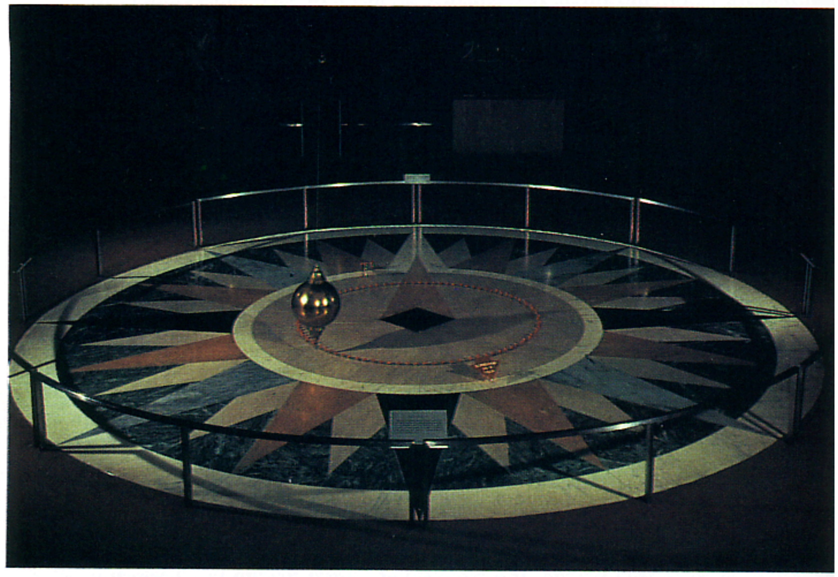
13

Oscillatory Motion



The Foucault pendulum at the Smithsonian Institution in Washington, D.C. This type of pendulum was first used by the French physicist Jean Foucault to verify the earth's rotation experimentally. During its swinging motion, the pendulum's plane of oscillation appears to rotate, as the bob successively knocks over the red indicators arranged in a horizontal circle. In reality, the pendulum's plane of motion is fixed in space, while the earth rotates beneath the swinging pendulum. (Courtesy of the Smithsonian Institution)

The main objectives of the previous chapters was to discover that the motion of a body can be predicted if the initial conditions describing its state of motion and the external forces acting on it are known. If a force varies in time, the velocity and acceleration of the body will also change with time. A very special kind of motion occurs when the force on a body is proportional to the displacement of the body from equilibrium. If this force always acts toward the equilibrium position of the body, a repetitive back-and-forth motion will result about this position. The motion is an example of what is called *periodic* or *oscillatory* motion.

You are most likely familiar with several examples of periodic motion, such as the oscillations of a mass on a spring, the motion of a pendulum, and the vibrations of a stringed musical instrument. The number of systems that exhibit oscillatory motion is extensive. For example, the molecules in a solid oscillate about their equilibrium positions; electromagnetic waves, such as light waves, radar, and radio waves, are characterized by oscillating electric and magnetic field vectors; and in alternating-current circuits, voltage, current, and electrical charge vary periodically with time.

Most of the material in this chapter deals with *simple harmonic motion*. For this type of motion, an object oscillates between two spatial positions for an indefinite period of time, with no loss in mechanical energy. In real mechanical systems, retarding (or frictional) forces are always present. Such forces reduce the mechanical energy of the system as motion progresses, and the oscillations are said to be *damped*. If an external driving force is applied such that the energy loss is balanced by the energy input, we call the motion a *forced oscillation*.

13.1 SIMPLE HARMONIC MOTION

A particle moving along the x axis is said to exhibit **simple harmonic motion** when x, its displacement from equilibrium, varies in time according to the relationship

Displacement versus time for simple harmonic motion

$$x = A\cos(\omega t + \delta) \tag{13.1}$$

where A, ω, and δ are constants of the motion. In order to give physical significance to these constants, it is convenient to plot x as a function of t, as in Figure 13.1. First, we note that A, called the **amplitude** of the motion, is simply the *maximum displacement* of the particle in either the positive or negative x direction. The constant ω is called the *angular frequency* (defined in Eq. 13.4). The constant angle δ is called the **phase constant** (or phase angle) and along with the amplitude A is determined uniquely by the initial displacement and velocity of the particle. The constants δ and A tell us what the displacement was at time $t = 0$. The quantity $(\omega t + \delta)$ is called the **phase** of the motion and is useful in comparing the motions of two systems of particles. Note that the function x is periodic and repeats itself when ωt increases by 2π radians.

