Modeling Marine Magnetic Anomalies

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SIO 234: Geodynamics

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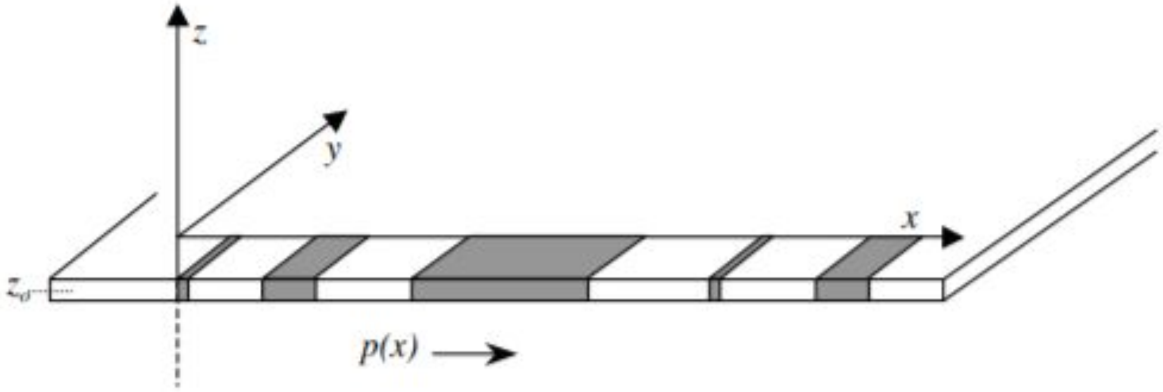
Objectives

- Derive magnetic anomaly model in relation to the distance from a spreading ridge axis

- Plot observed data and find a best fit model for magnetic anomalies at the Pacific Antarctic Rise and Mid Atlantic Ridge

Assumptions

- Constant spreading rate (m/my)
- Symmetry spreading about the spreading axis
- Constant ocean depth



Magnetics Methodology

- Total field magnetometer which records: the earth's magnetic field $\boldsymbol{B}$ and the magnetization $\boldsymbol{M}$ of earth materials.

- $\boldsymbol{M} = M_I + M_R = \chi H + M_R$
- Measured Magnetics (nT)= $\boldsymbol{M} + \boldsymbol{B} - \boldsymbol{B}$
  Anomalies

- Varies from ~30,000nT at equator to ~60,000nT at the poles
- Typical magnetic anomalies vary by ~300nT
- Large difference in intensity allows for some simplification as ΔB

Magnetic Anomaly Model Derivation

- $A(k, z) = C\mu_{\circ} 2\pi|k|e^{-2\pi[k]z}e^{i\theta sgn(k)}p(k)$

- A = scalar magnetic anomaly
- C = constant
- $\mu_{\circ}$ = magnetic permeability (4π E-7)
- k = wavenumber ($1/\lambda$)
- θ = skewness
- z = depth (m)
- $p(k)$ = Fourier Transform of magnetic timescale

Note:

- $sgn(k) = \begin{cases} 1 \text{ if } x > 0 \\ 0 \text{ if } x = 0 \\ -1 \text{ if } x < 0 \end{cases}$
- $k = -(\frac{nx}{2} : \frac{nx}{2} - 1)/L = (\frac{nx}{2} : \frac{nx}{2} - 1)/v \cdot dt$

Magnetics Derivation

- $\vec{B} = B_E + \Delta B$
- Scalar Magnetometer: $|\vec{B}|$=($|\vec{B}_E|^2 + 2B_E \cdot \Delta B + |\Delta B|^2$)^(1/2)
- $|\vec{B}|$=$|\vec{B}_E|(1 + \frac{2\Delta B \cdot \vec{B}_E}{|\vec{B}_E|^2}) \cong |\vec{B}_E|(1 + \frac{\Delta B \cdot \vec{B}_E}{|\vec{B}_E|^2})$
- Large difference in intensity allows for some simplification as $\Delta B^{(1/2)} \cong 0$ so:
- $A = |\vec{B}| - |\vec{B}_E| \equiv \frac{\boldsymbol{\Delta B \cdot \vec{B}_E}}{\boldsymbol{\vec{B}_E}}$
- Where A is a scalar magnetic anomaly.

