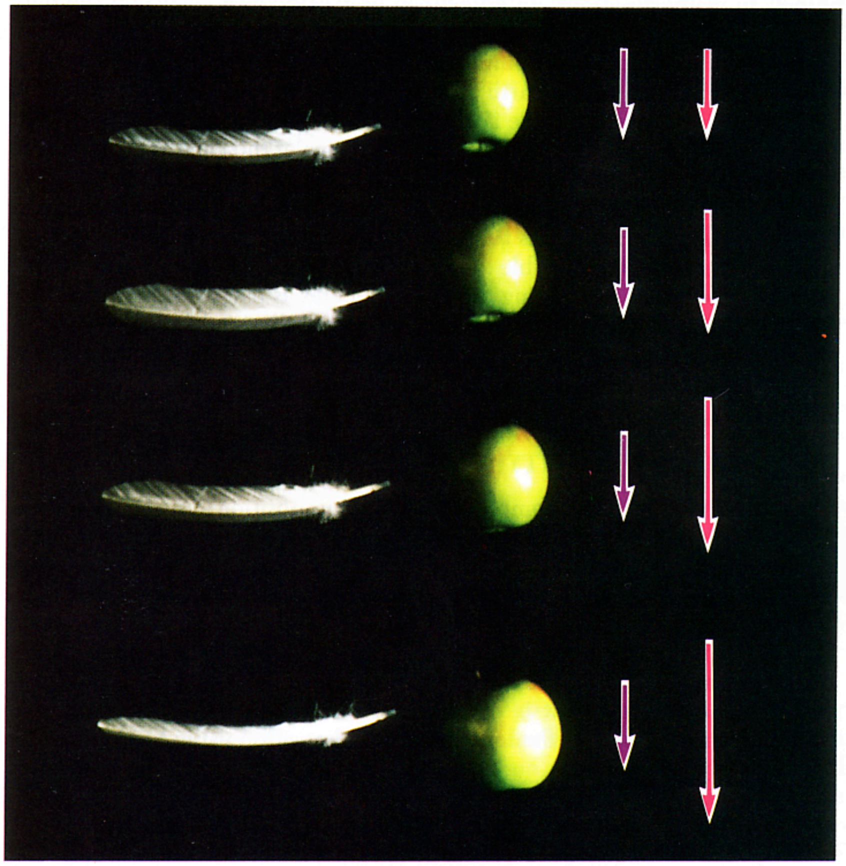
3

Motion in One Dimension



An apple and a feather, released from rest in a 4-ft vacuum chamber, fall at the same rate, regardless of their mass. Neglecting air resistance, all objects fall to the earth with the same acceleration of magnitude 9.8 m/s² as indicated by the violet arrows in this multiflash photograph. The velocity of the two objects increases linearly with time as indicated by the series of red arrows.

Dynamics is concerned with the study of the motion of an object and the relation of this motion to such physical concepts as force and mass. It is convenient to describe motion using the concepts of space and time, without regard to the causes of the motion. This portion of mechanics is called *kinematics*. In this chapter we shall consider motion along a straight line, that is, one-dimensional motion. In the next chapter we shall extend our discussion to two-dimensional motion. Starting with the concept of displacement discussed in the previous chapter, we shall define velocity and acceleration. Using these concepts, we shall proceed to study the motion of objects undergoing constant acceleration. The subject of *dynamics*, which is concerned with the causes of motion and relationships between motion, forces, and the properties of moving objects, will be discussed in Chapters 5 and 6.

From everyday experience we recognize that motion represents the continuous change in the position of an object. The movement of an object through space may be accompanied by the rotation or vibration of the object. Such motions can be quite complex. However, it is sometimes possible to simplify matters by temporarily neglecting the internal motions of the moving object. In many situations, an object can be treated as a *particle* if the only motion being considered is one of translation through space. An idealized particle is a mathematical point with no size. For example, if we wish to describe the motion of the earth around the sun, we can treat the earth as a particle and obtain reasonable accuracy in a prediction of the earth's orbit. This approximation is justified because the radius of the earth's orbit is large compared with the dimensions of the earth and sun. On the other hand, we could not use the particle description to explain the internal structure of the earth and such phenomena as tides, earthquakes, and volcanic activity. On a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles. However, the particle description of the gas molecules is generally inadequate for understanding those properties of the gas that depend on the internal motions of the gas molecules, namely, rotations and vibrations.