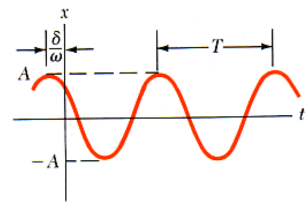


Figure 13.1 Displacement versus time for a particle undergoing simple harmonic motion. The amplitude of the motion is A and the period is T.

The **period**, T, is the time for the particle to go through one full cycle of its motion. That is, the value of x at time t equals the value of x at time $t + T$. We can show that the period of the motion is given by $T = 2\pi/\omega$ by using the fact that the phase increases by 2π radians in a time T:

$$\omega t + \delta + 2\pi = \omega(t + T) + \delta$$

Hence, $\omega T = 2\pi$ or

$$T = \frac{2\pi}{\omega} \tag{13.2}$$

Period

The inverse of the period is called the **frequency** of the motion, f. The frequency represents the *number of oscillations the particle makes per unit time:*

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \tag{13.3}$$

Frequency

The units of f are cycles/s, or hertz (Hz).

Rearranging Equation 13.3 gives

$$\omega = 2\pi f = \frac{2\pi}{T} \tag{13.4}$$

Angular frequency

The constant ω is called the **angular frequency** and has units of rad/s. We shall discuss the geometric significance of ω in Section 13.4.

We can obtain the velocity of a particle undergoing simple harmonic motion by differentiating Equation 13.1 with respect to time:

Velocity in simple harmonic motion

$$v = \frac{dx}{dt} = -\omega A \sin(\omega t + \delta) \quad (13.5)$$

The acceleration of the particle is given by dv/dt:

Acceleration in simple harmonic motion

$$a = \frac{dv}{dt} = -\omega^2 A \cos(\omega t + \delta) \quad (13.6)$$

Since $x = A\cos(\omega t + \delta)$, we can express Equation 13.6 in the form

$$a = -\omega^2 x \quad (13.7)$$

From Equation 13.5 we see that since the sine and cosine functions oscillate between ± 1, the extreme values of v are equal to $\pm\omega A$. Equation 13.6 tells us that the extreme values of the acceleration are $\pm\omega^2 A$. Therefore, the *maximum* values of the velocity and acceleration are given by

Maximum values of velocity and acceleration in simple harmonic motion

$$v_{max} = \omega A \quad (13.8)$$

$$a_{max} = \omega^2 A \quad (13.9)$$

Figure 13.2a represents the displacement versus time for an arbitrary value of the phase constant. The projection of a point moving with uniform

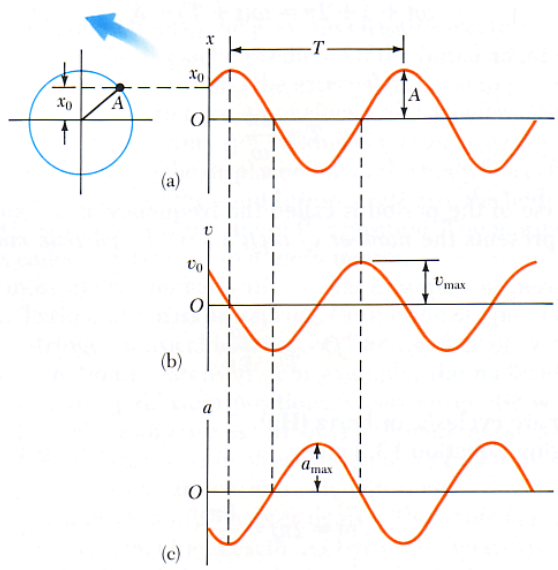


Figure 13.2 Graphical representation of simple harmonic motion: (a) the displacement versus time, (b) the velocity versus time, and (c) the acceleration versus time. Note that the velocity is 90° out of phase with the displacement and the acceleration is 180° out of phase with the displacement.

circular motion on a reference circle of radius A also moves in sinusoidal fashion. This will be discussed in more detail in Section 13.5.