Scalar Potential and Magnetization

- Magnetic anomaly (ΔB) is the negative gradient of the magnetic potential ($-\nabla U$).
- $\boldsymbol{\Delta B} = -\boldsymbol{\nabla U}$
- Where $\nabla U^{\,2} = 0$ where $z \neq z_o$; and $\nabla U^{\,2} = \mu_o \nabla \cdot \vec{M}$ where $z = z_o$
- Scalar potential $U(x, y, z)$; Magnetization $\vec{M}(x, y, z)$
- $\vec{M}(x, y, z) = (M_x \hat{\imath} + M_y \hat{\jmath} + M_z \hat{k}) \cdot p(k) \cdot \delta(z - z_o)$
- The potential satisfies Poisson's equation within the source layer and Laplace's equation above it.

- Magnetic source does not vary with the y direction (parallel to the ridge so we can cancel this term:

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial y^2}+\frac{\partial^2 U}{\partial z^2}=\mu_o\left[\frac{\partial}{\partial x}M_x p(x)\,\delta(z-z_o)+\cancel{\frac{\partial}{\partial y}M_y p(x)\,\delta(z-z_o)}+\frac{\partial}{\partial z}M_z p(x)\,\delta(z-z_o)\right]$$

- So this simplifies down to:

$$\frac{\partial^2 U}{\partial x^2}+\frac{\partial^2 U}{\partial z^2}=\mu_o\left[\frac{\partial}{\partial x}M_x p(x)\,\delta(z-z_o)+\frac{\partial}{\partial z}M_z p(x)\,\delta(z-z_o)\right]$$

- Next: apply boundary conditions to perform the Fourier Transform:
- $\lim_{x\to\infty} U(x)=0$ and $\lim\limits_{z\to\infty} U(z)=0$

Basic Double Fourier Transform:

- $F(k) = \iint_{-\infty}^{+\infty} f(x) e^{(-i2\pi(k\cdot x))} d^2 x$ OR: $F(k) = \mathfrak{I}_2[f(x)]$

- <u>Applied</u>

➢ in x-direction: $\mathfrak{I}_2[dU/dx] = \mathbf{i2}\boldsymbol{\pi k_x} \cdot \boldsymbol{\mathfrak{I}_2}\,[\boldsymbol{U}]$

➢ in z-direction: $\int_{-\infty}^{+\infty} \delta(z - z_o) \cdot e^{(-i2\pi k_z z)} dz \equiv \boldsymbol{e^{(-i2\pi k_z z_o)}}$

- FT result: $-[(i2\pi k_z)^2 + (i2\pi k_x)^2] \cdot U(k_x, k_z) = \mu_\circ p(k_x) e^{(-i2\pi k_z z_o)} \cdot (i2\pi \boldsymbol{k} \cdot \boldsymbol{M})$

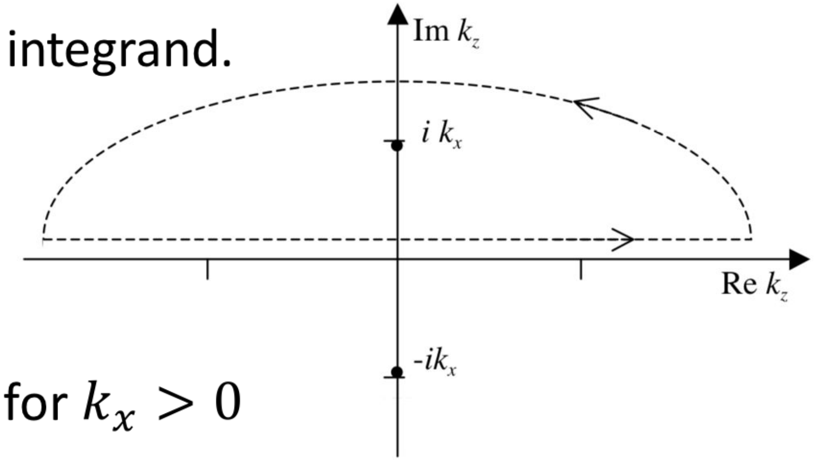
- Solve for $U(k_x, k_z)$: $\frac{-i\mu_o}{2\pi} p(k_x)(\overrightarrow{k \cdot} \vec{M}) \frac{e^{(-i2\pi k_z z_o)}}{({k_x}^2+{k_z}^2)}$

