LAKE LOADING FLEXURE

SIO 234

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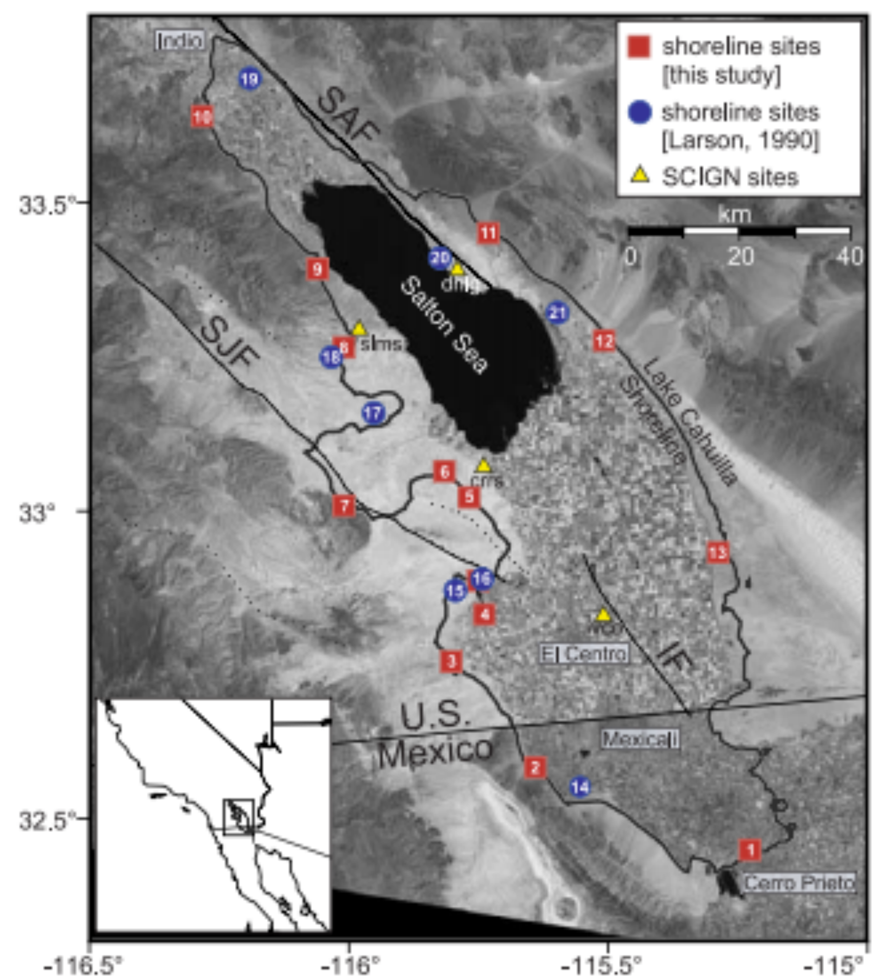
Dara Goldberg and Jessie Saunders

Lake Loading Problem

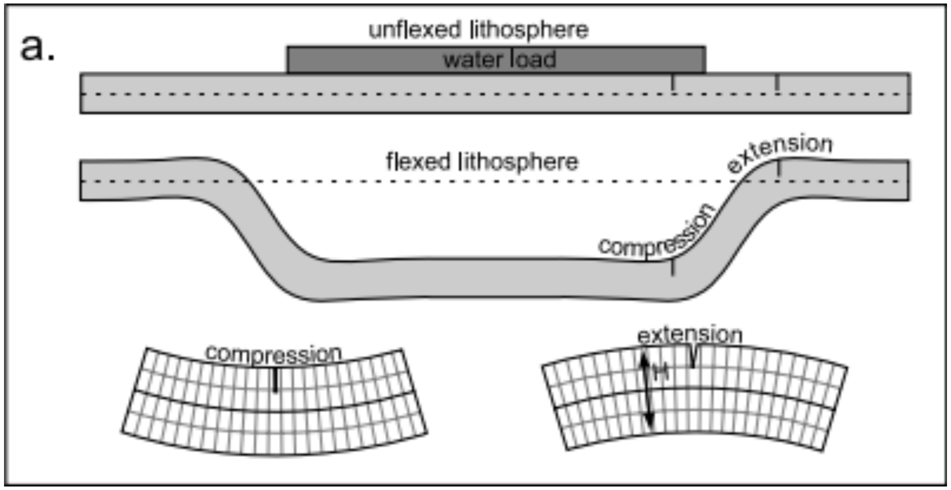
- Changes in water level in lake or oceans changes stress on surrounding rocks
  - Due to lake loading and pore pressure
- If large enough, could trigger earthquakes
- Luttrell et al. (2007)
  - Investigated stress changes in southern San Andreas region due to lake level changes of Lake Cahuilla