The velocity and acceleration versus time curves are illustrated in Figures 13.2b and 13.2c. These curves show that the phase of the velocity differs from the phase of the displacement by $\pi/2$ rad, or 90°. That is, when x is a maximum or a minimum, the velocity is zero. Likewise, when x is zero, the speed is a maximum. Furthermore, note that the phase of the acceleration differs from the phase of the displacement by π radians, or 180°. That is, when x is a maximum, a is a maximum in the opposite direction.

As we stated earlier, the solution $x = A\cos(\omega t + \delta)$ is a general solution of the equation of motion, where the phase constant δ and the amplitude A must be chosen to meet the initial conditions of the motion. The phase constant is important when comparing the motion of two or more oscillating particles. Suppose that the initial position x_0 and initial velocity v_0 of a single oscillator are given, that is, at $t = 0$, $x = x_0$ and $v = v_0$. Under these conditions, the equations $x = A\cos(\omega t + \delta)$ and $v = -\omega A \sin(\omega t + \delta)$ give

$$x_0 = A\cos\delta \qquad \text{and} \qquad v_0 = -\omega A \sin\delta$$

Dividing these two equations eliminates A, giving

$$\frac{v_0}{x_0} = -\omega\tan\delta$$

$$\tan\delta = -\frac{v_0}{\omega x_0} \tag{13.10a}$$

The phase angle δ and amplitude A can be obtained from the initial conditions

Furthermore, if we take the sum $x_0{}^2 + \left(\frac{v_0}{\omega}\right)^2 = A^2\cos^2\delta + A^2\sin^2\delta$ and solve for A, we find that

$$A = \sqrt{x_0{}^2 + \left(\frac{v_0}{\omega}\right)^2} \tag{13.10b}$$

Thus, we see that δ and A are known if x_0, ω, and v_0 are specified. We shall treat a few specific cases in the next section.

We conclude this section by pointing out the following important properties of a particle moving in simple harmonic motion:

1. The displacement, velocity, and acceleration all vary sinusoidally with time but are not in phase, as shown in Figure 13.2.
2. The acceleration of the particle is proportional to the displacement, but in the opposite direction.
3. The frequency and the period of motion are independent of the amplitude.

Properties of simple harmonic motion

EXAMPLE 13.1 An Oscillating Body □
A body oscillates with simple harmonic motion along the x axis. Its displacement varies with time according to the equation

$$x = (4.0\text{ m})\cos\left(\pi t + \frac{\pi}{4}\right)$$

where t is in s, and the angles in the parentheses are in radians. (a) Determine the amplitude, frequency, and period of the motion.

By comparing this equation with the general relation for simple harmonic motion, $x = A\cos(\omega t + \delta)$, we see that $A = 4.0$ m and $\omega = \pi$ rad/s; therefore we find $f = \omega/2\pi = \pi/2\pi = 0.50\text{ s}^{-1}$ and $T = 1/f = 2.0$ s.

(b) Calculate the velocity and acceleration of the body at any time t.

$$v = \frac{dx}{dt} = -4.0 \sin\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t)$$

$$= -(4\pi \text{ m/s}) \sin\left(\pi t + \frac{\pi}{4}\right)$$

$$a = \frac{dv}{dt} = -4\pi \cos\left(\pi t + \frac{\pi}{4}\right) \frac{d}{dt}(\pi t)$$

$$= -(4\pi^2 \text{ m/s}^2) \cos\left(\pi t + \frac{\pi}{4}\right)$$

(c) Using the results to (b), determine the position, velocity, and acceleration of the body at $t = 1$ s.

