Overview

- Pacific plate is subducting under Japan.
- This is some of the oldest oceanic crust in the world
- Old Crust = Cold and Strong

http://2.bp.blogspot.com/-jjZfHSZ9MwU/UQCekP1UDql/AAAAAAAAADLo/2xlmv28FAbE/s1600/quake,+pacific+plate.jpg
Trenches Around the Pacific Plate

http://www.shorstmeyer.com/msj/geo130/slideShows/RingofFire.gif
Problem Intro

ON THE APPLICABILITY OF A UNIVERSAL ELASTIC TRENCH PROFILE

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The paper that the problem is based off of, notice last author.
More Intro

- Trying to get the same model for various plates
- Modern day research is going on now in this area (Garcia and Peterson, lecture Nov 21)
- 40 years later, still unanswered questions
- Reproduce figure 3
Derivation

\[ D \frac{d^4 w}{dx^4} + S \frac{d^2 w}{dx^2} + kw = 0 \]  \hspace{2cm} (1)

\[ w = A \sin \left[ \frac{x}{\alpha} \left( 1 + \epsilon \right)^{\frac{1}{2}} \right] \exp \left[ -\frac{x}{\alpha} \left( 1 - \epsilon \right)^{\frac{1}{2}} \right] \]  \hspace{2cm} (2)
Derivation

Boundary Conditions:

\[ x \to \infty, w = 0 \quad x = 0, w = 0 \]

Educated Guess:

\[ w = Ce^{\gamma x} \]
Derivation

\[ \frac{d^2 w}{dx^2} = C \gamma^2 e^{\gamma x} \]

\[ \frac{d^4 w}{dx^4} = C \gamma^4 e^{\gamma x} \]

\[ DC \gamma^4 e^{\gamma x} + SC \gamma^2 e^{\gamma x} + kCe^{\gamma x} = 0 \]

\[ D \gamma^4 + S \gamma^2 + k = 0 \]

\[ \gamma^4 + \frac{S}{D} \gamma^2 + \frac{k}{D} = 0 \]
Substitutions

\[ \alpha^4 = \frac{k}{D} \]

\[ \epsilon = \frac{S}{2(Dk)^{\frac{1}{2}}} \]

\[ \frac{k}{D} = \frac{4}{\alpha^4} \]

\[ \frac{S}{D} = \frac{4\epsilon}{\alpha^2} \]

\[ \gamma^4 + \frac{4\epsilon}{\alpha^2} \gamma^2 + \frac{4}{\alpha^4} = 0 \]
Quadratic Formula

\[ \gamma^2 = \frac{-4\epsilon \alpha^{-2} \pm \sqrt{(4\epsilon \alpha^{-2})^2 - 4(4\alpha^{-4})}}{2} \]

\[ \gamma^2 = \frac{-4\epsilon \alpha^{-2} \pm \sqrt{(\epsilon^2 - 1)(16\alpha^{-4})}}{2} \]

\[ \gamma^2 = 2\alpha^{-2} \left[ -\epsilon \pm (\epsilon^2 - 1)^{\frac{1}{2}} \right] \]
Quadratic Formula

\[ \gamma^2 = 2\alpha^{-2} \left[ -\epsilon \pm i \left( 1 - \epsilon^2 \right)^{\frac{1}{2}} \right] \]

\[ \gamma^2 = 2\alpha^{-2} \left[ \left( \frac{1 - \epsilon}{2} \right)^{\frac{1}{2}} \pm i \left( \frac{1 + \epsilon}{2} \right)^{\frac{1}{2}} \right]^2 \]

\[ \gamma^2 = \alpha^{-2} \left[ (1 - \epsilon)^{\frac{1}{2}} \pm i (1 + \epsilon)^{\frac{1}{2}} \right]^2 \]

\[ \gamma = \pm \alpha^{-1} \left[ (1 - \epsilon)^{\frac{1}{2}} \pm i (1 + \epsilon)^{\frac{1}{2}} \right] \]
Solving for \( w \)