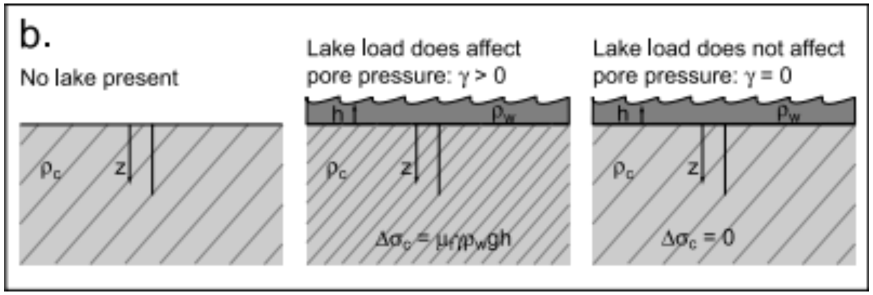


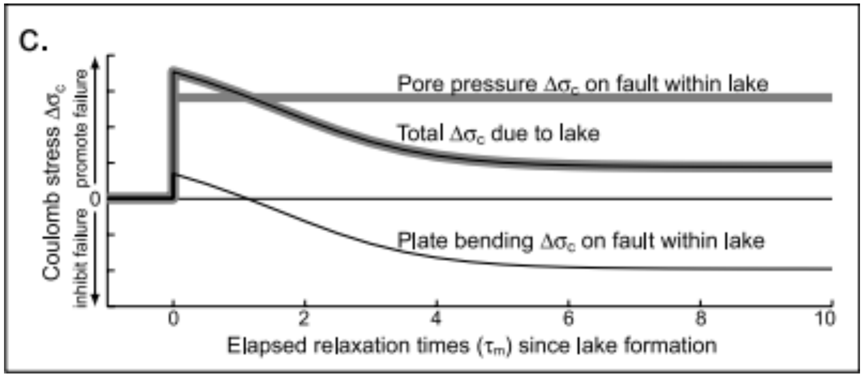
Outline of Luttrell et al.

- Created 3-D model of elastic plate
- Tested model against 2-D elastic plate model
- Constrained model parameters to fit Lake Cahuilla system
- Calculated stress change in southern San Andreas region for past 1300 years of lake history

Change in stress due to lake loading: 2-D elastic plate model







Derivation of vertical displacement

We begin with the solution for a line load, equation 3.130 from Turcotte and Schubert:

$$w(x) = \frac{V_0 \alpha^3}{8D} e^{\frac{-x}{\alpha}} \left[cos\left(\frac{x}{\alpha}\right) + sin\left(\frac{x}{\alpha}\right)\right]$$

$$W(x) = \frac{V_0 \alpha^3}{8D} \int_{-\infty}^{0} e^{\frac{-x}{\alpha}} \left[cos\left(\frac{x}{\alpha}\right) + sin\left(\frac{x}{\alpha}\right)\right] dx + \int_{0}^{x} e^{\frac{-x}{\alpha}} \left[cos\left(\frac{x}{\alpha}\right) + sin\left(\frac{x}{\alpha}\right)\right] dx$$

Let $u = \frac{x}{\alpha}$ and $du = \frac{1}{\alpha} dx$

$$W(x) = \frac{V_0 \alpha^3}{8D} \alpha \int_{-\infty}^{0} e^{-|u|} [cos(|u|) + sin(|u|)] du + \int_{0}^{x} e^{-|u|} [cos(|u|) + sin(|u|)] du$$

$$W(x) = \frac{V_0 \alpha^4}{8D} ([-e^{-|u|} cos(|u|)]_{-\infty}^{0} + \int_{0}^{x} e^{-|u|} cos(|u|) du + \int_{0}^{x} e^{-|u|} sin(|u|) du)$$

