Elastic-viscoelastic correspondence principal

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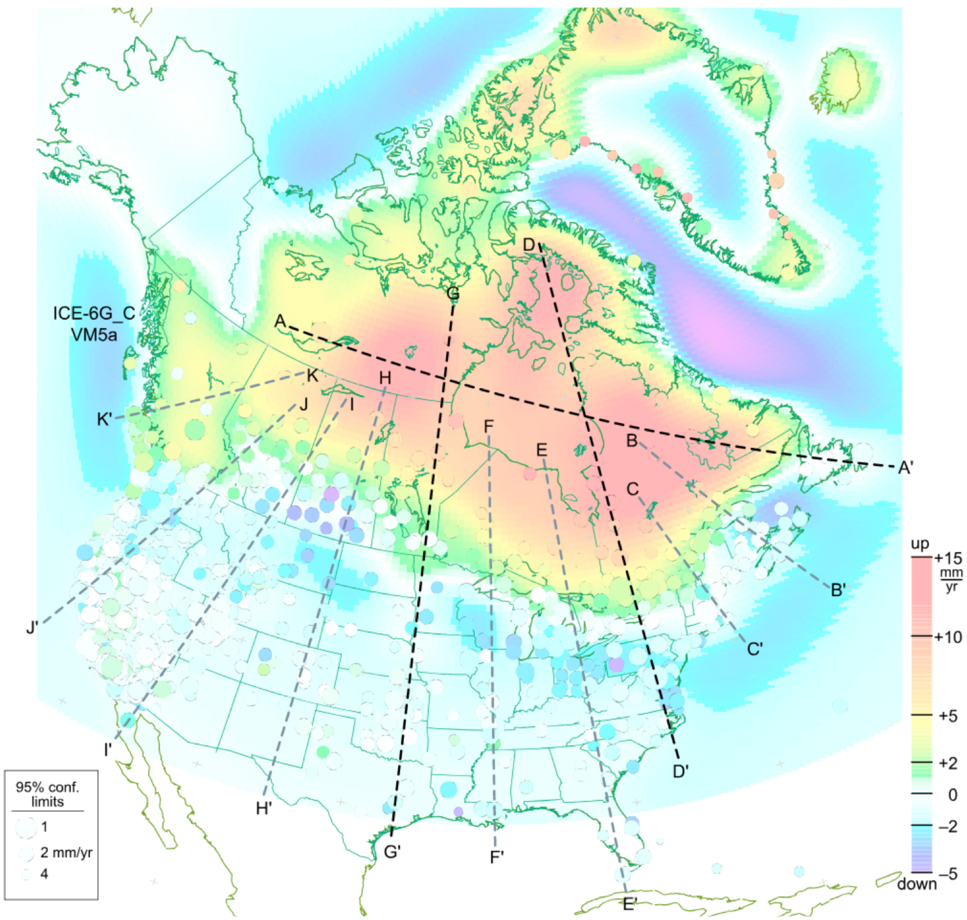
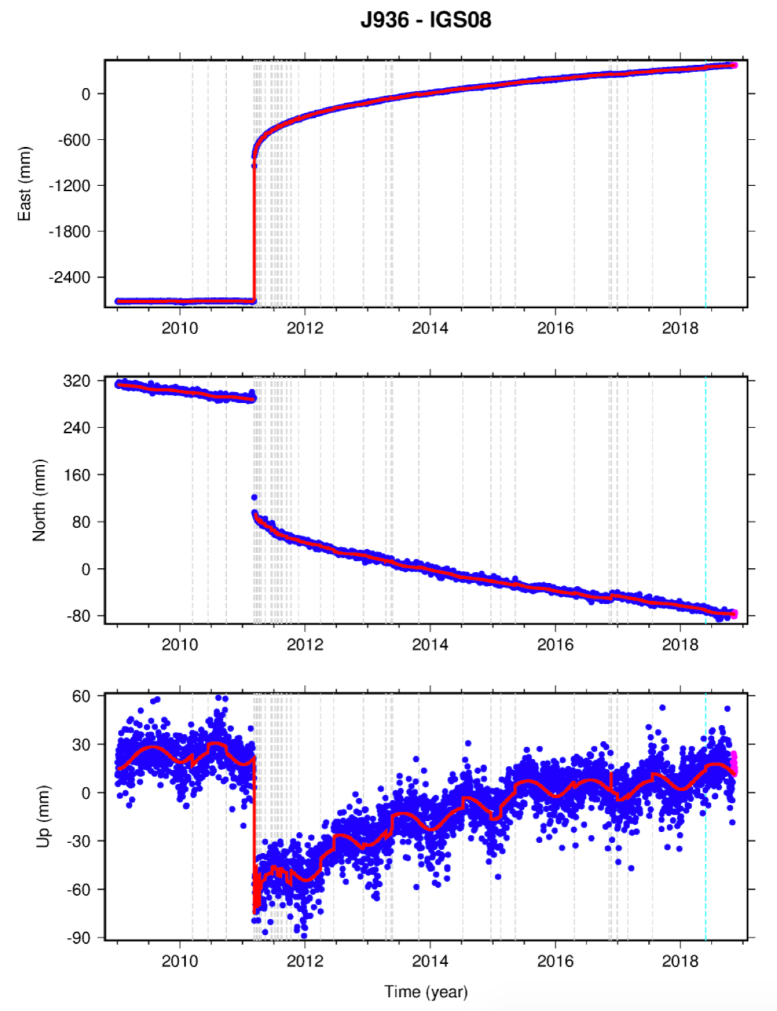


