Heat Flow Across the San Andreas Fault

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Calculating Shear Stress on the San Andreas Fault

\[ \tau(z) = f \sigma_n \]

Where \( \sigma_n = \rho_c g z \)

| \( \tau(z) \) | shear stress on a locked fault |
| \( \sigma_n \) | normal force |
| \( f \) | coefficient of static friction |
| \( \rho_c \) | crustal density |
| \( g \) | acceleration due to gravity |
| \( z \) | depth of seismogenic zone |
Calculating Shear Stress on the San Andreas Fault

San Andreas Fault Assumptions

\[ f = 0.6 \]
\[ \rho_c = 2600 \text{ kg m}^{-3} \]
\[ g = 9.8 \text{ m s}^{-2} \]
\[ z = 12 \text{ km} \]

\[ \tau(z) = 183 \text{ MPa} \]

PROBLEM – Average stress drop on a fault during an actual earthquake \( \leq 10 \text{ MPa} \)
Calculating Shear Stress on the San Andreas Fault

Considerations

Coefficients of Static Friction

\( f \) varies with depth

Pore fluid pressure

\[
\sigma = \sigma_n - \sigma_{\text{pore fluid pressure}} \quad \text{so} \quad \tau(z) = f (\rho_c - \rho_w)g z
\]
Calculating Shear Stress

Shear Stress on the San Andreas Fault

- Ignoring pore fluid pressure
- Including pore fluid pressure
- Average stress drop during an earthquake

Shear Stress (MPa) vs Coefficient of static friction
Derivation of a Line Source of Heat to get Heat Flow Anomaly

• We want to model the heat flow off of the San Andreas Fault as a line source of heat that dissipates as one moves away from the fault

• We need to get an equation that can be used to calculate the heat flow anomaly for different coefficients of static friction given an equation for a temperature anomaly

• Method
  • Start with an equation for a temperature anomaly and take a Fourier Transform
  • Take the inverse Fourier Transform in two dimensions to receive an easier temperature function to work with
  • Use the relationship between temperature and heat flow to get our heat flow anomaly equation
Derivation of Heat Flow from a Line Source

Given:

$$\nabla^2 T = \frac{1}{k} Q(x, z) = \frac{1}{k} \delta(x) \delta(z + a)$$

$$\nabla^2 T = \frac{d^2 T}{dx^2} + \frac{d^2 T}{dz^2}$$

$$T(x, 0) = 0$$

$$\lim_{|z| \to \infty} T(x, z) = 0$$

$$\lim_{|x| \to \infty} T(x, z) = 0$$

T = Temperature Anomaly  
k = Thermal Conductivity = 3.3W/mK  
Q = Heat Generation
Step 1: Take Fourier Transform with Respect to x and z

\[
\frac{d^2 T}{dx^2} + \frac{d^2 T}{dz^2} = \frac{1}{k} \delta(x) \delta(z + a)
\]

- Use the derivative property of Fourier Transforms:
  \[\mathcal{F}\left[\frac{\partial f}{\partial x}\right] = i2\pi k F(k)\]

\[
[(i2\pi k_x)^2 + (i2\pi k_z)^2] [T(k_x, k_z)] = f_t(\delta(x) \delta(z + a))
\]

- Use property for the transform of a delta function:
  \[F[\delta(x - x_0)] = e^{-i2\pi f(x_0)}\]

\[
4\pi^2 (k_x^2 + k_z^2) T(k_x, k_z) = \int_{-\infty}^{\infty} \delta(x) e^{i2\pi k_x x} dk_x \int_{-\infty}^{\infty} \delta(z + a) e^{i2\pi k_z (z + a)} dk_z
\]

- Which when simplified equals:
  \[-4\pi^2 (k_x^2 + k_z^2) T(k_x, k_z) = 1 \cdot e^{-i2\pi k_z a}\]

\[
T(k_x, k_z) = \frac{e^{-i2\pi k_z a}}{-4\pi^2 (k_x^2 + k_z^2)}
\]

- Rearranging for T yields:
Step 2: Take the Inverse Fourier Transform with Respect to $k_z$

$$ T(k_x, k_z) = \frac{e^{-i2\pi k_z a}}{-4\pi^2 (k_x^2 + k_z^2)} $$

Note that: $(k_x^2 + k_z^2) = (k_z + ik_x)(k_z - ik_x)$

$$ T(k_x, z) = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} \frac{e^{i2\pi k_z(z+a)}}{(k_z + ik_x)(k_z - ik_x)} dk_z $$
Cauchy Residue Theorem

- If a function is analytic, integrating around complex poles on a closed loop will equal zero

\[ \oint f(z)\,dz = 0 \]

- If we have complex poles in the denominator of the function, the Cauchy Residue theorem states

\[ \oint \frac{f(z)}{z - z_0} = i2\pi \, f(z_0) \]
Step 2: Take the Inverse Fourier Transform with Respect to $k_z$

$$ T(k_x, z) = \frac{-1}{4\pi^2} \int_{-\infty}^{\infty} e^{i2\pi k_z(z+a)} \frac{e^{ik_xz}}{(k_z + ik_x)(k_z - ik_x)} dk_z $$