Derivation of vertical displacement

$$W(x) = \frac{V_0\alpha^4}{8D}([-e^{-|u|}cos(|u|)]_{-\infty}^{0} + \int_0^x e^{-|u|}cos(|u|)du + \int_0^x e^{-|u|}sin(|u|)du)$$

Integrate by parts: $w = e^{-|u|}$; $dw = -e^{-|u|}$; $v = -cos(|u|)$; $dv = sin(|u|)du$

$$W(x) = \frac{V_0\alpha^4}{8D}(1 + \int_0^x e^{-|u|}cos(|u|)du - e^{-|u|}cos(|u|) - \int_0^x e^{-|u|}cos(|u|)du$$

$$W(x) = \frac{V_0\alpha^4}{8D}(1 - e^{-|u|}cos(|u|))$$

$$W(x) = \frac{V_0\alpha^4}{8D}\left(1 - e^{\frac{-x}{\alpha}}cos\left(\frac{-x}{\alpha}\right)\right)$$

Derivation of vertical displacement

$$W(x) = \frac{V_0 \alpha^4}{8D} \left(1 - e^{\frac{-x}{\alpha}} cos\left(\frac{-x}{\alpha}\right)\right)$$

We recognize that if x were negative, the entire expression for W would be negative as well, thus we rewrite as:

$$W(x) = \frac{V_0 \alpha^4}{8D} \left(1 - e^{\frac{-|x|}{\alpha}} cos\left(\frac{-|x|}{\alpha}\right)\right) * sign(x)$$

Derivation of horizontal stress

$$\sigma_{xx} = \frac{E}{1-\nu^2}\epsilon_{xx}$$

$$\epsilon_{xx} = -z\frac{d^2W}{dx^2}$$

$$\frac{dW(x)}{dx} = \frac{V_0\alpha^4}{8D}\left[\frac{1}{\alpha}e^{\frac{-x}{\alpha}} sin\left(\frac{-x}{\alpha}\right) + \frac{1}{\alpha}e^{\frac{-x}{\alpha}} cos\left(\frac{-x}{\alpha}\right)\right]$$

$$\frac{dW(x)}{dx} = \frac{V_0\alpha^3}{8D}\left[e^{\frac{-x}{\alpha}} sin\left(\frac{-x}{\alpha}\right) + e^{\frac{-x}{\alpha}} cos\left(\frac{-x}{\alpha}\right)\right]$$

Derivation of horizontal stress

$$\frac{d^2W(x)}{dx^2} = \frac{V_0\alpha^3}{8D}\left[\frac{1}{\alpha}e^{\frac{-x}{\alpha}}cos\left(\frac{-x}{\alpha}\right) - \frac{1}{\alpha}e^{\frac{-x}{\alpha}}sin\left(\frac{-x}{\alpha}\right) - \frac{1}{\alpha}e^{\frac{-x}{\alpha}}sin\left(\frac{-x}{\alpha}\right) - \frac{1}{\alpha}e^{\frac{-x}{\alpha}}cos\left(\frac{-x}{\alpha}\right)\right]$$

$$\frac{d^2W(x)}{dx^2} = \frac{V_0\alpha^3}{8D}\left[-\frac{2}{\alpha}e^{\frac{-x}{\alpha}}sin\left(\frac{-x}{\alpha}\right)\right]$$

$$\frac{d^2W(x)}{dx^2} = \frac{-V_0\alpha^2}{4D}\left[e^{\frac{-x}{\alpha}}sin\left(\frac{-x}{\alpha}\right)\right]$$

Derivation of horizontal stress

$$\sigma_{xx} = \frac{E}{1-\nu^2}\epsilon_{xx}$$

$$\epsilon_{xx} = -z\frac{d^2W}{dx^2}$$

$$\sigma_{xx} = \frac{E}{1-\nu^2}\frac{V_0\alpha^2 z}{4D}\left[e^{\frac{-x}{\alpha}} sin\left(\frac{-x}{\alpha}\right)\right]$$

Where $$D = \frac{Eh^3}{12(1-\nu^2)}$$

$$\sigma_{xx} = \frac{E}{1-\nu^2}\frac{V_0\alpha^2 z}{4}\frac{12(1-\nu^2)}{Eh^3}\left[e^{\frac{-x}{\alpha}} sin\left(\frac{-x}{\alpha}\right)\right]$$

