Fluid mechanics of a surfboard

SIO 87

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Outline

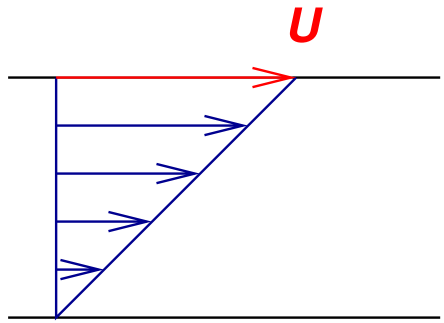
1. What is fluid mechanics?
2. Dimensional analysis
3. Drag
4. Fluid-structure interactions
5. Previous work
6. Conclusions

Health warning

I won't actually describe the fluid mechanics of a surfboard. That's very difficult. But we'll learn about fluid mechanics.

What is fluid mechanics?

A fluid is a substance that deforms continuously when subjected to a shear stress.



stress or force per unit area: $\tau = \mu \dfrac{\mathrm{d}u}{\mathrm{d}y}.$

Examples of fluids: water, air, liquid helium, shampoo, blood, the Earth's mantle, the sun's interior, etc. . .

Use continuum mechanics: talk about the value at all the points in the fluid of quantities such as density, velocity, temperature, etc. . .

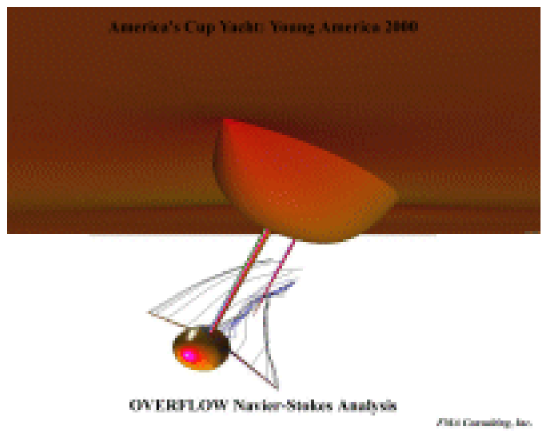
Difficult because unlike particle or rigid-body dynamics, we have to describe the fluid everywhere. Also nonlinear.

Where, how and when?

Oceanography, meteorology, astrophysics, chemical engineering, aeronautical engineering, mechanical engineering, etc. . .

Field goes back to Newton, Euler, Laplace, Lagrange, Cauchy, Kirchoff, Helmholtz, Green, Airy, Kelvin, Stokes, and many other famous physicists and mathematicians.

Big expansion in 20th century with development of aircraft. Advent of computers has led to CFD: Computational Fluid Dynamics. Used to design racing cars, aircraft, America's Cup yachts, etc. . .



America's Cup design from Pointwise – `http://www.pointwise.com`.

Also new interest in biological applications.

Underlying principles

Governing equations very complex. Basic principles might be familiar from physics classes.

1. Conservation of mass
2. Newton's Second Law
3. First Law of Thermodynamics
4. equation of state: e.g. perfect gas
5. constitutive relation: e.g. Newtonian fluid
6. other conservation laws: e.g. salinity, entropy, etc...

We'll talk about some simple physical ideas that are often used in fluid dynamics.

Dimensional analysis

The governing equations of physics relate quantities with the same dimensions.

Buckingham's Π theorem relates dimensionless quantities that tell us everything about the system.

Example 1: waves on deep water – this means the depth of the water can :be taken to be infinite. Dimensional quantities are
ω: frequency $[T^{-1}]$
λ: wavelength $[L]$
g: acceleration due to gravity $[LT^{-2}]$.
Then $\omega^2\lambda/g$ is a universal number.

Example 2: waves on water with depth h. Now there is a relation between $\omega^2\lambda/g$ and λ/h, so

$$\omega^2 = \frac{g}{\lambda}F(\lambda/h).$$

Have to solve full problem to get actual solution (see Lecture 2).

Nondimensional numbers

Dimensional analysis leads to numbers whose value characterizes the flow. Look at only three here:

Reynolds number:

$$\mathrm{Re} = \frac{UL}{\nu};$$

ratio of inertial to viscous forces.