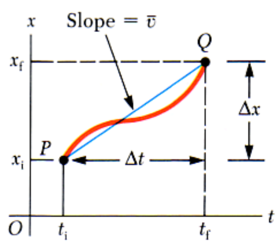


Figure 3.1 Position-time graph for a particle moving along the x axis. The average velocity $\bar{v}$ in the interval $\Delta t = t_f - t_i$ is the slope of the straight line connecting the points P and Q.

3.1 AVERAGE VELOCITY

The motion of a particle is completely known if its position in space is known at all times. Consider a particle moving along the x axis from point P to point Q. Let its position at point P be x_i at some time t_i, and let its position at point Q be x_f at time t_f. (The indices i and f refer to the initial and final values.) At times other than t_i and t_f, the position of the particle between these two points may vary as in Figure 3.1. Such a plot is often called a *position-time graph*. In the time interval $\Delta t = t_f - t_i$, the displacement of the particle is $\Delta x = x_f - x_i$. (Recall that the displacement is defined as the change in the position of the particle, which equals its final minus its initial position value.)

The x-component of the **average velocity** of the particle, $\bar{v}$, is defined as the ratio of its displacement, Δx, and the time interval, Δt:

Average velocity

$$\bar{v} \equiv \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} \tag{3.1}$$

From this definition, we see that the average velocity has the dimensions of length divided by time, or m/s in SI units and ft/s in conventional units. The average velocity is *independent* of the path taken between the points P and Q. This is true because the average velocity is proportional to the displacement, Δx, which in turn depends only on the initial and final coordinates of the particle. It therefore follows that if a particle starts at some point and returns to the same point via any path, its average velocity for this trip is zero, since its displacement along such a path is zero. The displacement should not be confused with the distance traveled, since the distance traveled for any motion is clearly nonzero. Thus, average velocity gives us no details of the motion between points P and Q. (How we evaluate the velocity at some instant in time is discussed in the next section.) Finally, note that the average velocity in one dimension can be positive or negative, depending on the sign of the displacement. (The time interval, Δt, is always positive.) If the coordinate of the particle increases in time (that is, if $x_f > x_i$), then Δx is positive and $\bar{v}$ is positive. This corresponds to a velocity in the positive x direction. On the other hand, if the coordinate decreases in time ($x_f < x_i$), Δx is negative; hence $\bar{v}$ is negative. This corresponds to a velocity in the negative x direction.

The average velocity can also be interpreted geometrically by drawing a straight line between the points P and Q in Figure 3.1. This line forms the hypotenuse of a triangle of height Δx and base Δt. The slope of this line is the ratio $\Delta x/\Delta t$. Therefore, we see that the *average* velocity of the particle during the time interval t_i to t_f is equal to the "slope" of the straight line joining the initial and final points on the space-time graph. (The word *slope* will often be used when referring to the graphs of physical data. Regardless of what data are plotted, the word *slope* will represent the ratio of the change in the quantity represented on the vertical axis to the change in the quantity represented on the horizontal axis.)

EXAMPLE 3.1 Calculate the Average Velocity
A particle moving along the x axis is located at $x_i = 12$ m at $t_i = 1$ s and at $x_f = 4$ m at $t_f = 3$ s. Find its displacement and average velocity during this time interval.

Solution The displacement is given by

$$\Delta x = x_f - x_i = 4\text{ m} - 12\text{ m} = -8\text{ m}$$

The average velocity is

$$\bar{v} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{t_f - t_i} = \frac{4\text{ m} - 12\text{ m}}{3\text{ s} - 1\text{ s}} = -\frac{8\text{ m}}{2\text{ s}} = -4\text{ m/s}$$

Since the displacement and average velocity are negative for this time interval, we conclude that the particle has moved to the left, toward decreasing values of x.