Derivation of horizontal stress

$$\sigma_{xx} = \frac{3V_0\alpha^2}{h^2}\frac{z}{h}\left[e^{\frac{-x}{\alpha}} sin\left(\frac{-x}{\alpha}\right)\right]$$

Again, we recognize that if x were negative, the entire expression for σ_{xx} would be negative as well, thus we rewrite as:

$$\sigma_{xx} = \frac{3V_0\alpha^2}{h^2}\frac{z}{h}\left[e^{\frac{-|x|}{\alpha}} sin\left(\frac{-|x|}{\alpha}\right)\right] * sign(x)$$

2-D Model Results

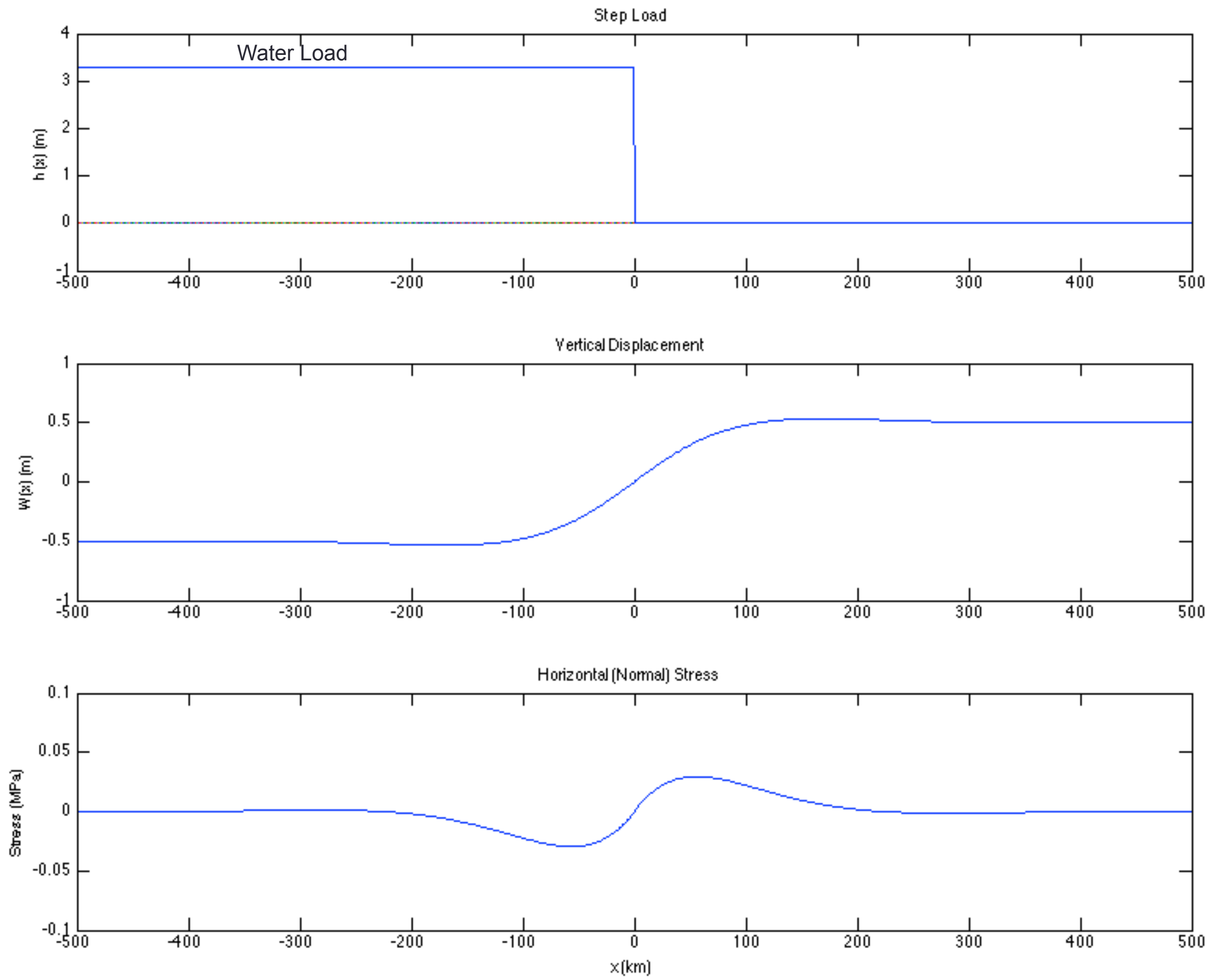
Young's Modulus, $E = 70e^9$
Poisson's Ratio, $\nu = 0.25$
Mantle Density, $\rho_m = 3300kg/m^3$
Water Density, $\rho_w = 1000kg/m^3$
Elastic Plate Thickness, $H = 30e^3$
Evaluated at seismogenic zone depth, $z = 5km$