Noting that the angles in the trigonometric functions are in radians, we get at $t = 1$ s

$$x = (4.0 \text{ m}) \cos\left(\pi + \frac{\pi}{4}\right) = (4.0 \text{ m}) \cos\left(\frac{5\pi}{4}\right)$$

$$= (4.0 \text{ m})(-0.707) = -2.83 \text{ m}$$

$$v = -(4\pi \text{ m/s}) \sin\left(\frac{5\pi}{4}\right) = -(4\pi \text{ m/s})(-0.707) = 8.89 \text{ m/s}$$

$$a = -(4\pi^2 \text{ m/s}^2) \cos\left(\frac{5\pi}{4}\right) = -(4\pi^2 \text{ m/s}^2)(-0.707)$$

$$= 27.9 \text{ m/s}^2$$

(d) Determine the maximum speed and maximum acceleration of the body.

From the general relations for v and a found in (b), we see that the maximum values of the sine and cosine functions are unity. Therefore, v varies between $\pm 4\pi$ m/s, and a varies between $\pm 4\pi^2$ m/s^2. Thus, $v_{max} = 4\pi$ m/s and $a_{max} = 4\pi^2$ m/s^2. The same results are obtained using $v_{max} = \omega A$ and $a_{max} = \omega^2 A$, where $A = 4.0$ m and $\omega = \pi$ rad/s.

(e) Find the displacement of the body between $t = 0$ and $t = 1$ s.

The x coordinate at $t = 0$ is given by

$$x_0 = (4.0 \text{ m}) \cos\left(0 + \frac{\pi}{4}\right) = (4.0 \text{ m})(0.707) = 2.83 \text{ m}$$

In (c), we found that the coordinate at $t = 1$ s was -2.8 m; therefore the displacement between $t = 0$ and $t = 1$ s is

$$\Delta x = x - x_0 = -2.83 \text{ m} - 2.83 \text{ m} = -5.66 \text{ m}$$

Because the particle's velocity changes sign during the first second, the magnitude of Δx is *not* the same as the distance traveled in the first second.

(f) What is the phase of the motion at $t = 2$ s?

The phase is defined as $\omega t + \delta$, where in this case $\omega = \pi$ and $\delta = \pi/4$. Therefore, at $t = 2$ s, we get

$$\text{Phase} = (\omega t + \delta)_{t=2} = \pi(2) + \pi/4 = 9\pi/4 \text{ rad}$$

13.2 MASS ATTACHED TO A SPRING

In Chapter 7 we introduced the physical system consisting of a mass attached to the end of a spring, where the mass is free to move on a horizontal, frictionless surface (Fig. 13.3). We know from experience that such a system will oscillate back and forth if disturbed from the equilibrium position $x = 0$, where the spring is unstretched. If the surface is frictionless, the mass will exhibit simple harmonic motion. One possible experimental arrangement that clearly demonstrates that such a system exhibits simple harmonic motion is illustrated in Figure 13.4, in which a mass oscillating vertically on a spring has a marking pen attached to it. While the mass is in motion, a sheet of paper is moved horizontally as shown, and the marking pen traces out a sinusoidal pattern. We can understand this qualitatively by first recalling that when the mass is displaced a small distance x from equilibrium, the spring exerts a force on m given by **Hooke's law**,

$$F = -kx \tag{13.11}$$

where k is the force constant of the spring. We call this a **linear restoring force** since it is linearly proportional to the displacement and is always directed toward the equilibrium position, *opposite* to the displacement. That is, when the mass is displaced to the right in Figure 13.3, x is positive and the restoring force is to the left. When the mass is displaced to the left of $x = 0$, then x is negative and F is to the right. If we now apply Newton's second law to the motion of m in the x direction, we get

$$F = -kx = ma$$

$$a = -\frac{k}{m}x \tag{13.12}$$

that is, *the acceleration is proportional to the displacement of the mass from equilibrium and is in the opposite direction.* If the mass is displaced a maximum distance $x = A$ at some initial time and released from rest, its *initial* acceleration will be $-kA/m$ (that is, it has its extreme negative value). When it passes through the equilibrium position, $x = 0$ and its acceleration is zero. At this instant, its velocity is a maximum. It will then continue to travel to the left of equilibrium and finally reach $x = -A$, at which time its acceleration is kA/m (maximum positive) and its velocity is again zero. Thus, we see that the mass will oscillate between the turning points $x = \pm A$. In one full cycle of its motion, the mass travels a distance $4A$.