3.2 INSTANTANEOUS VELOCITY

We would like to be able to define the velocity of a particle at a particular instant of time, rather than just over a finite interval of time. The velocity of a particle at any instant of time, or at some point on a space-time graph, is called the **instantaneous velocity.** This concept is especially important when the average velocity in different time intervals is *not constant.*

Consider the motion of a particle between the two points P and Q on the space-time graph shown in Figure 3.2. As the point Q is brought closer and closer to the point P, the time intervals ($\Delta t_1, \Delta t_2, \Delta t_3, \ldots$) get progressively smaller. The average velocity for each time interval is the slope of the appropriate dotted line in Figure 3.2. As the point Q approaches P, the time interval approaches zero, but at the same time the slope of the dotted line approaches that of the blue line tangent to the curve at the point P. The slope of the line tangent to the curve at P is defined to be the *instantaneous velocity* at the time t_i. In other words,

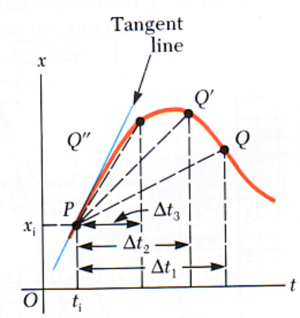


Figure 3.2 Position-time grap for a particle moving along the axis. As the time intervals startin at t_i get smaller and smaller, th average velocity for that interv: approaches the slope of the lin tangent at P. The instantaneous ve locity at P is defined as the slope (the blue tangent line at the time t

the instantaneous velocity, v, equals the limiting value of the ratio $\Delta x/\Delta t$ as Δt approaches zero[1]:

[1] Note that the displacement, Δx, also approaches zero as Δt approaches zero. However, as Δx and Δt become smaller and smaller, the ratio $\Delta x/\Delta t$ approaches a value equal to the *true* slope of the line tangent to the x versus t curve.

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} \tag{3.2}$$

In the calculus notation, this limit is called the *derivative* of x with respect to t, written dx/dt:

Definition of the derivative

$$v \equiv \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt} \tag{3.3}$$

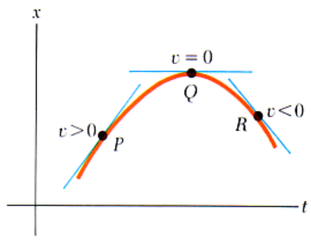


Figure 3.3 In the position-time graph shown here, the velocity is positive at P, where the slope of the tangent line is positive; the velocity is zero at Q, where the slope of the tangent line is zero; and the velocity is negative at R, where the slope of the tangent line is negative.

The instantaneous velocity can be positive, negative, or zero.

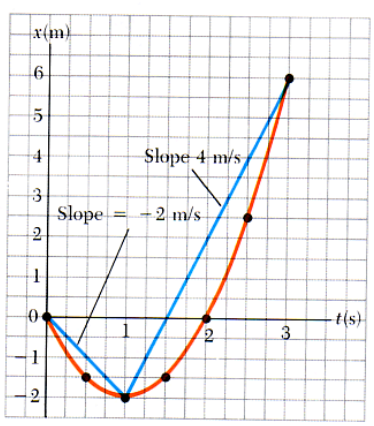
When the slope of the space-time graph is positive, such as at the point P in Figure 3.3, v is positive. At point R, v is negative since the slope is negative. Finally, the instantaneous velocity is zero at the peak Q (the turning point), where the slope is zero. *From here on, we shall usually use the word* velocity *to designate instantaneous velocity.*

The **instantaneous speed** of a particle is defined as the magnitude of the instantaneous velocity vector. Hence, by definition, *speed* can never be negative.

It is also possible to find the displacement of a particle if its velocity is known as a function of time using a mathematical technique called integration. Because this procedure may not be familiar to many students, the topic is treated in Section 3.6, which is optional, for general interest and for those courses that cover this material.

EXAMPLE 3.2 Average and Instantaneous Velocity □

A particle moves along the x axis. Its x coordinate varies with time according to the expression $x = -4t + 2t^2$, where x is in m and t is in s. The position-time graph for this motion is shown in Figure 3.4. Note that the particle first moves in the negative x direction for the first second of motion, stops instantaneously at $t = 1$ s, and then heads back in the positive x direction for $t > 1$ s. (a) Determine the displacement of the particle in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.



t(s)	x(m)
0	0
0.5	−1.5
1	−2
1.5	−1.5
2	0
2.5	2.5
3	6

Figure 3.4 (Example 3.2) Position-time graph for a particle having an x coordinate that varies in time according to $x = -4t + 2t^2$. Note that $\bar{v}$ is *not* the same as $v = -4 + 4t$.