$$D = \frac{EH^3}{12(1-\nu)}$$

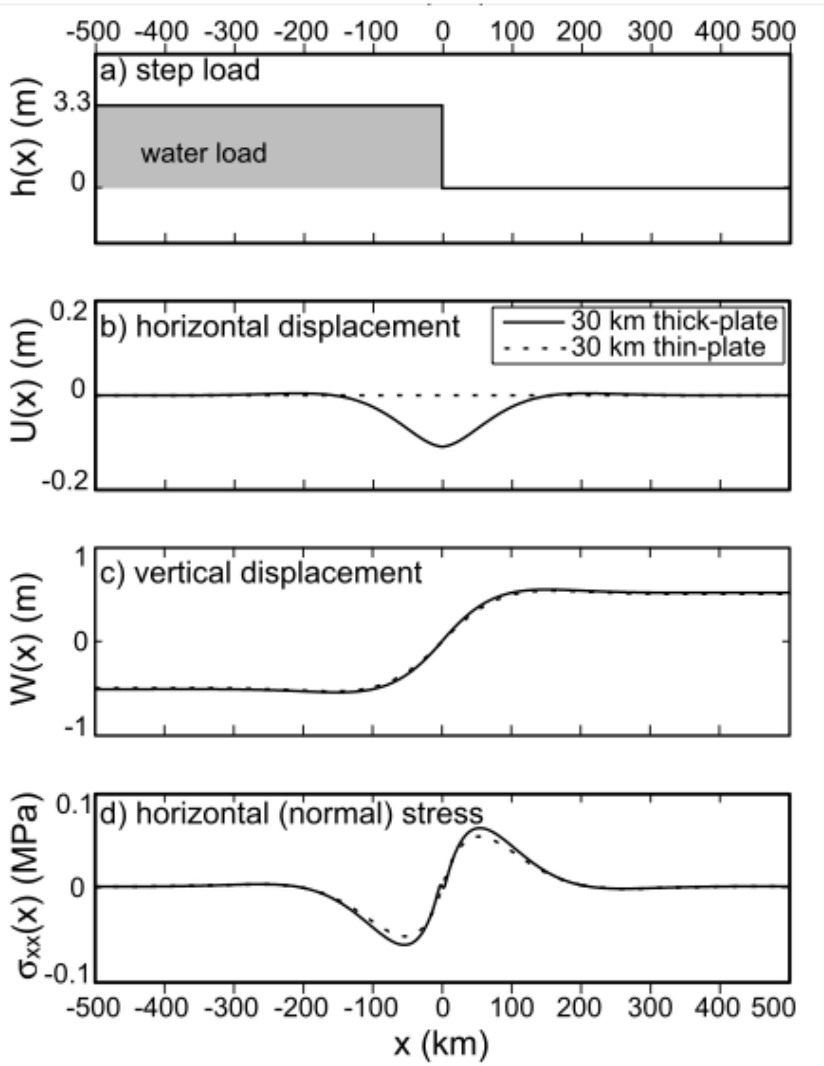
$$\alpha = \frac{4D}{(g\rho_m)^{1/4}}$$