- Inverse FT (Cauchy Residue Thrm):

$$U(k_x, k_z) = \frac{\mu_o}{2\pi i} p(k_x) \int_{-\infty}^{+\infty} \frac{(\overrightarrow{k \cdot} \vec{M}) \cdot e^{(i2\pi k_z (z-z_o))}}{({k_x}^2 + {k_z}^2)} dk_z$$

Cauchy Residue Theorem

- To solve integral, calculate the poles of the integrand.
- Factor: $k_x{}^2 + k_z{}^2 = (k_z + ik_x)(k_z - ik_x)$



- Solve over closed loops:
- $\oint(\quad)dkz = \frac{e^{(-i2\pi k_x(z-z_o))}}{(2ik_x)}(k_z M_x + ik_x M_{z)}$ for $k_x > 0$
- $\oint(\quad)dkz = \frac{e^{(+i2\pi k_x(z-z_o))}}{(2ik_x)}(k_z M_x - ik_x M_{z)}$ for $k_x < 0$

Cauchy Residue Theorem (cont’d)

- Combine integrands and drop k subscripts:
- $U(k, z\ ) = \frac{\mu_o}{2} p(\mathrm{k}) \cdot e^{(-2\pi|k|(z-z_o)}\ (M_z - i\,\frac{k}{|k|} M_x)$
- To further simplify assume spreading ridge is located at Earth’s magnetic pole, the dipolar field lines will be parrallel to z-axis…thus, no x-component!
- $U(k, z\ ) = \frac{\mu_o M_z}{2} p(\mathrm{k}) \cdot e^{(-2\pi|k|(z-z_o)}$

Calculate Magnetic Anomaly:

- Recall: $\Delta B = -\nabla U$
- Substitute values evaluated at U:
- $\Delta B = (-\mathrm{i}2\pi k, 2\pi|k|)\mathrm{U}(\mathrm{k}, \mathrm{z})$
- Recall: $A = \left|\vec{B}\right| - \left|\vec{B}_E\right| \equiv \frac{\Delta B \cdot \vec{B}_E}{\vec{B}_E}$
- Since only the z-component of earth's magnetic field is nonzero due to our assumptions, the anomaly simplifies down to:
- $A(k, z) = \frac{\mu_o M_z}{2} p(\mathrm{k}) \cdot e^{(-2\pi|k|(z-z_o))}$ →
- $\boldsymbol{A(k, z) = C\mu_{\circ} 2\pi|k| e^{-2\pi[k]z} e^{i\theta sgn(k)} p(k)}$

Matlab Code – initial constants

```
% halfrate in m/my; dt is in my, L =v*dt is in meters, z = 3000 m

  v = 50000.;

  dt = 20;

  L = v*dt;

  z = 3000.;

  theta = 0.* pi / 180.;

  theta = 0.* pi / 180.;

  nx = 2048;

  dx = L/(nx-1);

  nx2 = nx/2;

C=1;

mu_o= (4*pi)*10^(-7);
```

Matlab Code

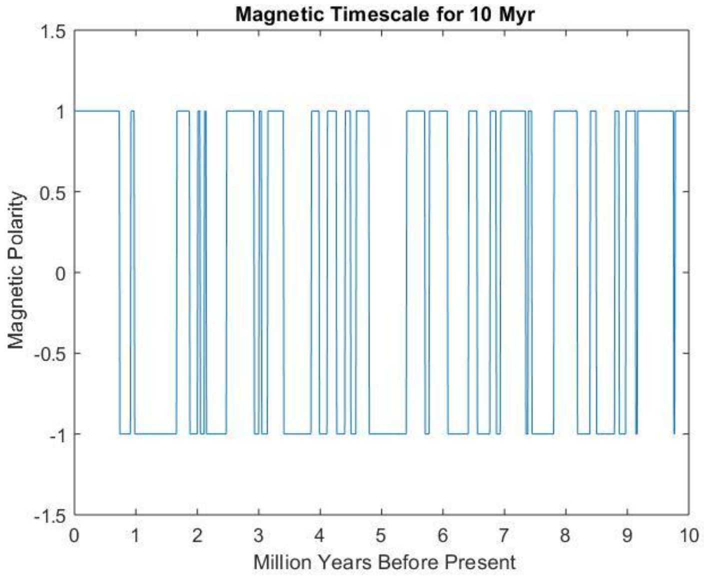
```
%reflect the magnetic timescale about the spreading ridge axis

[ndat,mdat]=size(t);

  mt =-fliplr(t(2:ndat)')';

  mp = fliplr(p(2:ndat)')';

  time=[mt;t];

  polarity=[mp;p];
```