\[
w = Ce^{\gamma x}
\]

\[
w = C \exp \left( \pm \frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} \pm i (1 + \epsilon)^{\frac{1}{2}} \right] \right)
\]

\[
w = C_1 \exp \left( \frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} + i (1 + \epsilon)^{\frac{1}{2}} \right] \right) + C_2 \exp \left( \frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} - i (1 + \epsilon)^{\frac{1}{2}} \right] \right) + C_3 \exp \left( -\frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} + i (1 + \epsilon)^{\frac{1}{2}} \right] \right) + C_4 \exp \left( -\frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} - i (1 + \epsilon)^{\frac{1}{2}} \right] \right)
\]
Applying Boundary Conditions

\[ BC1 \Rightarrow C_1 = C_2 = 0 \]

\[ w = C_3 \exp \left( -\frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} + i (1 + \epsilon)^{\frac{1}{2}} \right] \right) + C_4 \exp \left( -\frac{x}{\alpha} \left[ (1 - \epsilon)^{\frac{1}{2}} - i (1 + \epsilon)^{\frac{1}{2}} \right] \right) \]

\[ w = \exp \left[ -\frac{x}{\alpha} (1 - \epsilon)^{\frac{1}{2}} \right] \left( C_3 \exp \left[ i \frac{x}{\alpha} (1 + \epsilon)^{\frac{1}{2}} \right] + C_4 \exp \left[ -i \frac{x}{\alpha} (1 + \epsilon)^{\frac{1}{2}} \right] \right) \]
Applying Boundary Conditions

\[ BC2 \Rightarrow C_3 = -C_4 = A \]

\[
\begin{align*}
  w &= A \exp \left[ -\frac{x}{\alpha} \left( 1 - \epsilon \right)^{\frac{1}{2}} \right] 
         \left( \exp \left[ i \frac{x}{\alpha} \left( 1 + \epsilon \right)^{\frac{1}{2}} \right] - \exp \left[ -i \frac{x}{\alpha} \left( 1 + \epsilon \right)^{\frac{1}{2}} \right] \right) \\
  &= A \exp \left[ -\frac{x}{\alpha} \left( 1 - \epsilon \right)^{\frac{1}{2}} \right] \sin \left[ \frac{x}{\alpha} \left( 1 + \epsilon \right)^{\frac{1}{2}} \right]
\end{align*}
\]
Fig. 3. Graphs of the non-dimensional deflection of a thin elastic plate and the associated non-dimensional bending moment and shear force.

\[ x_b = \frac{\pi \alpha}{4} = \frac{\pi}{4} \left( \frac{4D}{k} \right)^{\frac{1}{2}} \]

\[ \ddot{w} = 2^\frac{1}{2} \sin \left( \frac{\pi x}{4} \right) \exp \left[ \frac{\pi}{4} \left( 1 - x^2 \right) \right] \]
Figure 3

\[ \bar{M} = \frac{2^{\frac{3}{2}} \pi^2}{8} \cos\left(\frac{\pi \bar{x}}{4}\right) \exp\left(\frac{\pi}{4} (1 - \bar{x})\right) \]  

(10)
\[ \bar{Q} = -\frac{2^{\frac{1}{2}} \pi^3}{32} \left[ \cos\left(\frac{\pi \bar{x}}{4}\right) + \sin\left(\frac{\pi \bar{x}}{4}\right) \right] \exp\left[\frac{\pi}{4}(1 - \bar{x})\right] \] (11)
Some Real Data

The Peru Chile Trench

http://www.shorstmeyer.com/msj/geo130/slide_shows/RingofFire.gif
Conclusions of The Paper

- The model and the data fit well
- This paper is only valid for this trench
- Other trenches might have stronger/older plates
- Knowing the exact forces on plates is hard
- Horizontal forces *waves hand*