Froude number:

$$\mathrm{Fr} = \frac{U^2}{gL};$$

ratio of inertial force to gravitational force (lecture 2).

Mach number:

$$\mathrm{Ma} = \frac{U}{c};$$

where c is the speed of sound; ratio of fluid speed to sound speed.

Quantities such as U are typical values for the situation under consideration.

Similarity

Consider two different systems A and B.

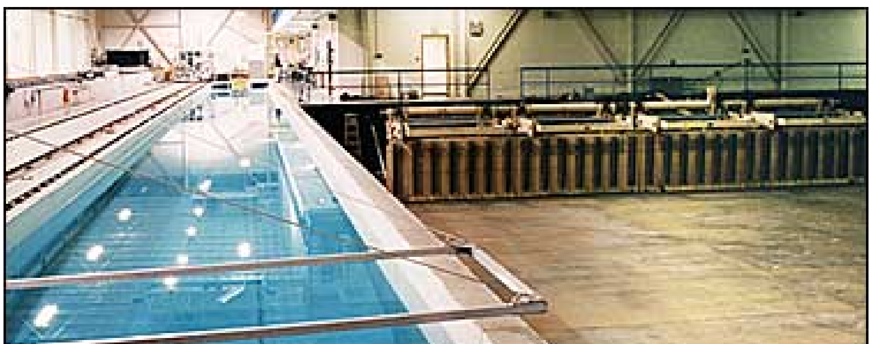
Geometric similarity: shapes of bodies in A and B same.

Dynamic similarity: values of all non-dimensional parameters the same.

Complete similarity: both of the above.

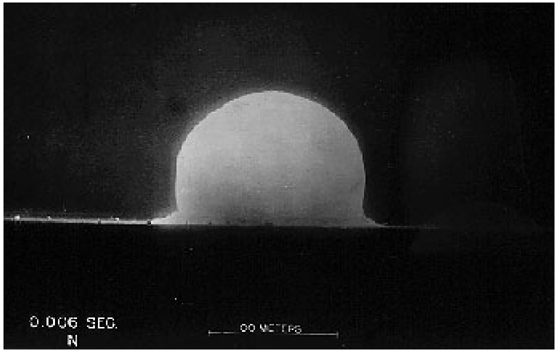
With complete similarity, can apply results from system A to system B just by rescaling.

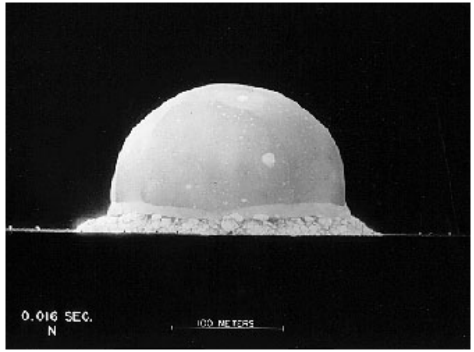
Difficult in general: keeping Re the same in a wind tunnel by raising U is likely to increase Ma. Similar problems for ship model testing with Fr.

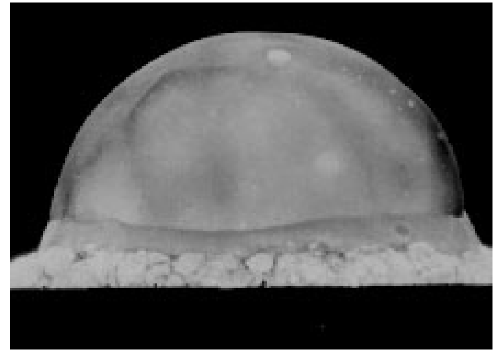


Vizon SciTec's Ocean Engineering Centre's ship model testing facility (picture from `http://www.vizonscitec.com/services/shipdynamics/default.htm`).

Taylor's solution







Pictures from `http://mt.sopris.net/mpc/military/trinity.htm`.

5:29:45 am on July 16, 1945. How do you calculate the yield of the device (top secret) from the photographs?

G. I. Taylor argued that the spread of the front r ($[L]$) is characterized only by the amount of energy released e ($[ML^2T^{-2}]$), the density of the fluid ρ ($[ML^{-3}]$), and time t ($[T]$). So $r \propto (et^2/\rho)^{1/5}$.