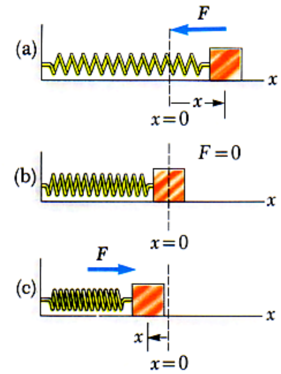


Figure 13.3 A mass attached to a spring on a frictionless surface exhibits simple harmonic motion. (a) When the mass is displaced to the right of equilibrium, the displacement is positive and the acceleration is negative. (b) At the equilibrium position, $x = 0$, the acceleration is zero but the speed is a maximum. (c) When the displacement is negative, the acceleration is positive.

We shall now describe the motion in a quantitative fashion. This can be accomplished by recalling that $a = dv/dt = d^2x/dt^2$. Thus, we can express Equation 13.12 as

$$\frac{d^2x}{dt^2} = -\frac{k}{m}x \tag{13.13}$$

If we denote the ratio k/m by the symbol ω^2,

$$\omega^2 = k/m \tag{13.14}$$

then Equation 13.13 can be written in the form

$$\frac{d^2x}{dt^2} = -\omega^2 x \tag{13.15}$$

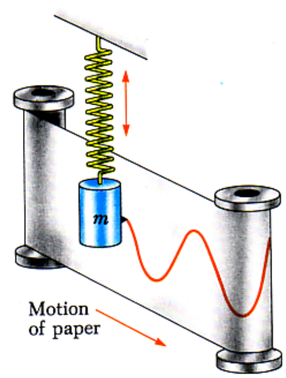


Figure 13.4 An experimental apparatus for demonstrating simple harmonic motion. A pen attached to the oscillating mass traces out a sine wave on the moving chart paper.

What we now require is a solution to Equation 13.15, that is, a function $x(t)$ that satisfies this second-order differential equation. The nature of such a solution $x(t)$ as an algebraic relationship is that it reduces the differential equation to an identity. However, since Equations 13.15 and 13.7 are equivalent, we see that the solution must be that of simple harmonic motion:

$$x(t) = A\cos(\omega t + \delta)$$

To see this explicitly, note that if

$$x = A\cos(\omega t + \delta)$$

then

$$\frac{dx}{dt} = A\frac{d}{dt}\cos(\omega t + \delta) = -\omega A\sin(\omega t + \delta)$$

$$\frac{d^2x}{dt^2} = -\omega A\frac{d}{dt}\sin(\omega t + \delta) = -\omega^2 A\cos(\omega t + \delta)$$

Comparing the expressions for x and d^2x/dt^2, we see that $d^2x/dt^2 = -\omega^2 x$ and Equation 13.15 is satisfied.

The following general statement can be made based on the above discussion:

> Whenever the force acting on a particle is linearly proportional to the displacement and in the opposite direction, the particle will exhibit simple harmonic motion.

We shall give additional physical examples in subsequent sections.

Since the period is given by $T = 2\pi/\omega$ and the frequency is the inverse of the period, we can express the period and frequency of the motion for this system as

Period and frequency for mass-spring system

$$T = \frac{2\pi}{\omega} = 2\pi\sqrt{\frac{m}{k}} \tag{13.16}$$

$$f = \frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{k}{m}} \tag{13.17}$$

That is, the period and frequency depend *only* on the mass and on the force constant of the spring. As we might expect, the frequency is larger for a stiffer spring and decreases with increasing mass.

