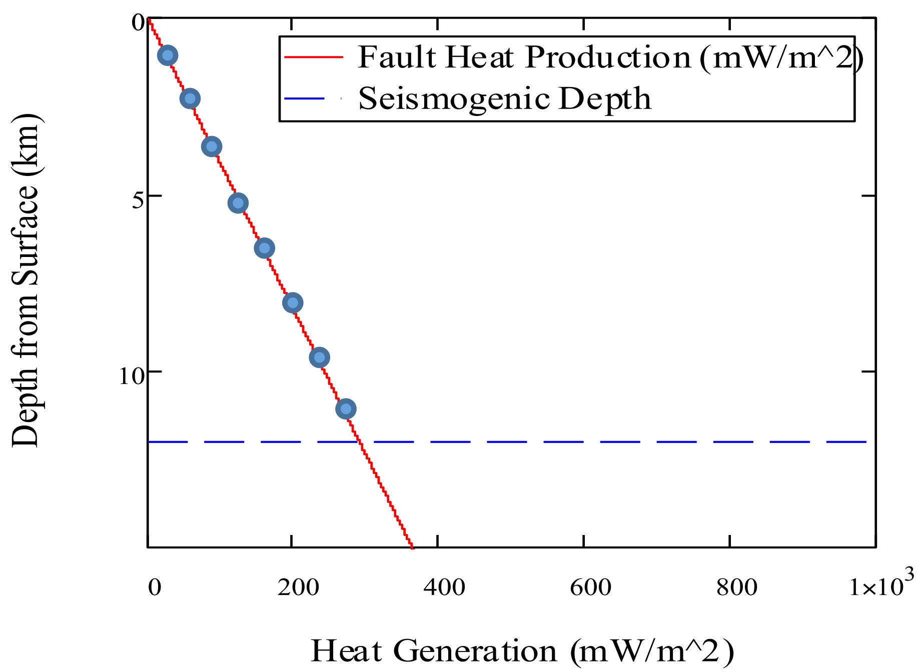
Use method of images to satisfy boundary condition:

$$T(x,z) = \frac{1}{2\pi}\frac{q_{source}}{k}\left\{\ln\left[x^2 + (z-a)^2\right]^{\frac{1}{2}} - \ln\left[x^2 + (z+a)^2\right]^{\frac{1}{2}}\right\}$$

This solution will provide the scalar temperature field from the line-source. We can use this to determine heat-flux at the surface.

Temperature Distribution

- Using solution for a line source, can solve for temperature distribution due to fault heat production
- First approximation: discretize into a few line sources



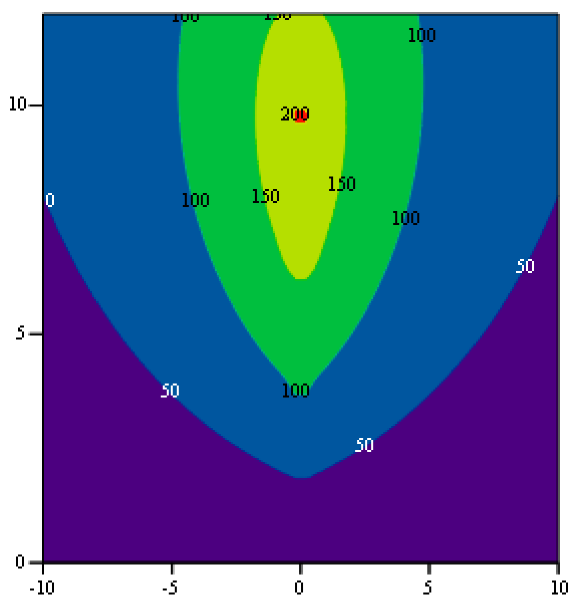
$$q^{i}_{source} = \int_{d_0}^{d_1} \tau(z)\,dz$$

Temperature is then the sum of temperature fields due to each line source:

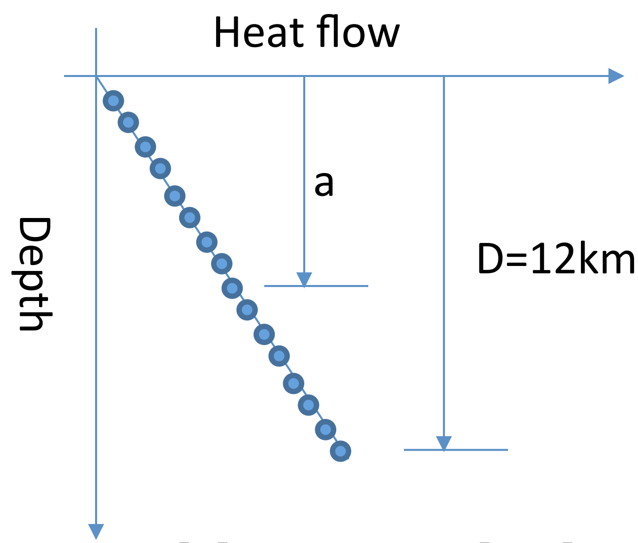
$$T_{tot}(x,z) := \sum_{i=1}^{n} T(x, z, a_i)$$

Temperature Distribution

- Temperature distribution from discretized sources:



- Sum of infinitesimally small line sources can be described by an integral

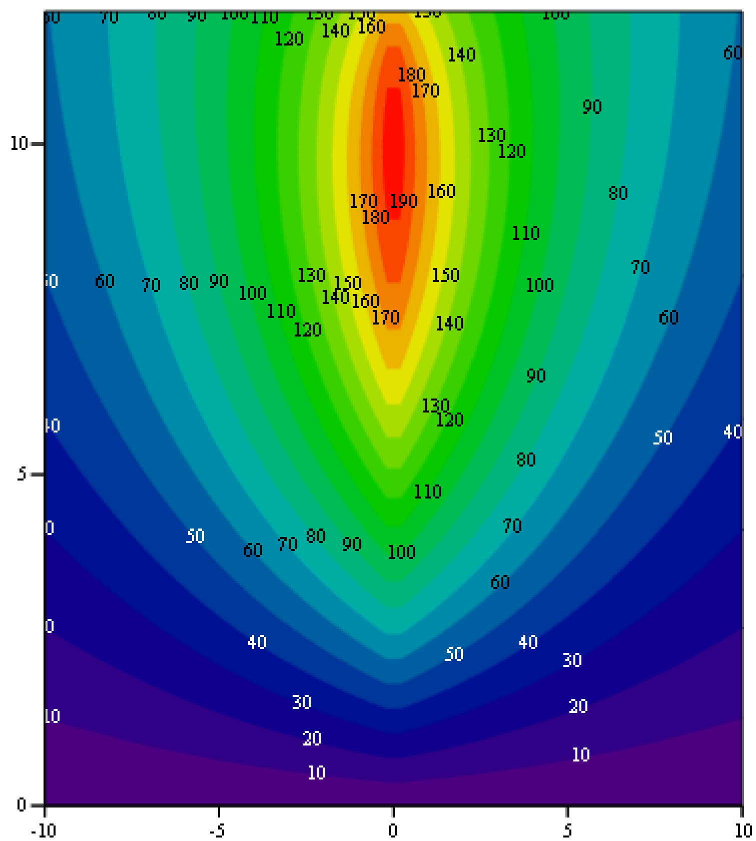


$$T_{gen}(a,x,z) := \frac{1}{2\pi}\cdot\frac{q(a)}{k}\cdot\left[\ln\left[\left[(x)^2+(z+a)^2\right]^{\frac{1}{2}}\right]-\ln\left[\left[(x)^2+(z-a)^2\right]^{\frac{1}{2}}\right]\right]$$

$$T_{tot}(x,z) := \int_0^D T_{gen}(a,x,z)\,da$$

Temperature Distribution

- Integral can be evaluated analytically or numerically:



- Highest temperatures are a few km above depth of the seismogenic zone
- Temperature map meets required boundary conditions:

$$T(x,0) = 0$$

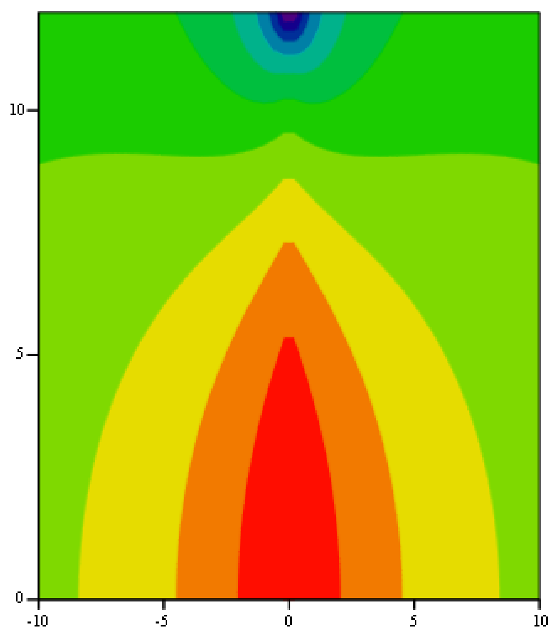
$$\lim_{|z|\to\infty} T(x,z) = 0$$

$$\lim_{|x|\to\infty} T(x,z) = 0$$

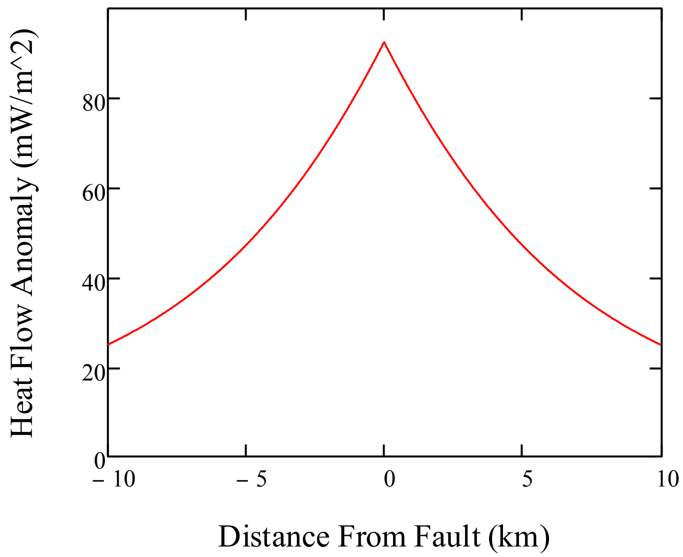
Surface Heat Flow

- Surface heat flow can be found using Fourier's law:

$$\vec{q} = -k\nabla T$$



z-direction



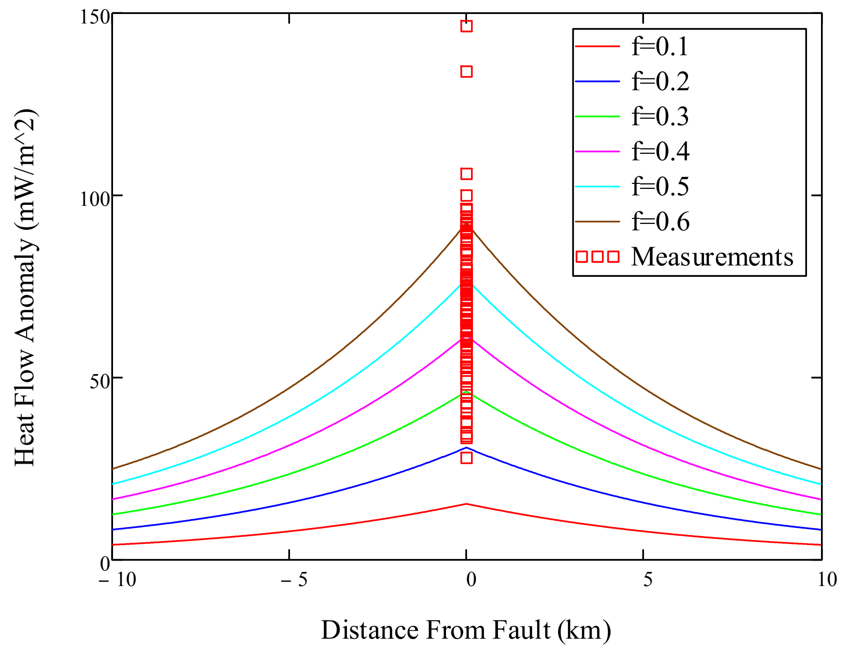
z-direction (surface, f=0.6)