In the first time interval, we set $t_i = 0$ and $t_f = 1$ s. Since $x = -4t + 2t^2$, we get for the first displacement

$$\Delta x_{01} = x_f - x_i$$
$$= [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$$= -2 \text{ m}$$

Likewise, in the second time interval we can set $t_i = 1$ s and $t_f = 3$ s. Therefore, the displacement in this interval is

$$\Delta x_{13} = x_f - x_i$$
$$= [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]$$
$$= 8 \text{ m}$$

Note that these displacements can also be read directly from the position-time graph (Fig. 3.4).

(b) Calculate the average velocity in the time intervals $t = 0$ to $t = 1$ s and $t = 1$ s to $t = 3$ s.

In the first time interval, $\Delta t = t_f - t_i = 1$ s. Therefore, using Equation 3.1 and the results from (a) gives

$$\bar{v}_{01} = \frac{\Delta x_{01}}{\Delta t} = \frac{-2 \text{ m}}{1 \text{ s}} = \boxed{-2 \text{ m/s}}$$

Likewise, in the second time interval, $\Delta t = 2$ s; therefore

$$\bar{v}_{13} = \frac{\Delta x_{13}}{\Delta t} = \frac{8 \text{ m}}{2 \text{ s}} = \boxed{4 \text{ m/s}}$$

These values agree with the slopes of the lines joining these points in Figure 3.4.

(c) Find the instantaneous velocity of the particle at $t = 2.5$ s.

By measuring the slope of the position-time graph at $t = 2.5$ s, we find that $v = 6$ m/s. (You should show that the velocity is -4 m/s at $t = 0$ and zero at $t = 1$ s.) Do you see any symmetry in the motion? For example, does the speed ever repeat itself?[2]

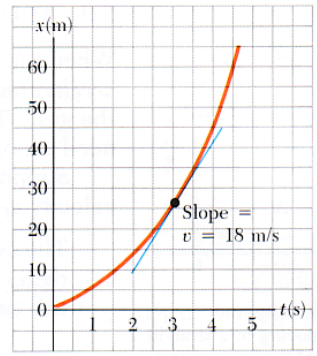


Figure 3.5 (Example 3.3) Position-time graph for a particle having an x coordinate that varies in time according to $x = 3t^2$. Note that the instantaneous velocity at $t = 3$ s equals the slope of the blue line tangent to the curve at this point.

EXAMPLE 3.3 The Limiting Process

The position of a particle moving along the x axis varies in time according to the expression $x = 3t^2$, where x is in m, 3 is in m/s², and t is in s. Find the velocity at any time.

Solution The position-time graph for this motion is shown in Figure 3.5. We can compute the velocity at any time t by using the definition of the instantaneous velocity. If the initial coordinate of the particle at time t is $x_i = 3t^2$, then the coordinate at a later time $t + \Delta t$ is

$$x_f = 3(t + \Delta t)^2 = 3[t^2 + 2t\,\Delta t + (\Delta t)^2]$$
$$= 3t^2 + 6t\,\Delta t + 3(\Delta t)^2$$

Therefore, the displacement in the time interval Δt is

$$\Delta x = x_f - x_i = 3t^2 + 6t\,\Delta t + 3(\Delta t)^2 - 3t^2$$
$$= 6t\,\Delta t + 3(\Delta t)^2$$

The average velocity in this time interval is

$$\bar{v} = \frac{\Delta x}{\Delta t} = 6t + 3\,\Delta t$$

To find the instantaneous velocity, we take the limit of this expression as Δt approaches zero. In doing so, we see that the term $3\,\Delta t$ goes to zero, therefore

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \boxed{6t \text{ m/s}}$$

Notice that this expression gives us the velocity at *any* general time t. It tells us that v is increasing linearly in time. It is then a straightforward matter to find the velocity at some specific time from the expression $v = 6t$. For example, at $t = 3$ s, the velocity is $v = 6(3) = 18$ m/s. Again, this can be checked from the slope of the graph (the blue line) at $t = 3$ s.