It is interesting to note that a mass suspended from a vertical spring attached to a fixed support will also exhibit simple harmonic motion. Although there is a gravitational force to consider in this case, the equation of motion still reduces to Equation 13.15, where the displacement is measured from the equilibrium position of the suspended mass. The proof of this is left as a problem (Problem 56).

Special Case I In order to better understand the physical significance of our solution of the equation of motion, let us consider the following special case. Suppose we pull the mass from equilibrium by a distance A and release it from rest from this stretched position, as in Figure 13.5. We must then require that our solution for $x(t)$ obey the *initial conditions* that at $t = 0$, $x_0 = A$ and $v_0 = 0$. These conditions will be met if we choose $\delta = 0$, giving $x = A\cos\omega t$ as our solution. Note that this is consistent with $x = A\cos(\omega t + \delta)$, where $x_0 = A$ and $\delta = 0$. To check this, we see that the solution $x = A\cos\omega t$ satisfies the condition that $x_0 = A$ at $t = 0$, since $\cos 0 = 1$. Thus, we see that A and δ contain the

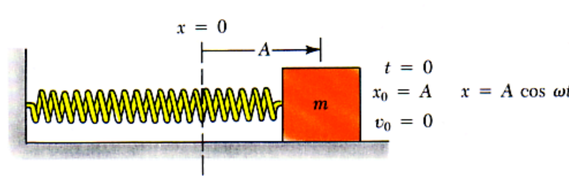


Figure 13.5 A mass-spring system that starts from rest at $x_0 = A$. In this case, $\delta = 0$, and so $x = A\cos\omega t$.

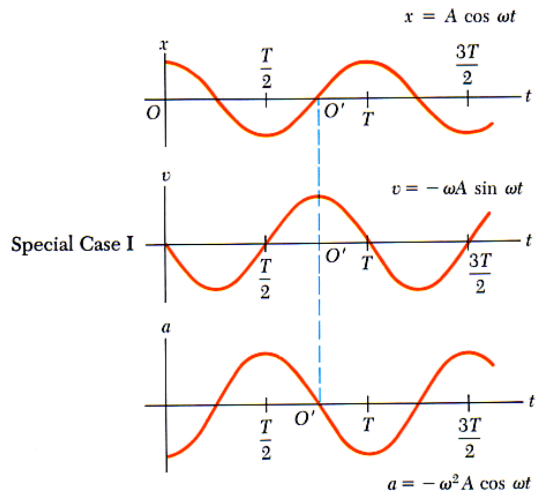


Figure 13.6 Displacement, velocity, and acceleration versus time for a particle undergoing simple harmonic motion under the initial conditions that at $t = 0$, $x_0 = A$ and $v_0 = 0$.

information on initial conditions. Now let us investigate the behavior of the velocity and acceleration for this special case. Since $x = A \cos \omega t$

$$v = \frac{dx}{dt} = -\omega A \sin \omega t$$

and

$$a = \frac{dv}{dt} = -\omega^2 A \cos \omega t$$

From the velocity expression $v = -\omega A \sin \omega t$, we see that at $t = 0$, $v_0 = 0$, as we require. The expression for the acceleration tells us that at $t = 0$, $a = -\omega^2 A$. Physically this makes sense, since the force on the mass is to the left when the displacement is positive. In fact, at this position $F = -kA$ (to the left), and the initial acceleration is $-kA/m$.

We could also use a more formal approach to show that $x = A \cos \omega t$ is the correct solution by using the relation $\tan \delta = -v_0/\omega x_0$ (Eq. 13.10a). Since $v_0 = 0$ at $t = 0$, $\tan \delta = 0$ and so $\delta = 0$.

The displacement, velocity, and acceleration versus time are plotted in Figure 13.6 for this special case. Note that the acceleration reaches extreme values of $\pm\omega^2 A$ when the displacement has extreme values of $\pm A$. Furthermore, the velocity has extreme values of $\pm\omega A$, which both occur at $x = 0$. Hence, the quantitative solution agrees with our qualitative description of this system.