Excellent agreement. Led to trouble with authorities.

Drag

Drag is simply the force on a body in a flow in the direction of the flow.
Dimensional analysis: body of typical size L, flow velocity U; fluid density ρ, kinematic viscosity ν (dimensions $[L^2T^{-1}]$), gravitational acceleration g, sound speed c.

$$\text{Drag} = \tfrac{1}{2}\rho U^2 F(\text{Re}, \text{Fr}, \text{Ma}).$$

Numbers for surfboard

Wave speed from Lecture 2: $U = 10\,\text{m s}^{-1}$ (average).
Length of surfboard: 2 m.
Kinematic viscosity of water $\nu = 10^{-6}\,\text{m}^2\,\text{s}^{-1}$.
$g = 10\,\text{m s}^{-2}$
Speed of sound in water $c = 1500\,\text{m s}^{-1}$.

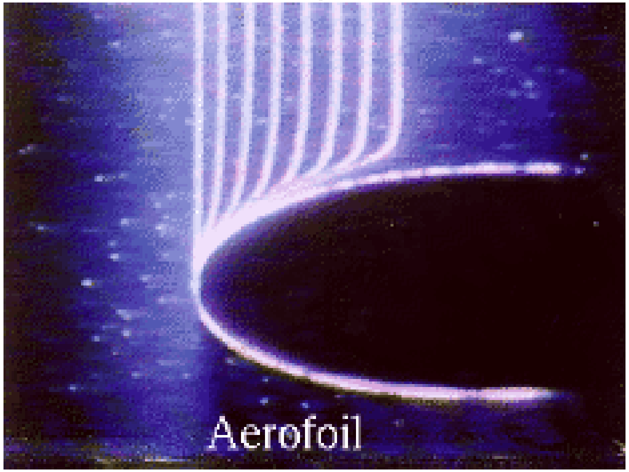
$$\text{Re} = 20,000,000; \qquad \text{Fr} = 5; \qquad \text{Ma} = 0.007.$$

High-Reynolds number, incompressible flow with gravity waves.

High-Reynolds number flow

Arguably the most technologically important question of the 20th century in fluid mechanics. 18–19th century ideas of inviscid flow led to d'Alembert's paradox of zero drag. Not too surprising. . .

Resolved by Prandtl's boundary-layer theory (1906): there is a thin layer near the boundary of objects where viscosity is important.



Picture from Iowa Institute of Hydraulic Research, University of Iowa.

Nice online course at `http://cvu.strath.ac.uk/courseware/calf/CALF/index/nindx.html`.

Boundary-layer equations not entirely satisfactory, especially with turbulence and free surface. Need to use CFD. Very expensive computationally.

Fluid-structure interaction

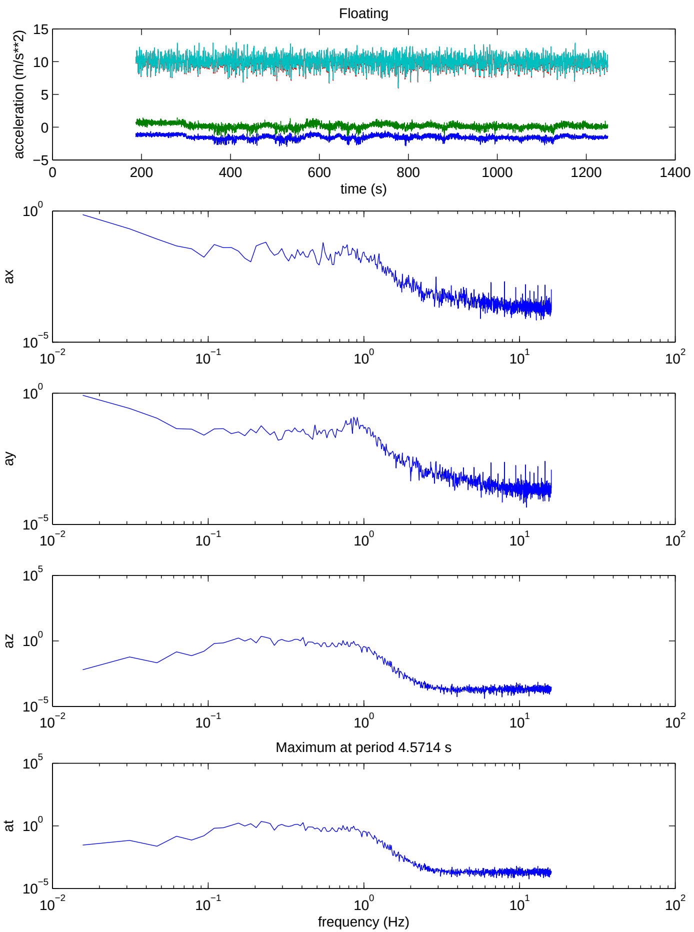
One of the major areas of current research is fluid-structure interaction. Rather than just looking at an airfoil in a wind tunnel, study how flow affects motion of body and vice versa.