The limiting process can also be examined numerically. For example, we can compute the displacement and average velocity for various time intervals beginning at $t = 3$ s, using the expressions for Δx and $\bar{v}$. The results of such calculations are given in Table 3.1. Notice that as the time intervals get smaller and smaller, the average velocity more nearly approaches the value of the instantaneous velocity at $t = 3$ s, namely, 18 m/s.

TABLE 3.1 Displacement and Average Velocity for Various Time Intervals for the Function $x = 3t^2$ (the intervals begin at $t = 3$ s)

Δt (s)	Δx (m)	$\Delta x/\Delta t$ (m/s)
1.00	21	21
0.50	9.75	19.5
0.25	4.69	18.8
0.10	1.83	18.3
0.05	0.9075	18.15
0.01	0.1803	18.03
0.001	0.018003	18.003

[2] We could also use the rules of differential calculus to find the velocity from the displacement. That is, $v = \frac{dx}{dt} = \frac{d}{dt}(-4t + 2t^2) = 4(-1 + t)$ m/s. Therefore, at $t = 2.5$ s, $v = 4(-1 + 2.5) = 6$ m/s. A review of basic operations in the calculus is provided in Appendix B.6.

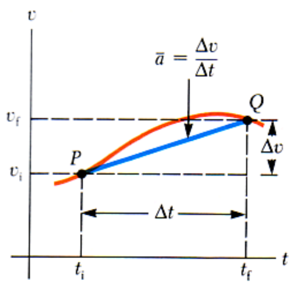


Figure 3.6 Velocity-time graph for a particle moving in a straight line. The slope of the line connecting the points P and Q is defined as the average acceleration in the time interval $\Delta t = t_f - t_i$.

3.3 ACCELERATION

When the velocity of a particle changes with time, the particle is said to be *accelerating*. For example, the speed of a car will increase when you "step on the gas." The car will slow down when you apply the brakes. However, we need a more precise definition of acceleration than this.

Suppose a particle moving along the x axis has a velocity v_i at time t_i and a velocity v_f at time t_f, as in Figure 3.6.

The **average acceleration** of the particle in the time interval $\Delta t = t_f - t_i$ is defined as the ratio $\Delta v/\Delta t$, where $\Delta v = v_f - v_i$ is the *change* in velocity in this time interval:

$$\bar{a} \equiv \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t} \tag{3.4}$$

Acceleration is a vector quantity having dimensions of length divided by $(\text{time})^2$, or L/T^2. Some of the common units of acceleration are meters per second per second (m/s^2) and feet per second per second (ft/s^2).

In some situations, the value of the average acceleration may be different over different time intervals. It is therefore useful to define the **instantaneous acceleration** as the limit of the average acceleration as Δt approaches zero. This concept is analogous to the definition of instantaneous velocity discussed in the previous section. If we imagine that the point Q is brought closer and closer to the point P in Figure 3.6 and take the limit of the ratio $\Delta v/\Delta t$ as Δt approaches zero, we get the *instantaneous acceleration:*

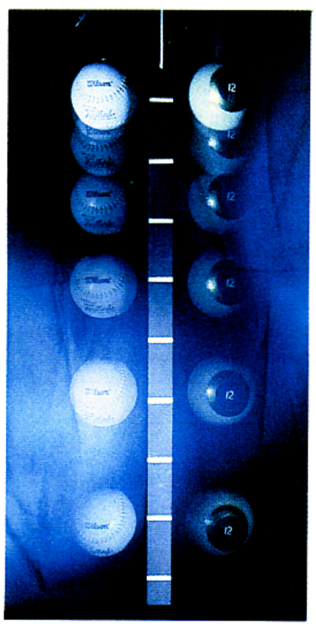
$$a \equiv \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} \tag{3.5}$$

That is, the instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity-time graph. One can interpret the derivative of the velocity with respect to time as the *time rate of change of velocity.* Again you should note that if a is positive, the acceleration is in the positive x direction, whereas negative a implies acceleration in the negative x direction. *From now on we shall use the term* acceleration *to mean instantaneous acceleration.* Average acceleration is seldom used in physics.