$$\text{vertical load, } V_0 = \rho_w g h$$

2-D Model Results



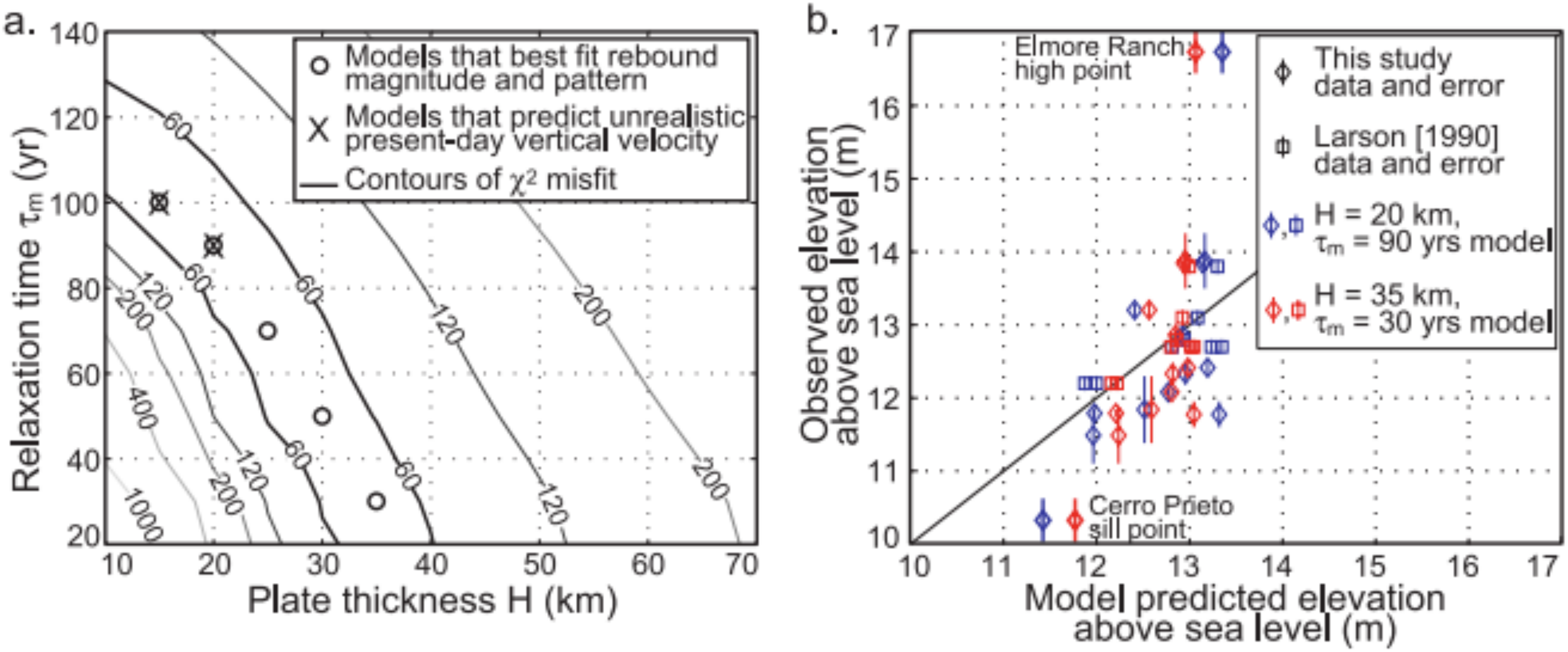
Model Comparison



Luttrell et al. (2007)