Many biological and technological applications: insect flight, fish swimming, microbe, blood flow, etc. . .

Can look at kinematic or dynamic cases: body motion can be prescribed or be affected by forces.

Dynamics of body can be very important, e.g. elasticity of arterial wall, properties of flapping flag. Influence of wake can be very important. Sébastien Michelin currently working on vortex shedding effects on deformable bodies (see video).

Intrinsic dynamics of surfboard: Normal modes, as found using accelerometer. Probably important for "feel" of board.



Floating
acceleration (m/s**2)
time (s)
ax
ay
az
Maximum at period 4.5714 s
at
frequency (Hz)

Literature

1. *The Surf Skimmer*, Edge 1968, Am. J. Physics: simple treatment of the surf skimmer.

2. *Hydrodynamics of surfboards*, Paine 1974, Ph.D. Thesis University of Sydney. `http://www1.tpgi.com.au/users/mpaine/thesis.html`. Probably the most detailed treatment.

3. *A stationary oblique breaking wave for laboratory testing of surfboards*, Hornung & Killen 1976, J. Fluid Mech.: long experimental treatment.

4. *Surf-skimmer planing hydrodynamics*, Tuck & Dixon 1989, Journal of Fluid Mechanics: detailed treatment of surf skimmer. Hard.

5. *Surfing on solitary waves*, Vanden-Broeck & Keller 1989, J. Fluid Mech.: flat plate on single wave.

6. *How to ride a wave: mechanics of surfing, Sugimoto 1998, SIAM Rev.:* simple analysis. Unconvincing?

7. *Surf Physics*, Edge 2001, The Physics Teacher: good introductory article on surfing in general. Available at `http://ojps.aip.org/tpt/`.

Paine's thesis work

Taken from website:

1. riding waves with an instrumented surfboard (a pitot tube and pressure gauge for speed measurement) - measured speeds ranged from 5 to 10 m/s
2. observing surfboard riding and taking speed measurements from the shore (similar results)
3. reviewing ocean wave theory - deep ocean waves, solitary (cnoidal) waves and breaking waves
4. developing an early (punch card!) computer model of a shoaling breaking wave
5. reviewing and adapting theory for planing water craft - including stability problems (porpoising) and lift/drag/trim data from seaplane research.
6. designing, constructing and operating an experiment which generated a standing wave in a flume tank. The resulting wave was about 200mm high (flume 1.2m wide, flow rate 0.37 cumecs)
7. constructing model surfboards and experimenting with them on the standing wave.
8. (unsuccessfully) trying to model a human spring mass system to dampen the instability of the models (this was essentially a 2D flow - it turns out that a 3D

flow would probably have eliminated the instability - see the paper by Hornung and Killen).

9. recommending some design changes for surfboards: sharp trailing edges and a stepped front to eliminate the strong tendency of the convex front of a surfboard to nosedive (I later built such a surfboard - although it performed as expected on a wave its non-planing drag was high and this made it much more difficult to paddle than a conventional surfboard and this made it impractical)

Conclusions

1. Fluid dynamics difficult and interesting subject
2. Some simple physical ideas can give insight
3. Fluid dynamics of surfboard still poorly understood: high-Re number flow on interface between air and water over unsteady turbulent breaking wave. . .
4. Lots left to do
5. Senior Research Projects?