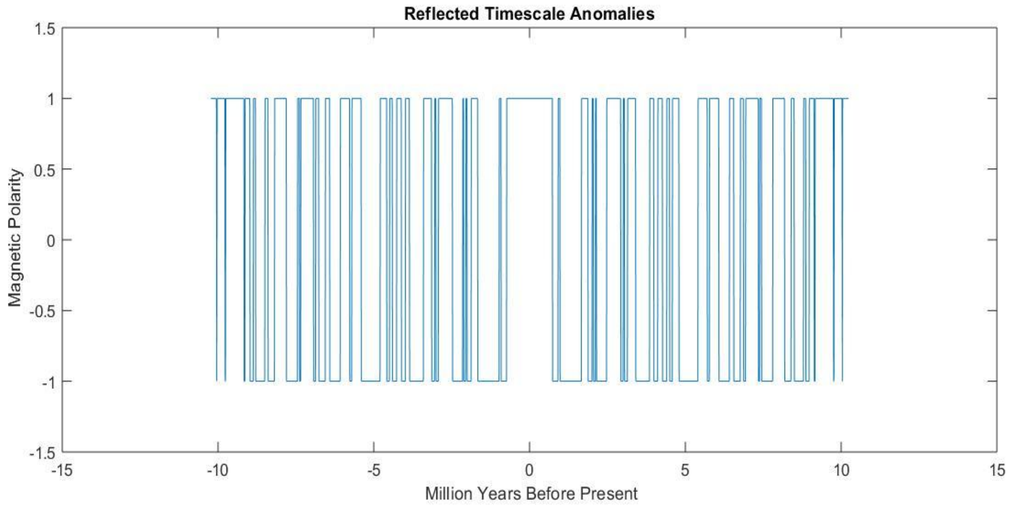
```
% subset of timescale for near ridge magnetics

  [ndat,mdat] = size(time)

nmid = ndat/2;

ntime=2048;

  ntime2=ntime/2;

  stime = time((nmid-ntime2+1):(nmid+ntime2),1);

  spolr =
polarity((nmid-ntime2+1):(nmid+ntime2),1);
```

Matlab Code

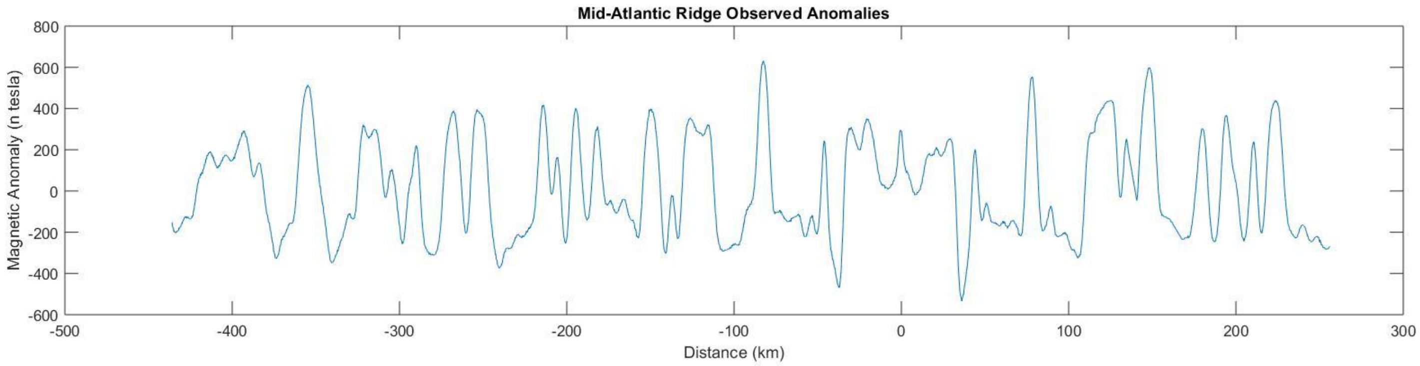
```
%create model anomalies

%Fourier transform of geomagnetic timescale p(k)

fourier_p=fftshift(fft(spolr));


%calculate Fourier transform of magnetic anomalies

A=abs(k).*exp(abs(k).*z*-2*pi).*exp(sign(k).*1i*theta).*fourier_p*(C*mu_o*2*pi);


%overlay observed and model

AA=fftshift(ifft(fftshift(A)));

plot(sdist_PAR,smagobsPAR,stime,AA);
```