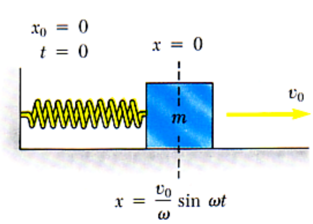


Figure 13.7 The mass-spring system starts its motion at the equilibrium position, $x_0 = 0$ at $t = 0$. If its initial velocity is v_0 to the right, its x coordinate varies as $x = \frac{v_0}{\omega} \sin \omega t$.

Special Case II Now suppose that the mass is given an initial velocity v_0 to the *right* at the unstretched position of the spring so that at $t = 0$, $x_0 = 0$ and $v = v_0$ (Fig. 13.7). Our particular solution must now satisfy these initial conditions. Since the mass is moving toward positive x values at $t = 0$, and $x_0 = 0$ at $t = 0$, the solution has the form $x = A \sin \omega t$.

Applying $\tan \delta = -v_0/\omega x_0$ and the initial condition that $x_0 = 0$ at $t = 0$ gives $\tan \delta = -\infty$ or $\delta = -\pi/2$. Hence, the solution is $x = A\cos(\omega t - \pi/2)$, which can be written $x = A \sin \omega t$. Furthermore, from Equation 13.10b we see that $A = v_0/\omega$; therefore we can express our solution as

$$x = \frac{v_0}{\omega} \sin \omega t$$

The velocity and acceleration in this case are given by

$$v = \frac{dx}{dt} = v_0 \cos \omega t$$

$$a = \frac{dv}{dt} = -\omega v_0 \sin \omega t$$

This is consistent with the fact that the mass always has a maximum speed at $x = 0$, while the force and acceleration are zero at this position. The graphs of these functions versus time in Figure 13.6 correspond to the origin at O'. What would be the solution for x if the mass is initially moving to the left in Figure 13.7?

EXAMPLE 13.2 That Car Needs a New Set of Shocks
A car of mass 1300 kg is constructed using a frame supported by four springs. Each spring has a force constant of 20 000 N/m. If two people riding in the car have a combined mass of 160 kg, find the frequency of vibration of the car when it is driven over a pot hole in the road.

Solution We shall assume that the weight is evenly distributed. Thus, each spring supports one fourth of the load. The total mass supported by the springs is 1460 kg, and therefore each spring supports 365 kg. Hence, the frequency of vibration is

$$f = \frac{1}{2\pi}\sqrt{\frac{k}{m}} = \frac{1}{2\pi}\sqrt{\frac{20\ 000\ \text{N/m}}{365\ \text{kg}}} = \boxed{1.18\ \text{Hz}}$$

Exercise 1 How long does it take the car to execute two complete vibrations?
Answer 1.70 s.

EXAMPLE 13.3 A Mass-Spring System
A mass of 200 g is connected to a light spring of force constant 5 N/m and is free to oscillate on a horizontal, frictionless surface. If the mass is displaced 5 cm from equilibrium and released from rest, as in Figure 13.5, (a) find the period of its motion.

This situation corresponds to Case I, where $x = A\cos \omega t$ and $A = 5 \times 10^{-2}$ m. Therefore,

$$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{5\ \text{N/m}}{200 \times 10^{-3}\ \text{kg}}} = 5\ \text{rad/s}$$

Therefore

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{5} = \boxed{1.26\ \text{s}}$$

(b) Determine the maximum speed of the mass.

$$v_{max} = \omega A = (5\ \text{rad/s})(5 \times 10^{-2}\ \text{m}) = \boxed{0.250\ \text{m/s}}$$

(c) What is the maximum acceleration of the mass?

$$a_{max} = \omega^2 A = (5\ \text{rad/s})^2(5 \times 10^{-2}\ \text{m}) = \boxed{1.25\ \text{m/s}^2}$$

(d) Express the displacement, speed, and acceleration as functions of time.