Since $v = dx/dt$, the acceleration can also be written

$$a = \frac{dv}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2} \tag{3.6}$$

That is, the acceleration equals the *second derivative* of the coordinate with respect to time.



A multiflash photograph of a freely falling baseball (mass 0.23 kg) and shotput (mass 5.4 kg) taken at a flash rate of 1/15 s. The spacing between markers is 10 cm. Note that the two objects fall at the same rate. Why is this so, in view of the fact that they have different masses? (Courtesy of Henry Leap)

Figure 3.7 shows how the acceleration-time curve can be derived from the velocity-time curve. In these sketches, the acceleration at any time is simply the slope of the velocity-time graph at that time. Positive values of the acceleration correspond to those points where the velocity is increasing in the positive x direction. The acceleration reaches a maximum at time t_1, when the slope of the velocity-time graph is a maximum. The acceleration then goes to zero at time t_2, when the velocity is a maximum (that is, when the velocity is

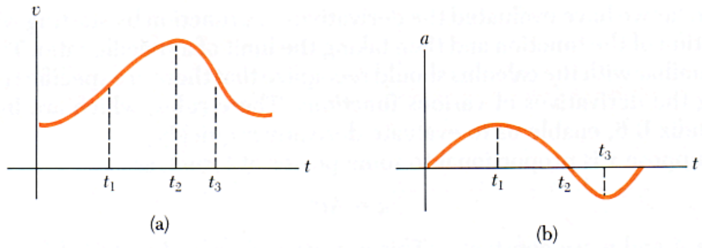


Figure 3.7 The instantaneous acceleration can be obtained from the velocity-time graph (a). At each instant, the acceleration in the a versus t graph (b) equals the slope of the line tangent to the v versus t curve.

momentarily not changing and the slope of the v versus t graph is zero). Finally, the acceleration is negative when the velocity in the positive x direction is decreasing in time.

EXAMPLE 3.4 Average and Instantaneous Acceleration

The velocity of a particle moving along the x axis varies in time according to the expression $v = (40 - 5t^2)$ m/s, where t is in s. (a) Find the average acceleration in the time interval $t = 0$ to $t = 2$ s.

The velocity-time graph for this function is given in Figure 3.8. The velocities at $t_i = 0$ and $t_f = 2$ s are found by substituting these values of t into the expression given for the velocity:

$$v_i = (40 - 5t_i^2) \text{ m/s} = [40 - 5(0)^2] \text{ m/s} = 40 \text{ m/s}$$

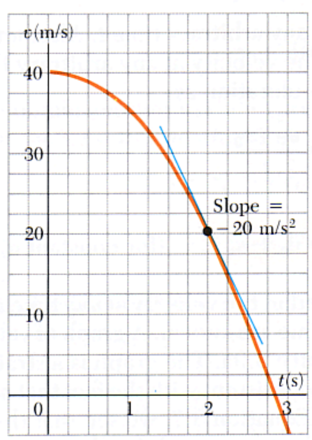


Figure 3.8 (Example 3.4) The velocity-time graph for a particle moving along the x axis according to the relation $v = (40 - 5t^2)$ m/s. Note that the acceleration at $t = 2$ s is equal to the slope of the blue tangent line at that time.

$$v_f = (40 - 5t_f^2) \text{ m/s} = [40 - 5(2)^2] \text{ m/s} = 20 \text{ m/s}$$

Therefore, the average acceleration in the specified time interval $\Delta t = t_f - t_i = 2$ s is given by

$$\bar{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{(20 - 40) \text{ m/s}}{(2 - 0) \text{ s}} = \boxed{-10 \text{ m/s}^2}$$

The negative sign is consistent with the fact that the slope of the line joining the initial and final points on the velocity-time graph is negative.

(b) Determine the acceleration at $t = 2$ s.