$$ \int \frac{f(z)}{z - z_0} = i2\pi f(z_0) $$

If $k_z = ik_x$, considering $k_x > 0$ and $z > -a$

$$ T(k_x, z) = (i2\pi) \frac{-1 e^{i2\pi k_x(z+a)}}{4\pi^2 (ik_x + ik_x)} $$

$$ T(k_x, z) = \frac{-1 e^{-2\pi k_x(z+a)}}{4\pi k_x} $$

If $k_z = -ik_x$, considering $k_x < 0$ and $z > -a$

$$ T(k_x, z) = (i2\pi) \frac{-1 e^{-i2\pi k_x(z+a)}}{4\pi^2 (-ik_x - ik_x)} $$

$$ T(k_x, z) = \frac{1 e^{2\pi k_x(z+a)}}{4\pi k_x} $$

Combine the last two equations using the absolute value of $k_x$

$$ T(k_x, z) = \frac{-1 e^{-2\pi |k_x|(z+a)}}{4\pi \frac{|k_x|}{k_x}} $$
Step 3: Take the Inverse Fourier Transform with Respect to $k_x$

$$T(x, z) = \frac{-1}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-2\pi |k_x|(z+a)}}{|k_x|} e^{i2\pi k_xx} \, dk_x$$

- Using the derivative property of Fourier Transforms

$$\frac{\partial T(x, z)}{\partial z} = i2\pi k_z \frac{-1}{4\pi} \int_{-\infty}^{\infty} \frac{e^{-2\pi |k_x|(z+a)}}{|k_x|} e^{i2\pi k_xx} \, dk_x$$

$$\frac{\partial T(x, z)}{\partial z} = \frac{1}{2} \int_{-\infty}^{\infty} e^{-2\pi |k_x|(z+a)} e^{i2\pi k_xx} \, dk_x$$

$$\frac{\partial T(x, z)}{\partial z} = \frac{-(z + a)}{4\pi[x^2 + (z + a)^2]}$$
Step 3: Take the Inverse Fourier Transform with Respect to $k_x$

• Integrating with respect to $z$

$$T(x, z) = \frac{-1}{4\pi} \ln \left( \frac{1}{[x^2 + (z + a)^2]^\frac{1}{2}} \right)$$

• Because this does not satisfy our boundary conditions, we must add a heat sink at $z = a$, giving us our final equation that satisfies all boundary conditions.

$$T(x, z) = \frac{-1}{4\pi} \left[ \ln \left( \frac{1}{[x^2 + (z + a)^2]^\frac{1}{2}} \right) - \ln \left( \frac{1}{[x^2 + (z - a)^2]^\frac{1}{2}} \right) \right]$$

$$T(x, z) = \frac{-1}{4\pi} \ln \left( \frac{(x^2 + (z + a)^2)}{(x^2 + (z - a)^2)} \right)^\frac{1}{2}$$
Step 4, take our temperature anomaly and find heat flow

- We have temperature in terms of $z$ and $x$, now we need to find the heat flow $q$ by integrating with respect to $a$

$$q(x, z, a) = -k \frac{\partial T}{\partial z} = \frac{1}{4\pi} \left( \frac{z + a}{x^2 + a^2} - \frac{z - a}{x^2 + a^2} \right)$$

- To satisfy our boundary conditions, we need to set $z$ equal to zero, thus we get

$$q(x, a) = \frac{1}{4\pi} \frac{2a}{x^2 + a^2}$$
Aside: Green’s Function

• Now we must use a Green’s function to evaluate heat flow over our heat source by using the following relation:

\[ q(x) = \int q(x, a)f(a)da \]

• For us, \( f(a) \) is the heat generated due to frictional heating, which we also have:

\[ f(a) = v \cdot \tau = v\mu \rho_c ga \]

• Where \( v \) is the mean velocity of the plate: San Andreas Fault: 35mm/year
Step 4, take our temperature anomaly and find heat flow

- Substitute $q(x,a)$ and $f(a)$ into our equation for $q(x)$ and get the following:

$$q(x) = \frac{2\nu\mu\rho_c g}{4\pi} \int_{a_1}^{a_2} \frac{a^2}{x^2 + a^2} \, da$$

- Finally, use the integral identity for an arc tangent function to get the final expected surface heat flow anomaly!

$$q(x) = \frac{2\nu\mu\rho_c g}{4\pi} \left[ a - x\tan^{-1}\left(\frac{a}{x}\right) \right] \bigg|_{a_1}^{a_2}$$
Observed Heat Flow Measurements

Mean Surface Heat Flow
- 73 mW m$^{-2}$

Average Maximum Anomaly
– 94 mW m$^{-2}$

Observed heat flow
By Lachenbruch & Sass (1980)
Expected Heat Flow Anomaly

\[ \rho_c = 2600 \text{ kg m}^{-3} \]
\[ g = 9.8 \text{ m s}^{-2} \]
\[ a = 12 \text{ km} \]
\[ v = 35 \text{ mm/year} \]
Expected Heat Flow Anomaly (including pore fluid pressure)

\[ \rho = 1600 \text{ kg m}^{-3} \]
\[ g = 9.8 \text{ m s}^{-2} \]
\[ a = 12 \text{ km} \]
\[ v = 35 \text{ mm/year} \]
Expected Heat Flow Anomaly (including hydrothermal circulation)

\[ \rho_c = 2600 \text{ kg m}^{-3} \]
\[ g = 9.8 \text{ m s}^{-2} \]
\[ a = 4 - 12 \text{ km} \]
\[ v = 35 \text{ mm/year} \]
Discussion

• No evidence for significant hydrothermal circulation along the San Andreas (Lachenbruch & Sass 1980, Turcotte et al 1980)

• Error may be associated with coefficient of static friction
  • Evidence for talc found in serpentinite could account for low $\mu$ for creeping portions of SAF (Moore & Rymer, 2007)

• Still a large debate as to whether the San Andreas Fault is a strong or weak fault (Saffer et al 2003)
References