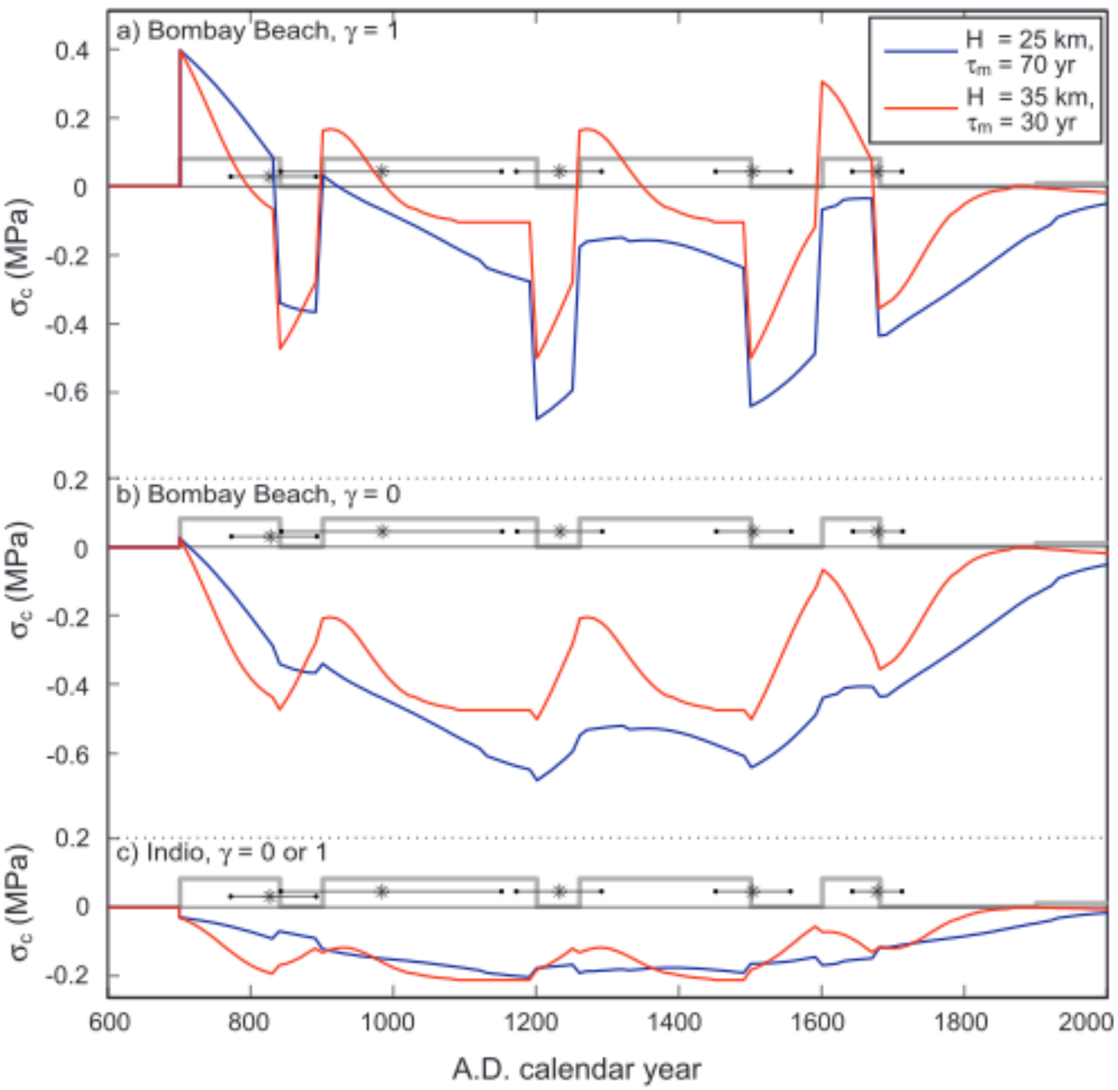
Constraining 3-D model parameters

- Used current elevation of observed ancient shoreline to constrain plate thickness and relaxation time



- Range of best fitting models based on current rebound velocity measurements: H=25km, τ=70yr – H=35km, τ=30 yr
- Calculated stress change due to lake history for both models with and without pore pressure effects.

Results



Luttrell et al. (2007)

- Expected perturbations of ± 0.4 – 0.6 MPa
- Changes smaller and less sudden without pore pressure effects (~0.2 MPa)
- ~10 times smaller than tectonic loading between major earthquakes
- Main differences in models:
  - Magnitudes higher for shorter relaxation time
  - Broader plate deformation effect for thicker plate

Could these stress changes trigger earthquakes?

- Calculated stress changes are of comparable size to other effects suggested to trigger events