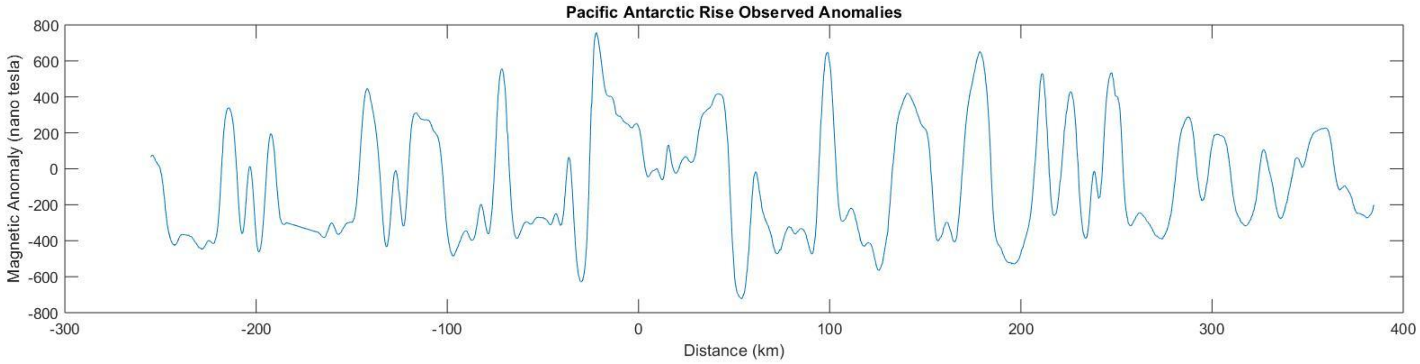
Magnetic Timescale



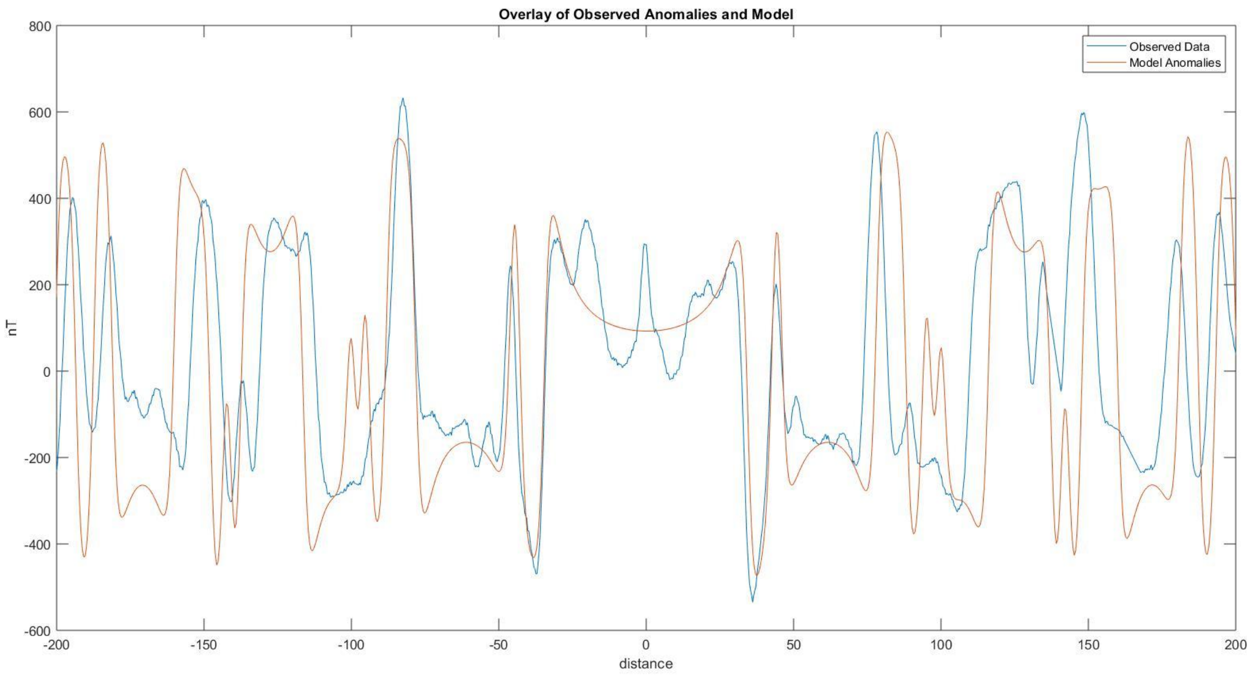


Observed Anomalies – near spreading ridge





Model Fitting – Pacific Antarctic Rise



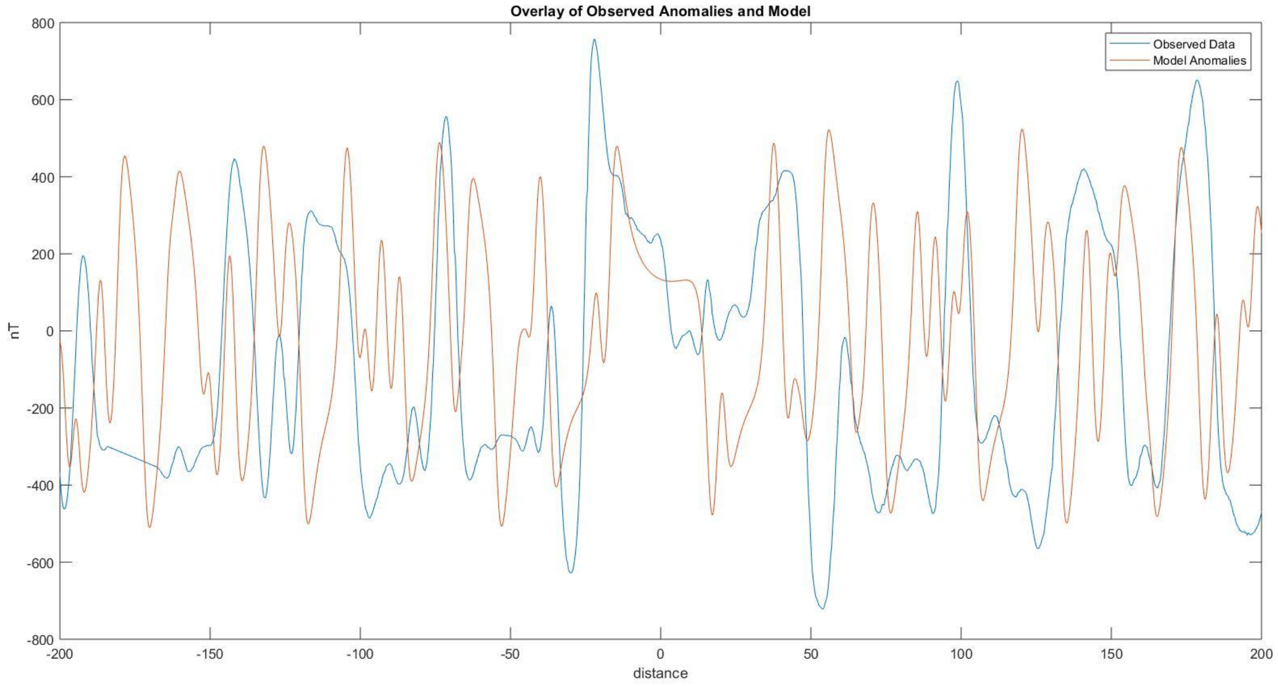
Skewness = 5 degrees

Ocean Depth = 3000 m

Spreading rate = 47,000 m/my

Constant = 2.75*10^12

Model Fitting – Mid Atlantic Ridge



Skewness = 40 degrees

Ocean Depth = 3000 m

Spreading rate = 22,000 m/my

Constant = 2.5*10^12

Conclusion

Problems fitting the data:

It was difficult finding an optimal combination of spreading rate and skewness that best fit the anomalies.

The possibility that different combinations of variables result in a model that fits the data well creates a non-unique solution.

Possibly a better model would involve changing these parameters with time.

Spreading rate range estimates and skewness:

PAR: 45,000-47,000 m/my, 5-10 degrees

MAR: 22,500-23,500 m/my, 30-40 degrees

Acknowledgements

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