Constraints on **Yield Strength** in the Oceanic Lithosphere derived from Observations of Flexure

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@Geodynamics - Homework 6 Group G
Bending of the plate

- The deflection of the plate $w$ satisfies the PDE:

$$\frac{d^2M}{dx^2} - N\frac{d^2w}{dx^2} - \Delta \rho gw = 0$$

- The fiber stress is caused by the bending moment

$$M = \int_0^H \sigma_f(z - z_n)dz$$

(+) stress - tensional
(-) stress - compressional
$z$ - positive downward
$w$ - positive upward

(Turcotte and Schubert, 2014)
Imagine a totally elastic world...

- Bending moment is expressed by

\[ M(x) = -DK(x) \]

- **D** flexure rigidity, the mechanical stiffness of the plate, is controlled by effective elastic thickness (**Te**) of the plate.

- **K(x)** is the curvature of the plate

\[ D = \frac{ETe^3}{12(1 - \nu^2)} \]

\[ K(x) = \frac{d^2w}{dx^2} \]

(H = 40km  
\nu = 0.25  
E = 8E+10)  

(McNutt and Menard, 1982)
What is yield strength?

-The maximum stress that can be applied to a material without causing **plastic deformation**.
More realistic case

- Real earth materials do have a **finite strength**
- Plate behaves elastically up to the yield stress, then additional strain causes **no increase** in stress
- Finite strength **reduces** effective elastic thickness of the plate
- Saturated moment-curvature curve

(McNutt and Menard, 1982)
Yield strength in the oceanic lithosphere is depth dependent. Constrained by results of rock experiments under various temperature, pressure, and strain rate conditions. For the modeling of oceanic plate flexure, only three regimes are considered, **brittle**, **semi-brittle**, and **ductile**.
Brittle & semi-brittle regime

- Uppermost, cool regions of the lithosphere
- Strength increases with **overburden pressure**
- Insensitive to temperature, strain rate, and rock type
- Assuming that rocks fail by movement along localized **fractures**

**Byerlee’s law**

\[
\tau = 0.85 \bar{\sigma}_n, \quad 3 < \bar{\sigma}_n < 200 \text{MPa}
\]

\[
\tau = 60 + 0.6 \bar{\sigma}_n, \quad 200 \text{MPa} < \bar{\sigma}_n
\]

**Relationship between principle stresses** and stresses on the fault

\[
\begin{align*}
\bar{\sigma}_n &= \frac{1}{2} (\sigma_1 + \sigma_3) + \frac{1}{2} (\sigma_1 - \sigma_3) \cos (2\theta) \\
\tau &= \frac{1}{2} (\sigma_1 - \sigma_3) \sin (2\theta)
\end{align*}
\]

**Yield strength in terms of principle stresses**

\[
\bar{\sigma}_1 \approx 5 \bar{\sigma}_3, \quad \bar{\sigma}_3 < 110 \text{MPa}
\]

\[
\bar{\sigma}_1 \approx 3.1 \bar{\sigma}_3 + 210, \quad \bar{\sigma}_3 > 110 \text{MPa}
\]

*(Brace and Kohlstedt, 1980)*
Ductile regime

- Dominated by elevated temperature
- Yield strength is insensitive to pressure
- Dominant mechanism for failure is ductile flow

(+ stress - tensional
- stress - compressional

(McNutt and Menard, 1982)
Moment-Curvature Formulation

A **rheologically independent** measurement of moment:

\[ M(x_0) = \int_{x_0}^{\infty} \Delta \rho g w(x)(x - x_0)dx + Nw(x_0) \]

Physically, this formula sums up the torques about \( x_0 \)

Letting the first zero-crossing point as \( x_0 \),
the formula is simplified to a form that is only based on observed \( w(x) \):

\[ M(x_0) = \int_{x_0}^{\infty} \Delta \rho g w(x)(x - x_0)dx \]

(McNutt and Menard, 1982)

**Rheologically dependent** measurement

\[ M = \int_0^{H} \sigma_f(z - z_n)dz \]
Data parameterization

Theory predicts that the deflection will be the form of a damped, sinusoidal function:

\[ w(x) = A \exp\left(\frac{-x}{\alpha}\right) \sin\left(\frac{x}{\alpha}\right) \]

Fitting the data to determine 2 unknown parameters.

Such that, one could translate the observations into bending moment, curvature, and strain rate at \( x=0 \) in terms of \( x_b \) and \( w_b \).

(Caldwell et al., 1976; McNutt and Menard, 1982; Turcotte and Schubert, 2014)
Application to trench data

Measured from bathymetry profile and/or free-air gravity anomaly

McNutt and Menard, 1982

Table 2. Trench profiles.

<table>
<thead>
<tr>
<th>No.</th>
<th>Location</th>
<th>Profile</th>
<th>Age (Myr)</th>
<th>Velocity (mm yr⁻¹)</th>
<th>Strain rate x 10⁻¹⁶ s⁻¹</th>
<th>( \omega_b ) (m)</th>
<th>( X_b ) (km)</th>
<th>Curvature x 10⁻⁷ m⁻¹</th>
<th>Moment x 10¹⁶ N</th>
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Weaker lithosphere

- The yield envelope with 70 km depth of base produces a moment that is too large to explain the data points.
- It implies the lithosphere is **weaker** than the laboratory yield strength.

Two possible solutions:

1. Decrease the depth of the base of the yield envelope.
2. Increase the slope of the yield envelope by adding pore-fluid pressure.
Possible solution 1: Decreasing the depth of base

$H = 70 \text{ km} \rightarrow H = 40 \text{ km}$

Age-dependent depth

McNutt and Menard, 1982
Possible solution 2: Invoking pore fluid pressure

H = 70 km (Laboratory depth)
Larger slope -> weaker lithosphere
Age dependence of mechanical thickness

(Burov, 2011)
References