Surface Heat Flow - Measurements

- Measurements of heat flow above San Andreas have been compiled by Lachenbruch and Sass (1980)
  - Measurements are from geographically large area
  - Study from 1973 found San Andreas is within a ~100km-wide band of high heat flow, but there is no thermal anomaly along main heat trace
  - Data have significant scatter: μ=69.8 mW/m^2 σ=19.8 mW/m^2
- Heat flow values are inconsistent with reasonable value of 0.6 assumed for f

Surface Heat Flow - Measurements

- Coefficient of friction of 0.4 is consistent with the mean of the data, 0.3 approaches 84th percentile of the data

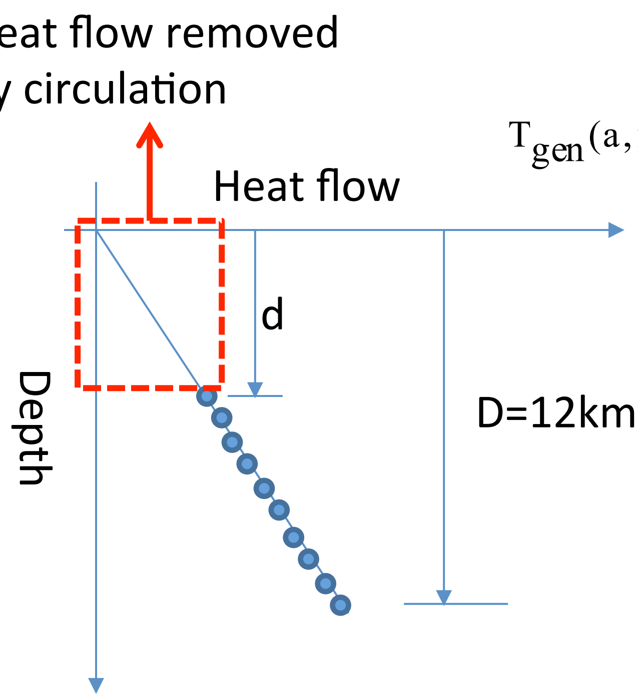


Surface Heat Flow - Discussion

- Turcotte et al (1980), using a more sophisticated model with brittle and plastic zones determined that a coefficient of friction of 0.05 would be consistent with no measurable heat flow anomaly
  - Coefficient of friction would have to be applicable to depths of 25 to 30 km
- Turcotte et al propose three possible explanations for this
  - Hydrostatic pressure is nearly equal to the lithostatic pressure (might give a coefficient of friction as low as 0.05, but this situation is deemed unlikely)
  - Proposed model is inaccurate – perhaps strain in fault system occurs over a broad region?
  - Measurements have failed to identify the heat flow anomaly (groundwater effects) – also dismissed as unlikely

Surface Heat Flow - Discussion

- Can determine whether hydrothermal circulation is even a plausible theory to explain lack of thermal anomaly by removing heat sources to depth d:

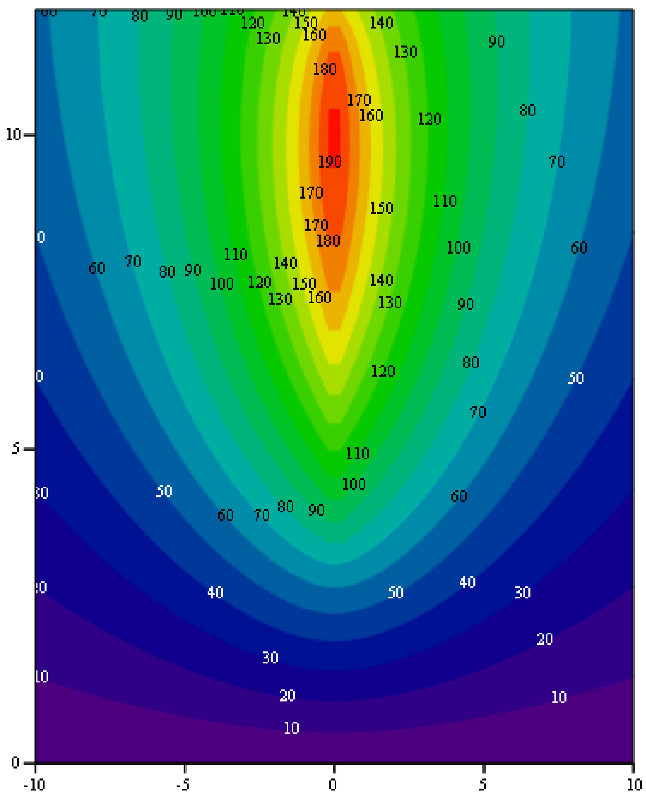


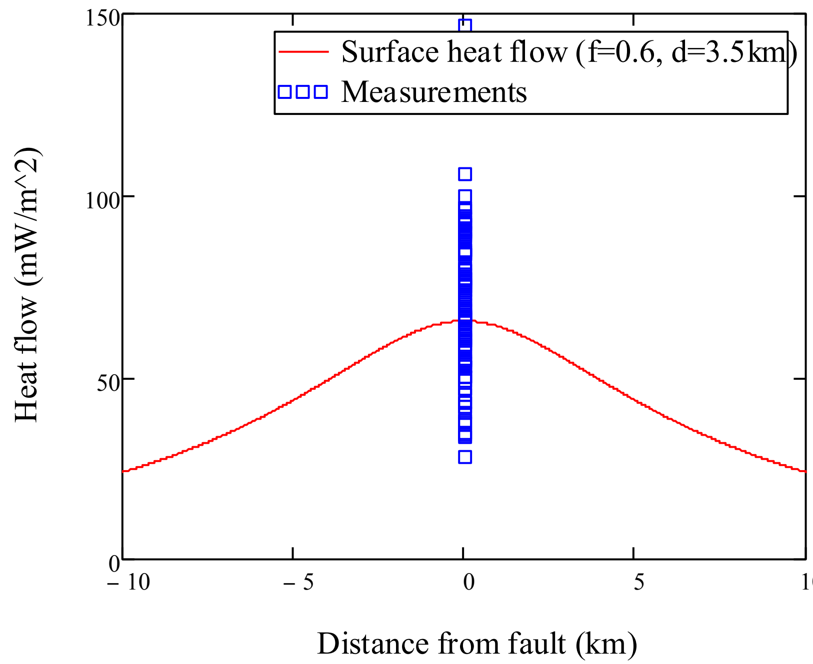
$$T_{gen}(a,x,z) := \frac{1}{2\pi}\cdot\frac{q(a)}{k}\cdot\left[\ln\left[\left[(x)^2+(z+a)^2\right]^{\frac{1}{2}}\right]-\ln\left[\left[(x)^2+(z-a)^2\right]^{\frac{1}{2}}\right]\right]$$

$$T_{tot}(x,z) := \int_0^D T_{gen}(a,x,z)\,da$$

Surface Heat Flow - Discussion

- Temperature and vertical heat flow calculations indicate match to measured values is possible with a depth of heat flow "deletion" to about 3.5 km with f=0.6:





Conclusions

- Solution to infinite plane source of heat with functional dependence on z is possible using solution to line source
- Based on reasonable assumptions regarding coefficient of friction, shear stress on the fault, and plate velocities, San Andreas should produce a measureable heat anomaly. However, no such anomaly has been measured.
- Possible explanations include:
  - High hydrostatic stresses which reduce the coefficient of friction
  - Hydrothermal effects which dilute or spread the heat anomaly
  - Model errors
- Based on the simple calculations in this presentation, low coefficient of friction and hydrothermal circulation appear to be plausible explanations for the lack of a measured heat anomaly