Peltier et al., 2015, JGR



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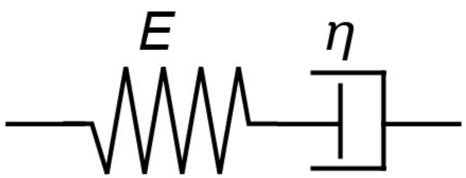
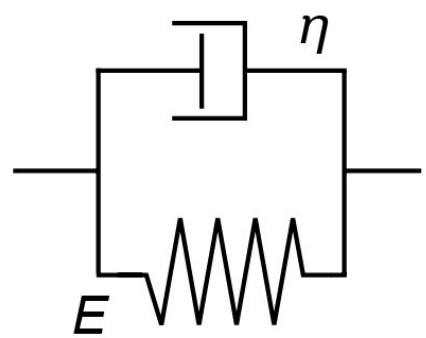




1D stress-strain for elastic and viscous materials

$$\sigma_E = E\epsilon$$
$$\sigma_v = \eta\dot{\epsilon}$$

$$\sigma_T = \sigma_E = \sigma_V$$
$$\epsilon_T = \epsilon_E + \epsilon_V$$

$$\frac{d\epsilon_T}{dt} = \frac{d}{dt}(\epsilon_E + \epsilon_V)$$

$$\dot{\epsilon} = \frac{\sigma}{\eta} + \frac{\dot{\sigma}}{E}$$

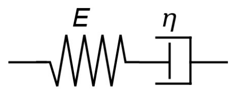
$$\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\dot{\epsilon}$$

Just like electric circuits!
Series vs Parallel

$$\sigma_T = \sigma_E + \sigma_V$$
$$\epsilon_T = \epsilon_E = \epsilon_V$$

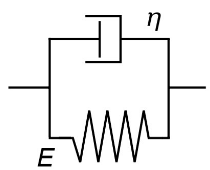
$$\sigma_T = \eta\dot{\epsilon} + E\epsilon$$
$$\sigma = E\epsilon + \eta\dot{\epsilon}$$

Maxwell



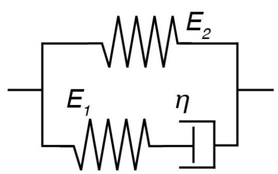
$$\sigma + \frac{\eta}{E}\dot{\sigma} = \eta\dot{\epsilon}$$

Kelvin-Voigt



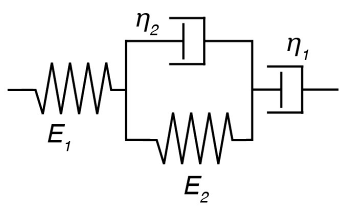
$$\sigma = E\epsilon + \eta\dot{\epsilon}$$

Standard linear solid



$$\sigma + \frac{\eta}{E_1}\dot{\sigma} = E_2 + \frac{\eta(E_1 + E_2)}{E1}\dot{\epsilon}$$

Burgers



$$\sigma + (\frac{\eta_1}{E_1} + \frac{\eta_1}{E_2} + \frac{\eta_2}{E_2}) + \frac{\eta_1\eta_2}{E_1E_2}\ddot{\sigma} = \eta_1\dot{\epsilon} + \frac{\eta_1\eta_2}{E_2}\ddot{\epsilon}$$

5 steps to compute a viscoelastic solution

1) Compute the elastic solution for the equation of motion (conservation of linear momentum and Hooke's Law);

2) Take the Laplace transform of the elastic solution;

3) Take the Laplace transform of the viscoelastic model of choice, and solve for the transformed elastic modulus, or "effective modulus";

4) Replace the original elastic modulus in the transformed elastic solution with the new effective modulus;

5) Inverse transform the Laplacian domain solution back to time/space domain (Bromwich integral)