The velocity at time t is given by $v_i = (40 - 5t^2)$ m/s, and the velocity at time $t + \Delta t$ is given by

$$v_f = 40 - 5(t + \Delta t)^2 = 40 - 5t^2 - 10t\,\Delta t - 5(\Delta t)^2$$

Therefore, the change in velocity over the time interval Δt is

$$\Delta v = v_f - v_i = [-10t\,\Delta t - 5(\Delta t)^2] \text{ m/s}$$

Dividing this expression by Δt and taking the limit of the result as Δt approaches zero, we get the acceleration at *any* time t:

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\,\Delta t) = -10t \text{ m/s}^2$$

Therefore, at $t = 2$ s, we find that

$$a = (-10)(2) \text{ m/s}^2 = \boxed{-20 \text{ m/s}^2}$$

This result can also be obtained by measuring the slope of the velocity-time graph at $t = 2$ s. Note that the acceleration is not constant in this example. Situations involving constant acceleration will be treated in the next section.

So far we have evaluated the derivatives of a function by starting with the definition of the function and then taking the limit of a specific ratio. Those of you familiar with the calculus should recognize that there are specific rules for taking the derivatives of various functions. These rules, which are listed in Appendix B.6, enable us to evaluate derivatives quickly.

Suppose x is proportional to some power of t, such as

$$x = At^n$$

where A and n are constants. (This is a very common functional form.) The derivative of x with respect to t is given by

$$\frac{dx}{dt} = nAt^{n-1}$$

Applying this rule to Example 3.3, where $x = 3t^2$, we see that $v = dx/dt = 6t$, in agreement with our result of taking the limit explicitly. Likewise, in Example 3.4, where $v = 40 - 5t^2$, we find that $a = dv/dt = -10t$. (Note that the rate of change of any constant quantity is zero.)

3.4 ONE-DIMENSIONAL MOTION WITH CONSTANT ACCELERATION

If the acceleration of a particle varies in time, the motion can be complex and difficult to analyze. A very common and simple type of one-dimensional motion occurs when the acceleration is constant, or uniform. Because the acceleration is constant, the average acceleration equals the instantaneous acceleration. Consequently, the velocity increases or decreases at the same rate throughout the motion.

If we replace $\bar{a}$ by a in Equation 3.4, we find that

$$a = \frac{v_f - v_i}{t_f - t_i}$$

For convenience, let $t_i = 0$ and t_f be any arbitrary time t. Also, let $v_i = v_0$ (the initial velocity at $t = 0$) and $v_f = v$ (the velocity at any arbitrary time t). With this notation, we can express the acceleration as

$$a = \frac{v - v_0}{t}$$

or

Velocity as a function of time

$$v = v_0 + at \qquad \text{(for constant } a\text{)} \qquad (3.7)$$

This expression enables us to predict the velocity at *any* time t if the initial velocity, acceleration, and elapsed time are known. A graph of velocity versus time for this motion is shown in Figure 3.9a. The graph is a straight line the slope of which is the acceleration, a, consistent with the fact that $a = dv/dt$ is a constant. From this graph and from Equation 3.7, we see that the velocity at any time t is the sum of the initial velocity, v_0, and the change in velocity, at. The graph of acceleration versus time (Fig. 3.9b) is a straight line with a slope of zero, since the acceleration is constant. Note that if the acceleration were negative (the particle is slowing down), the slope of Figure 3.9a would be negative.

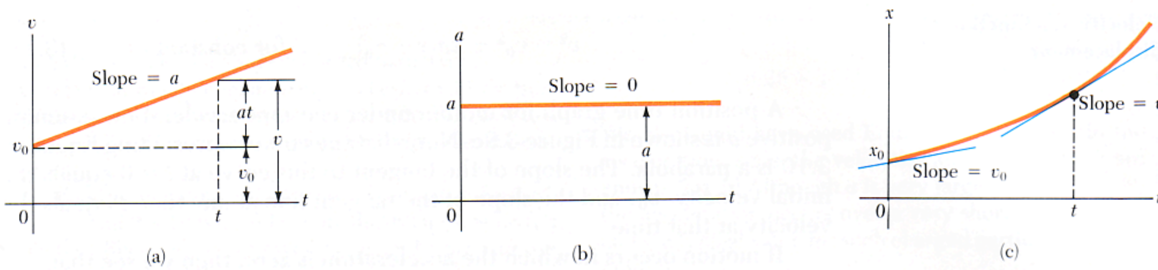


Figure 3.9 A particle moving along the x axis with constant acceleration a; (a) the velocity-time graph, (b) the acceleration-time graph, and (c) the space-time graph.

