HOMEWORK 8
(due December 04)

1) Turcotte and Schubert, problem 5-25.

2) MATLAB - same as HW1. The fourier transform of the derivative of a function is equal to $i2\pi k$ times the fourier transform of the original function (see Bracewell, p. 117) where k is the wavenumber (1/wavelength), and i is the square root of -1. Show that this relationship also holds for a discrete time series by carrying out the operations on the computer. Use the first difference formula to compute the derivative of the geoid height profile. Also compute the derivative by multiplication in the fourier domain. Apply a phase shift to the FFT derivative so it will be aligned with the first difference derivative. Compare results. Obtain the data at:
ftp://topex.ucsd.edu/pub/class/geodynamics/hw1

3) Assuming the lithosphere responds as thin elastic plate when it is subjected to topographic loads, one can calculate a linear relationship between gravity anomaly and topography in the Fourier transform domain.

$$\Delta g(\mathbf{k}) = 2\pi G(\rho_c - \rho_w) e^{-2\pi|\mathbf{k}|s} \left\{ 1 - \left[1 + \frac{D(2\pi|\mathbf{k}|)^4}{g(\rho_m - \rho_c)} \right]^{-1} e^{-2\pi|\mathbf{k}|d} \right\} T(\mathbf{k}).$$

The flexural rigidity D is

$$D = \frac{Eh^3}{12(1-\nu^2)}$$

G	-	gravitational constant	6.67×10^{-11} m^3 kg^{-1} s^{-2}
g	-	acceleration of gravity	9.82 m s^{-2}
ρ_w	-	seawater density	1025 kg m^{-3}
ρ_c	-	crustal density	2800 kg m^{-3}
ρ_m	-	mantle density	3330 kg m^{-3}
s	-	mean water depth	(average of topography, must be positive)
d	-	crustal thickness	6000 m
h	-	elastic plate thickness	(0 - 50,000 m)
E	-	Young's modulus	6.5×10^{10} N m^{-2}
ν	-	Poisson's ratio	0.25

Plot this transfer function (i.e., $\Delta g / T$ for wave numbers $|\mathbf{k}|$ ranging from 0 to 10^{-4} m^{-1}. Use elastic thicknesses of 0 m and 30000 m. Why does the transfer function approach zero at high wave numbers? Why does it approach zero at low wave numbers?

Explain what happens when the elastic thickness is zero. For zero elastic thickness, what is the relationship between topography (above the base level) and the total crustal thickness?

Using this transfer function Q(|k|), and the topography given in the computer file ftp://topex.ucsd.edu/pub/class/geodynamics/hw6, calculate a model gravity anomaly profile for h=0. The basic procedure is to take the Fourier transform of the topography, multiply by the transfer function Q(|k|) and inverse Fourier transform the result.

Compare this model gravity profile with the observed gravity profile. Increase the elastic thickness until the model gravity profile matches the observed gravity profile. How does this value of elastic thickness compare with the value found by Watts, JGR, v. 83, 5989-6004, 1978? How old was the lithosphere when this seamount formed?