Shear stress elastic solution $$u(x) = \frac{\sigma}{2\mu} x$$

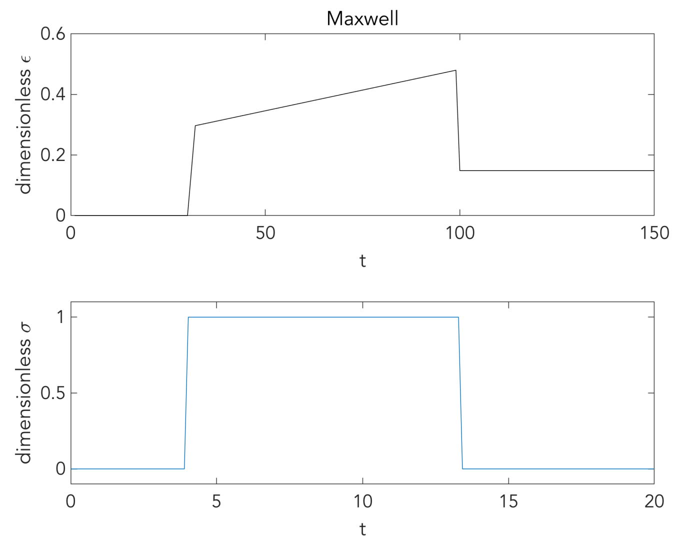
Laplace transform elastic solution $$u(s) = \frac{\sigma}{2s\mu} x$$

Laplace transform Maxwell stress-strain relationship $$\bar{\epsilon} = \frac{\bar{\sigma} s}{2s\mu^{-1} + \eta^{-1}}$$

Rearrange… looks like elastic! $$\bar{\sigma} = \frac{2s\bar{\epsilon}}{\mu s^{-1} + \eta^{-1}} = 2\bar{\epsilon}\frac{s}{\mu s^{-1} + \eta^{-1}}$$

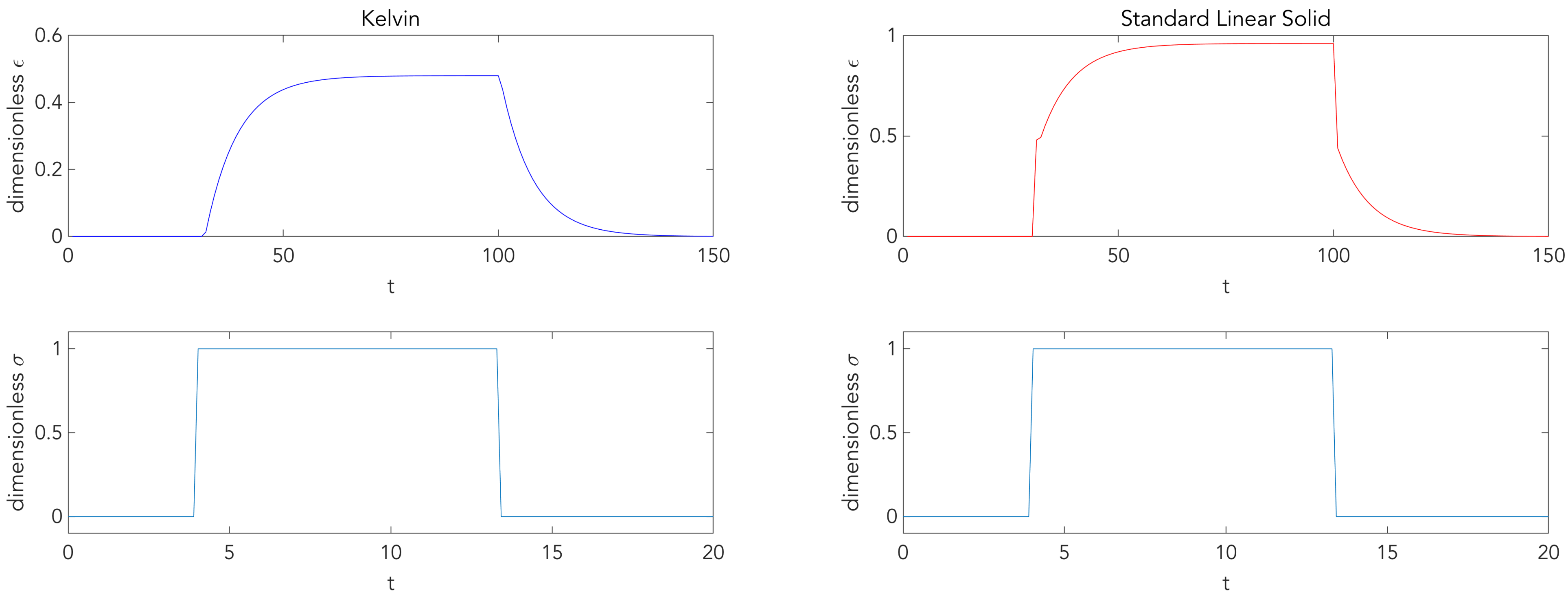
Rearrange for new elastic modulus $$\bar{\mu} = \frac{s}{\mu s^{-1} + \eta^{-1}}$$

Plug it to Laplacian domain Elastic solution $$u(s) = \frac{\sigma}{2s\bar{\mu}} x = \frac{\sigma x (\mu s^{-1} + \eta^{-1})}{2s^2}$$



Inverse Laplace transform results in viscoelastic solution!

$$u(x) = \frac{\sigma x}{2\mu}(1 + \frac{t\mu}{\eta})$$


Kelvin
Standard Linear Solid
dimensionless ϵ
dimensionless σ
t
0
0.2
0.4
0.6
0.5
1
50
100
150
5
10
15
20