The expression $x = A\cos \omega t$ is our special solution for Case I, and so we can use the results from (a), (b), and (c) to get

$$x = A\cos \omega t = \boxed{(0.05\ \text{m}) \cos 5t}$$

$$v = -\omega A \sin \omega t = \boxed{-(0.25\ \text{m/s}) \sin 5t}$$

$$a = -\omega^2 A \cos \omega t = \boxed{-(1.25\ \text{m/s}^2) \cos 5t}$$

13.3 ENERGY OF THE SIMPLE HARMONIC OSCILLATOR

Let us examine the mechanical energy of the mass-spring system described in Figure 13.6. Since the surface is frictionless, we expect that the total mechanical energy is conserved, as was shown in Chapter 8. We can use Equation 13.5 to express the kinetic energy as

$$K = \tfrac{1}{2}mv^2 = \tfrac{1}{2}m\omega^2 A^2 \sin^2(\omega t + \delta) \qquad (13.18)$$

Kinetic energy of a simple harmonic oscillator

The elastic potential energy stored in the spring for any elongation x is given by $\tfrac{1}{2}kx^2$. Using Equation 13.1, we get

$$U = \tfrac{1}{2}kx^2 = \tfrac{1}{2}kA^2 \cos^2(\omega t + \delta) \qquad (13.19)$$

Potential energy of a simple harmonic oscillator

We see that K and U are *always* positive quantities. Since $\omega^2 = k/m$, we can express the *total energy* of the simple harmonic oscillator as

$$E = K + U = \tfrac{1}{2}kA^2[\sin^2(\omega t + \delta) + \cos^2(\omega t + \delta)]$$

But $\sin^2\theta + \cos^2\theta = 1$, where $\theta = \omega t + \delta$; therefore this equation reduces to

$$E = \tfrac{1}{2}kA^2 \qquad (13.20)$$

Total energy of a simple harmonic oscillator

That is,

the energy of a simple harmonic oscillator is a constant of the motion and proportional to the square of the amplitude.

In fact, the total mechanical energy is just equal to the maximum potential energy stored in the spring when $x = \pm A$. At these points, $v = 0$ and there is no kinetic energy. At the equilibrium position, $x = 0$ and $U = 0$, so that the total energy is all in the form of kinetic energy. That is, at $x = 0$, $E = \tfrac{1}{2}mv^2_{\text{max}} = \tfrac{1}{2}m\omega^2 A^2$.

Plots of the kinetic and potential energies versus time are shown in Figure 13.8a, where we have taken $\delta = 0$. In this situation, both K and U are always

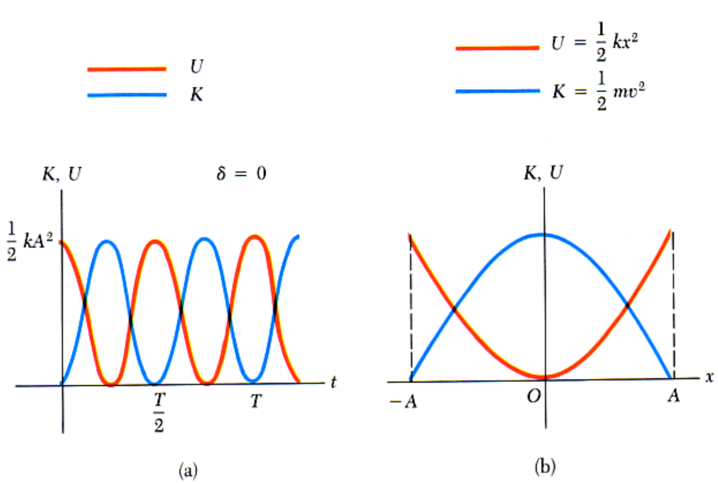


Figure 13.8 (a) Kinetic energy and potential energy versus time for a simple harmonic oscillator with $\delta = 0$. (b) Kinetic energy and potential energy versus displacement for a simple harmonic oscillator. In either plot, note that $K + U =$ constant.