- Could trigger on faults that are near critically stressed

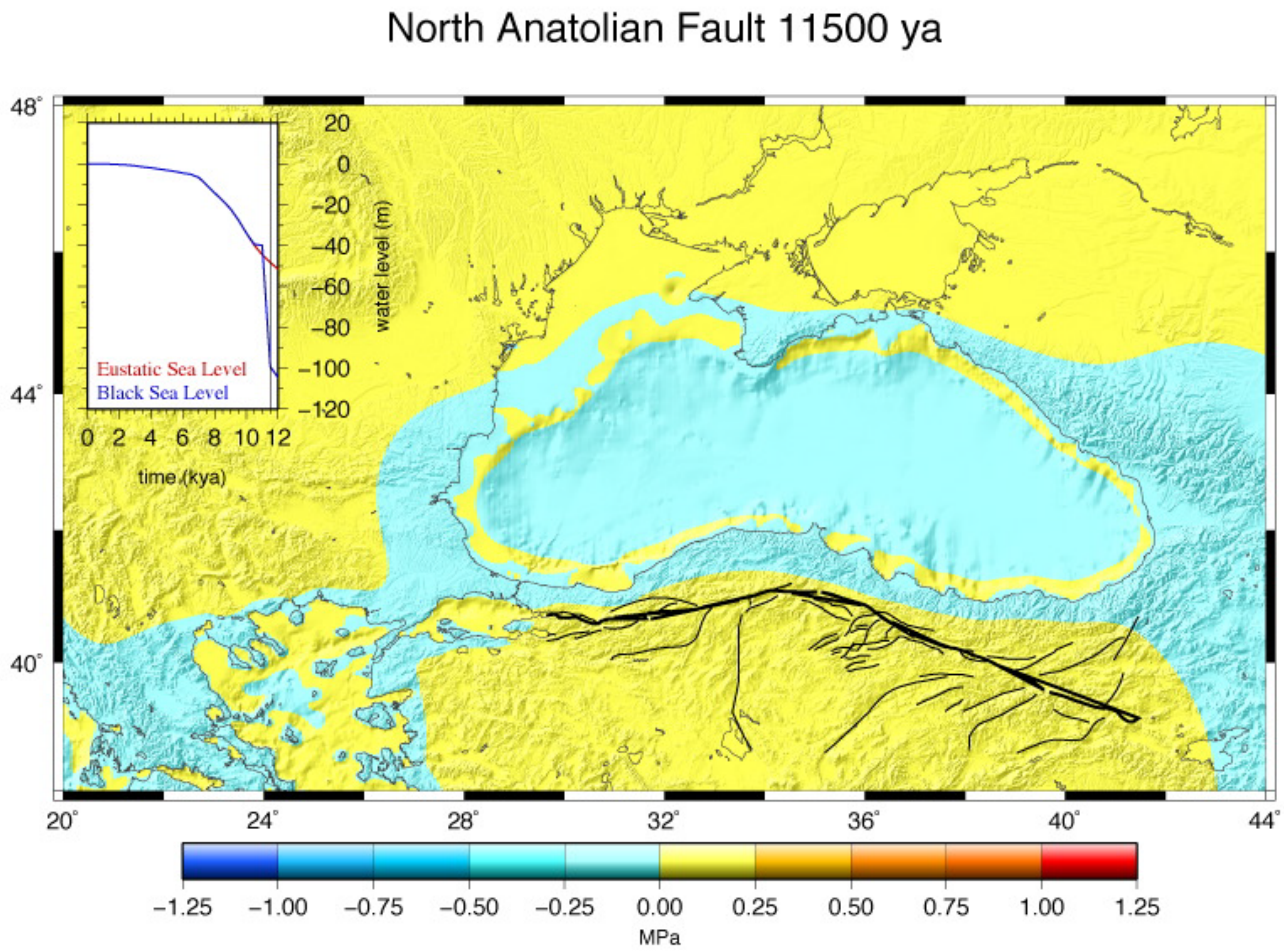
- 4/5 of last events are near times of lake level change

- However, large error bars in the timing of past lake level changes and past earthquake events

Ocean Loading Problem

- Most affected areas are those where tectonic plate boundary coincides with coastline
- Rapid sea level rise at the Black Sea following last glacial maximum
  - Rise of ~70m within a few years to a few hundred years
- Black Sea filling increases Coulomb stress along the North Anatolian Fault by ~75kPa

North Anatolian Fault



References

- Luttrell, K., D. Sandwell, B. Smith-Konter, B. Bills, and Y. Bock (2007). Modulation of the earthquake cycle at the southern San Andreas fault by lake loading. Journal of Geophysical Research. 112, B08411, doi: 10.1029/2006JB004752.

- Luttrell, K., and D. Sandwell (2010). Ocean Loading Effects on Stress at Near Shore Plate Boundary Fault Systems. Journal of Geophysical Research. 115, B08411, doi:10.1029/2009JB006541.