Because the velocity varies linearly in time according to Equation 3.7, we can express the average velocity in any time interval as the arithmetic mean of the initial velocity, v_0, and the final velocity, v:

$$\bar{v} = \frac{v_0 + v}{2} \qquad \text{(for constant } a\text{)} \qquad (3.8)$$

Note that this expression is only useful when the acceleration is constant, that is, when the velocity varies linearly with time.

We can now use Equations 3.1 and 3.8 to obtain the displacement as a function of time. Again, we choose $t_i = 0$, at which time the initial position is $x_i = x_0$. This gives

$$\Delta x = \bar{v}\,\Delta t = \left(\frac{v_0 + v}{2}\right) t$$

or

$$x - x_0 = \tfrac{1}{2}(v + v_0)t \qquad \text{(for constant } a\text{)} \qquad (3.9)$$

We can obtain another useful expression for the displacement by substituting Equation 3.7 into Equation 3.9:

$$x - x_0 = \tfrac{1}{2}(v_0 + v_0 + at)t$$

$$x - x_0 = v_0 t + \tfrac{1}{2}at^2 \qquad \text{(for constant } a\text{)} \qquad (3.10)$$

The validity of this expression can be checked by differentiating it with respect to time, to give

$$v = \frac{dx}{dt} = \frac{d}{dt}\left(x_0 + v_0 t + \frac{1}{2}at^2\right) = v_0 + at$$

Finally, we can obtain an expression that does not contain the time by substituting the value of t from Equation 3.7 into Equation 3.9. This gives

$$x - x_0 = \tfrac{1}{2}(v_0 + v)\left(\frac{v - v_0}{a}\right) = \frac{v^2 - v_0^2}{2a}$$

Velocity as a function of displacement

$$v^2 = v_0^2 + 2a(x - x_0) \qquad \text{(for constant } a\text{)} \qquad (3.11)$$

A position-time graph for motion under constant acceleration assuming positive a is shown in Figure 3.9c. Note that the curve representing Equation 3.10 is a parabola. The slope of the tangent to this curve at $t = 0$ equals the initial velocity, v_0, and the slope of the tangent line at any time t equals the velocity at that time.

If motion occurs in which the acceleration is *zero*, then we see that

$$\left.\begin{aligned} v &= v_0 \\ x - x_0 &= vt \end{aligned}\right\} \text{ when } a = 0$$

That is, when the acceleration is zero, the velocity is a constant and the displacement changes linearly with time.

Equations 3.7 through 3.11 are five *kinematic expressions that may be used to solve any problem in one-dimensional motion with constant acceleration.* Keep in mind that these relationships were derived from the definition of velocity and acceleration, together with some simple algebraic manipulations and the requirement that the acceleration be constant. It is often convenient to choose the initial position of the particle as the origin of the motion, so that $x_0 = 0$ at $t = 0$. In such a case, the displacement is simply x.

The four kinematic equations that are used most often are listed in Table 3.2 for convenience.

The choice of which kinematic equation or equations you should use in a given situation depends on what is known beforehand. Sometimes it is necessary to use two of these equations to solve for two unknowns, such as the displacement and velocity at some instant. For example, suppose the initial velocity, v_0, and acceleration, a, are given. You can then find (1) the velocity after a time t has elapsed, using $v = v_0 + at$, and (2) the displacement after a time t has elapsed, using $x - x_0 = v_0t + \frac{1}{2}at^2$. You should recognize that the quantities that vary during the motion are velocity, displacement, and time.

You will get a great deal of practice in the use of these equations by solving a number of exercises and problems. Many times you will discover that there is more than one method for obtaining a solution.

TABLE 3.2 Kinematic Equations for Motion in a Straight Line Under Constant Acceleration

Equation	Information Given by Equation
$v = v_0 + at$	Velocity as a function of time
$x - x_0 = \frac{1}{2}(v + v_0)t$	Displacement as a function of velocity and time
$x - x_0 = v_0t + \frac{1}{2}at^2$	Displacement as a function of time
$v^2 = v_0^2 + 2a(x - x_0)$	Velocity as a function of displacement

Note: Motion is along the x axis. At $t = 0$, the position of the particle is x_0 and its velocity is v_0.