Yield Strength of the Outer Rise

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Overview

- Introduction
- Importance of Yield Strength
- Moment-Curvature Relationship
- Elastic Thickness versus Mechanical Thickness
- Conclusion
Introduction

The outer rise is a topographic high, which is a flexural response to the downward deflection of the subducting plate.
Introduction

- The outer rise height can range from 0-1.3 km.
- The outer rise region contains two types of bending-induced faults, due to the bending forces that can break the surface of the plate.

Mofjeld et al., 2004
Introduction

Tonga Trench, Massell
Real earth materials do have a finite strength. Stress difference are linearly proportional to distance from the neutral axis in the elastic plate. The plate behaves elastically up to the yield stress, at which point the plate fails. Additional strain causes no increases in stress.
Yield Envelope

Three zones of rock behavior:

1. Brittle zone
   Upper cool lithosphere. Governed by brittle failure. Strength increases with overburden pressure but is insensitive to temperature, strain rate and rock type.

2. Semi-brittle zone
   Both brittle and ductile processes occur, usually not included in the yield envelope.

3. Ductile zone
   Lower hot lithosphere. Governed by ductile flow. Strength is insensitive to pressure effects but decreases with decreasing strain rate.
Moment-Curvature Relationship

The moment is defined by the vertical integral of the fibre stress $\sigma_f$ weighted by the distance from the neutral plane of bending at a depth $z_n$:

$$M = \int_0^H \sigma_f (z - z_n) dz$$  \hspace{1cm} (1)

For a thin elastic plate

$$M(x) = -DK(x)$$  \hspace{1cm} (2)

where

- Flexural rigidity $D = ET_e^3 / 12(1-v^2)$  \hspace{1cm} (3)
- Curvature of plate $K(x) = dw^2 / dx^2$  \hspace{1cm} (4)
The homogeneous equation describing the bending of a plate subjected only to mechanical equilibrium:

\[
d^2M/dx^2 - Nd^2w/dx^2 - \Delta \rho gw = 0
\]  \hspace{1cm} (5)

Integrating the equation above twice yields

\[
M(x_0) = \int_{x_0}^{\infty} \Delta \rho gw(x)(x - x_0)dx + Nw(x_0)
\]  \hspace{1cm} (6)

This equation sums up the torques about \(x_0\) caused by the weight of the deformed plate and the axial load.

Subduction zone for an arbitrary \(x_0\); \(x_0\) lies at the first zero crossing, therefore: \(Nw(x_0) = 0\) (McNutt & Menard, 1982)
Moment-Curvature Relationship

\[ M = \int_0^H \sigma_f(z - z_n)dz \]  

(1)

Extremely depends on rheology. Specify the curvature \( K \) at \( x_0 \), which then determines the stress differences in the non-yielded portion. Second derivative of \( w \), the curvature, is unstable.

\[ M(x_0) = \int_{x_0}^{\infty} \Delta \rho gw(x)(x - x_0)dx + Nw(x_0) \]  

(6)

Rheologically independent. Select first zero crossing where \( w(x_0) = 0 \), so use observed quantities, either the bathymetric profile \( w \) or the free air gravity anomaly \( \Delta \rho gw \), to determine \( M(x_0) \). Integration is stable.
## Example Problem

### Trench Profile

<table>
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<tr>
<th>No.</th>
<th>Location</th>
<th>Profile</th>
<th>Age (Myr)</th>
<th>Velocity (mm yr⁻¹)</th>
<th>Strain rate ( \times 10^{-15} ) s⁻¹</th>
<th>( \nu_b ) m</th>
<th>( X_h ) km</th>
<th>Curvature ( \times 10^{-7} ) m⁻¹</th>
<th>Moment ( \times 10^{14} ) N</th>
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Example Problem

Yield Envelope

Moment-Curvature Relationship
Correspondence between elastic-plate thickness and lithospheric mechanical thickness is based on the requirement that both support identical bending moments at a specified curvature. (Mueller & Phillips, 1995)
Elastic Thickness ($T_e$)

- Dependent on the plate curvature and is always less than the mechanical thickness. (Mueller & Phillips, 1995)
- Two methods in estimating the elastic thickness (Mueller & Phillips, 1995):
  1) $T_e + (0.01369 m^{-1/3})(x_b - x_{zc})^{4/3}$
     $x_b =$ location of the outer rise
     $x_{zc} =$ location of the first zero crossing
  2) Finding the minimum misfit of analytical models of elastic-plate flexure with varying input parameters with respect to the observed bathymetry.

Sample parameters used to calculate elastic thickness.
(Mueller & Phillips, 1995)
Mechanical Thickness ($T_m$)

- The thickness of the plate
- Greatest depth at which the lithosphere possesses an significant long-term strength. (Mueller & Phillips, 1995)
- The mechanical thickness is influenced by the age of the oceanic lithosphere, plate curvature, and finite yield strength. (McNutt & Menard, 1982)
  - Mechanical thickness is underestimated if the age is only considered.
Conclusion

- The plate behaves elastically up to the yield stress, at which point the plate fails.
- Plate curvature and finite yield strength are important factors in calculating an elastic and mechanical thickness.
Reference


Thank you!
Questions